Time-symmetric formulation of quantum mechanics

Greg Dmochowski
Group meeting
November 24, 2010

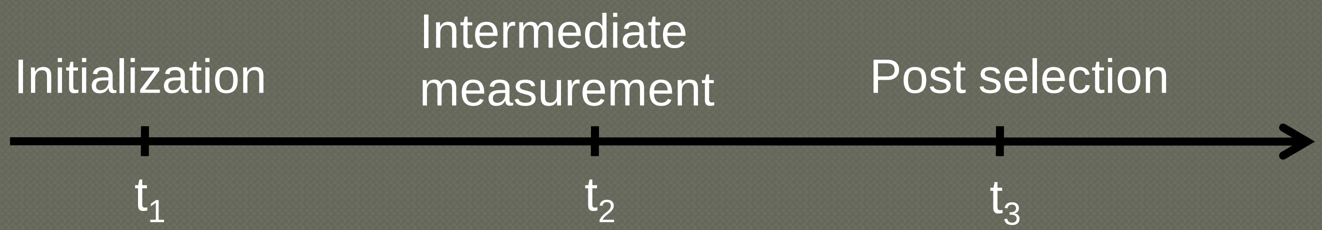
Overview

1. Pre- and post selection
2. (Strong and weak) Measurement theory
3. Three box problem
4. The flow of time

Pre- and postselection I

- Pre-selection = state preparation
- Post-selection = ignoring data that doesn't fit your cause

- e.g. An ensemble of N particles on which you make measurements at times t_1, t_2, t_3



  - Split ensemble into sub-ensembles based on the outcome of the final measurement
  - Statistical distribution of results at t_2 will differ between the various sub-ensembles
  - They claim, then, that measurement at t_2 depends not only on preparation but also on post-selected event

Pre- and postselection II

- Obviously, statistics of different (pre and) post selected ensembles differ – conditional probability!
- But in QM, there is genuinely new information from the post-selection, unlike in classical cases.
- “We argue therefore that pre- and postselected ensembles should be considered *the* fundamental quantum ensembles”

So what?

- Their view: The post selection influences the intermediate measurement
- Another view: Despite the randomness of quantum measurements, there is nevertheless some causal connection between the intermediate measurement and a final measurement

Any observable difference between the two interpretations?

Conditional probabilities

$$P(A) = \langle |A\rangle \langle A| \rangle$$

$$P(A\&B) = \langle |A\rangle \langle A| \, |B\rangle \langle B| \rangle$$

$$P(A\&B) = P(A|B)P(B)$$

$$P(A|B) = \frac{P(A\&B)}{P(B)}$$

$$\begin{aligned} P(A|f) &= \frac{\langle |f\rangle \langle f| \, |A\rangle \langle A| \rangle}{\langle |f\rangle \langle f| \rangle} \\ &= \frac{\langle i \,|f\rangle \langle f| \, |A\rangle \langle A| \, i\rangle}{\langle i \,|f\rangle \langle f| \, i\rangle} \\ &= \frac{\langle f|A\rangle \langle A|i\rangle}{\langle f|i\rangle} \end{aligned}$$

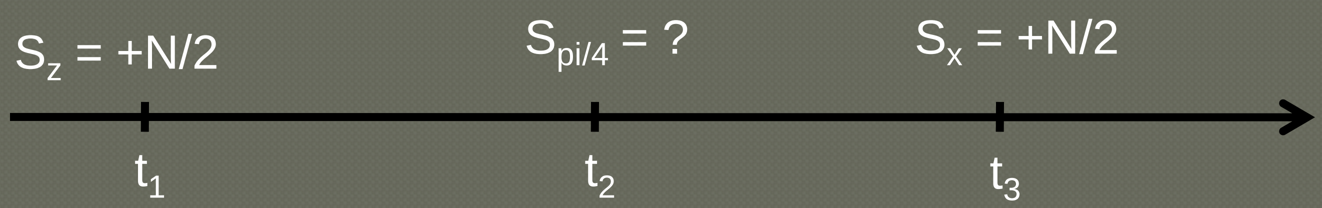
$$\langle A\rangle_{wk} = \frac{\langle f|A\rangle \langle A|i\rangle}{\langle f|i\rangle}$$

Non-trivial consequences?

- Prepare (pre-select) all N particles in $S_z = ½$
- Post-select on particles with $S_x = ½$

- What happens if we measure at intermediate time:

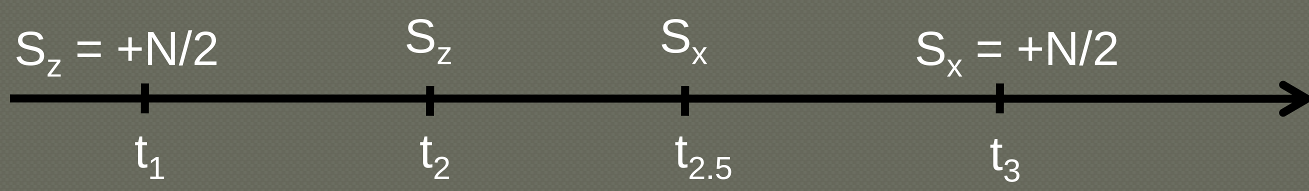
$$S_{pi/4} = (S_x + S_z)/\sqrt{2}$$



- Are S_z and S_x both well-defined at t_2?

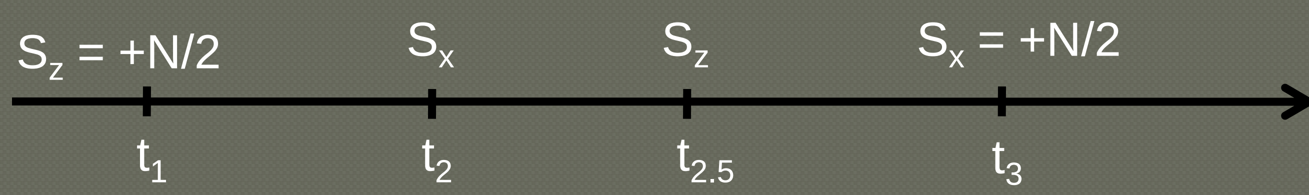
Non-trivial consequences?

- Is that the same as:



Would you get $(N/2 + N/2)/\sqrt{2} = \sqrt{2}(N/2)$??

- Or is it the same as:



Measurement theory

- We want to measure some observable *A* of a system
- Treat measuring device (i.e. a 'pointer') quantum mechanically
  - Position, *X*, of pointer describes measurement result

$$\frac{d\langle X\rangle}{dt} \propto A$$

$$\frac{d\langle X\rangle}{dt} = \frac{i}{\hbar}\langle[H,X]\rangle$$

$$H_{int} = gAP$$

$$[H_{int}, X] = -i\hbar g A$$

$$\frac{d\langle X\rangle}{dt} = gA$$

Back action

$$\frac{d\langle B\rangle}{dt} = \frac{i}{\hbar}\langle[H,B]\rangle$$

$$[H_{int}, B] = -i\hbar g P$$

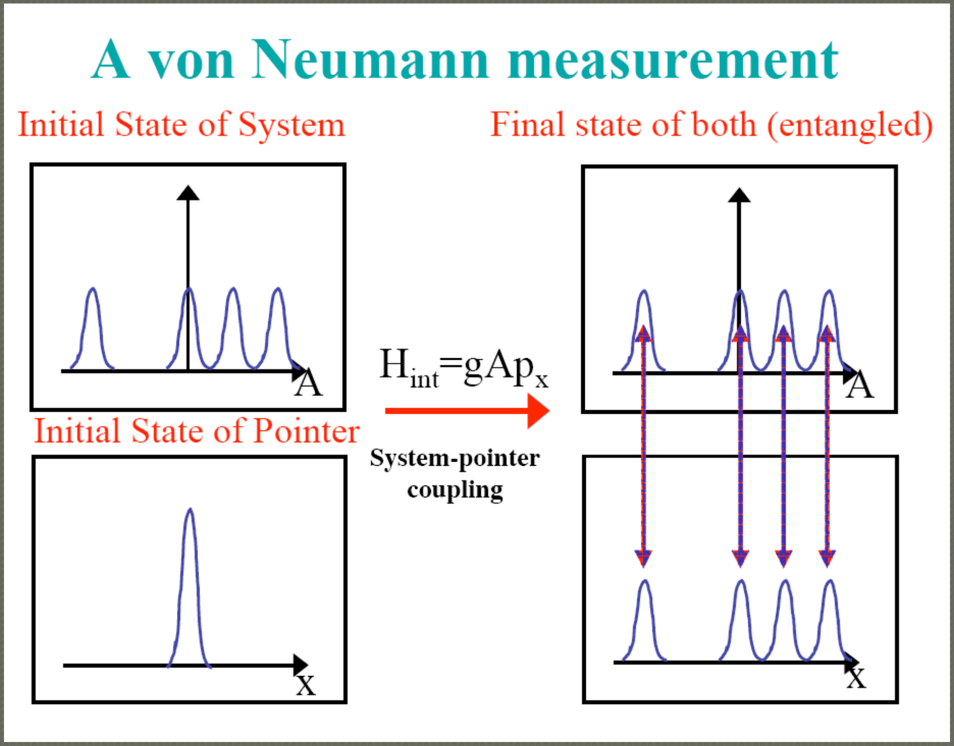
$$\frac{d\langle B\rangle}{dt} = gP$$

But P is very uncertain due to measurement of X!

Strong measurements

$$\frac{d\langle X\rangle}{dt} = gA$$

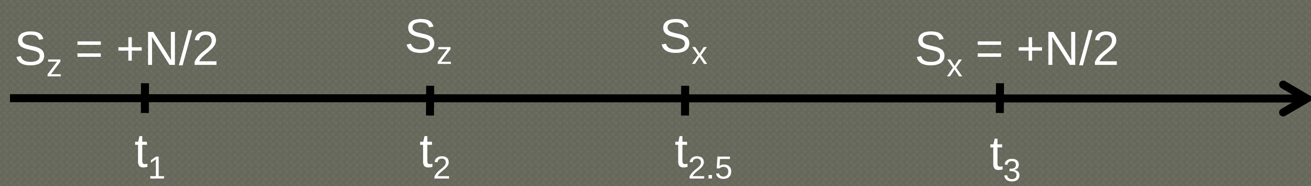
$$\frac{d\langle B\rangle}{dt} = gP$$



- Strong measurement (gA >> Δx) moves pointer a lot (relative to its uncertainty)
- i.e. Pointer position, X, very certain; large back-action

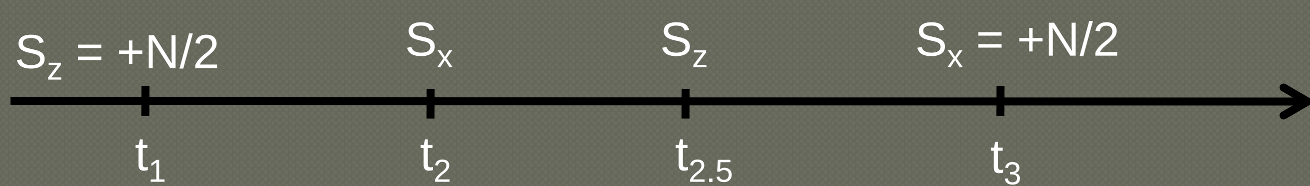
Non-trivial consequences?

- Is that the same as:



Would you get $(N/2 + N/2)/\sqrt{2} = \sqrt{2}(N/2)$??

- Or is it the same as:

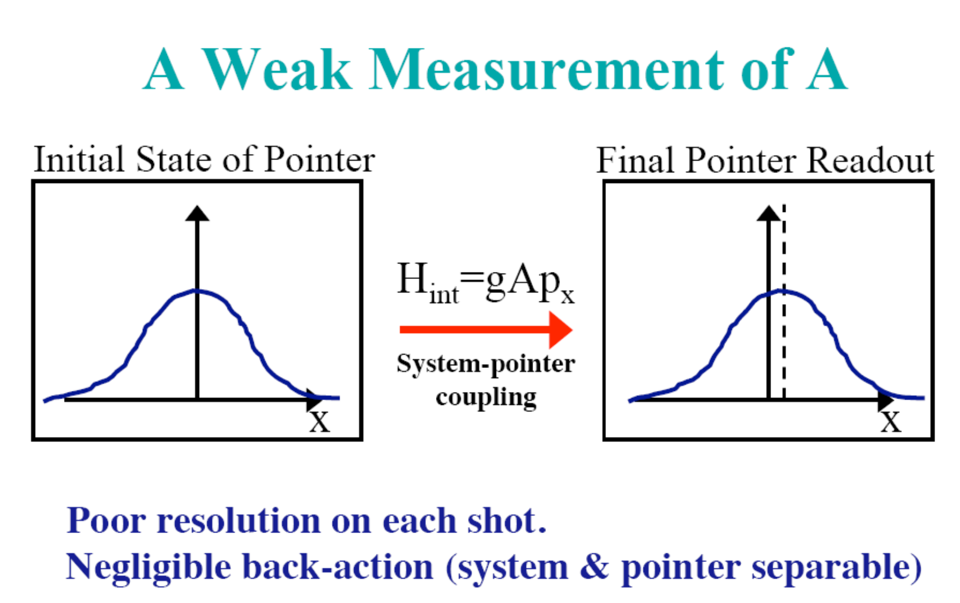


Weak measurements

- Turn g down a lot to make it a weak measurement (gA << Δx)

$$\frac{d\langle X\rangle}{dt} = gA$$

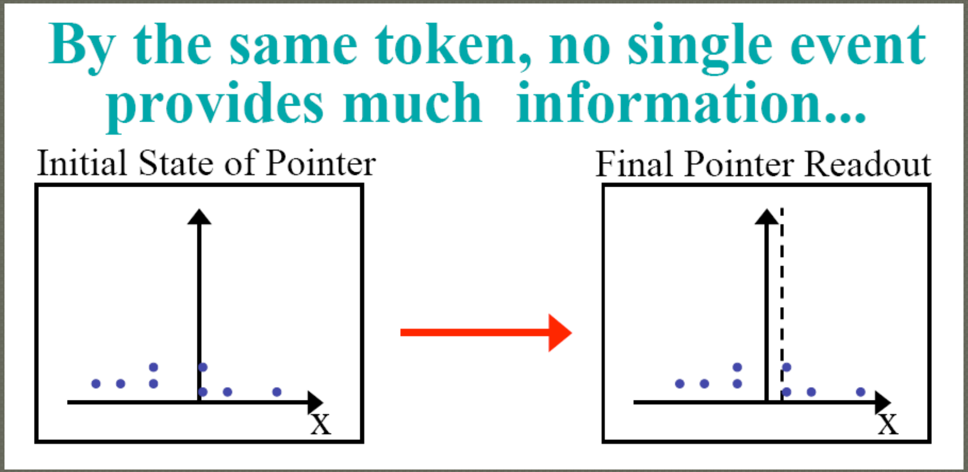
$$\frac{d\langle B\rangle}{dt} = gP$$

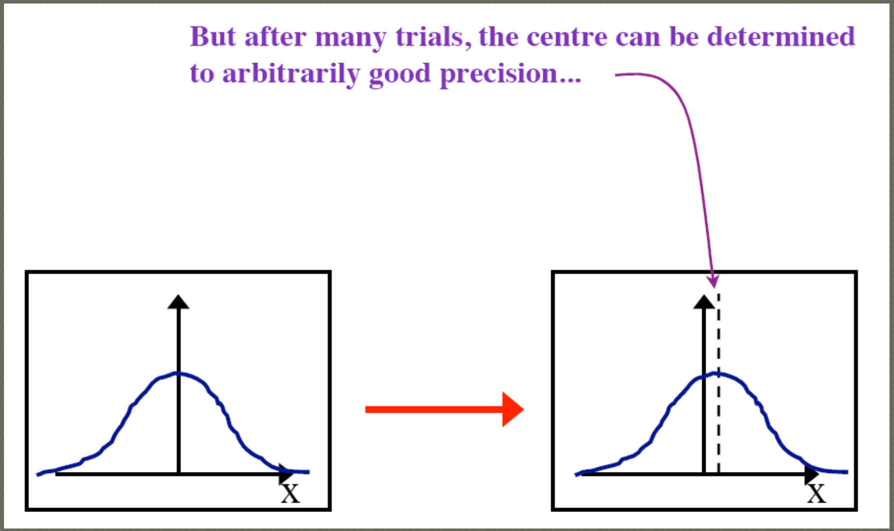


Large position uncertainty = small momentum uncertainty

And small back action

Weak measurements





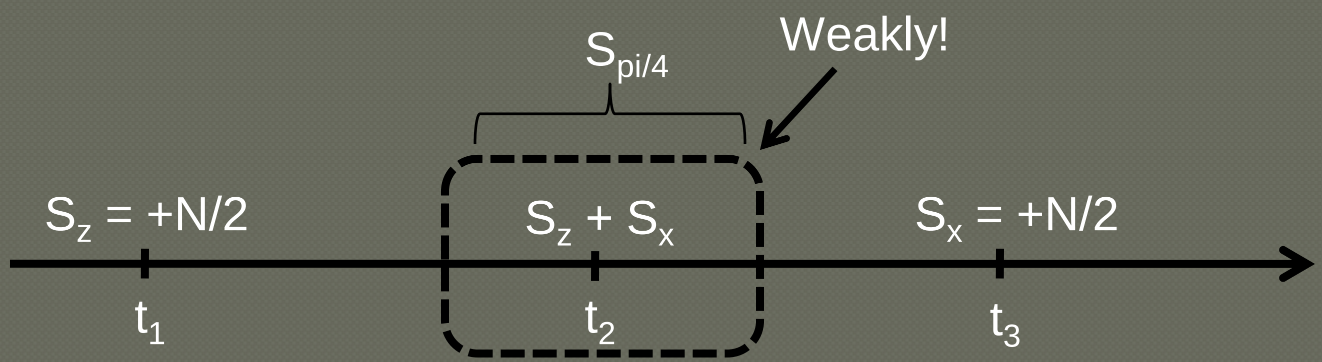
So:

Weak measurements allow us to obtain just a wee bit of information without bugging up our system.

With enough measurements, this wee bit becomes arbitrarily informative.

Weak measurements

- So, what if we measure $S_{pi/4}$ weakly?



- Lack of back-action preserves states
  - 'When post selection succeeds' weak measurement result will be:

$$S_{pi/4} = (S_x + S_z)/\sqrt{2} = \sqrt{2}(N/2)\ !!$$

$$\langle A \rangle_{wk} = \frac{\langle f|A\rangle \langle A|i\rangle}{\langle f|i\rangle}$$

Game of errors

- But how can we get this ‘inadmissible’ measurement result?
- No longer are we doing strong measurements – eigenvalue formalism doesn’t apply?
- Imperfect measurement can point, in error, to a value outside of the eigenvalue spectrum
- ‘one can show that if nondisturbance is to be achieved, larger errors *must* be possible’ (‘Superoscillations’)
- If post selection succeeds, $S_{pi/4}$ must have yielded the inadmissible value

Three points about this result

1. This strange weak value is precisely the value obtained via ‘intuition’.

2. It always occurs when the post selection succeeds.

3. It is not limited to any particular kind of measurement.
   - For that pre and post selection, anything that interacts with the system will ‘feel’ this strange weak value

Three box problem

- Prepare a single particle in state

$$(1 + 2 + 3)/\sqrt{3}$$

- Post-select on finding particle in final state

$$(1 + 2 - 3)/\sqrt{3}$$

- What if we were to look into box 1 at intermediate time?
  - If it's not there, our state must collapse to $(2+3)/\sqrt{2}$
  - But there is no overlap with the post-selected final state. Therefore, they claim that looking into box one must yield the particle there.
  - But the same argument holds for box 2!
- Do we conclude that there is a particle in box 1 and box 2?

Three box problem

- That was for a strong measurement, which necessarily disturbs the state.
  - Therefore, you cannot actually perform both intermediate measurements and get the paradoxical result.
- Try weak measurements, where you won't disturb the state
- Prepare large number N particles in initial state and post-select on those found in the desired final state
- If we measure boxes 1 and 2 weakly, we apparently will get $N \pm \sqrt{N}$ in each box
- Therefore, there must be $-N$ particles in box 3.

The flow of time

- These effects may be explained/computed with the standard view of forward-flowing time
  - Complicated, though! Superoscillations.
- Simpler explanation (apparently):
  - Both time boundary conditions (per and post selection) affect the intermediate measurement.
- Discretize time and associate with each moment two Hilbert spaces
  - One for states ‘flowing forward’ and one for states ‘flowing backwards’
- Time evolution is now seen as correlations between the forward flowing and backward flowing states

A note about counterfactuals

- (simple) Conditional:

  If A, then B

- Counterfactual conditional:

  If it were that A, then it would be that B

- In philosophy, the truth-ness of the counterfactual is debated based on the similarity between the actual world (in which A isn't true) and the counterfactual world (in which A is presumed to be true)
- In such analysis, it is assumed that if the counterfactual world is identical to the actual world, then the outcomes will be the same
- But randomness inherent in QM spoils this approach.
- So what *can* be said about counterfactuals? .... I don't know.