Self contained quantum heat engines

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The smallest possible heat engines

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We construct the smallest possible self contained heat engines; one composed of only two qubits, the other of only a single qutrit. The engines are self-contained as they do not require external sources of work and/or control. They are able to produce work which is used to continuously lift a weight. Despite the dimension of the engine being small, it is still able to operate at the Carnot efficiency.

arXiv:1010.6029v1 [quant-ph] 28 Oct 2010

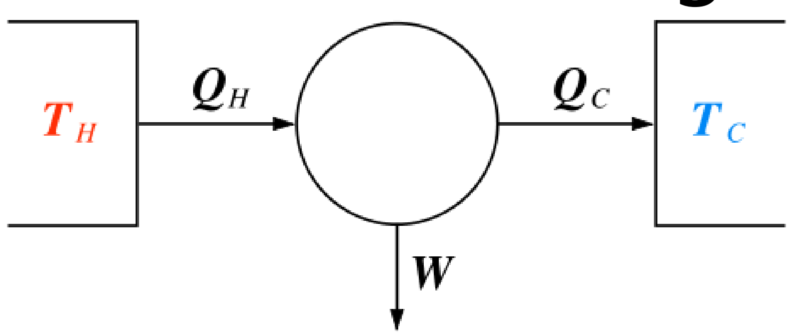
Punch line

Quantum thermodynamic

The definitions of work given so far have been applicable and very well suited to this situation. Our focus is however different; we are interested here in *self contained* heat engines, where there is no external work or control – the only external interaction being with thermal reservoirs. The previous definitions of work therefore do not apply directly to our situation, and an alternative must be found.

Outline

- Classical thermodynamics
  - Thermodynamic processes
  - Classical Carnot cycle
- Quantum thermodynamics
  - Definitions
  - Quantum Carnot cycle
- Self contained heat engines



Thermodynamic processes

1st law of thermodynamics $dU = đQ + đW$

System + thermal bath

Isothermal process
Heat absorbed or released
Work done
INV: U, T VAR: P, V

Isochoric process
Heat absorbed or released
No work done
INV: V VAR: P, T

Adiabatic process
No heat exchange
Work done
VAR: P, T, V

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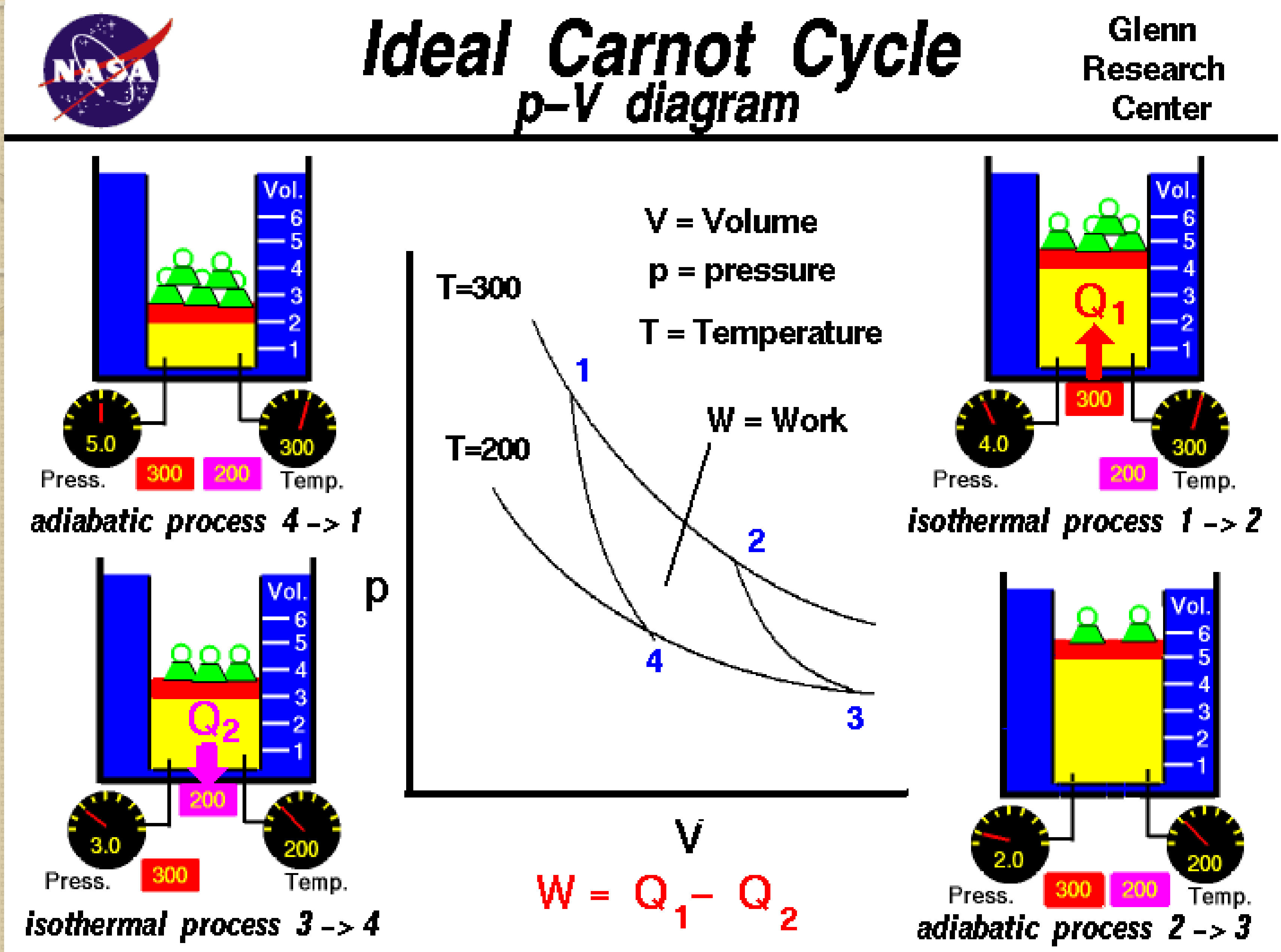


http://www.grc.nasa.gov/WWW/K-12/airplane/carnot.html

Quantum system?

Quantum states

Quantum system?

Quantum states

Pure state, Mixed state

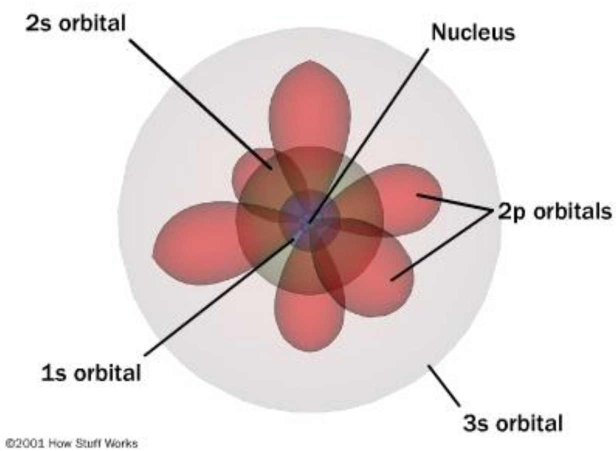
Quantum system?

Quantum states

Pure state, Mixed state

Wave functions

$$\Psi =$$



http://science.howstuffworks.com/atom9.htm

Quantum system?

Quantum states

Pure state, Mixed state

Wave functions

Density operator

$\rho =$

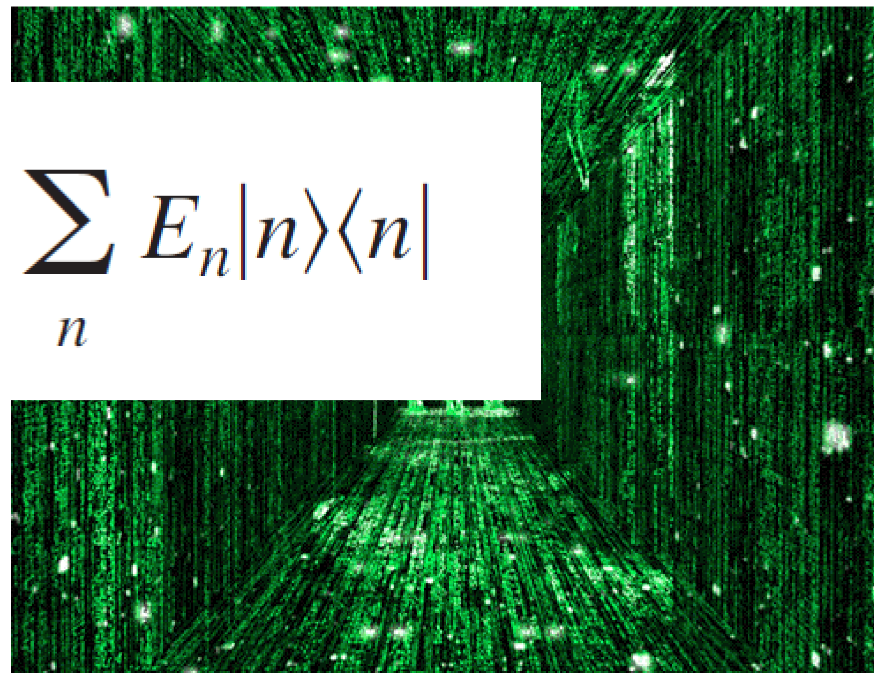
Quantum system?

Quantum states

Pure state, Mixed state

Wave functions

Density operator



Hamiltonian $H = \sum_n E_n |n\rangle\langle n|$

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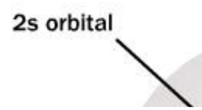
Quantum system?

Quantum states

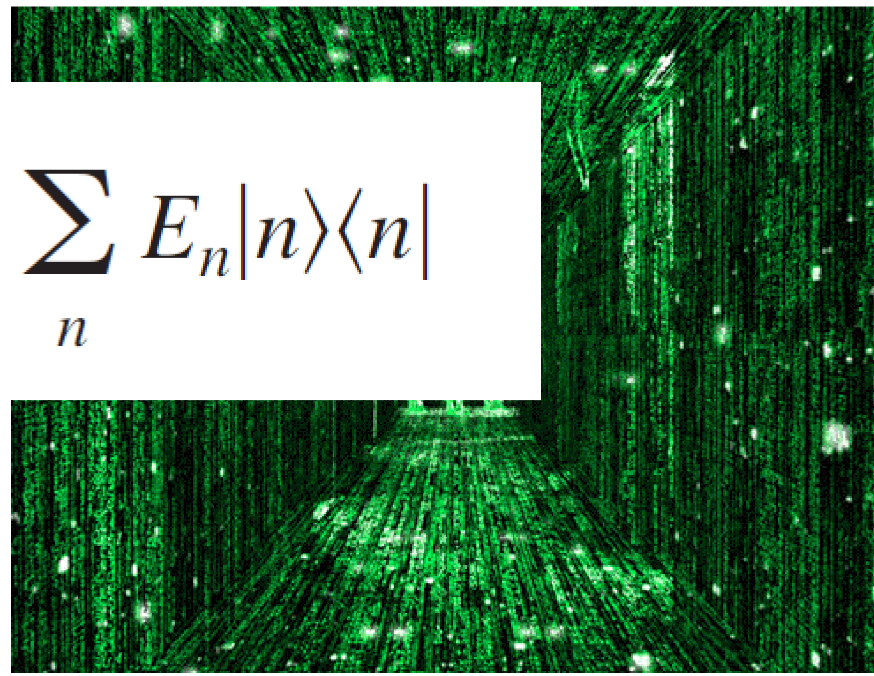
Pure state, Mixed state

Wave functions



Quantum process

Density operator



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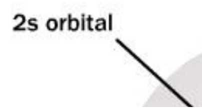

Quantum process

Density operator

Schrödinger equation

Hamiltonian $H =$

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

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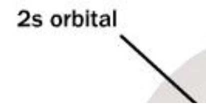
Quantum system?

Quantum states

Pure state, Mixed state

Wave functions



Quantum process

Density operator

Schrödinger equation

Hamiltonian H

Heisenberg's equation

$$i\hbar\frac{\partial}{\partial t}\rho(t) = [H, \rho(t)]$$

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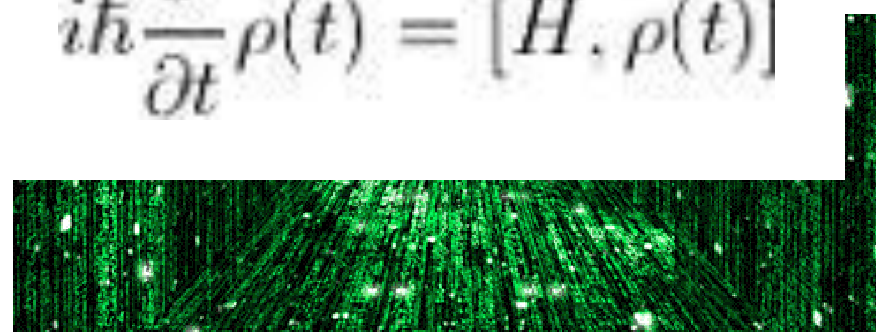
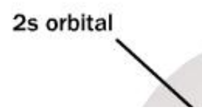


Quantum system?

Quantum states

Pure state, Mixed state

Wave functions



Quantum process

Density operator

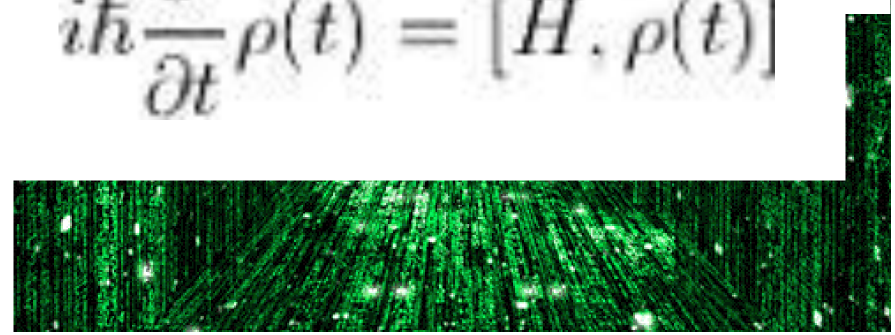
Schrödinger equation

Hamiltonian H

Heisenberg's equation

$$i\hbar\frac{\partial}{\partial t}\rho(t) = [H, \rho(t)]$$

Environment?
heat bath

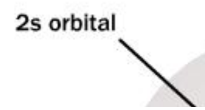


Quantum system?

Quantum states

Pure state, Mixed state

Wave functions



Quantum process

Density operator

Schrödinger equation

Hamiltonian H

Heisenberg's equation

Master equation

Environment?
heat bath

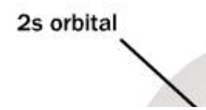
$$\partial_t \tilde{\rho} = -\frac{i}{\hbar}\left[\tilde{H}, \tilde{\rho}\right] + \Gamma \mathcal{D}[\sigma]\tilde{\rho}.$$

Quantum system?

Quantum states

Pure state, Mixed state

Wave functions



Quantum process

Density operator

Schrödinger equation

Hamiltonian H

Heisenberg's equation

Master equation

Environment?
heat bath

Superoperator ( )

Temperature & Thermal equilibrium state

$$\rho_T := e^{-H/kT}/tr(e^{-H/kT})$$

Hamiltonian $H = \sum_n E_n |n\rangle\langle n|$

ρ_{nn} = C exp{-E_n/kT}

ρ_{nm} = 0

Effective temperature?

How about Work & Heat?

$$U = \langle E \rangle = \mathrm{Tr}\{\hat{H}\hat{\rho}\}$$

$$\frac{d}{dt}\langle E \rangle = \mathrm{Tr}\left\{ \frac{d}{dt}\hat{H}\hat{\rho} \right\} + \mathrm{Tr}\left\{ \hat{H}\frac{d}{dt}\hat{\rho} \right\}$$

$$\frac{d}{dt}W = \mathrm{Tr}\left\{ \frac{d}{dt}\hat{H}\hat{\rho} \right\} = \sum_i \dot{E}^i p^i$$

Work

$$\frac{d}{dt}Q = \mathrm{Tr}\left\{ \hat{H}\frac{d}{dt}\hat{\rho} \right\} = \sum_i \dot{p}^i E^i$$

Heat

$$dU = đQ + đW$$

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How about Work & Heat?

Work: Change of eigenenergy, change of Hamiltonian
Heat: Change of population distribution

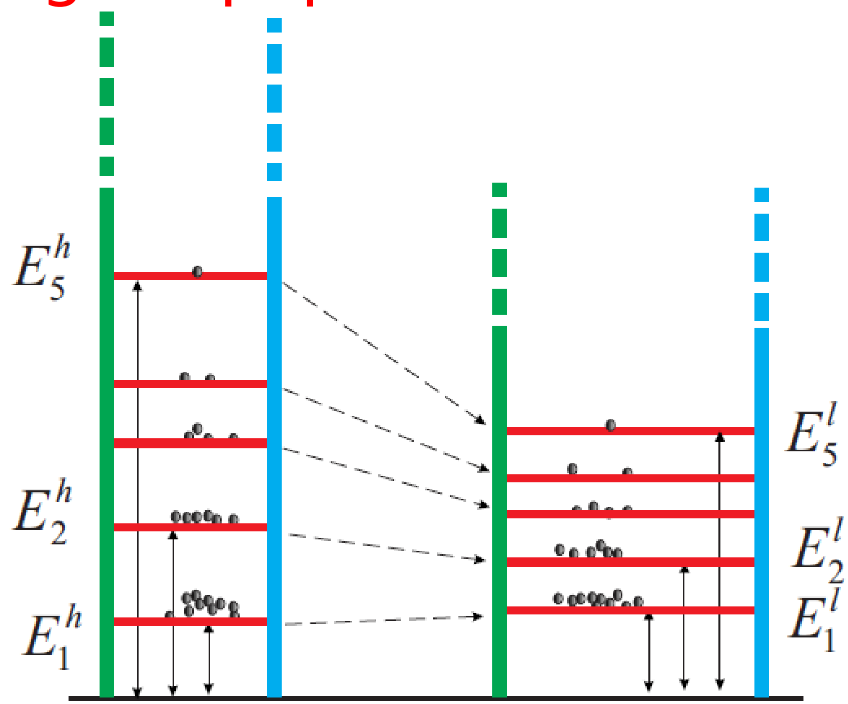

FIG. 1. (Color online) Schematic diagram of multilevel quantum system as the working substance for a QHE. E_n^h and E_n^l are the nth eigenenergy of the working substance in the two isochoric processes.

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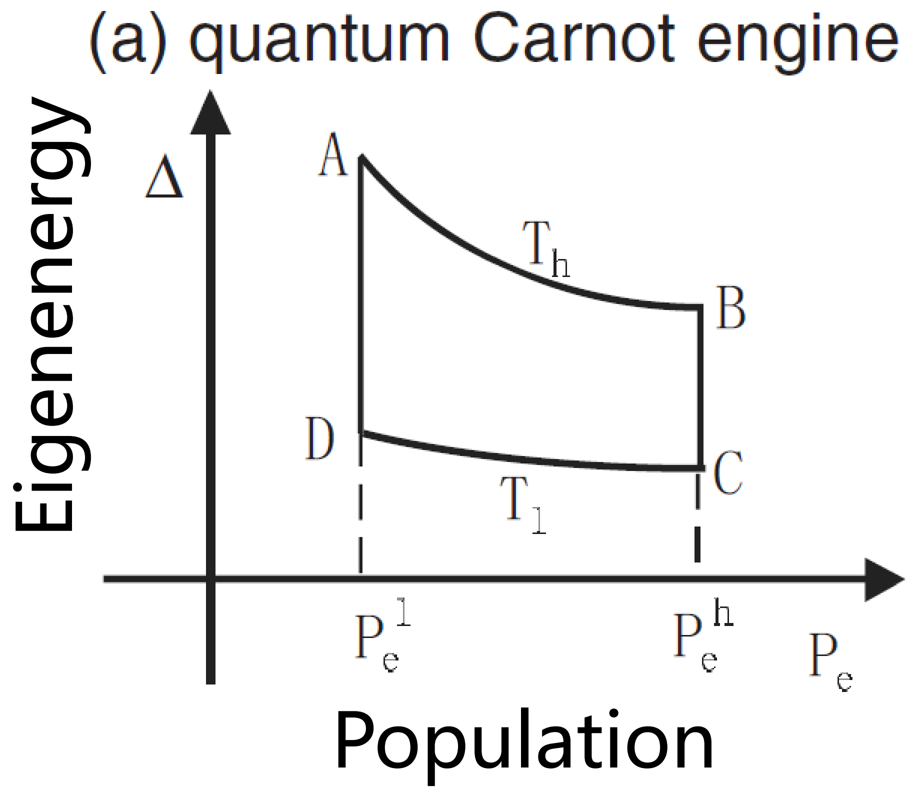
Quantum Carnot cycle

Two level system



Work

$$\sum_i \dot{E}^i p^i$$

External work (control) involved

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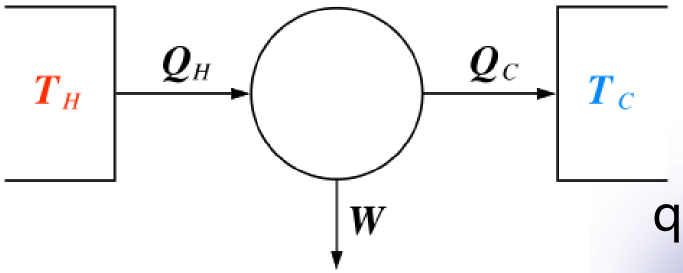
Self contained heat engine



$$\overline{r_1} = r_1 e^{-\frac{E_1}{kT_c}}$$

$$\overline{r_2} = r_2 e^{-\frac{E_2}{kT_h}}$$

$$E_2 - E_1 = \mathcal{E}$$

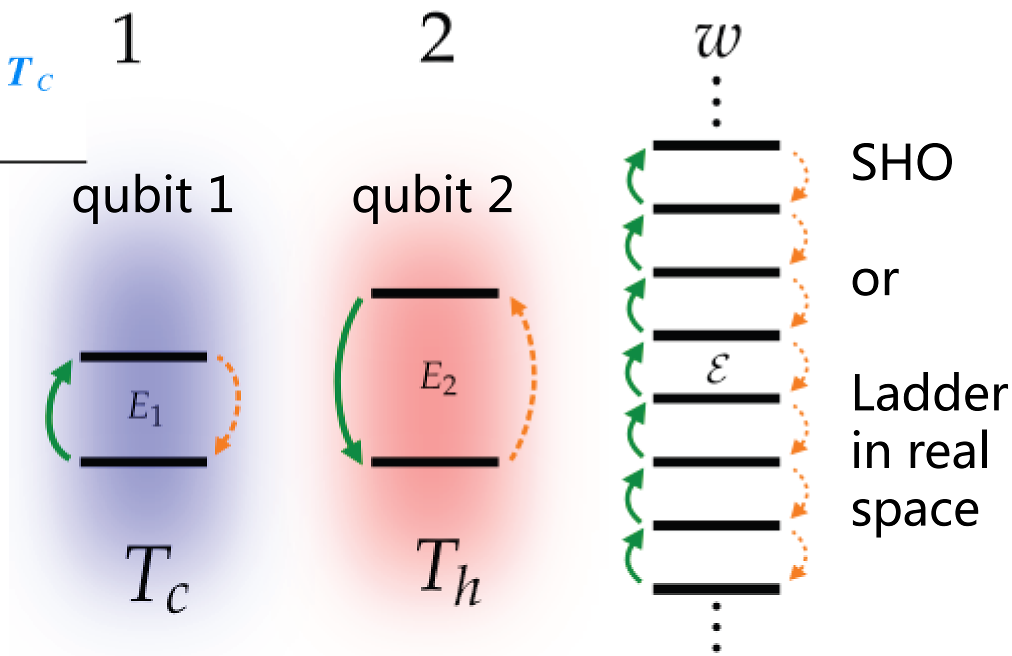
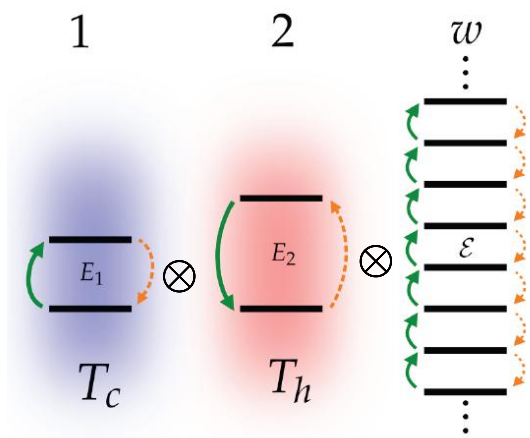


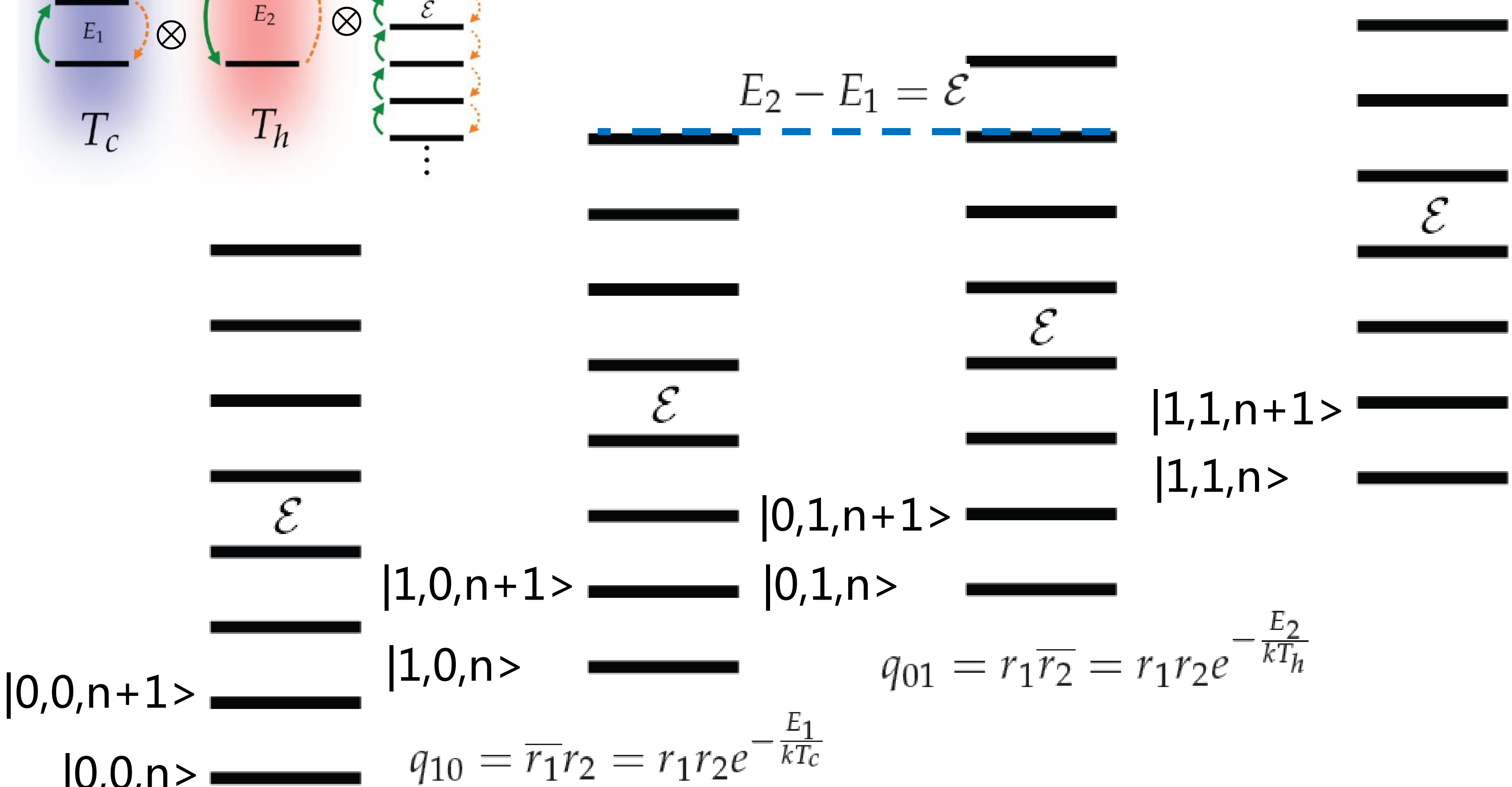
FIG. 1: **Schematic diagram of two-qubit heat engine.** Qubits 1 and 2, with energy level separations E_1 and E_2 are in contact with thermal reservoirs at temperatures T_c and T_h respectively. To this a weight is connected, with separation $\mathcal{E} = E_2 - E_1$. The particles interact with each other and the temperatures are chosen such that the transition where the weight is lifted (solid green arrows) is biased over the transition where the weight falls (dashed orange arrows).

Self contained heat engine



$$\frac{E_1}{T_c} > \frac{E_2}{T_h} \qquad q_{01} > q_{10}$$

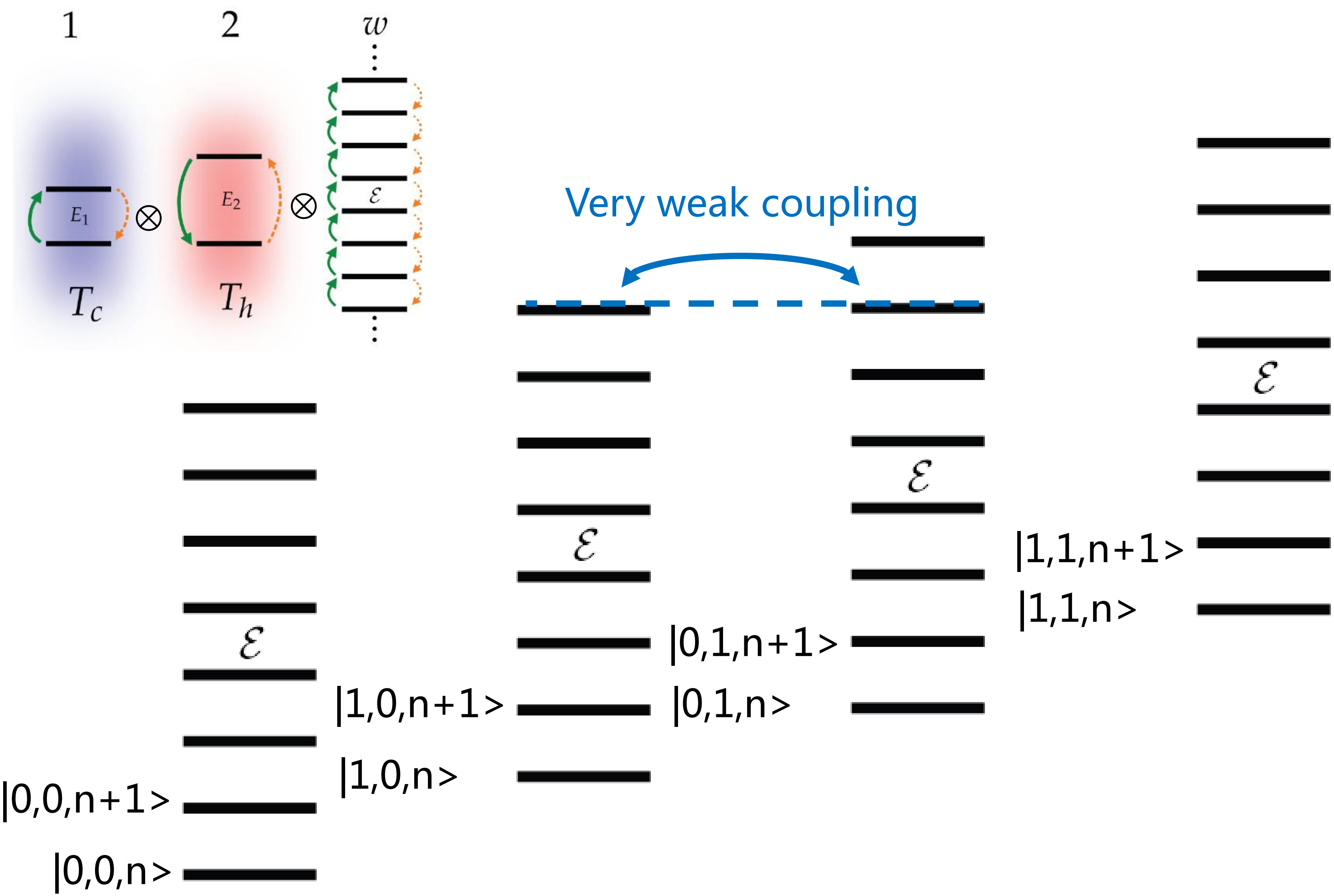
$$r_1 > r_2$$



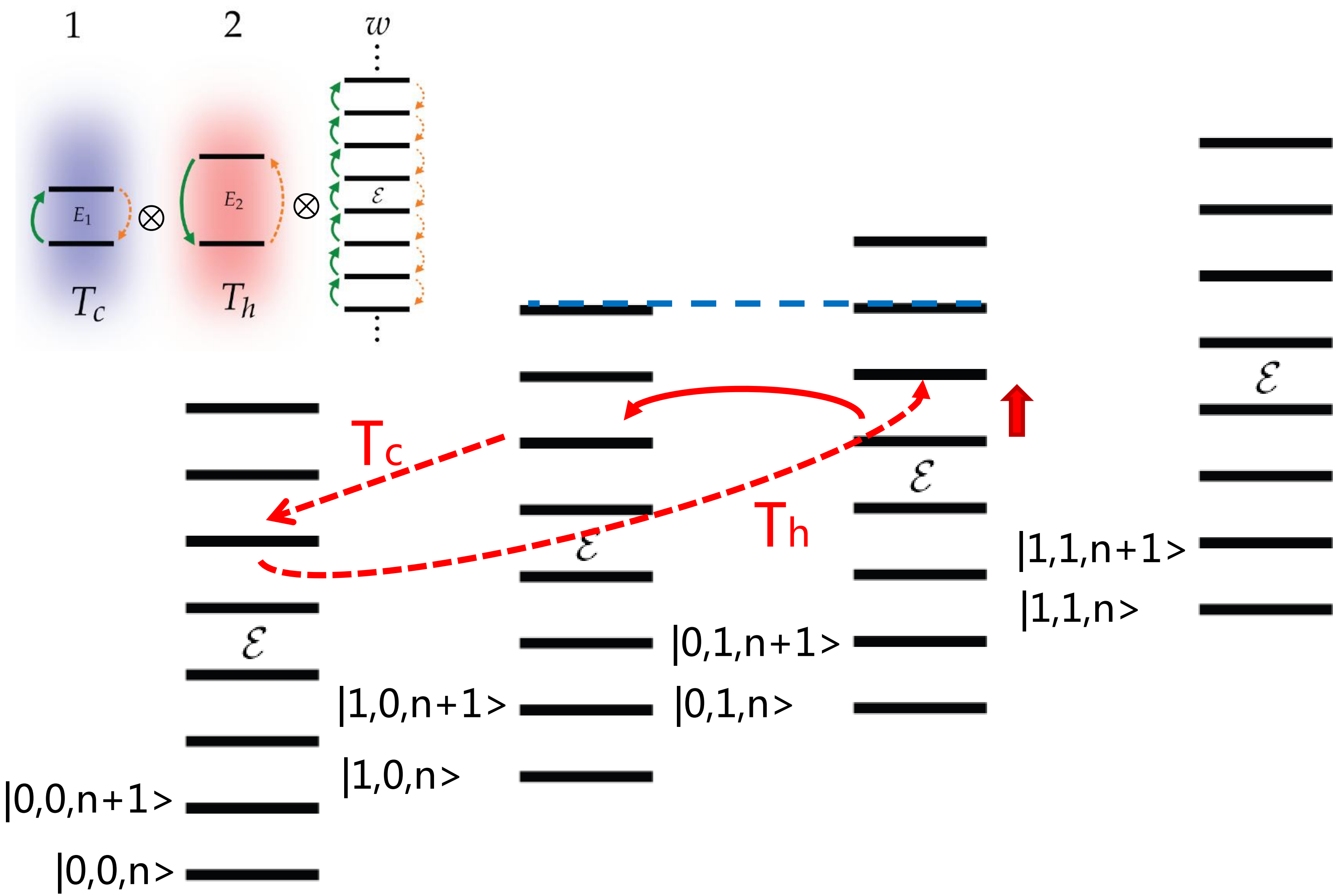
$$q_{01} = r_1\overline{r_2} = r_1 r_2 e^{-\frac{E_2}{kT_h}}$$

$$q_{10} = \overline{r_1} r_2 = r_1 r_2 e^{-\frac{E_1}{kT_c}}$$

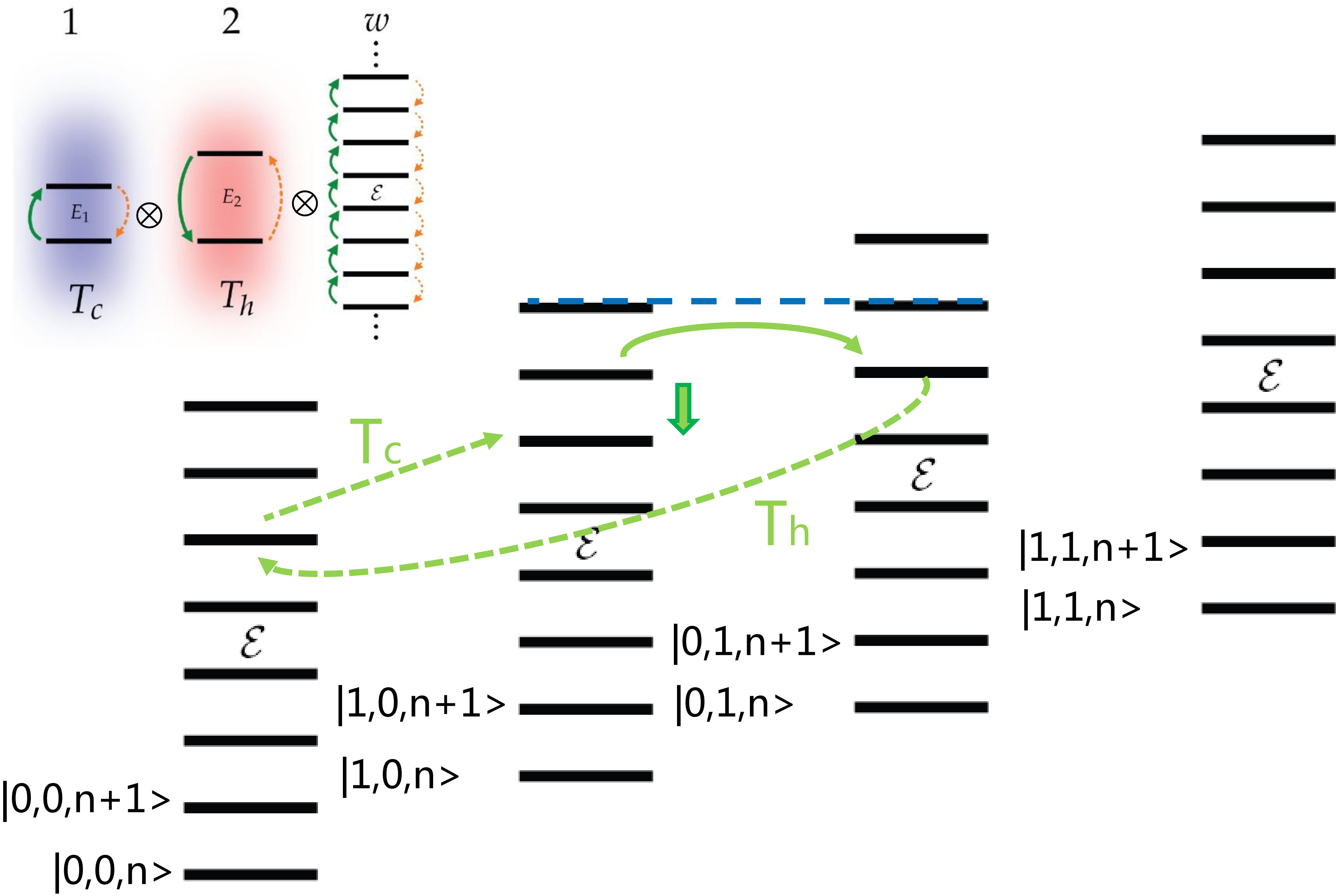
Self contained heat engine



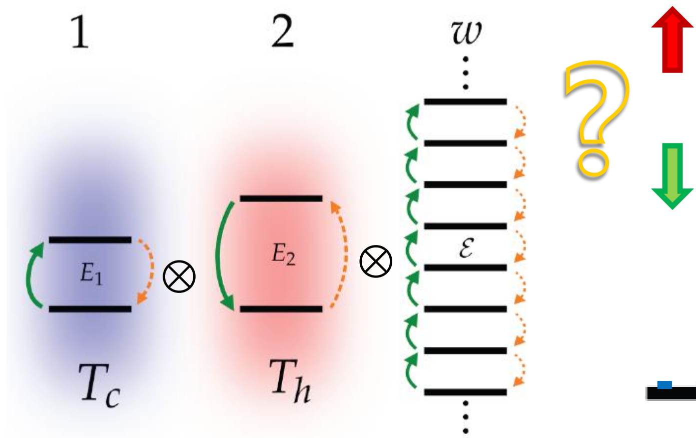
Self contained heat engine



Self contained heat engine

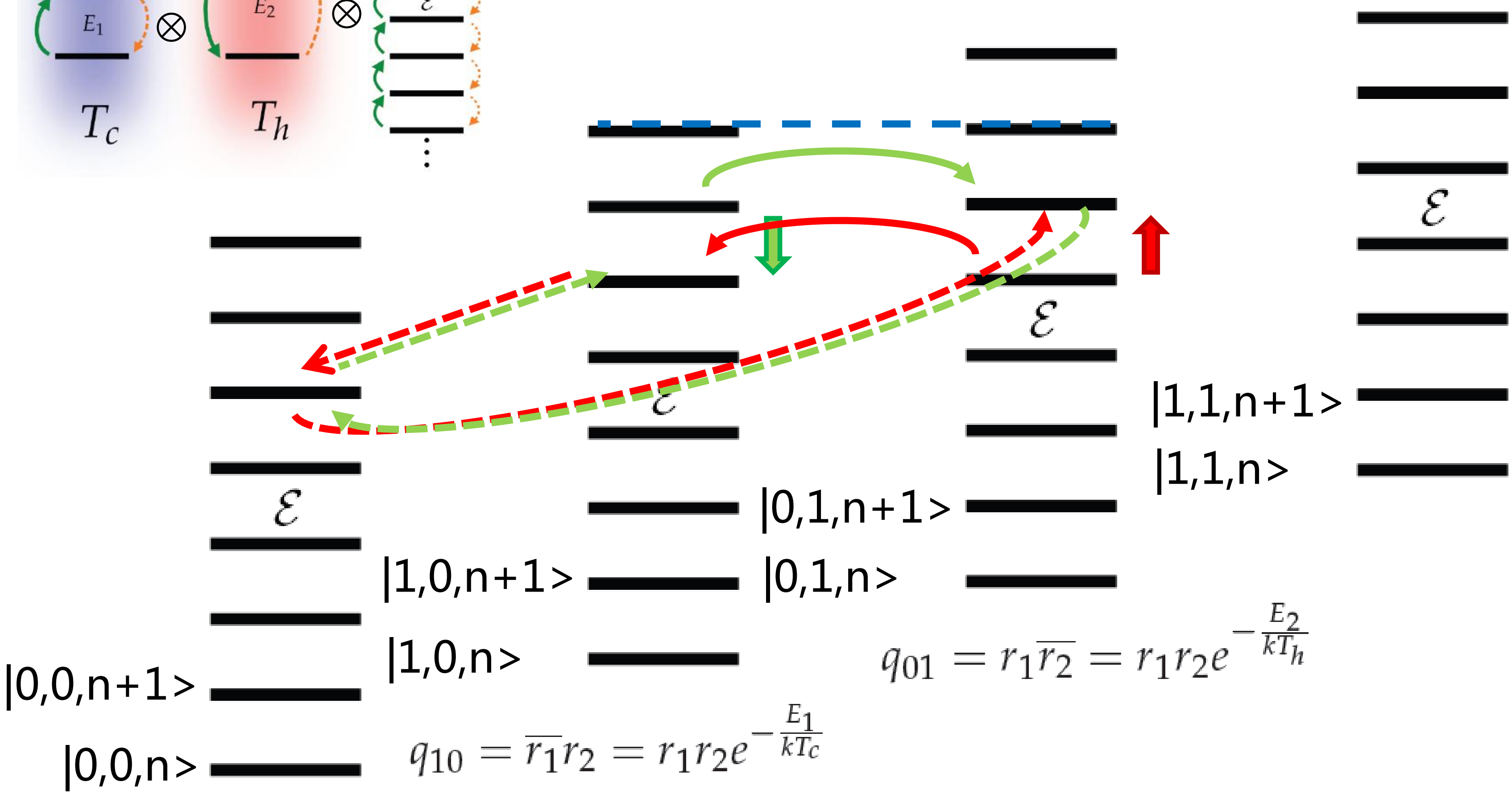


Self contained heat engine



i) population $q_{01} > q_{10}$

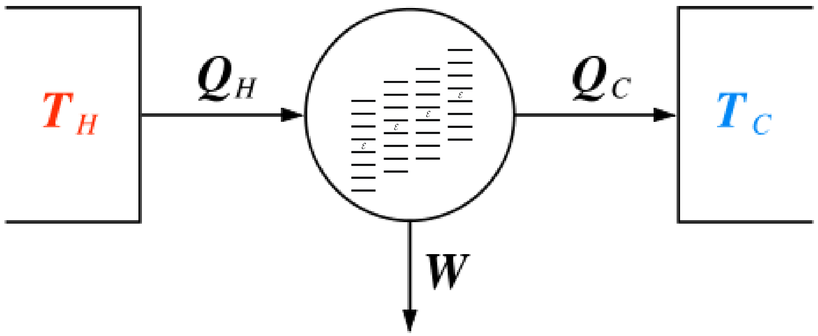
ii) probability $p_i \delta t$ the state of qubit i is reset to its thermal state



$$q_{01} = r_1 \overline{r_2} = r_1 r_2 e^{-\frac{E_2}{kT_h}}$$

$$q_{10} = \overline{r_1} r_2 = r_1 r_2 e^{-\frac{E_1}{kT_c}}$$

Self contained heat engine



Hamiltonian does not change!!

$$\sum_i \dot{E}^i p^i = 0$$

$$H_0 = E_1|1\rangle_1\langle 1| + E_2|1\rangle_2\langle 1| + \sum_{n=-\infty}^{\infty} n\mathcal{E}|n\rangle_w\langle n|$$

$$H_{int} = g\sum_{n=-\infty}^{\infty}\Big(|01,n\rangle\langle 10,n+1| + |10,n+1\rangle\langle 01,n|\Big)$$

$$\frac{\partial\rho}{\partial t} = -i[H_0 + H_{int},\rho] + \sum_{i=1}^{2} p_i(\tau_i \mathrm{Tr}_i\rho - \rho)$$

$$\frac{d}{dt}\langle E_w\rangle = \frac{d}{dt}\mathrm{Tr}(H_w\rho)$$

$$r_1 > r_2$$

$$\lim_{t\to\infty}\frac{d}{dt}\langle E_w\rangle = \frac{2\mathcal{E}g^2 p_1 p_2(r_1 - r_2)}{(p_1+p_2)(2g^2 + p_1 p_2)}$$

>0

Work and Heat?

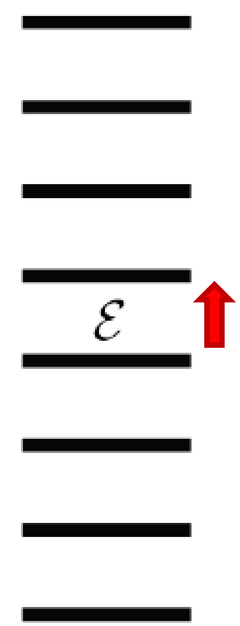
In this work we will use exactly the definition of work put forward by Carnot, namely:

"motive power (work) is the useful effect that a motor is capable of producing. This effect can always be likened to the elevation of a weight to a certain height." [3]

Work $\frac{d}{dt}\langle E_w \rangle = \frac{d}{dt}\mathrm{Tr}(H_w \rho)$

Heat current $\frac{d}{dt}Q_i = p_i \mathrm{Tr}\Big(H_i\big(\tau_i - \rho_i(t)\big)\Big)$

Population change



Efficiency

$$\eta^{Q} = \frac{\frac{d}{dt}\langle E_w \rangle}{\frac{d}{dt} Q_2} = \frac{E_2 - E_1}{E_2} = 1 - \frac{E_1}{E_2}$$

Carnot efficiency

$$\eta^{Q}_{\max} = 1 - \frac{T_c}{T_h}$$

when $\frac{E_1}{T_c} = \frac{E_2}{T_h}$

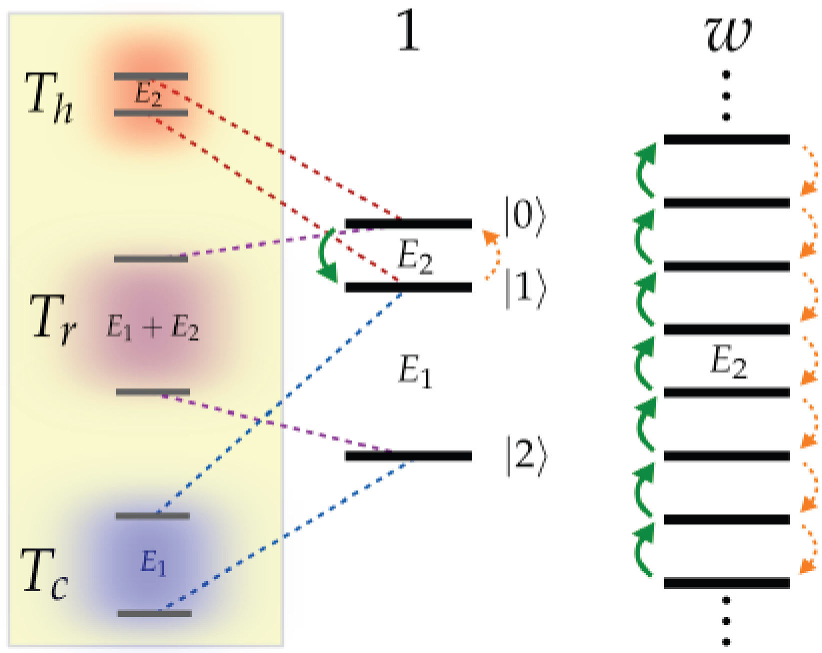


FIG. 2: **Schematic diagram of qutrit heat engine.** The qutrit is assumed to have each of its transitions at different temperatures. The temperature of the transitions is shown in the inset on the left. A weight is again connected, with energy level separation E_2, matching that of the separation of the two upper levels of the qutrit. The particles interact with each other and the temperatures are chosen such that the transition where the weight is lifted (solid green arrows) is biased over the transition where the weight falls (dashed orange arrows).

… and that's it