# (Selective and Efficient) Process Tomography

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Group Meeting

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# Quantum Processes

- How can we represent a quantum process that takes density matrices to density matrices?

$\rho_{in}$ ----> ???? ---> $\rho_{out}$

# Quantum Processes

- What properties should a quantum process have?
  - Must have the property

$$0 \le tr(\rho_{out}) \le 1$$

  - Must be a convex-linear map

$$\varepsilon\left(\sum_i p_i \rho_i\right) = \sum_i p_i \varepsilon(\rho_i)$$

  - Must be completely positive

$$\rho_{out} = \varepsilon(\rho_{in}) \ge 0 \quad \forall \rho_{in} \ge 0$$

# Quantum Processes

- The map ε satisfies all three properties iff:

$$\varepsilon(\rho) = \sum_i E_i \rho E_i^T$$

- Where $1 \leq i \leq d^2$ and $\sum_i E_i E_i^T \leq I$

(Proof of this and further discussion in Nielsen and Chuang)

# Quantum Processes

- Thus, if we know the operators $E_i$ , then we have characterized the process completely and can predict its action on any density matrix.

- This construction is not unique, however.

- Also, we don’t measure operators in the laboratory!

# Quantum Processes

- Thus, we write:

$$E_i = \sum_{m=1}^{d^2} e_{im} \overline{E_m}$$

- Where the $\overline{E_m}$ span the space of Hermitian dxd operators.

# Quantum Processes

- Then, our process takes the form:

$$\varepsilon(\rho)=\sum_{m,n=1}^{d^2}\overline{E_m}\rho\overline{E_m{}^T}\chi_{mn}$$

- Where everything is “fixed” except the matrix

$$\chi_{mn}=\sum_i e_{im}e_{in}^*$$

Which is positive Hermitian by construction

# Quantum Process Tomography

- So, send in $d^2$ linearly independent density matrices, and do state tomography out the other end.
- Then, we must invert our system of equations to determine the Chi matrix.

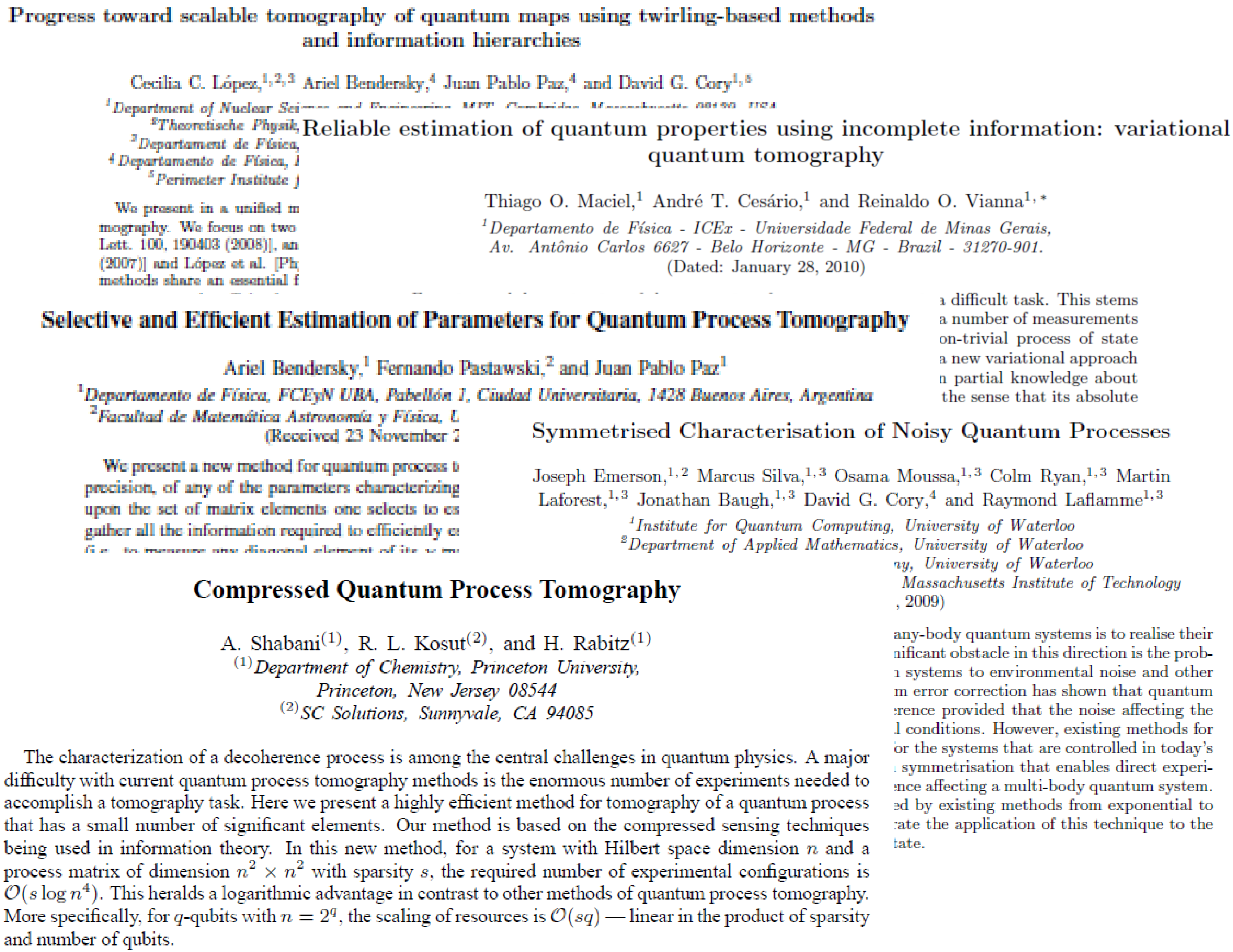
- Suffice it to say this is straightforward, and can be found again in Nielson and Chuang…

# Process Tomography

- In general, $\chi$ is a $d^4$x$d^4$ Hermitian matrix, which has $d^4$ parameters.

- For 2 qubits, this is 256 measurements!

- Is there a way we can do better than this?



# Progress toward scalable tomography of quantum maps using twirling-based methods and information hierarchies

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Lett. 100, 190403 (2008)], an
(2007)] and López et al. [Ph
methods share an essential f

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# Reliable estimation of quantum properties using incomplete information: variational quantum tomography

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(Dated: January 28, 2010)

a difficult task. This stems
a number of measurements
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a new variational approach
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# Selective and Efficient Estimation of Parameters for Quantum Process Tomography

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(Received 23 November 2

We present a new method for quantum process t
precision, of any of the parameters characterizing
upon the set of matrix elements one selects to es
gather all the information required to efficiently e

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# Symmetrised Characterisation of Noisy Quantum Processes

Joseph Emerson,[1,2] Marcus Silva,[1,3] Osama Moussa,[1,3] Colm Ryan,[1,3] Martin Laforest,[1,3] Jonathan Baugh,[1,3] David G. Cory,[4] and Raymond Laflamme[1,3]
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ny, University of Waterloo
Massachusetts Institute of Technology
, 2009)

any-body quantum systems is to realise their
nificant obstacle in this direction is the prob-
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or the systems that are controlled in today's
. symmetrisation that enables direct experi-
nce affecting a multi-body quantum system.
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ate the application of this technique to the
tate.

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# Compressed Quantum Process Tomography

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The characterization of a decoherence process is among the central challenges in quantum physics. A major difficulty with current quantum process tomography methods is the enormous number of experiments needed to accomplish a tomography task. Here we present a highly efficient method for tomography of a quantum process that has a small number of significant elements. Our method is based on the compressed sensing techniques being used in information theory. In this new method, for a system with Hilbert space dimension $n$ and a process matrix of dimension $n^2 \times n^2$ with sparsity $s$, the required number of experimental configurations is $\mathcal{O}(s \log n^4)$. This heralds a logarithmic advantage in contrast to other methods of quantum process tomography. More specifically, for $q$-qubits with $n = 2^q$, the scaling of resources is $\mathcal{O}(sq)$ — linear in the product of sparsity and number of qubits.

# The One Semi-Experimental Slide

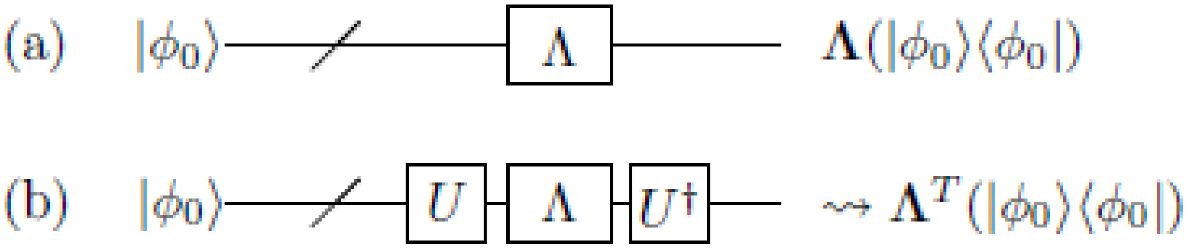

FIG. 1: Circuit representation of (a) the action of a map $\Lambda$; (b) the action of the map, now twirled by $U$.

# Back to theory…

- Normally,

$$\Lambda(\rho) = \sum_{l,l'=0}^{D^2-1} \chi_{l,l'} E_l \rho E_{l'}^\dagger$$

- Once we "twirl" the gate, which involves "averaging" over U:

$$\Lambda^{HT}(\rho) = \int dU U^\dagger \Lambda(U \rho U^\dagger) U$$

# The Haar Twirl

- Next, we add an extra unitary (E) to our system, and project back onto the original state, so that we are calculating:

$$\int d\psi \langle\psi|\mathbf{\Lambda}(E_l^{\dagger}|\psi\rangle\langle\psi|E_{l'})|\psi\rangle$$

- Where now $|\psi\rangle = U|\phi_0\rangle$

- And formally, the integral over psi is mathematically equivalent to taking the integral over U

# So what?!

- Well, in 1980, S. Samuel, J. Math. Phys. 21, 2695, calculated this integral for the Haar measure…

$$
\begin{aligned}
&\int dU\, \mathrm{Tr}[A_1 U^\dagger B_1 U A_2 U^\dagger B_2 U] \\
&= \frac{\mathrm{Tr}[A_1 A_2]}{D^2-1}\left(\mathrm{Tr}[B_1]\mathrm{Tr}[B_2] - \frac{\mathrm{Tr}[B_1 B_2]}{D}\right) \\
&+ \frac{\mathrm{Tr}[A_1]\mathrm{Tr}[A_2]}{D^2-1}\left(\mathrm{Tr}[B_1 B_2] - \frac{\mathrm{Tr}[B_1]\mathrm{Tr}[B_2]}{D}\right)
\end{aligned}
$$

# Again, so what?!

- All these traces can be calculated in terms of the Chi representation!
- Then:

$$\frac{D\chi_{l,l'} + \delta_{l,l'}}{D+1} = \int d\psi \langle\psi|\mathbf{\Lambda}(E_l^\dagger|\psi\rangle\langle\psi|E_{l'})|\psi\rangle$$

(The Haar Twirl)

C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, W. K. Wootters, Phys. Rev. A 54, 3824 (1996).

# But there is a problem!

- How exactly can we calculate the average of some function efficiently without taking an infinite number of measurements?
- One can show (rigorously!), that the FINITE number of measurements needed to estimate this average with a precision of epsilon and failure probability delta:

$$M \geq \ln(2/\delta)/(2\epsilon^2)$$

# The Haar Twirl

- This does not scale with the dimension of the system!

- This means we can efficiently estimate any diagonal element of the Chi matrix with a finite number of measurements.

# And…

- The Haar twirl actually defines a valid inner product over the state space!

$$\langle E_l, E_{l'}\rangle \equiv \int d\psi \langle\psi|\boldsymbol{\Lambda}(E_l^\dagger|\psi\rangle\langle\psi|E_{l'})|\psi\rangle$$

- This means the Cauchy-Shwarz inequality can be used:

$$|\chi_{l,l'}|^2 \leq \chi_{l,l}\ \chi_{l',l'} + \frac{\chi_{l,l} + \chi_{l',l'}}{D} + \frac{1}{D^2}$$

- Which provides a method for bounding the off-diagonal elements

# Summary

- They have provided a *selective* and *efficient* method for determining the diagonal elements of the Chi matrix for a given process.
- Also, the off diagonal elements are in effect bounded by the on-diagonal elements, making their estimation possible (in certain circumstances)

# Other neat algorithms involving twirling…

- Clifford twirl: Involves twirling over the set of Clifford states, which is finite
- MUB twirl: involves twirling over the MUBs, which are also finite
- Symplectic one qubit twirl (S1T): involves twirling over a finite set of single qubit rotations
- Ancilla-assisted twirling: Allows the off-diagonal elements of the Chi matrix to be determined, at the cost of an ancilla qubit

# Other neat algorithms involving twirling…

- Also: there is an algorithm for automatically selecting out the largest diagonal Chi matrix element which is fantastically complicated (and super cool!). Further reading can be found at:

- arXiv:1003.2444v2

Thank you!