Adding control to arbitrary quantum operations

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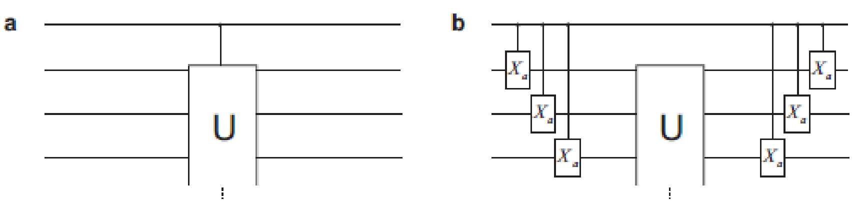
Quantum computers promise exponential power for particular tasks, however, the complexity of quantum algorithms remains a major technological challenge. We have developed and demonstrated an architecture independent technique for adding control qubits to arbitrary quantum operations (unitary or otherwise)—a key requirement in many quantum algorithms. The technique is independent of how the operation is done and does not even require knowledge of what the operation is. In this way the technical problems of how to implement a quantum operation and how to add a control are separated. The number of computational resources required is independent of the depth of the operation and increases only linearly with the number of qubits on which it acts. Our approach will significantly reduce the complexity of quantum computations such as Shor's factoring algorithm and the near-term prospect of quantum simulations. We use this new approach to implement a number of two-qubit photonic quantum gates in which the operation of the control circuit is completed independent of the choice of quantum operation.

Perhaps the most promising future application of quantum science is quantum information processing, which promises secure communication [1] and greatly increased speeds for



Lee Rozema
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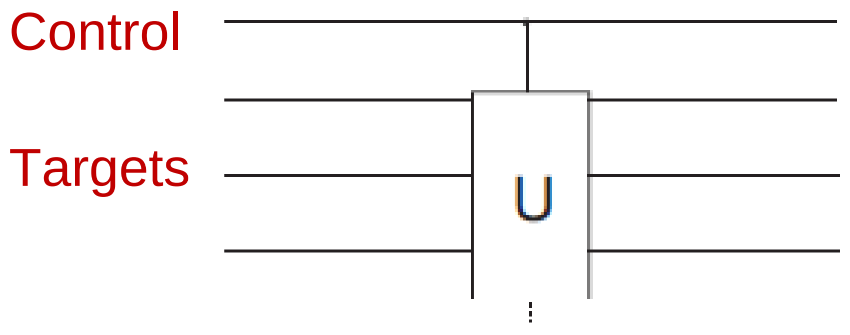
Outline

- Motivation
  - What is a controlled unitary and why do we want them?
  - Standard method
- New method
- Photonic new method
- Non-deterministic photonic new method
- Experimental non-deterministic photonic new method
- Results
- Conclusions

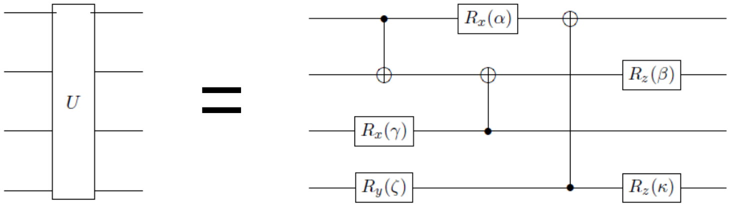
Controlled Unitaries

- Apply operation only if control qubit is |1>
- Need for Kitaev's phase estimation
  - therefore also in simulation and factoring

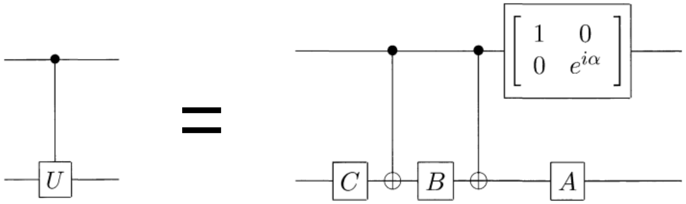


‘Standard’ Method

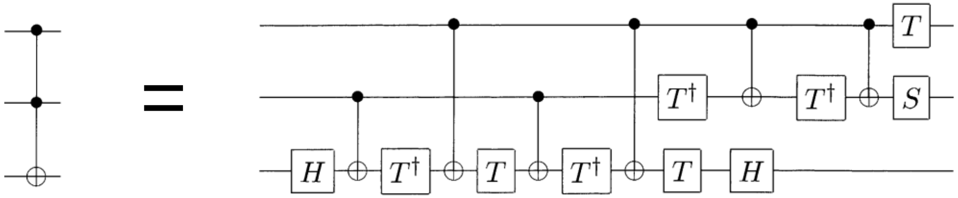
- Translate into U our universal gate set
  - C-NOTS and single qubit rotations



- Turn single qubit rotations into controlled rotations



- Turn C-NOTs into controlled C-NOTs (and back)



Drawbacks

Requires many gates

Requires prior knowledge of the unitary

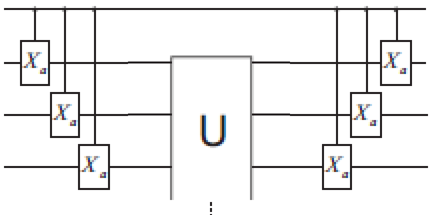
New Method

- Suppose we can implement U
- Expand Hilbert space with C-Xa gates
  - If control is |0> then:

|0> → |2> |2> → |0>

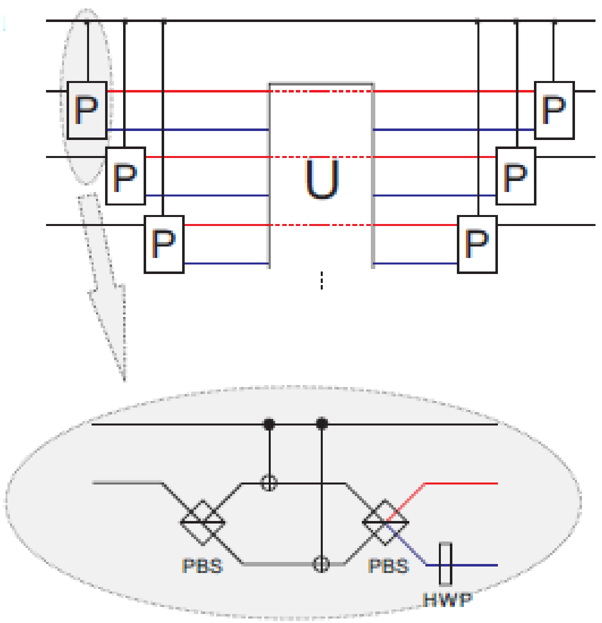
|1> → |3> |3> → |1>

- U acts on {|0>,|1>}; identity on {|2>,|3>};



Photonic C-U

- C-Xa gate → C-path gate
  - Send the photons around U



- C-Path using C-Nots

  (a|H> + b|V>)(c|H> + d|V>)

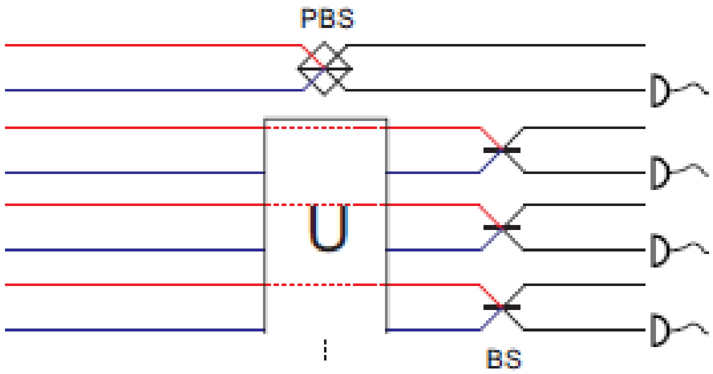
  a|H>(c|H> + d|V>) + b|V>(c|H> + d|V>)

- Entanglement between polarization of control and path of targets

Non-deterministic photonic C-U

- Start with entangled state:

(a|H>+ b|V>)|Ψ> + (a|H>+ b|V>)| Ψ >



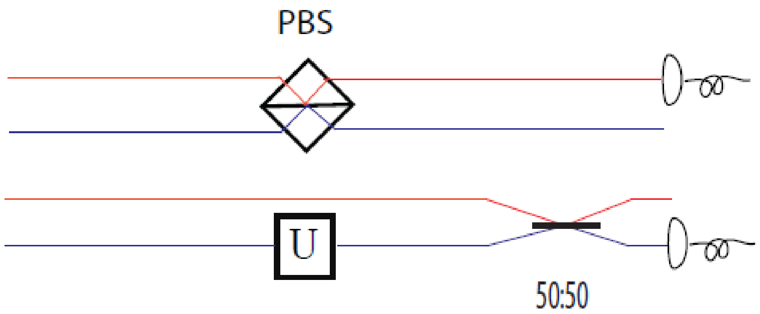
- Final C-Path → 50:50 BS

- Recall we want:

a|H>|Ψ> + b|V>U| Ψ >

Non-deterministic photonic C-U

2 – Photon Example



(a|H>+ b|V>)|Ψ> + (a|H>+ b|V>)| Ψ >

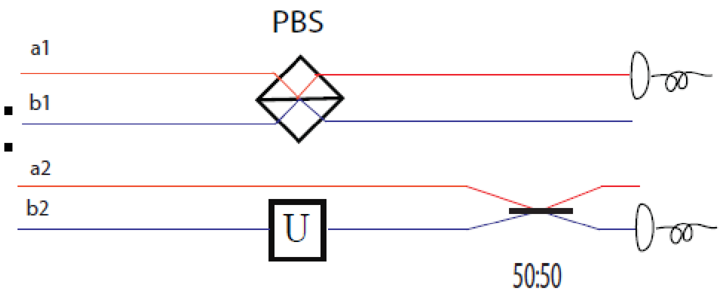
PBS → a|H>|Ψ> + b|V>| Ψ >

U → a|H>|Ψ> + b|V>U| Ψ >

50:50 → a|H>|Ψ> + b|V>U| Ψ >

Rearrange things a little

Recall we had:

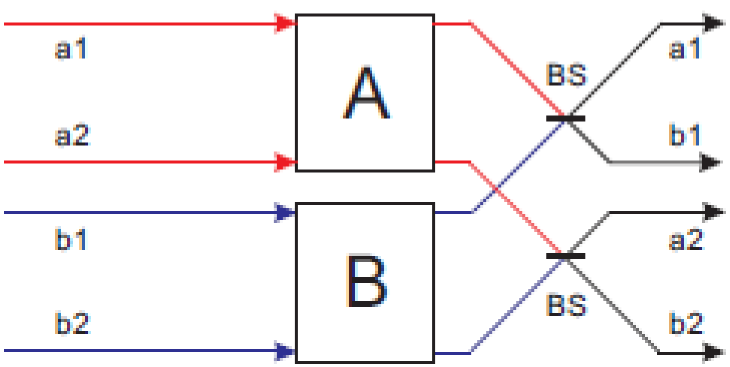


\# - photon

letter - path

Input: |Φ> | 0 > + |0> | Φ >

Output in a1 & a2: (A+B)|Φ>

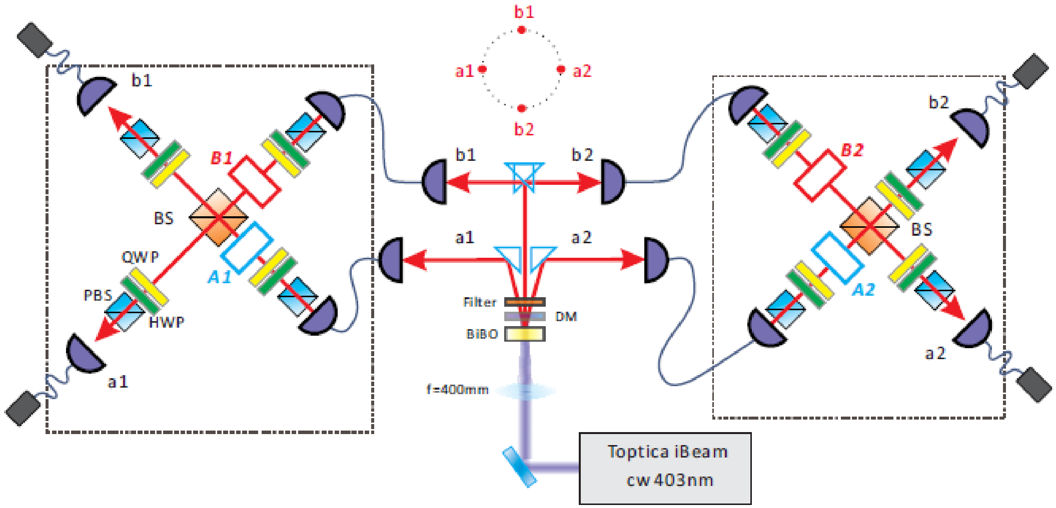


For a C-U:

A = |H><H| x **I**

B = |V><V| x **U**

Experimental non-deterministic 2 photon C-U



- Path entanglement from SPDC |HH> | 0 > + |0> | HH>

- For a C-U: A = |H><H| x **I**

  B = |V><V| x **U**

Process tomography shortcut

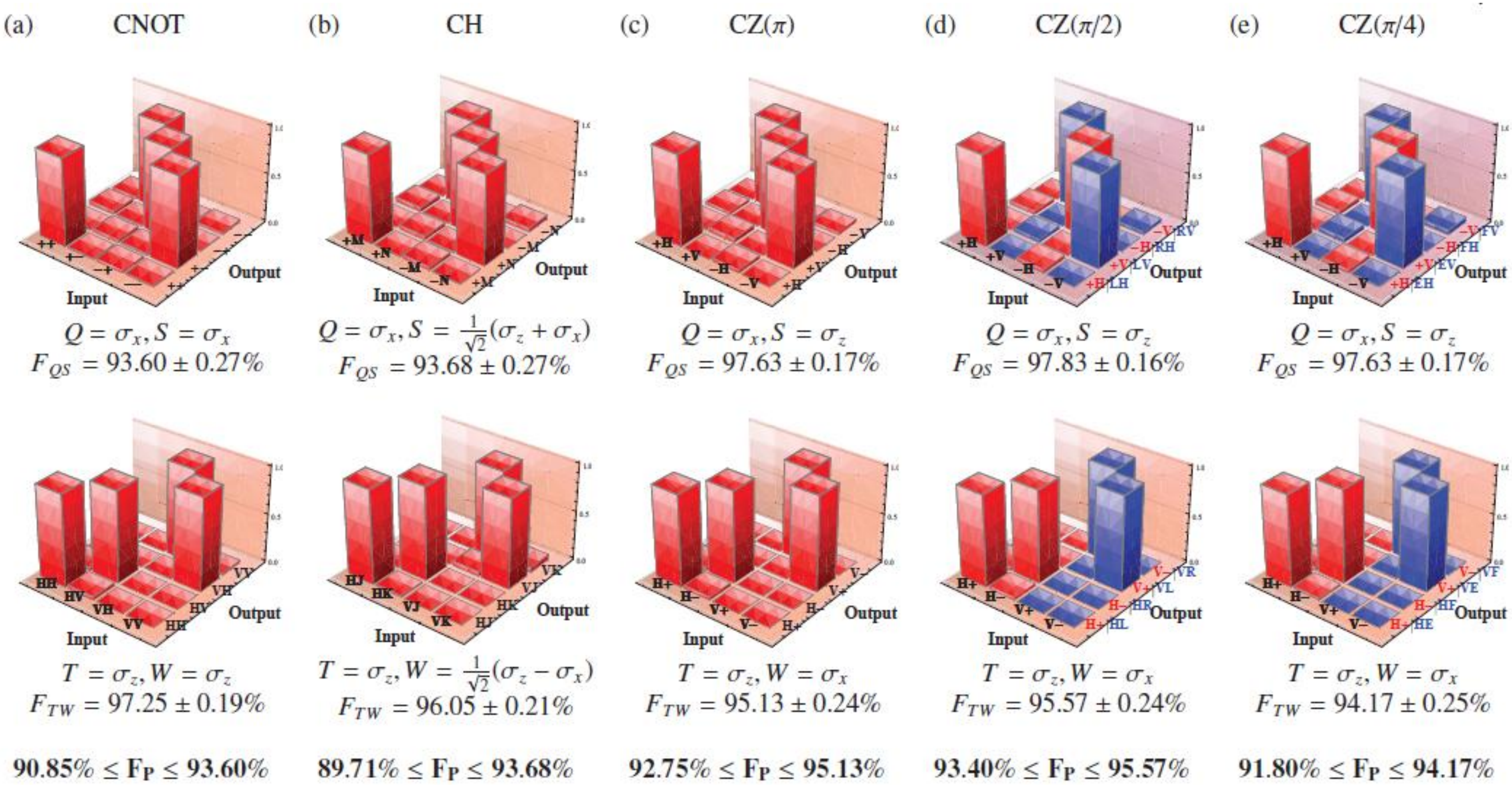
- Hofmann:

  "Any unitary transform is uniquely defined by its observable effects on two complementary sets of orthogonal states."

- From these effects we can put a bounds on the process fidelity

Hofmann, H.F. *Phys. Rev. Lett.* **94**, 160504 (2005).

Results



(a) CNOT
(b) CH
(c) CZ(π)
(d) CZ($\pi/2$)
(e) CZ($\pi/4$)
Input
Output
$Q = \sigma_x, S = \sigma_x$
$F_{QS} = 93.60 \pm 0.27\%$
$Q = \sigma_x, S = \frac{1}{\sqrt{2}}(\sigma_z + \sigma_x)$
$F_{QS} = 93.68 \pm 0.27\%$
$Q = \sigma_x, S = \sigma_z$
$F_{QS} = 97.63 \pm 0.17\%$
$Q = \sigma_x, S = \sigma_z$
$F_{QS} = 97.83 \pm 0.16\%$
$Q = \sigma_x, S = \sigma_z$
$F_{QS} = 97.63 \pm 0.17\%$
$T = \sigma_z, W = \sigma_z$
$F_{TW} = 97.25 \pm 0.19\%$
$T = \sigma_z, W = \frac{1}{\sqrt{2}}(\sigma_z - \sigma_x)$
$F_{TW} = 96.05 \pm 0.21\%$
$T = \sigma_z, W = \sigma_x$
$F_{TW} = 95.13 \pm 0.24\%$
$T = \sigma_z, W = \sigma_x$
$F_{TW} = 95.57 \pm 0.24\%$
$T = \sigma_z, W = \sigma_x$
$F_{TW} = 94.17 \pm 0.25\%$
$90.85\% \le F_P \le 93.60\%$
$89.71\% \le F_P \le 93.68\%$
$92.75\% \le F_P \le 95.13\%$
$93.40\% \le F_P \le 95.57\%$
$91.80\% \le F_P \le 94.17\%$

Conclusions

- Novel method for implementing controlled unitaries
- Resources required:
  - C-path gate (or C-NOT)
  - OR prior path entanglement
- Also used the gate as an entanglement filter (by making use of general A and B)