Itinerant Ferromagnetism in Ultracold Fermions

Group meeting, March 2, 2011

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Itinerant Ferromagnetism in Ultracold Fermions

- Magnetism and motivation
- Theoretical discussion
- MIT experiment
- …and controversy!
- Our experiment

Motivation

- What is the simplest Hamiltonian that can yield ferromagnetism?
  - Lattice ferromagnetism: Heisenberg Hamiltonian:

$$H = -\sum_{i,j} T_{i,j}\vec{S}_i \cdot \vec{S}_j \qquad E(\uparrow\uparrow) < E(\uparrow\downarrow)$$

  - Analyzed by Ising
    - No ferromagnetic transition in 1-D, but do have a ferromagnetic transition in any higher dimension

T = 0 T = 1 T = 2 T = 2.5 T = 3 T = 10

Motivation

- What is the simplest Hamiltonian that can yield ferromagnetism?
  - Itinerant ferromagnetism: Stoner Hamiltonian:

$$H = \sum_i \frac{\vec{P}^2}{2m} + \sum_{i,j} V(r_i, r_j)\vec{S}_i \cdot \vec{S}_j \qquad E(\uparrow\uparrow)<E(\uparrow\downarrow)$$

  - Remarkably, we are not sure if this should produce a ferromagnetic ground state

Motivation

- Ultracold gases can be used as quantum simulators
  - Quantum problems are inherently difficult due to the huge dimension of the space where solutions live and because of the statistics-enforced correlations between different particles.
- We use a cloud of ^{40}K atoms to simulate the Stoner Hamiltonian.

Basic theory: Stoner instability

- Stoner Hamiltonian for a system of free, interacting spin- ½ particles:

$$H = \sum_i \frac{\vec{P}^2}{2m} + \sum_{i,j} V(r_i, r_j) \vec{S}_i \cdot \vec{S}_j$$

- In the *local density approximation* (LDA), we assume the two spin states of the particles have densities $n_\uparrow$ and $n_\downarrow$ and that the interaction is short-range, i.e.

$$V(\vec{r}) = \frac{4\pi \hbar^2 a}{m} \delta(\vec{r})$$

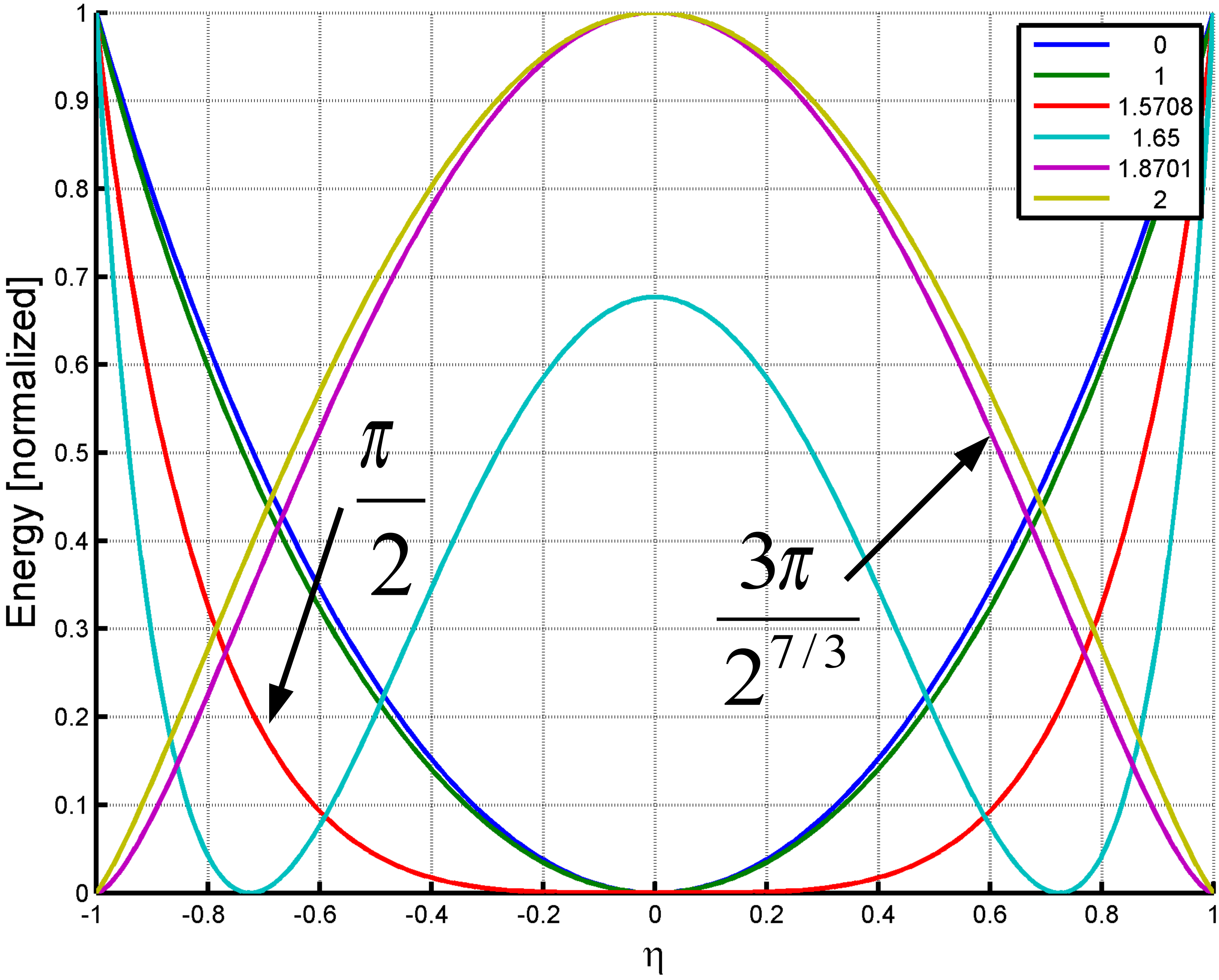
Basic theory: Stoner instability

- Then the energy density at T=0 equals

$$E = \frac{3}{5} E_F n \left[\underset{\text{Kinetic}}{\boxed{\left((1+\eta)^{5/3} + (1-\eta)^{5/3} \right)}} + \underset{\text{Potential}}{\boxed{\frac{20}{9\pi} k_F a (1+\eta)(1-\eta)}} \right]$$

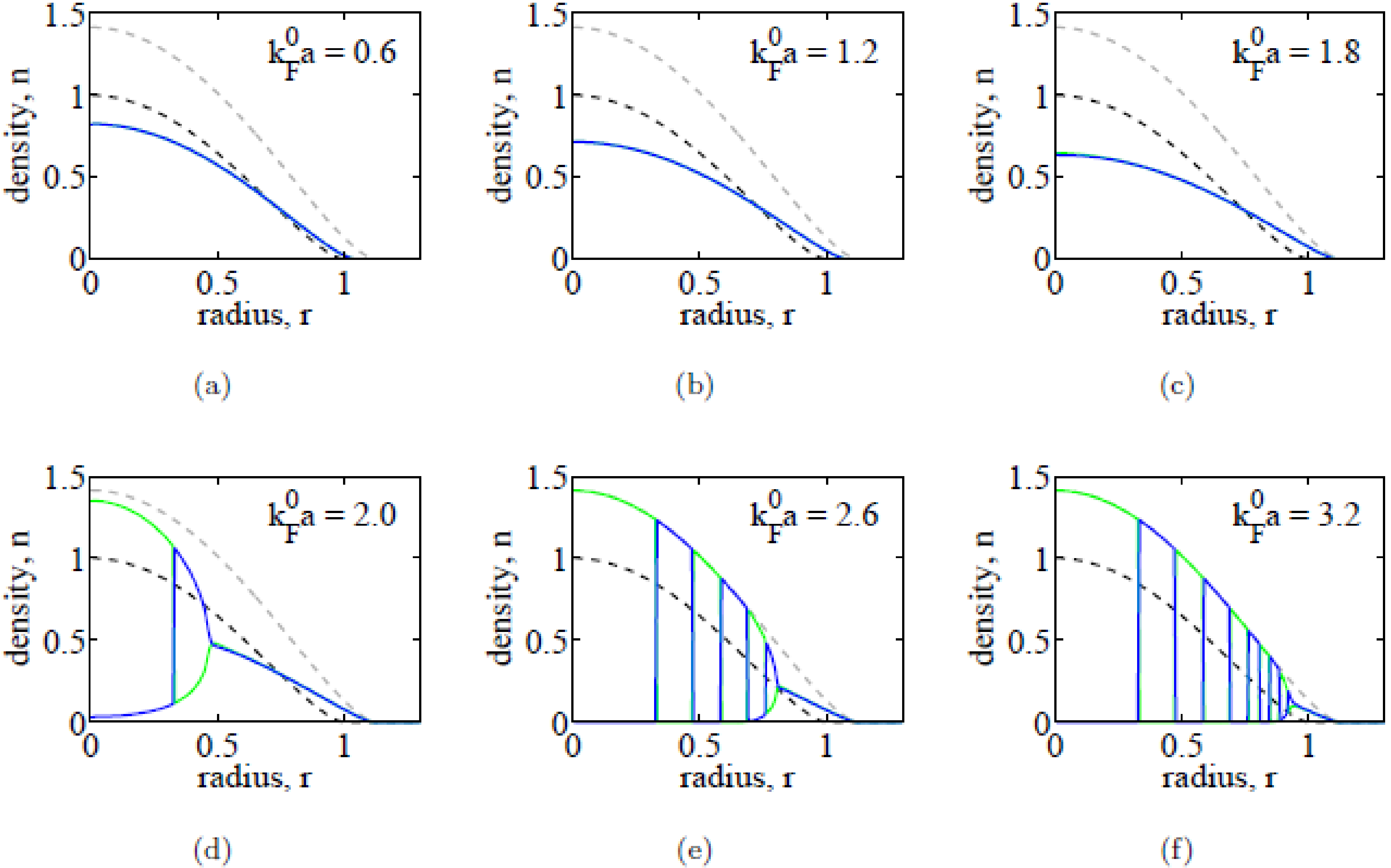
where

$$\eta = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$$


Energy [normalized]
η
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
0
1
1.5708
1.65
1.8701
2
$\frac{\pi}{2}$
$\frac{3\pi}{2^{7/3}}$

Basic theory: Stoner instability

- Expect ferromagnetism onset at $k_F a \sim 1.6$ and full ferromagnetism at $k_F a \sim 1.9$
- Unfortunately, at strong interaction strengths, the mean field treatment breaks down.



(a) (b) (c)

(d) (e) (f)

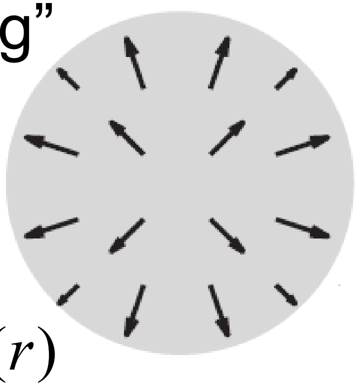
FIGURE 4.2: LDA density profiles for various interaction strengths. The numerical solutions of $n_\uparrow(r)$ (blue) and $n_\downarrow(r)$ (green) are shown for increasing interactions, with equal populations in each spin state ($N_\uparrow = N_\downarrow = N/2$; $m = 0$). Dashed black lines indicate the $k_F^0 a = 0$ noninteracting solution; grey dashed lines indicate the non-interacting solution for all particles in the same spin state ($N_\uparrow = N$; $N_\downarrow = 0$). Interaction parameters indicated in panels.

L. J. LeBlanc, “Exploring many-body physics with ultracold atoms” (PhD thesis), University of Toronto, (2010)

Intermediate theory: Spin textures

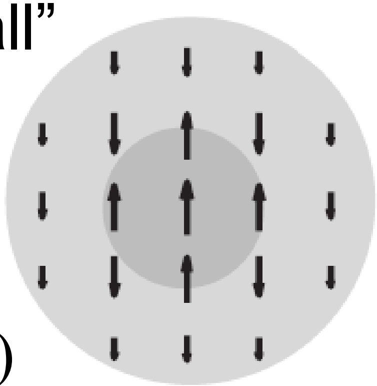
- Need to account for *kinetic and potential energy cost of wavefunction gradients*.
- Postulate a wavefunction ansatz, and calculate the energy!

"Hedgehog" state



$$\vec{m}(\vec{r}) = \begin{pmatrix} \cos\phi \, \sin\theta \\ \sin\phi \, \sin\theta \\ \cos\theta \end{pmatrix} m(r)$$

"Domain wall" state



$$\vec{m}(\vec{r}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} m(r)$$

L.J. LeBlanc, J.H. Thywissen, A.A. Burkov, and A. Paramekanti, "Repulsive Fermi gas in a harmonic trap: Ferromagnetism and spin textures" Phys. Rev. A 80 013607 (2009)

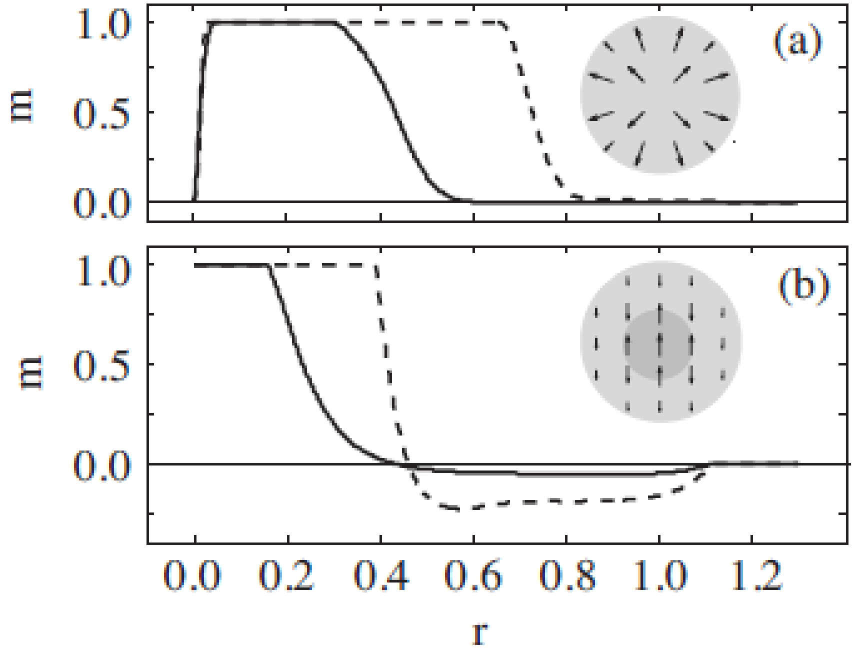


FIG. 4. (a) Magnetization profiles for the hedgehog state at $\lambda = 2.0$ (solid) and $\lambda = 2.4$ (dashed). (b) Magnetization profiles for the domain-wall state at $\lambda = 2.0$ (solid) and $\lambda = 2.4$ (dashed). The profiles have been calculated for 10^4 atoms in an isotropic trap. The hedgehog state has zero magnetization at the trap center while the domain-wall state magnetization gets suppressed around the domain wall but remains nonzero at the trap center. Insets indicate the schematic magnetization plot of the hedgehog state and the domain-wall state.

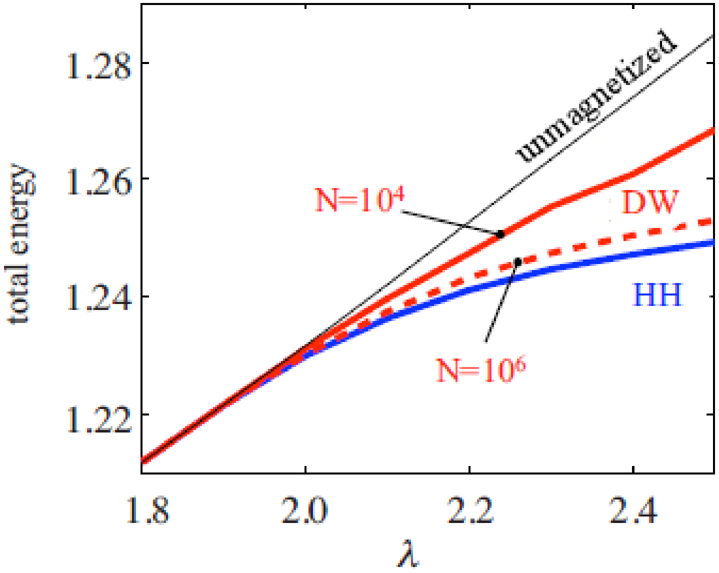


FIG. 3. (Color online) Dimensionless total energy, $\varepsilon_1 + \varepsilon_2$, shown as a function of interaction strength, λ, for an isotropic harmonic trap. DW indicates the energy of the domain-wall state for 10^4 atoms (solid) and 10^6 atoms (dashed). HH denotes $\varepsilon_1 + \varepsilon_2$ for the hedgehog state that is nearly identical for 10^4 and 10^6 atoms. Also shown (thin solid line, "unmagnetized") is ε_1, defined in Eq. (44), which depends only on the total density profile.

L.J. LeBlanc, J.H. Thywissen, A.A. Burkov, and A. Paramekanti, "Repulsive Fermi gas in a harmonic trap: Ferromagnetism and spin textures" Phys. Rev. A 80 013607 (2009)

Advanced theory: QMC simulations

- Can use Quantum Monte Carlo simulations to determine the equilibrium magnetization!

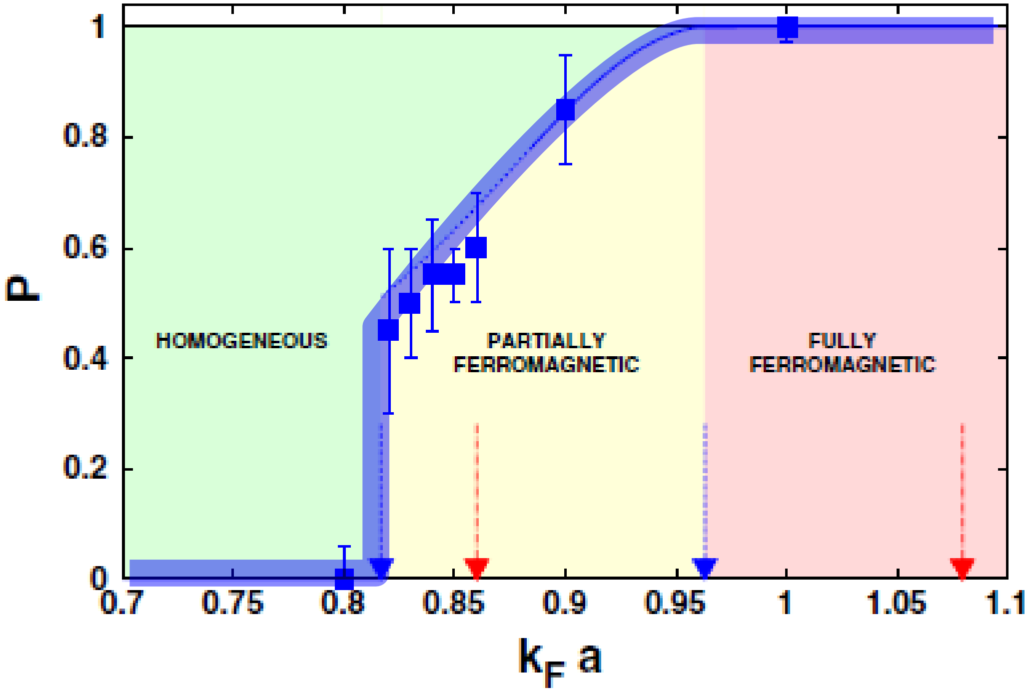


FIG. 1 (color online). Phase diagram of the HS gas in the interaction-polarization plane. The green region corresponds to the homogeneous phase. The other regions correspond to phase separated states with partially polarized domains (yellow) and fully ferromagnetic domains (pink). The (blue) symbols correspond to the minimum of the curve $E(P)$ and the solid-dashed line is the phase boundary determined from the equilibrium condition for pressure and chemical potentials. The blue and red arrows indicate the critical densities where χ diverges and full ferromagnetism sets in, respectively, for the HS and SW potential.

Fun note: simulations were done with 33 particles (and checked with some calculations with 81 particles)

S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, “Itinerant Ferromagnetism of a Repulsive Atomic Fermi Gas: A Quantum Monte Carlo Study” Phys. Rev. Lett. 105, 030405 (2010)

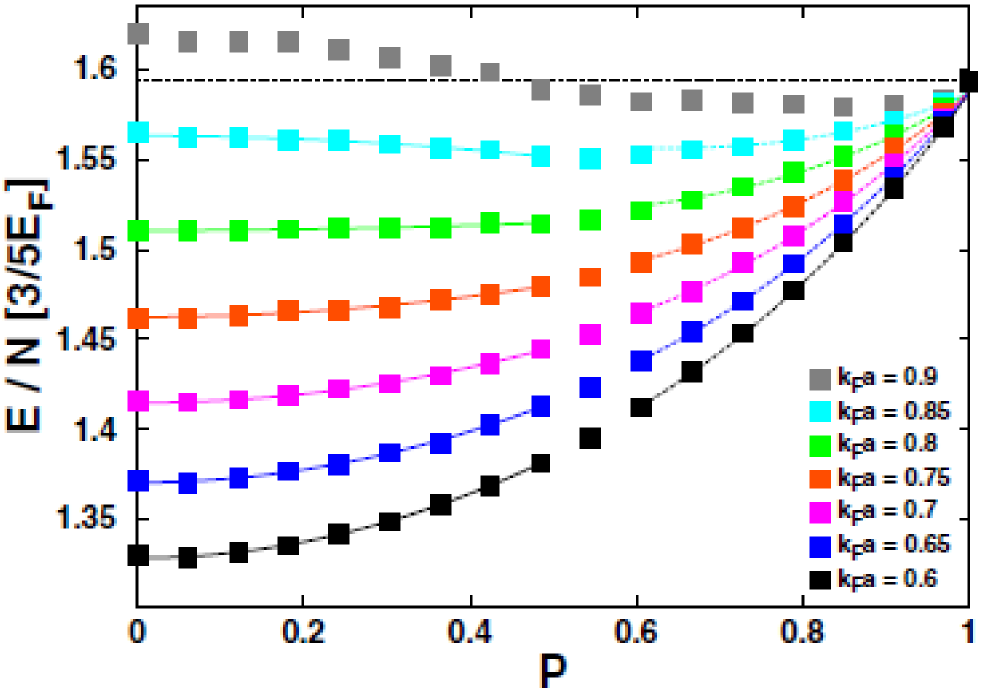
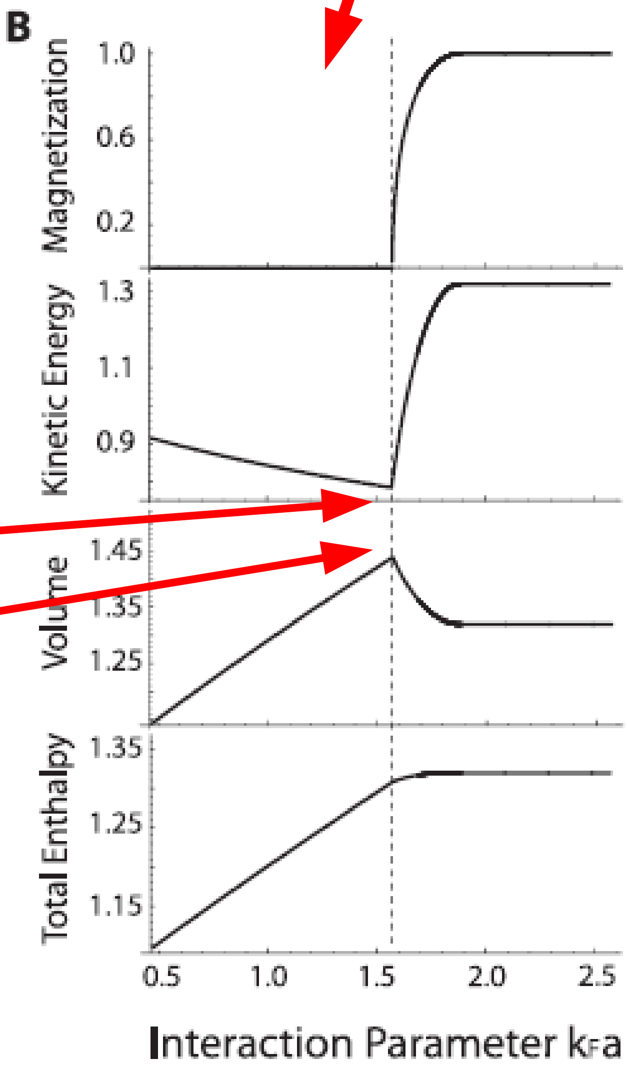


FIG. 3 (color online). Energy of the HS gas as a function of polarization for different values of $k_F a$. The symbols correspond to FN-DMC results, while the lines at small and large P correspond, respectively, to the fitted quadratic law and polaron energy functional of Eq. (4). The horizontal dotted line is the threshold energy E_{FF} of the fully ferromagnetic state.

The MIT experiment

- We expect the following signatures as the atoms transition into the ferromagnetic state:
  - Minimum of kinetic energy
  - Maximum of volume
  - Maximum of atom loss rate



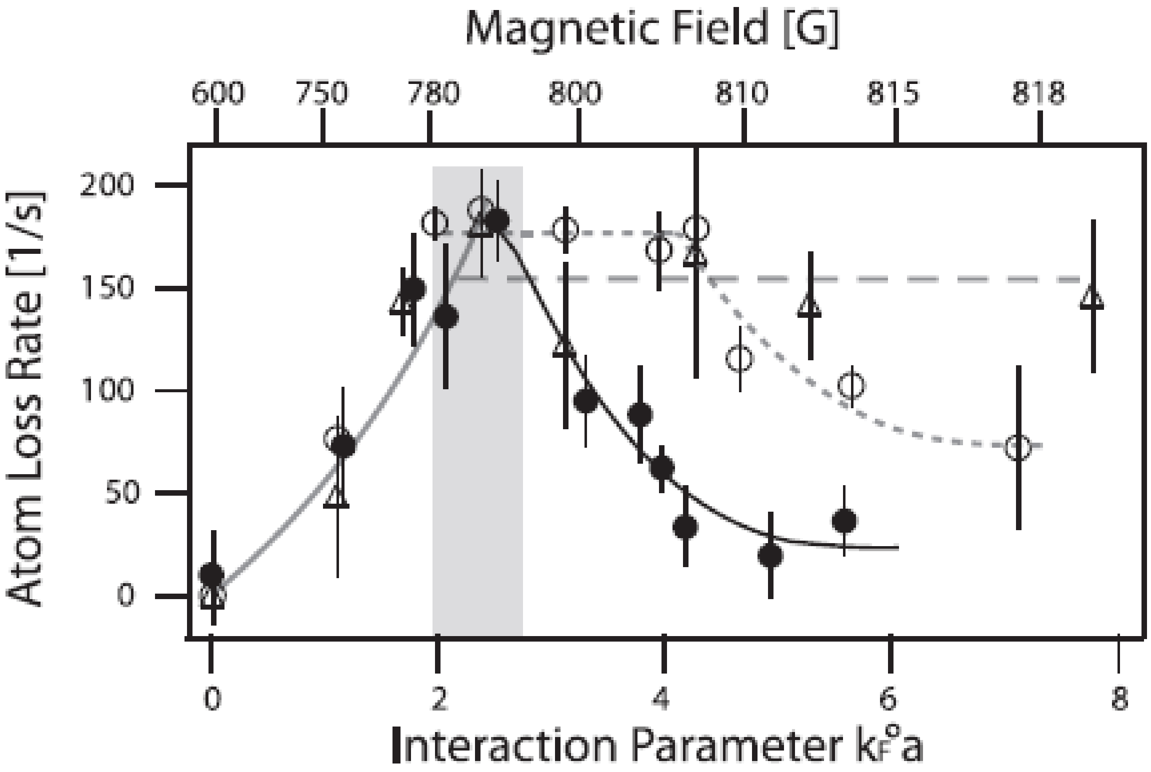
Mean field treatment

G.-B Jo, Y.-R Lee, J.-H Choi, C. A. Christensen, T.H. Kim, J.H. Thywissen, D.E. Pritchard, and W. Ketterle, “Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms” Science 325, 1521-1524 (2009)

The MIT experiment

- Some of these were observed [1]!

Fig. 2. Atom loss rate as a probe for local spin polarization, for different temperatures. $T/T_F = 0.55$ (triangles, dashed curve), $T/T_F = 0.22$ (open circles, dotted curve), and $T/T_F = 0.12$ (solid circles, solid black curve). The curves are guides to the eye, based on the assumption of a loss rate that saturates for increasing *a* in the normal state. The shaded area around the phase transition at $T/T_F = 0.12$ highlights the same region as in Figs. 3 and 4.



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1. G.-B Jo, Y.-R Lee, J.-H Choi, C. A. Christensen, T.H. Kim, J.H. Thywissen, D.E. Pritchard, and W. Ketterle, “Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms” Science 325, 1521-1524 (2009)

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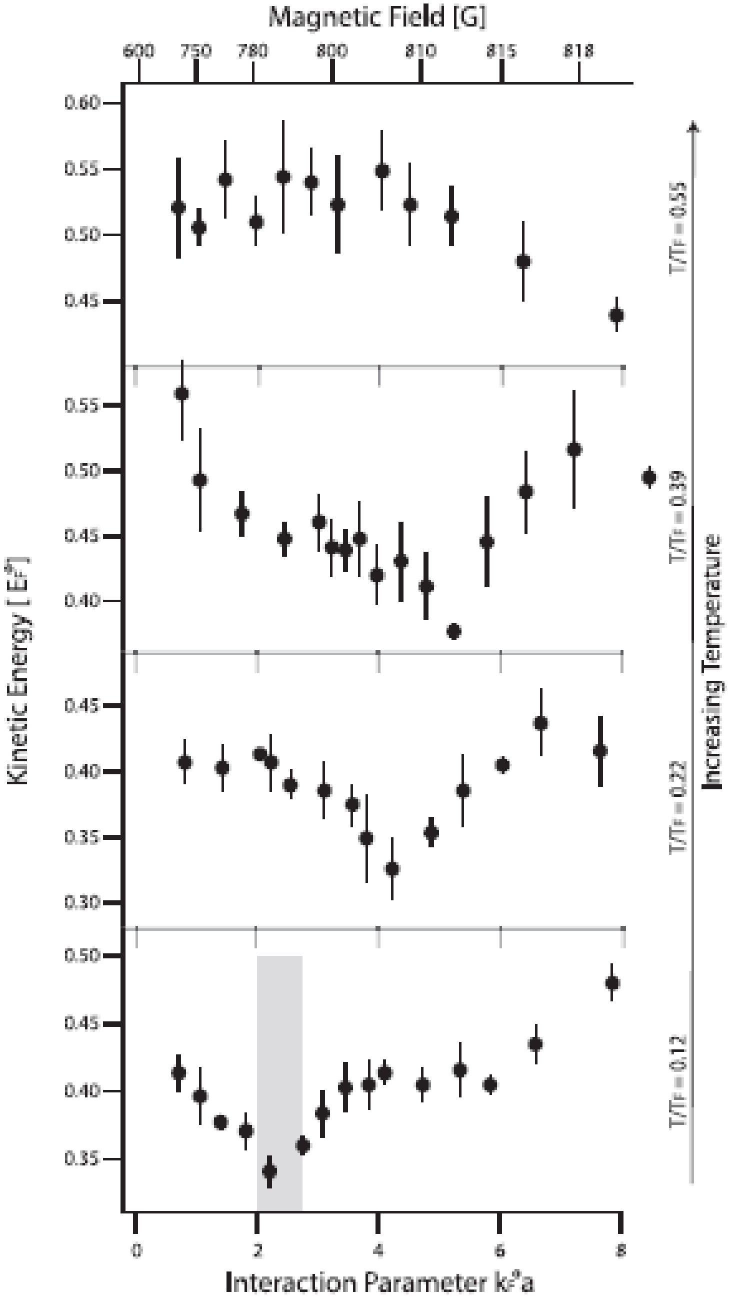


Fig. 3. Kinetic energy of a repulsively interacting Fermi gas determined for different interaction parameters $k_F^0 a$ and temperatures. The measured kinetic energy is normalized by the Fermi energy E_F^0 of the noninteracting Fermi gas at $T = 0$, calculated at the trap center with the same number of atoms per spin state. Each data point represents the average of three or four measurements.

1. G.-B Jo, Y.-R Lee, J.-H Choi, C. A. Christensen, T.H. Kim, J.H. Thywissen, D.E. Pritchard, and W. Ketterle, “Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms” Science 325, 1521-1524 (2009)

The MIT experiment

“In 2009, a team of MIT physicists demonstrated that a lithium gas cooled to less than one Kelvin can exhibit ferromagnetism.[6] The team cooled fermionic lithium-6 to less than 150 billionths of one Kelvin above absolute zero using infrared laser cooling. This demonstration is the first time that ferromagnetism has been demonstrated in a gas.”

… and the controversy

- Contrary to what Wikipedia will have you believe:

  > “In 2009, a team of MIT physicists demonstrated that a lithium gas cooled to less than one Kelvin can exhibit ferromagnetism.[6] The team cooled fermionic lithium-6 to less than 150 billionths of one Kelvin above absolute zero using infrared laser cooling. This demonstration is the first time that ferromagnetism has been demonstrated in a gas.”

not everyone was convinced!

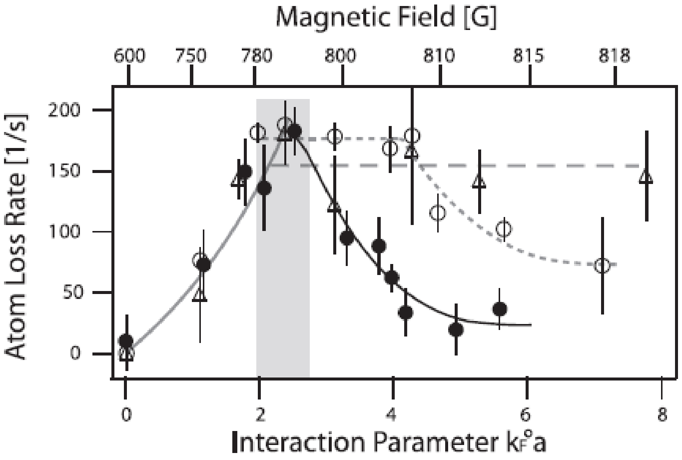
… and the controversy

- Observed and predicted transition temperatures differ greatly[1,2].
- Observed effects are not due to ferromagnetism!
  - Short-range correlations [3]
  - Molecular formation [4,5]
- Ferromagnetic state is unstable! [6]
- Experimental evidence against ferromagnetism [7]

1. R.A. Duine and A.H. MacDonald, “Itinerant Ferromagnetism in an Ultracold Atom Fermi Gas” Phys. Rev. Lett. 95, 230403 (2005)
2. S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, “Itinerant Ferromagnetism of a Repulsive Atomic Fermi Gas: A Quantum Monte Carlo Study” Phys. Rev. Lett. 105, 030405 (2010)
3. H. Zhai, "Correlated versus ferromagnetic state in repulsively interacting two-component Fermi gases", Phys. Rev. A 80, 051605(R) (2009)
4. D. Pekker, M. Babadi, R. Sensarma, N. Zinner, L. Pollet, M.W. Zwierlein, and E. Demler, "Competition between Pairing and Ferromagnetic Instabilities in Ultracold Fermi Gases near Feshbach Resonances“, Phys. Rev. Lett. 106, 050402 (2011)
5. S. Zhang, T.L. Ho, "Atom Loss Maximum in Ultra-cold Fermi Gases" arXiv:1102.5687 (2011)
6. X. Cui and H. Zhai, "Stability of a fully magnetized ferromagnetic state in repulsively interacting ultracold Fermi gases", Phys. Rev. A 81, 041602(R) (2010)
7. A. Sommer, M. Ku, G. Roati, and M.W. Zwierlein, "Universal Spin Transport in a Strongly Interacting Fermi Gas", arXiv:1101.0780 (2011)

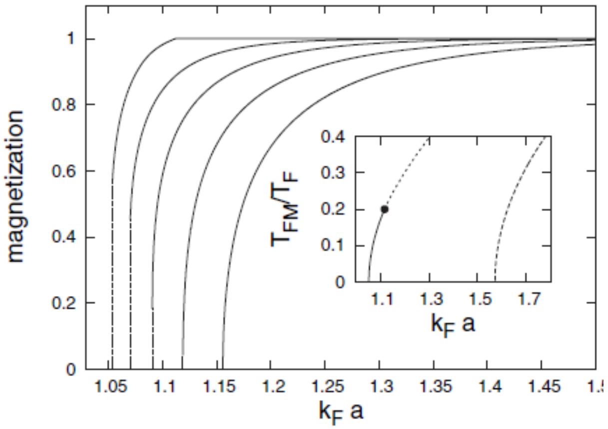
… and the controversy

- Observed and predicted transition temperatures differ greatly[1,2].

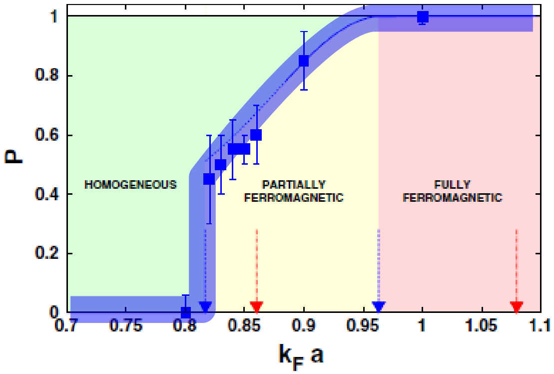


Measured $k_F a$ ~2.2

FIG. 1. Magnetization ξ as a function of $k_F a$, for various temperatures. From left to right $T/T_F = 0, 0.1, 0.15, 0.2, 0.25$. The dashed lines indicate magnetization jumps. The inset shows the critical temperature as a function of the gas parameter. The solid line indicates first-order transitions, and the dotted line second-order transitions. The dashed line is the Hartree-Fock theory result.



Second order corrections to mean-field: $k_F a$ ~1.1



Qunatum Monte Carlo: $k_F a$ ~0.9

1. R.A. Duine and A.H. MacDonald, “Itinerant Ferromagnetism in an Ultracold Atom Fermi Gas” Phys. Rev. Lett. 95, 230403 (2005)
2. S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, “Itinerant Ferromagnetism of a Repulsive Atomic Fermi Gas: A Quantum Monte Carlo Study” Phys. Rev. Lett. 105, 030405 (2010)

… and the controversy

- Short-range correlations:

  Rather than minimizing the energy by spatial segregation, the atoms can reduce their energy by being *locally anti-correlated*.

- "Short-range ferromagnetism"

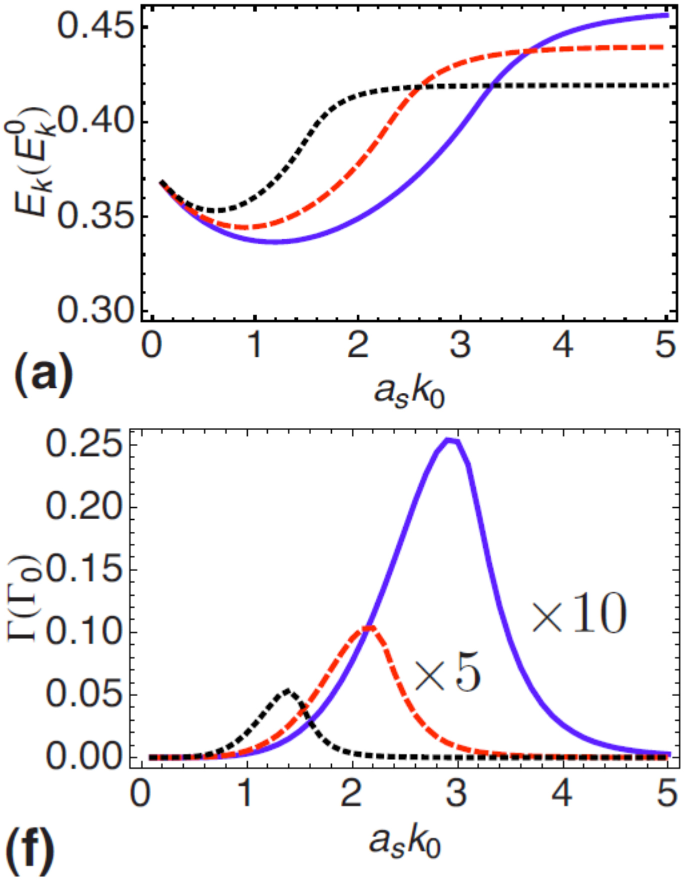


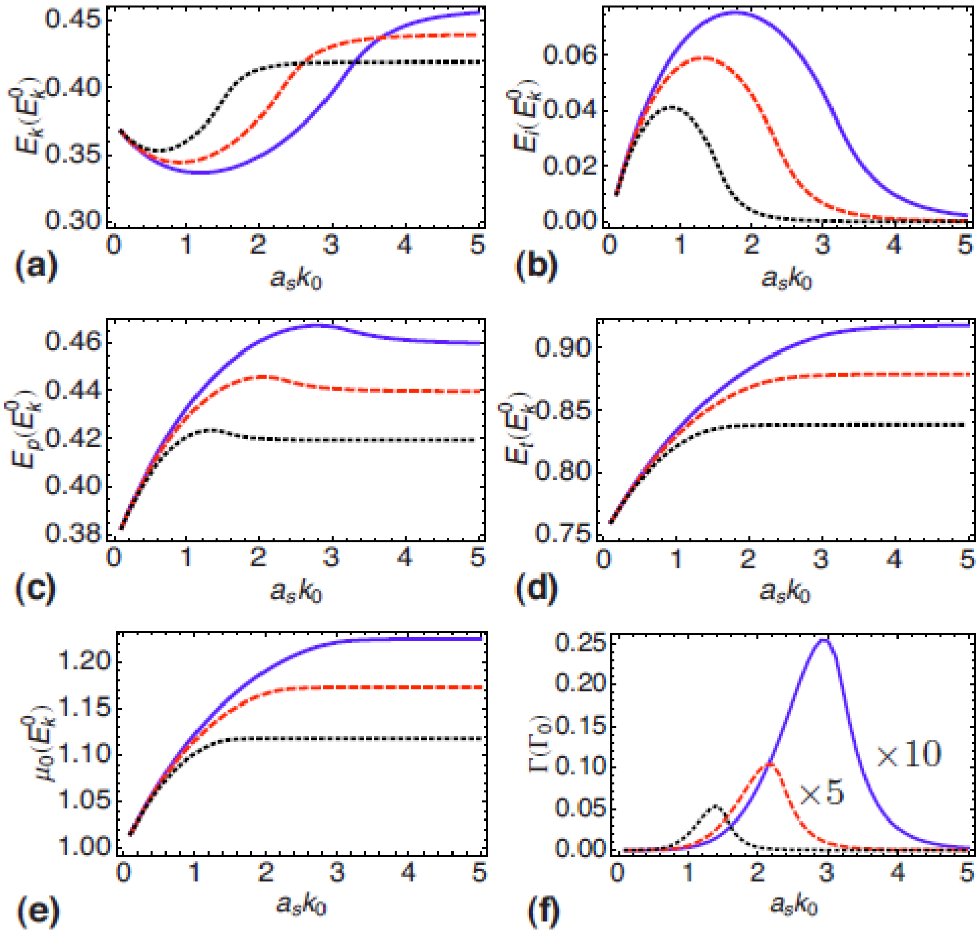
FIG. 2. (Color online) From (a)–(e) is the kinetic energy per particle $E_k=\mathcal{E}_{\text{kin}}/(2N)$, the interaction energy per particle $E_i=\mathcal{E}_{\text{int}}/(2N)$, the potential energy per particle $E_p=\mathcal{E}_{\text{pot}}/(2N)$, the total energy per particle $E_t=\mathcal{E}_{\text{tot}}/(2N)$, and the chemical potential μ_0 as a function of $a_s k_0$. k_0 is the Fermi momentum for free-Fermi gas at the center of the trap, and the energy unit is taken as $E_k^0=\hbar^2 k_0^2/(2m)$. (f) The three-body loss rate as a function of $a_s k_0$. $\alpha=1$ for blue solid line, $\alpha=0.75$ for red dashed line, and $\alpha=0.5$ for black dotted line.

H. Zhai, "Correlated versus ferromagnetic state in repulsively interacting two-component Fermi gases", Phys. Rev. A 80, 051605(R) (2009)

… and the controversy

- Short-range correlations:

  Rather than minimizing the energy by spatial segregation, the atoms can reduce their energy by being *locally anti-*



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H. Zhai, "Correlated versus ferromagnetic state in repulsively interacting two-component Fermi gases", Phys. Rev. A 80, 051605(R) (2009)

… and the controversy

- Molecular formation:
  - The dominant process is molecular formation, rather than the ferromagnetic transition, explaining observed signatures
  - However, this only explains the atom loss peak well

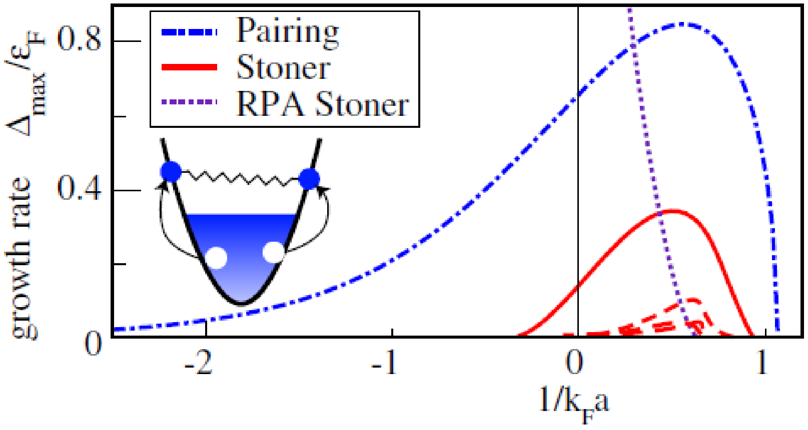
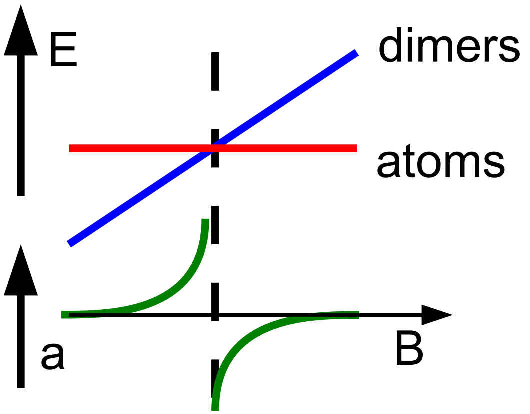


FIG. 1 (color online). Growth rate of the pairing and Stoner ferromagnetic instabilities after a quench as a function of the final interaction strength $1/k_Fa$. Final interactions with negative (positive) values of $1/k_Fa$ correspond to the BCS (BEC) side of the Feshbach resonance. The Stoner instability simultaneously occurs in multiple channels. The most unstable channel is indicated by the solid red line, the others by dashed red lines. The "RPA Stoner" instability corresponds to the RPA result with bare interactions (see text and Ref. [15]). Inset: Schematic diagram of the pair creation process showing the binding energy (spring) being absorbed by the Fermi sea (arrows).

D. Pekker, M. Babadi, R. Sensarma, N. Zinner, L. Pollet, M.W. Zwierlein, and E. Demler, "Competition between Pairing and Ferromagnetic Instabilities in Ultracold Fermi Gases near Feshbach Resonances", Phys. Rev. Lett. 106, 050402 (2011)

… and the controversy



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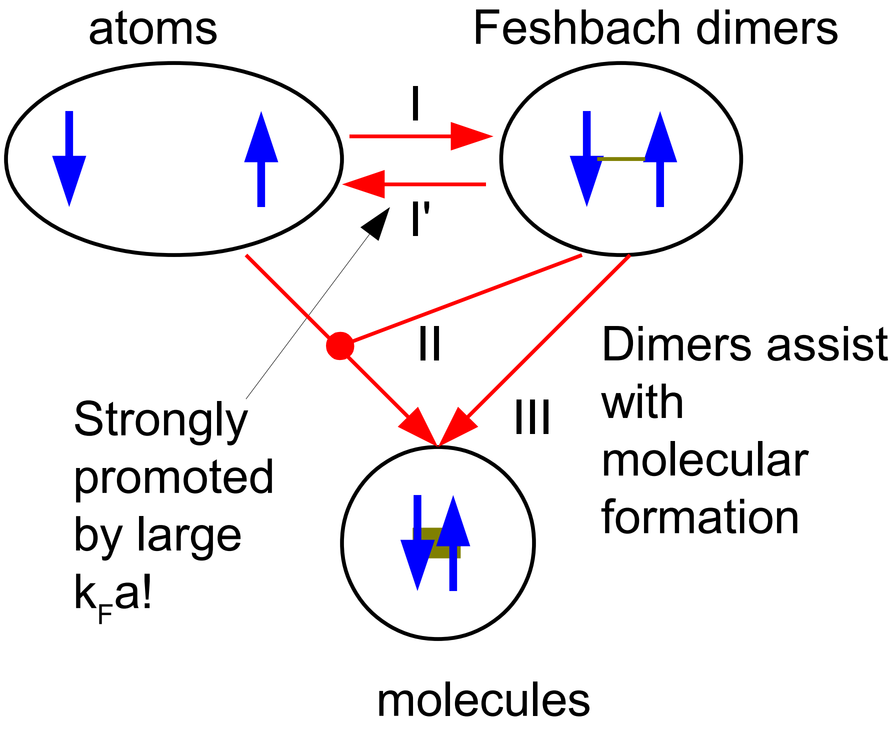


Strongly promoted by large $k_F a$!

Dimers assist with molecular formation

Two days ago!

S. Zhang, T.L. Ho, "Atom Loss Maximum in Ultra-cold Fermi Gases" arXiv:1102.5687 (2011)

… and the controversy

- Instability of the ferromagnetic state

Cost of adding a ↓ to a sea of ↓↓↓↓↓↓↓↓↓

Cost of adding a ↑ to a sea of ↓↓↓↓↓↓↓↓↓

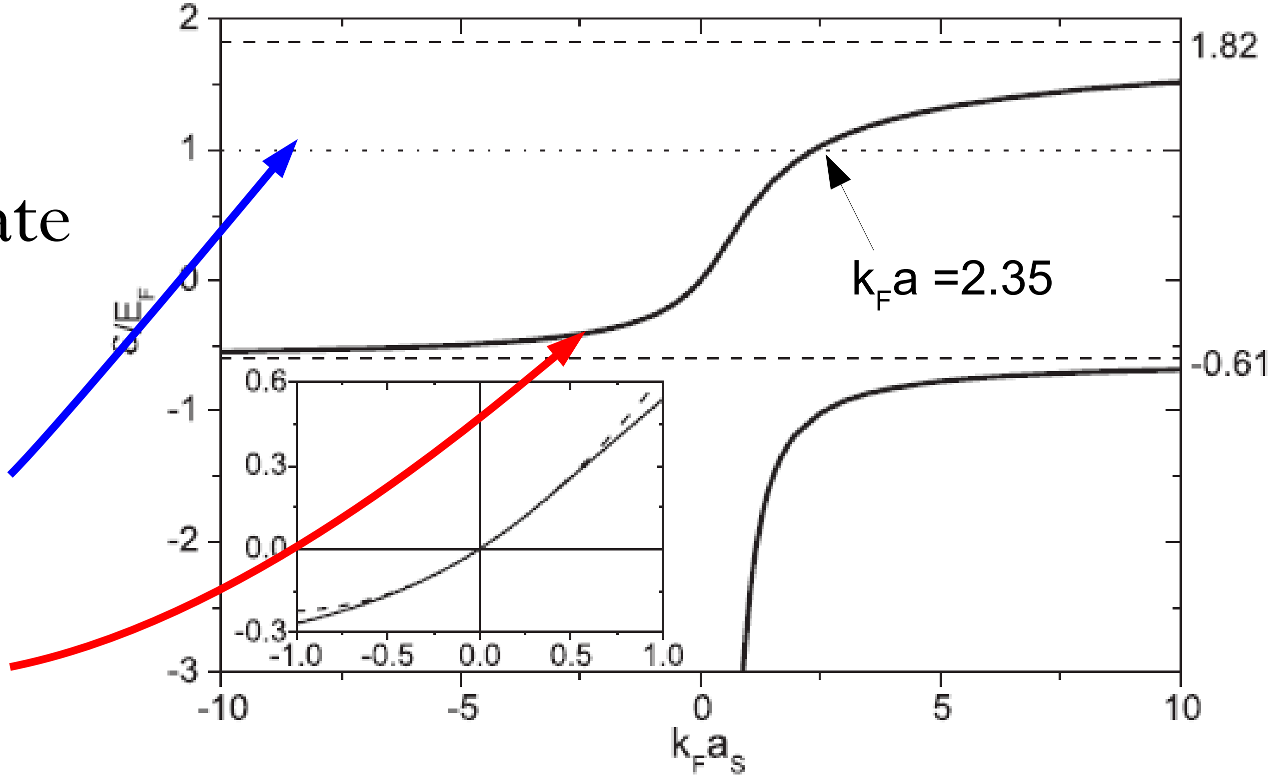
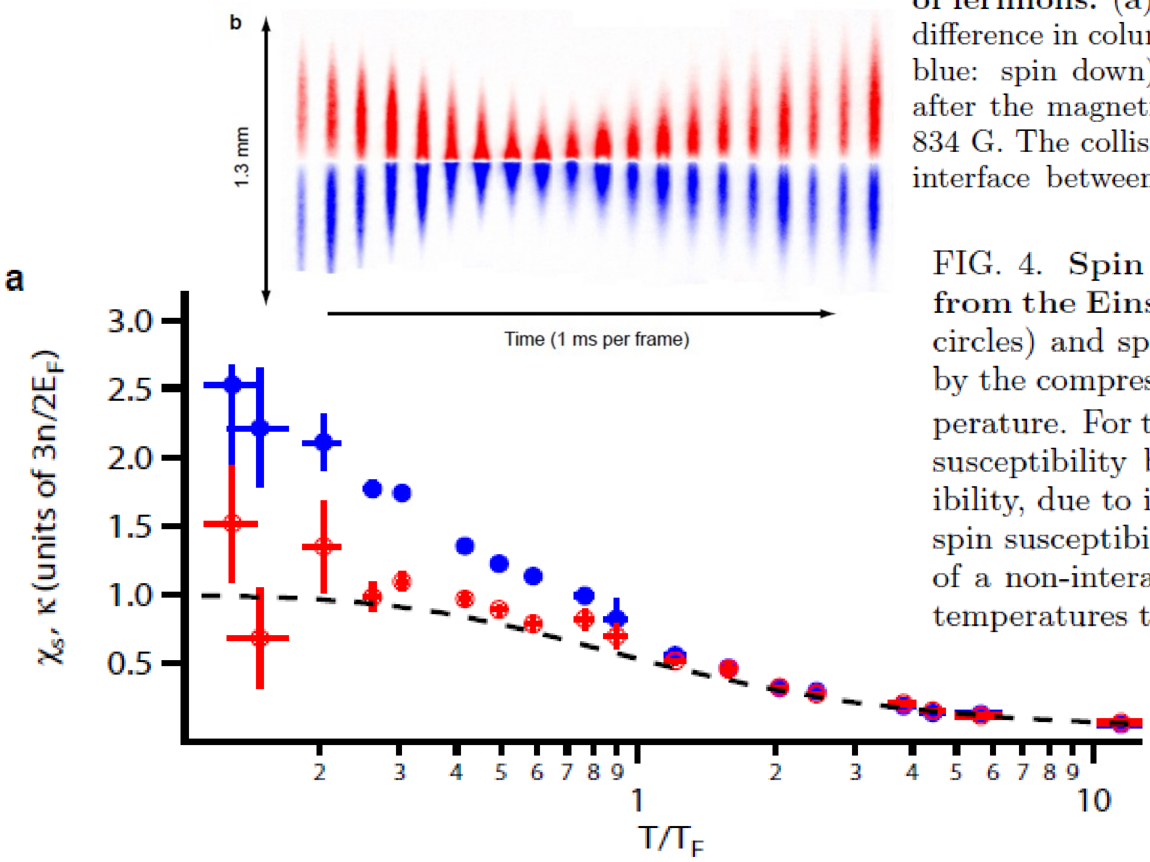


FIG. 3. $\mathcal{E}/E_F$ as a function of $k_F a_s$ for the continuum model with zero-range interactions. Dashed horizontal lines denote saturated values of $\mathcal{E}/E_F$ at unitarity. The inset shows the fit to the perturbation results (see text) for small $k_F a_s$.

X. Cui and H. Zhai, "Stability of a fully magnetized ferromagnetic state in repulsively interacting ultracold Fermi gases", Phys. Rev. A 81, 041602(R) (2010)

… and the controversy

- Experimental evidence against ferromagnetism:



of fermions. (a) shows the total column density and (b) the difference in column densities of the two clouds (red: spin up, blue: spin down), in 1 ms intervals during the first 20 ms after the magnetic field is set to the Feshbach resonance at 834 G. The collision leads to the formation of a high-density interface between the two spin states. (c) The separation

FIG. 4. **Spin susceptibility on resonance determined from the Einstein relation.** (a) Compressibility (solid blue circles) and spin susceptibility (open red circles) normalized by the compressibility $\frac{3n}{2E_F}$ of an ideal Fermi gas at zero temperature. For temperatures below the Fermi temperature, the susceptibility becomes suppressed relative to the compressibility, due to interactions between opposite-spin atoms. The spin susceptibility coincidentally matches the compressibility of a non-interacting Fermi gas (dashed line) in the range of temperatures that we could access. (b) Red circles: spin sus-

A. Sommer, M. Ku, G. Roati, and M.W. Zwierlein, "Universal Spin Transport in a Strongly Interacting Fermi Gas", arXiv:1101.0780 (2011)

… and the controversy

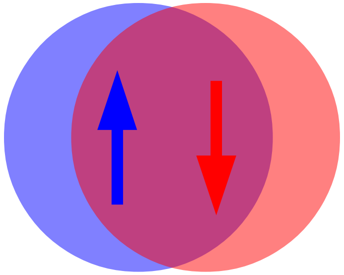
> “It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong”
>
> -Richard Feynman

Our experiment

- Implement Stringari's proposal and measure spin susceptibility [1].
- Try to replicate the MIT measurements [2].
- Investigate Ho's predictions [3].
- Check the "energy cost of spin gradient" model [4].

1. A. Recati and S. Stringari, "Spin Fluctuations, Susceptibility, and the Dipole Oscillation of a Nearly Ferromagnetic Fermi Gas", Phys. Rev. Lett. 106, 080402 (2011)
2. G.-B Jo, Y.-R Lee, J.-H Choi, C. A. Christensen, T.H. Kim, J.H. Thywissen, D.E. Pritchard, and W. Ketterle, "Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms" Science 325, 1521-1524 (2009)
3. S. Zhang, T.L. Ho, "Atom Loss Maximum in Ultra-cold Fermi Gases" arXiv:1102.5687 (2011)
4. L.J. LeBlanc, J.H. Thywissen, A.A. Burkov, and A. Paramekanti, "Repulsive Fermi gas in a harmonic trap: Ferromagnetism and spin textures" Phys. Rev. A 80 013607 (2009)

Our experiment

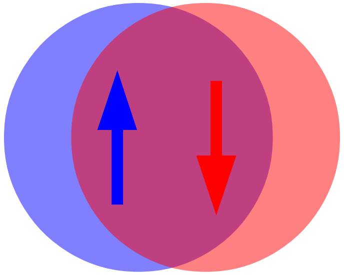


- Stringari's proposal: Take a spin-mixed cloud, separate the two spin components and let them oscillate relative to each other.
- Oscillation frequency and decay of oscillations, depend on the spin susceptibility.
- Therefore, should be able to detect ferromagnetism!

$$\boxed{\chi_m \to \infty}$$

A. Recati and S. Stringari, “Spin Fluctuations, Susceptibility, and the Dipole Oscillation of a Nearly Ferromagnetic Fermi Gas”, Phys. Rev. Lett. 106, 080402 (2011)

Our experiment



$$\omega^{2} \propto 1/\int \chi(n) z^{2} dr$$

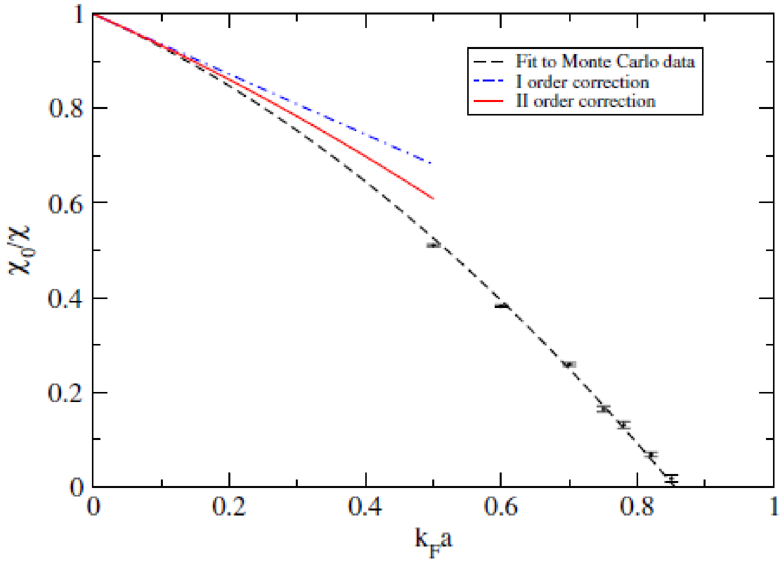


FIG. 1 (color online). Inverse spin susceptibility for an homogeneous Fermi gas as a function of the interaction parameter $k_F a$. Points: Monte Carlo data from Ref. [8]. Dashed (green) line: Fit to the Monte Carlo data. The dot-dashed (blue) and the continuous (red) lines are the first- and second-order expansion, respectively [see Eq. (8)].

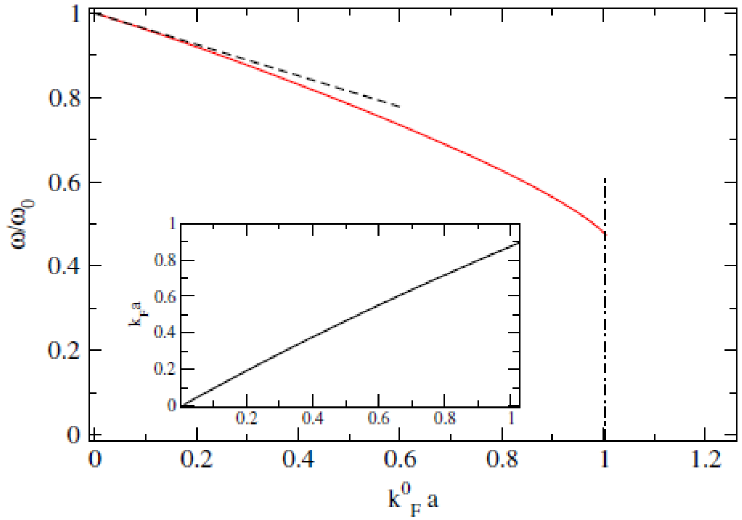


FIG. 2 (color online). Main panel: Spin dipole frequency as a function of the interaction parameter $k_F^0 a$ (see the text). The dashed line is the first-order expansion Eq. (13). The dash-dotted horizontal line is the position at which the susceptibility should diverge at the center of the cloud. Inset: The value of the interaction parameter $k_F a$ in the trap as a function of $k_F^0 a$. k_F is the Fermi momentum of the interacting cloud calculated at the center of the trap.

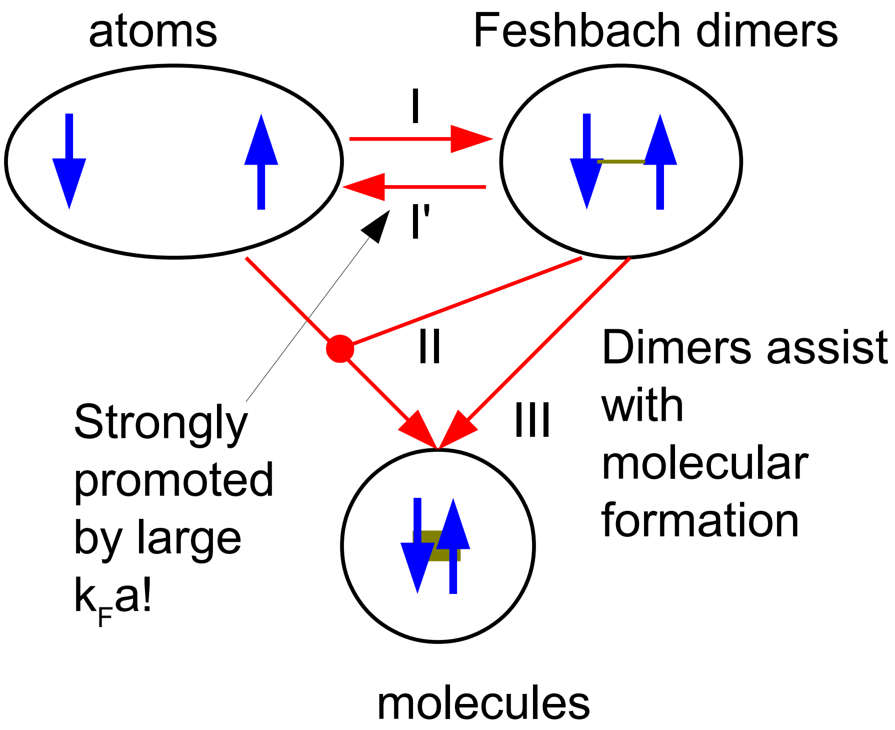
A. Recati and S. Stringari, “Spin Fluctuations, Susceptibility, and the Dipole Oscillation of a Nearly Ferromagnetic Fermi Gas”, Phys. Rev. Lett. 106, 080402 (2011)

Our experiment

- In practice, use ^{40}K; F=9/2 state with m_F=9/2 or 7/2
- Simultaneously apply a magnetic field and optical gradients, which exert different forces on the m_F = 7/2 and 9/2 components of the cloud, offsetting them from each other
- Science!

Our experiment

- Ho's argument implies that position of loss rate peak depends on total depth of trap (since this affects whether dimers can remain in trap)
- Can try to measure reaction rates for processes I, I', II, III.



S. Zhang, T.L. Ho, "Atom Loss Maximum in Ultra-cold Fermi Gases" arXiv:1102.5687 (2011)

Our experiment

- Energy of a spin gradient model: There is a “stiffness” associated with the magnetization of the cloud.
- Magnetized cloud will act like a torsion spring!
- An external magnetic field gradient can be used to pull on the spring.
- Magnetic domains in a ferromagnetic cloud may show a characteristic signature!

L.J. LeBlanc, J.H. Thywissen, A.A. Burkov, and A. Paramekanti, “Repulsive Fermi gas in a harmonic trap: Ferromagnetism and spin textures” Phys. Rev. A 80 013607 (2009)

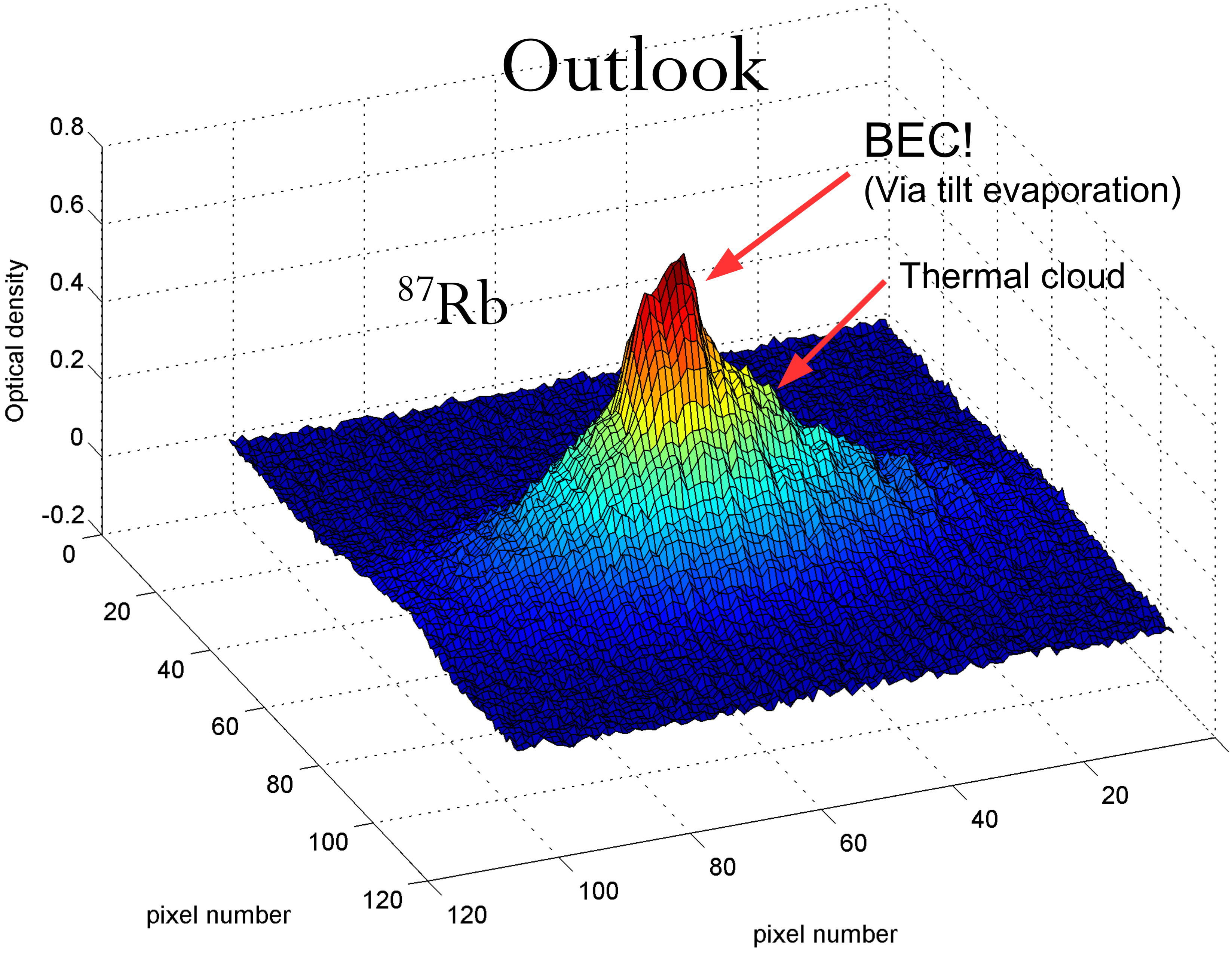
Our experiment

- Other signatures of ferromagnetism?

Outlook

- Now have up to ~50000 ^{40}K atoms at $T \sim 0.3 T_F$ in the dipole trap.
- As usual, “only a few technical issues remain to be resolved”.
- Answering the above questions should provide insight into Stoner-like Hamiltonians and settle the debates about ultracold fermion ferromagnetism.

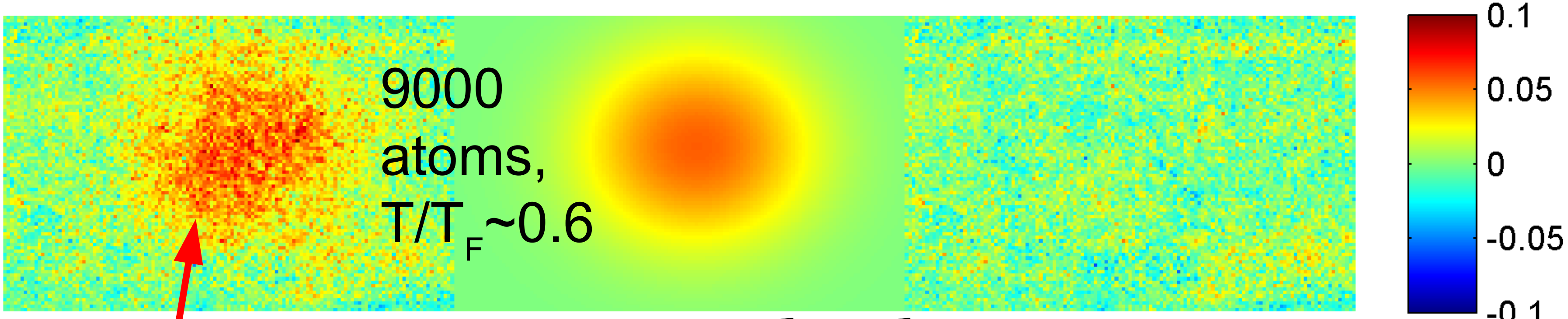
Outlook



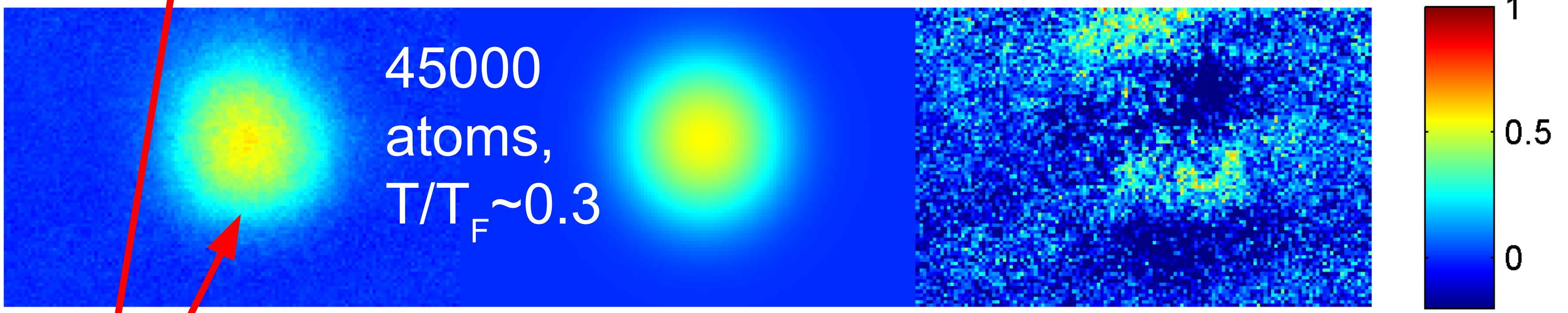
^{87}Rb

BEC!
(Via tilt evaporation)

Thermal cloud



Outlook



^{40}K

Today, 3:50PM: