# Efficient measurement of quantum dynamics via compressive sensing

**1 Undersample**

A camera or other device captures only a small, randomly chosen fraction of the pixels that normally comprise a particular image. This saves time and space.

**2 Fill in the dots**

An algorithm called $l_1$ minimization starts by arbitrarily picking one of the effectively infinite number of ways to fill in all the missing pixels.

**3 Add shapes**

The algorithm then begins to modify the picture in stages by laying colored shapes over the randomly selected image. The goal is to seek what's called sparsity, a measure of image simplicity.

**4 Add smaller shapes**

The algorithm inserts the smallest number of shapes, of the simplest kind, that match the original pixels. If it sees four adjacent green pixels, it may add a green rectangle there.

**5 Achieve clarity**

Iteration after iteration, the algorithm adds smaller and smaller shapes, always seeking sparsity. Eventually it creates an image that will almost certainly be a near-perfect facsimile of a hi-res one.

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Ardavan Darabi

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# Efficient Measurement of Quantum Dynamics via Compressive Sensing

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The resources required to characterize the dynamics of engineered quantum systems—such as quantum computers and quantum sensors—grow exponentially with system size. Here we adapt techniques from compressive sensing to exponentially reduce the experimental configurations required for quantum process tomography. Our method is applicable to processes that are nearly sparse in a certain basis and can be implemented using only single-body preparations and measurements. We perform efficient, high-fidelity estimation of process matrices of a photonic two-qubit logic gate. The database is obtained under various decoherence strengths. Our technique is both accurate and noise robust, thus removing a key roadblock to the development and scaling of quantum technologies.

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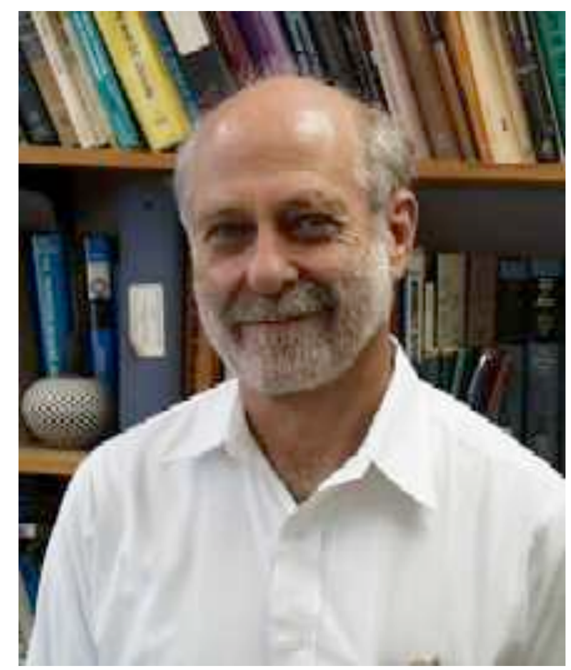


# Coming up...

- Quick Recap of tomography and stuff..
- Why process tomography is hard and how does compress sensing work?
- experimental data
- Conclusions

# Quantum Process Tomography

- Quick Recap of state tomography and Process Tomography ...

Density operators

Hermitian, semi-positive definite, unit trace

Consider a qubit:

3 real parameters and can be written as

$$\rho = \frac{tr(\rho)I + tr(\rho X)X + tr(\rho Y)Y + tr(\rho Z)Z}{2}$$

# Quantum Process Tomography

- This can be generalized for n qubits:

$$\rho = \sum_v \frac{tr(\sigma_{v_1} \otimes \sigma_{v_2} \ldots \otimes \sigma_{v_n} \rho) \sigma_{v_1} \otimes \sigma_{v_2} \ldots \otimes \sigma_{v_n}}{2^n}$$

- How to characterize the processes??

  Most intuitive answer:

  If the system has dimension d, choose $d^2$ pure states $|\psi_1\rangle$ , $|\psi_2\rangle$ ,...., and $|\psi_{d^2}\rangle$ such that $|\psi_i\rangle\langle\psi_i|$'s form a complete basis for state space...

  Mathematically this should be Enough!!

# Representations ....

- We'd like to find a useful representation of the process.. e.g.

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger$$

- Alternatively, one can fix a basis $E'_i$ and use the following representation:

$$E_i = \sum_i e_{im} E'_m$$

$$\mathcal{E}(\rho) = \sum_{m,n} E'_m \rho E'^\dagger_n \chi_{mn}$$

Where:

$$\chi_{mn} = \sum_i e_{im} e^*_{in}$$

Generally X has $d^4$-$d^2$ independent, real entries.

# What to remember:

Forgive the change in notation

$$\mathcal{S}(\rho) = \sum_{\alpha,\beta=1}^{d^2} \chi_{\alpha\beta}\Gamma_\alpha \rho \Gamma_\beta^\dagger$$

$\chi$ positive semidefinite, trace-preserving matrix often referred to as 'process matrix'.

$\{\Gamma_\alpha\}$ Form an orthonormal basis:

$$\mathrm{Tr}(\Gamma_\beta^\dagger \Gamma_\alpha) = \delta_{\alpha\beta}$$

$$\sum_{\alpha,\beta} \chi_{\alpha\beta}\Gamma_\beta^\dagger \Gamma_\alpha = I_d$$

# Nomenclature:

- s-sparse ... a vector is s-sparse if all its entries except at most s are zero.(not a property of vector but representation)
- $l_0$,$l_1$,$l_2$ norms...

$$\ell_0 \text{ norm} = \max\{|x_i|\}$$

$$\ell_1 \text{ norm} = \sum_i |x_i|$$

$$\ell_2 \text{ norm} : \sqrt{\sum |x_i|^2}$$

# Nomenclature:

cont'd


- Restricted Isometry Property:

$$(1-\delta_s)\|x\|_{\ell_2}^2 \leqslant \|\phi x\|_{\ell_2}^2 \leqslant (1+\delta_s)\|x\|_{\ell_2}^2$$

isometry constant $\delta_s$

# Statement of the problem:

We have a system prepared in $\rho_1, \ldots \rho_K$
Sent through the Quantum Process (Channel) and perform Measurements

$\{M_1, \ldots M_\ell\}$ (note: no completeness assumptions here).

We look at probabilities:

$$y = \begin{bmatrix} y_{M_1,\rho_1} \\ \vdots \\ y_{M_m,\rho_m} \end{bmatrix} = \Phi \vec{\chi}_0$$

→ This is the vectorized version of $\chi$

$\Phi$ is an $m \times d^4$ matrix and

$\mathrm{Tr}(\Gamma_\alpha \rho_i \Gamma_\beta^\dagger M_i)/\sqrt{m}$

The Question:
Is it possible to invert such a highly underdetermined system of equations?

$$y = \begin{bmatrix} y_{M_1, \rho_1} \\ \vdots \\ y_{M_m, \rho_m} \end{bmatrix} = \Phi \vec{X}_0$$

This is the vectorized version of $X$

$\Phi$ is an $m \times d^4$ matrix and

$$\mathrm{Tr}\left(\Gamma_\alpha \rho_i \Gamma_\beta^{\dagger} M_i\right)/\sqrt{m}$$

Here is where the compressed sensing comes into play:
Provided $\Phi$ satisfies certain conditions, it is possible...

# The conditions:

i) for all $s$-sparse $\chi_1(s)$, $\chi_2(s)$ process matrices

$$1 - \delta_s \leq \frac{\|\Phi\vec{\chi}_1(s) - \Phi\vec{\chi}_2(s)\|^2_{\ell_2}}{\|\vec{\chi}_1(s) - \vec{\chi}_2(s)\|^2_{\ell_2}} \leq 1 + \delta_s$$

ii) $\delta_{2s} < \sqrt{2} - 1$

iii) $m \geq C_0 s \log(d^4/s)$

Then one recovers the solutions by solving:

minimize $\|\vec{\chi}\|_{\ell_1}$ subject to $\|y - \Phi\vec{\chi}\|_{\ell_2} \leq \varepsilon$

positive-semidefinite and trace-preserving condition

$$\text{minimize } \|\vec{\chi}\|_{\ell_1} \text{ subject to } \|y - \Phi\vec{\chi}\|_{\ell_2} \leq \varepsilon$$

positive-semidefinite and trace-preserving condition

The conditions guarantee that the solution $\chi^\star$ satisfies:

$$\|\vec{\chi}^\star - \vec{\chi}_0\|_{\ell_2} \leq \frac{C_1}{\sqrt{s}}\|\vec{\chi}_0(s) - \vec{\chi}_0\|_{\ell_1} + C_2\varepsilon$$

# Quick Recap:

RIP: $$1-\delta_s \le \frac{\|\Phi\vec{\chi}_1(s)-\Phi\vec{\chi}_2(s)\|_{\ell_2}^2}{\|\vec{\chi}_1(s)-\vec{\chi}_2(s)\|_{\ell_2}^2} \le 1+\delta_s$$

Number of measurement settings:

$$m \ge C_0 s \log(d^4/s).$$

Optimization problem:

$$\text{minimize } \|\vec{\chi}\|_{\ell_1} \text{ subject to } \|y-\Phi\vec{\chi}\|_{\ell_2} \le \varepsilon$$

The Result:

$$\|\vec{\chi}^\star-\vec{\chi}_0\|_{\ell_2} \le \frac{C_1}{\sqrt{s}}\|\vec{\chi}_0(s)-\vec{\chi}_0\|_{\ell_1} + C_2\varepsilon$$

$C_0$, $C_1$, $C_2$ are constants on the order of $O(\delta_s)$

As an example, for n qubits:

$$m \ge C_0 s(4n\log 2 - \log s) = O(sn)$$

But wait, how can one choose the set of initial conditions and measurements such that the matrix satisfies all the conditions??

$$\rho_1, \ldots, \rho_K$$

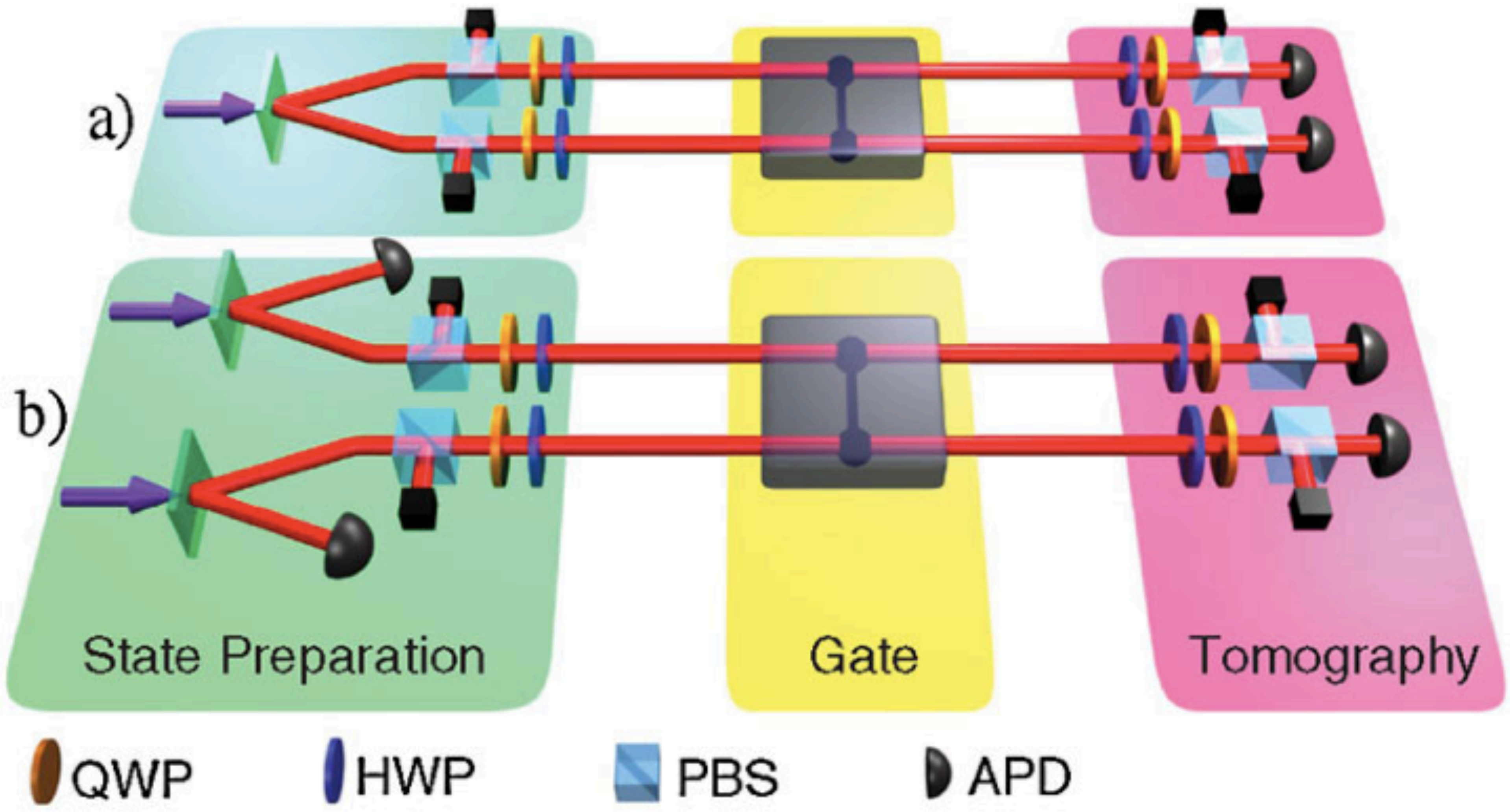
$$\{M_1, \ldots, M_\ell\} \xrightarrow{\text{yield good}} \Phi$$

It turns out that if they are chosen randomly, with a high probability they will satisfy the RIP condition!

Hence, a nearly sparse process can be recovered from exponentially fewer measurements!!!

# The Experiment

Put this to test against the Full-QPT



# How?

Take any pair of $\{|H\rangle, |V\rangle, |D\rangle, |R\rangle\}$ as input states.–

For each input measure any two combinations of

$\{|H\rangle, |V\rangle, |A\rangle, |D\rangle, |R\rangle, |L\rangle\}$ -- 36 all together!

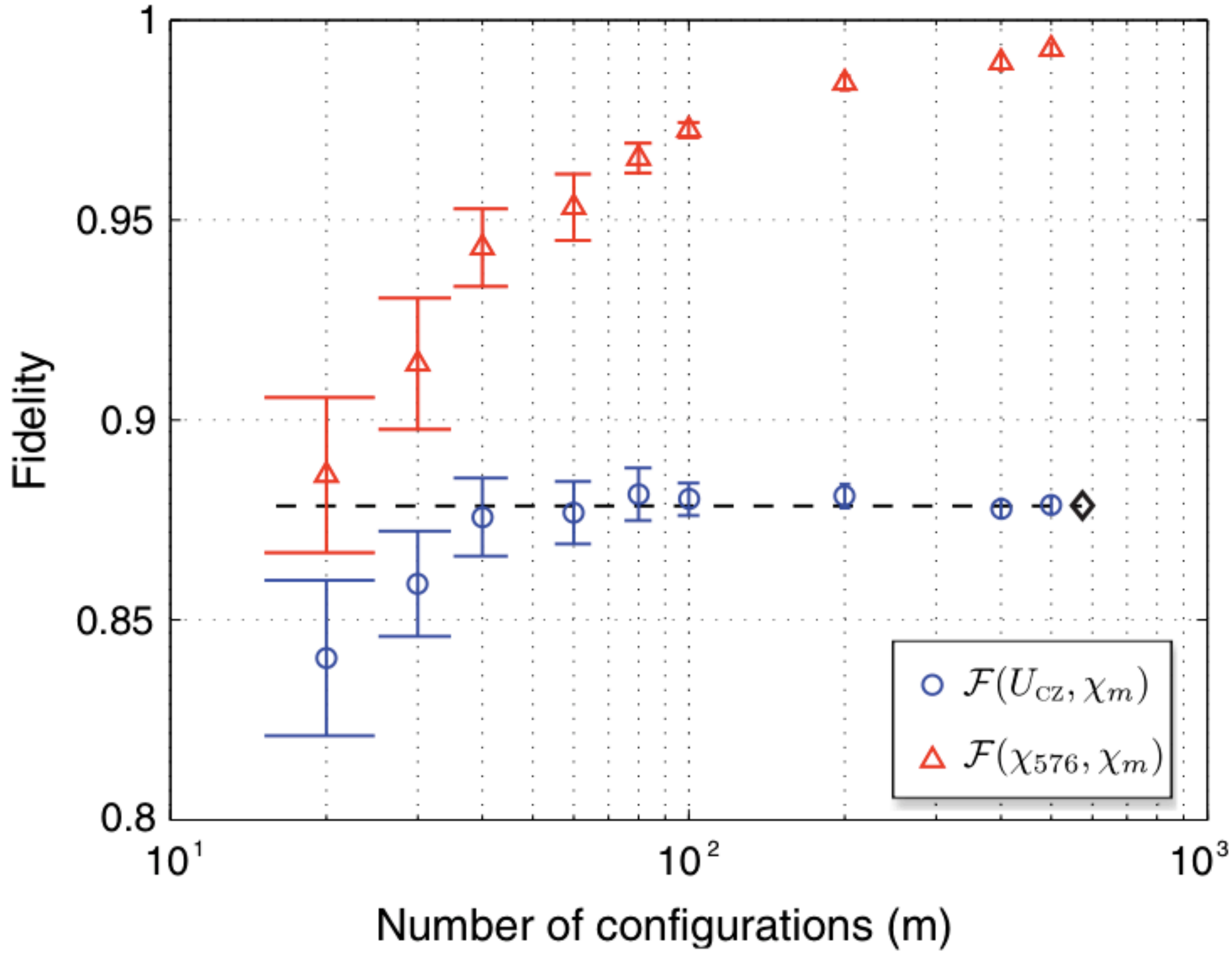
576 over-complete set of measurements!

$\chi_{576}$: best estimate of the process.–

Take any random subset the measurement data a and extract the process matrix

Why is this not "too" realistic??

# First Set of Experimental data:




Data here is randomly chosen from all the $X_{576}$ measurements ..

# More experimental Results...

For this next set of data 16 input states and 2 output measurements were chosen (m=32) .. (Not quite that randomly)

input $\{|H\rangle, |V\rangle, |D\rangle, |R\rangle\}^{\otimes 2}$

projector $\{|R\rangle|I\rangle, |I\rangle|R\rangle\}$

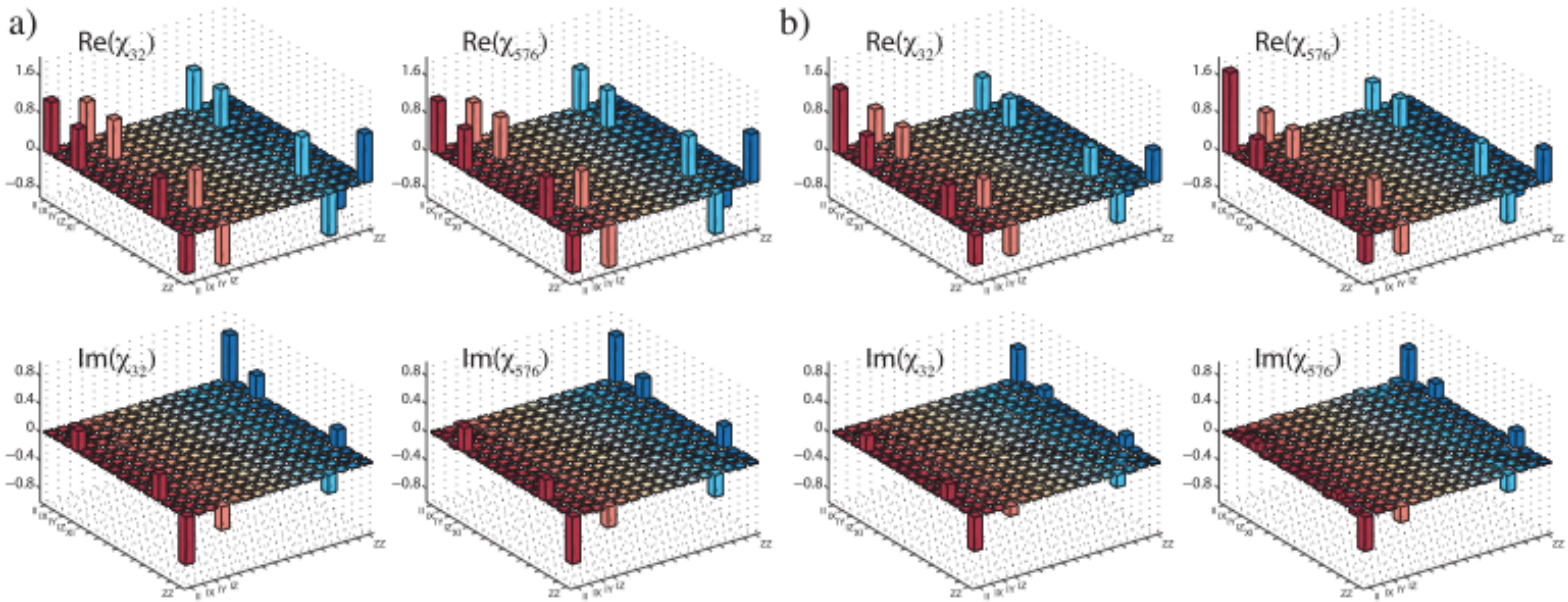



FIG. 3 (color online). Real and imaginary process matrix elements in the Pauli basis for the CQPT estimate $\chi_{32}$, 32 configurations (left) vs full data estimate $\chi_{576}$, 576 configurations (right) for (a) a low noise, two-photon experiment, $\mathcal{P} = 0.91$, and (b) a high-noise, four-photon experiment, $\mathcal{P} = 0.62$. The CQPT reconstructions have fidelities, $\mathcal{F}(\chi_{576}, \chi_{32})$, of 95% and 85%, respectively. The CQPT estimation accuracy is excellent for low noise, and reliable even for high noise, see [19] for more details.

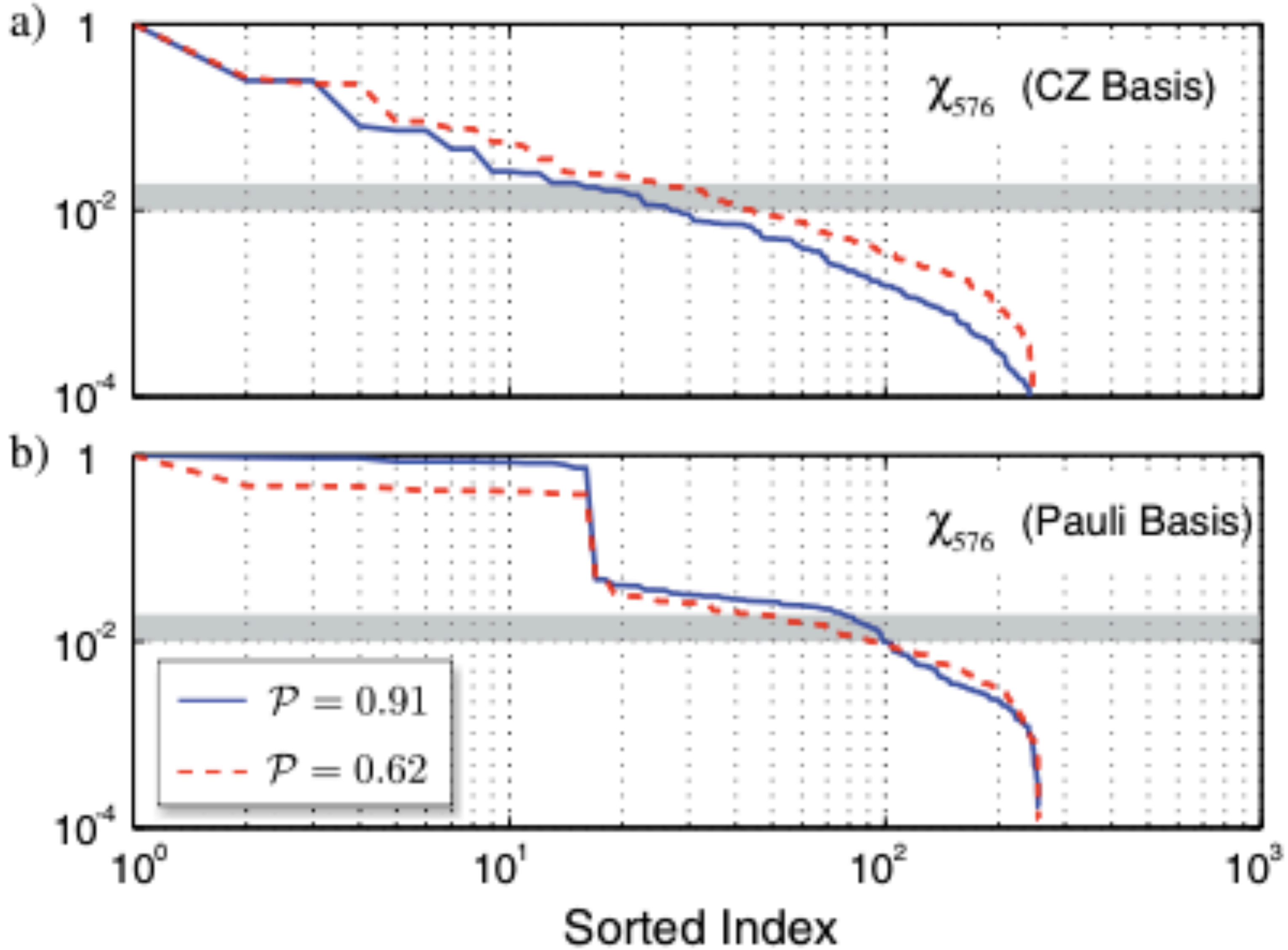



FIG. 4 (color online). Absolute values of the 256 process matrix elements of $\chi_{576}$ for our lowest and highest noise level, sorted by relative magnitude [with respect to the (1, 1) element] in the CZ basis (top) and the Pauli basis (bottom). The error threshold, which indicates the required number of configurations, is shown in grey.

# Summary:

- QPT for Sparse processes can be executed exponentially faster! (One may ask how does it depend on the rank, knowledge of process, basis, etc)
- This may enable us to perform QPT on larger systems(very broad range of applications)
- We still need to find the optimal settings + possibly faster methods for convex optimization and etc...