Non-Ground State Bose-Einstein Condensation

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Why do we care?

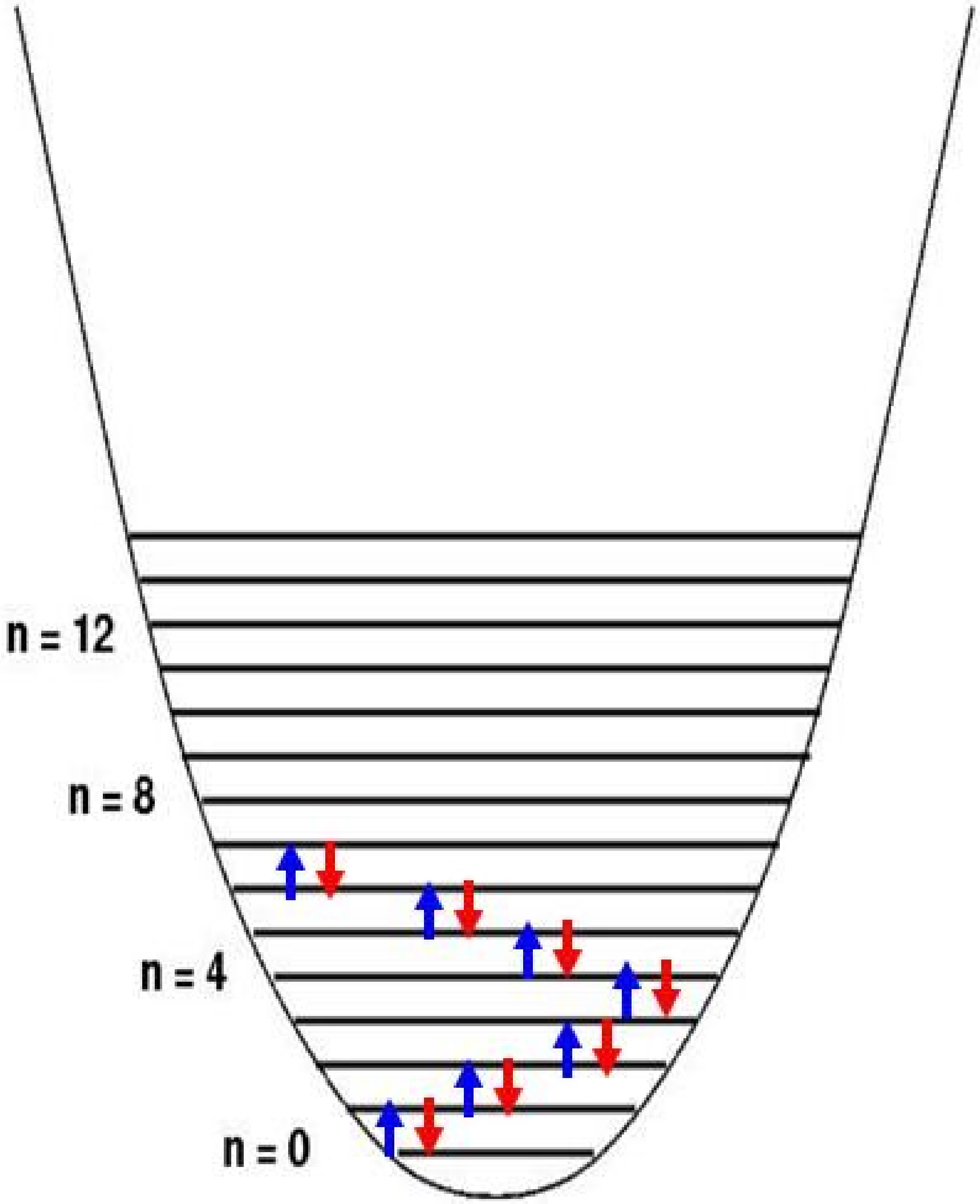
- New possibilities for understanding relaxation processes in the quantum degenerate regime
- Production of various spatial modes in a coherent beam of atoms
- And more..?

Main Idea

- BEC at ground state

➔ External pumping to an excited state
➔ Rabi oscillation between the two states
➔ Stop the external pumping when all atoms are moved to the excited state

However…



- Harmonic Potential levels are equally spaced!
- Coupling between the ground state and an excite state will disperse the population in many other states!

Non-linearity comes into play

- Atom-atom interaction introduces a non-linearity to the system

- Energy levels no longer equally spaced even in a harmonic trapping potential

Non-Ground-State BEC

$$H = \frac{-\hbar^2}{2m}\nabla^2 + U(r) + A\left|\varphi^2\right|$$

$$A = (N-1)4\pi\hbar^2 \frac{a}{m}$$

$$U(r) = \frac{1}{2}m(\omega_x{}^2 x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2)$$

- a=s-wave scattering length

Non-Ground-State BEC

$$H = \frac{1}{2}\sum_{i=1}^{3}\left(-\frac{\partial^2}{\partial x_i{}^2} + \lambda_i{}^2 x_i{}^2\right) + g\left|\Psi^2\right|$$

$$g = 4\pi(N-1)\frac{a}{l}$$

$$x_i = \frac{x}{l}$$

$$l = \sqrt{\frac{\hbar}{m\omega_o}}$$

$$\lambda_i = \frac{\omega_i}{\omega_o}$$

$$\omega_o = \sqrt{\omega_x \omega_y \omega_z}$$

$$\Psi = l^{3/2}\varphi$$

Eigenstates

$$H = \frac{1}{2}\sum_{i=1}^{3}\left(-\frac{\partial^2}{\partial x_i^{\,2}} + \lambda_i^{\,2} x_i^{\,2}\right) + g\left|\Psi^2\right|$$

Control Function

$$H_o = \frac{1}{2}\sum_{i=1}^{3}\left(-\frac{\partial^2}{\partial x_i^{\,2}} + u_i^{\,2} x_i^{\,2}\right)$$

$$E_n^{\,o} = \sum_{i=1}^{3} u_i\left(n_i + \frac{1}{2}\right)$$

$$\Delta H = \sum_{i=1}^{3}\left(\lambda_i^{\,2} - u_i^{\,2}\right)x_i^{\,2} + g\left|\Psi^2\right|$$

$$\Delta E_n = \left\langle \Psi_n \middle| \Delta H \middle| \Psi_n \right\rangle$$

Eigenstates

For what follows, it is useful to introduce the notation

$$g_n \equiv g J_n, \qquad J_n \equiv \prod_{i=1}^{3} J_{n_i}, \tag{43}$$

in which

$$J_{n_i} = \frac{(|\psi_{n_i}|^2, |\psi_{n_i}|^2)}{\sqrt{u_i(n)}} = \frac{1}{\pi(2^{n_i} n_i!)^2} \int_{-\infty}^{+\infty} H_{n_i}^4(x) \exp\left(-2x^2\right) dx,$$

where $H_{n_i}(x)$ is a Hermite polynomial and $u_i(n) \equiv u_i(\lambda, g, n)$.

Eq.(40), with notation (43), can be written as

$$E_n^{(1)}(\lambda, g, u) = \frac{1}{2} \sum_{i=1}^{3} \left(n_i + \frac{1}{2}\right)\left(u_i + \frac{\lambda_i^2}{u_i}\right) + \sqrt{u_1 u_2 u_3}\, g_n. \tag{44}$$

Condition (41) results in the equation

$$\left(n_i + \frac{1}{2}\right)(u_i^2 - \lambda_i^2) + u_i \sqrt{u_1 u_2 u_3}\, g_n = 0. \tag{45}$$

Eigenstates

- After some Math..

$$E_n^{\ 1} = \frac{1}{6}\sum_{i=1}^{3}\left(u_i + 5\frac{\lambda_i^{\ 2}}{u_i}\right)\left(n_i + \frac{1}{2}\right)$$

Resonant Pumping

$$V_p = V(r)\cos(\omega t)$$

- Detuning frequency $\Delta\omega=\omega-\omega_p$. We consider the case where $\Delta\omega$ is small such that the pumping does not influence the other states.

Resonant Pumping

$$\varphi(r,t)=\sum_n c_n(t)\varphi_n(r)\exp\left(\frac{-i}{\hbar}E_n t\right)$$

$$i\hbar\frac{\partial\varphi}{\partial t}=\left(\frac{-\hbar^2}{2m}\nabla^2+U(r)+A\left|\varphi^2\right|\right)\varphi$$

$$i\hbar\frac{dc_n}{dt}=\sum_m\left(V_{nm}\cos\omega t+\sum_{k(\neq n)}A_{nkm}|c_k|^2\right)c_m e^{i\omega_{nm}t},$$

Resonant Pumping

$$\frac{dC_o}{dt} = -i\alpha n_p C_o - \frac{i}{2}\beta \exp(i\Delta\omega t) C_p$$

$$\frac{dC_p}{dt} = -i\alpha n_o C_p - \frac{i}{2}\beta^* \exp(i\Delta\omega t) C_o$$

$$\frac{dC_i}{dt} = 0$$

$$\alpha = \frac{A}{\hbar}\int \left|\varphi_o\right|^2 \left|\varphi_p\right|^2 d^3 r$$

$$\beta = \frac{1}{\hbar}\int \varphi_o^* V(r) \varphi_p d^3 r$$

$$n_i = \left|C_i\right|^2$$

Rabi Oscillation

- Assuming the initial ground state population is 1..

$$n_o = 1 - \frac{|\beta|^2}{\Omega^2}\sin^2\left(\frac{\Omega t}{2}\right)$$

$$n_p = \frac{|\beta|^2}{\Omega^2}\sin^2\left(\frac{\Omega t}{2}\right)$$

where

$$\Omega = \sqrt{(\alpha(n_o - n_p) - \Delta\omega)^2 + |\beta|^2}$$

Population Dynamics

$$b = \frac{|\beta|}{\alpha}$$

$$\delta = \frac{\Delta\omega}{\alpha}$$

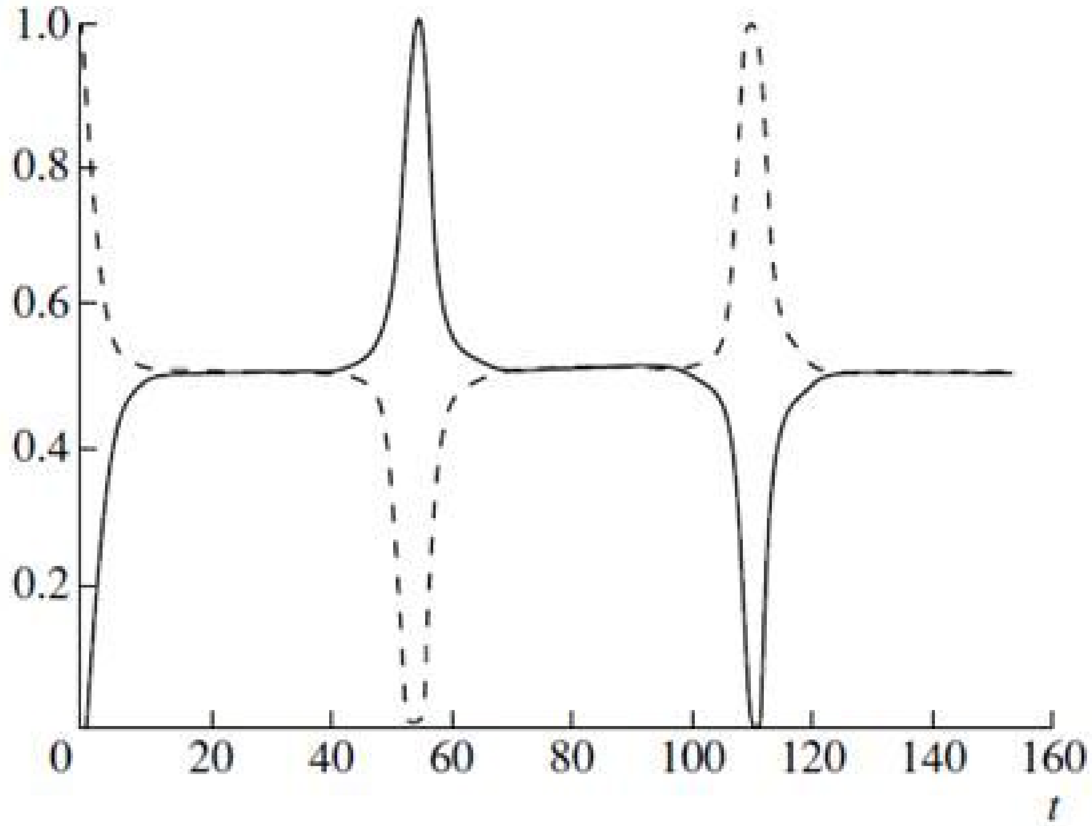


Fig. 3. The appearance of the upward cusps of $n_j(t)$ and of the downward cusps of $n_0(t)$ for b = 0.4999 and δ = 0.0001001.

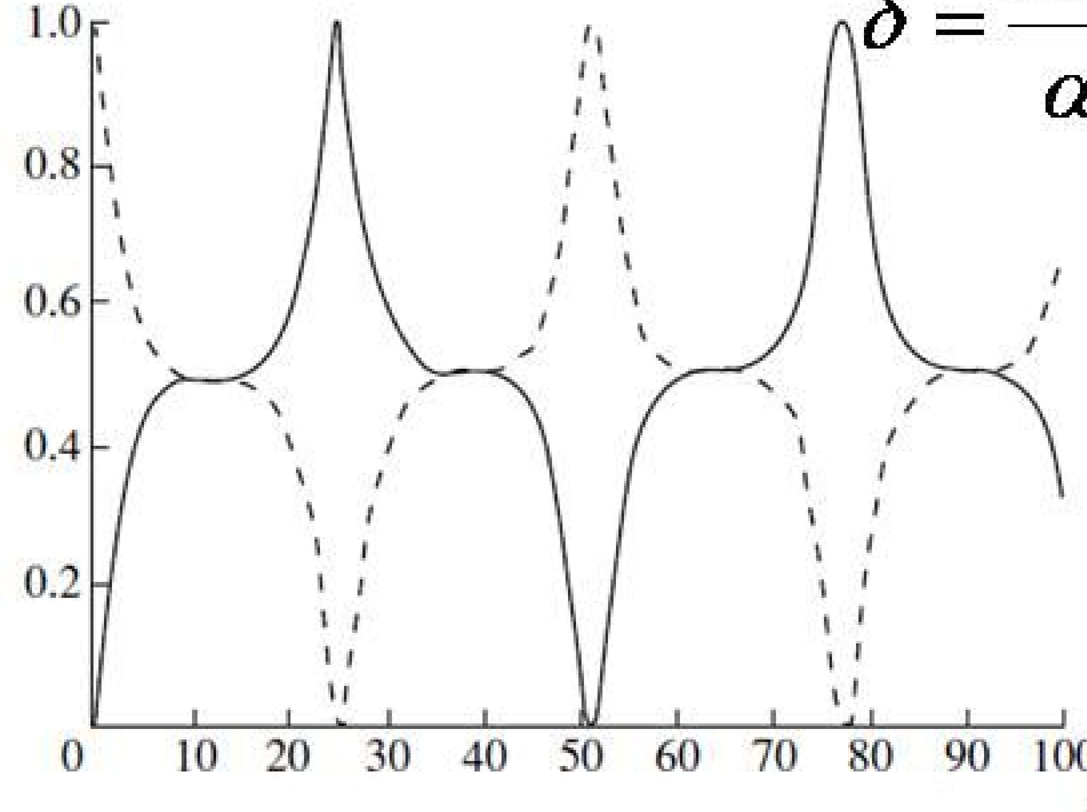


Fig. 4. Fractional populations versus time for $b = 0.4999$ and $\delta = 0.00011$.

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Population Dynamics

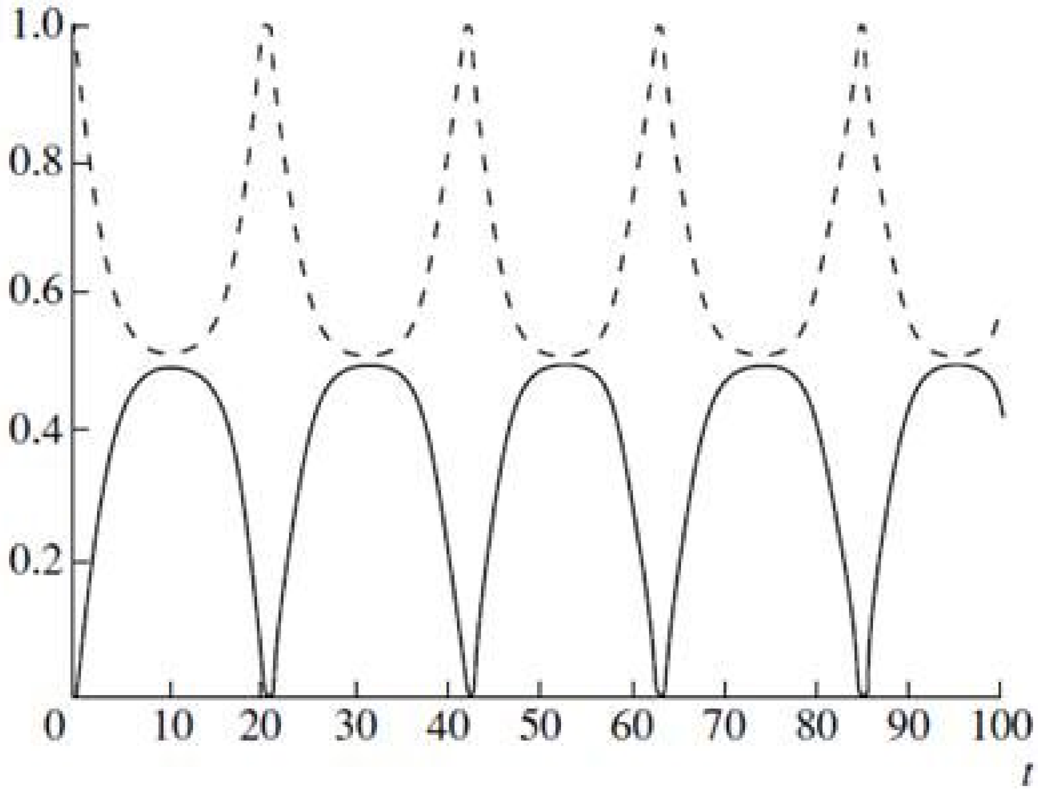

Fig. 1. The time dependence of the fractional populations $n_0(t)$ and $n_j(t)$ for $b = 0.4999$ and $\delta = 0$. Here and in all following pictures the dashed line corresponds to the ground-state population $n_0(t)$ and the solid line, to the excited-level population $n_j(t)$.

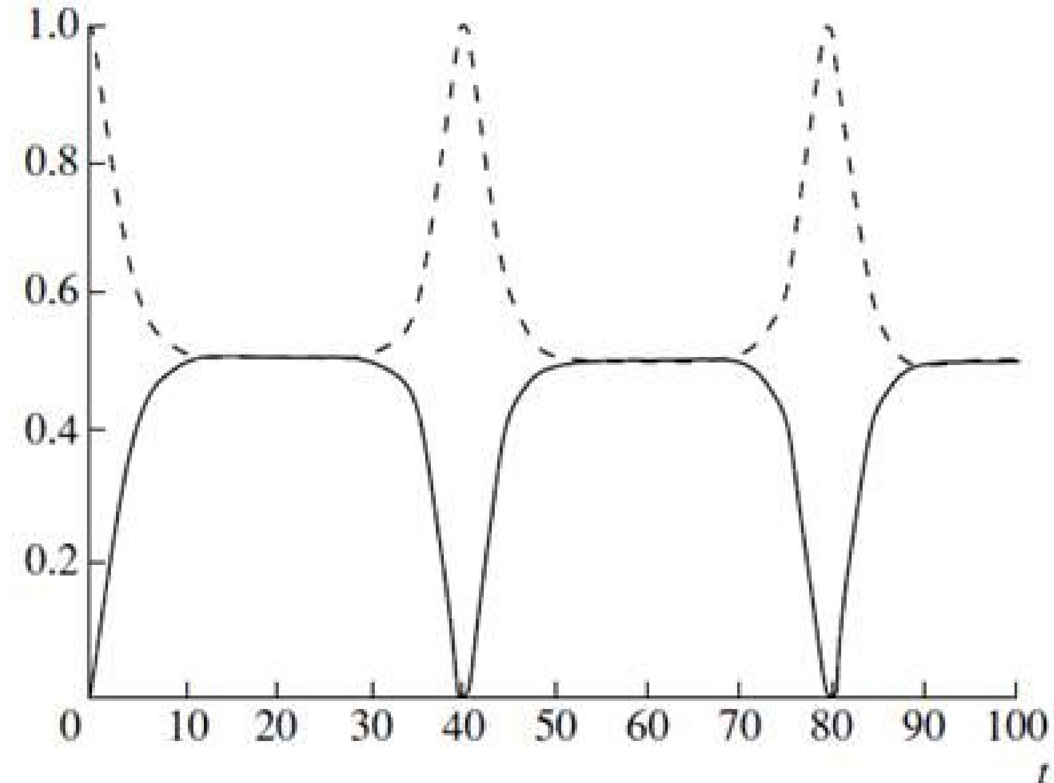

Fig. 2. Flattening of the fractional populations, with their oscillation period being doubled, at $b = 0.4999$ and $\delta = 0.0001$.

Summery

- Atomic interactions introduce non-linearity to a harmonic trapping potential and the energy levels are not equally spaced
- Manipulation of detuning and the transition amplitude allows a complete transition from the ground state to an excited state.