Work & Energy!!!

Outline

Work and Kinetic energy

Work done by a <u>net</u> force results in kinetic energy

Some examples: gravity, spring, friction

Work done by some (conservative) forces can be retrieved. This leads to the principle that energy is conserved

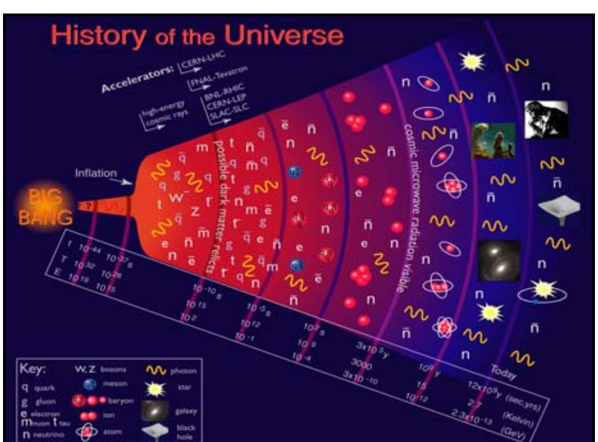
Conservation of Energy

The dependence of the conservative force on position is related to the position dependence of the PE

$F(x) = -d(U)/dx$

Potential energy

It is the energy stored by the object due to a work done by a conservative force (ex. Gravity & spring)



Energy

- Different Forms
  - **Kinetic Energy** **Motion**
  - **Potential Energy** **Stored**
  - **Heat** **Radiant**
  - **Mass ($E=mc^2$)** **Stored**
- Units Joules = kg m^2 / s^2

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Work & Energy

- One of the most important concepts in physics
  - Alternative approach to mechanics
- Many applications beyond mechanics
  - Thermodynamics (movement of heat)
  - Waves, E&M & Quantum mechanics...
- Very useful tools
  - Conserved in quite general situations!
  - You will learn new (sometimes much easier) ways to solve problems

Kinetic Energy

Work-Kinetic Energy Theorem

Change in KE≡ work done by all forces

$$\Delta K \equiv \Delta W$$

Work-Kinetic Energy Theorem

$$W = \int_{x_i}^{x_f} F.dx$$

$$= \int_{x_i}^{x_f} ma.dx$$

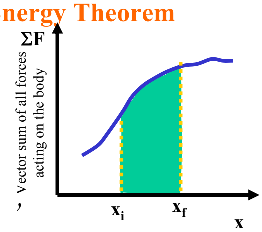
$$= \int_{x_i}^{x_f} m \frac{dv}{dt}.dx$$

$$= m\int_{v_i}^{v_f} v.dv$$

$= 1/2mv_f^2 - 1/2mv_i^2$

$= K_f - K_i$

$= \Delta K$



$\Delta K \equiv \Delta W$

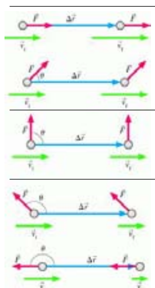
Work done by net force = change in KE

Work and the Direction of a Constant Force

Work done = F.d cos θ

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$= |A||B| Cos\,\alpha$$



Force and Displacement	θ	Work W	Sign	Energy Transfer
	0°	F(Δr)	+	Energy is transferred *into* the system. The particle speeds up. K increases
	< 90°	F(Δr)cos θ	+	
	90°	0	0	No energy is transferred. Speed and K are constant.
	> 90°	F(Δr)cos θ	–	Energy is transferred *out of* the system. The particle slows down. K decreases
	180°	−F(Δr)	–	

Gravitation and work

Work done by me (take up as +ve)

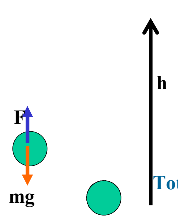
= F.(h) = mg(h) = mgh

Work done by gravity

= mg.(-h) = -mgh

Total work by ALL forces (ΔW) = 0

=ΔK



Lift mass m with constant velocity

Work done by ALL forces = change in KE

$W = \Delta K$

What happens if I let go?

Compressing a spring

Compress a spring by an amount x

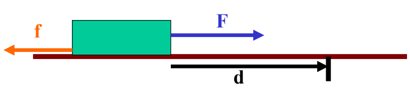
Work done by me $\int F dx = \int kx dx = 1/2 kx^2$

Work done by spring $\int -kx dx = -1/2 kx^2$

Total work done (ΔW) = **0** $= \Delta K$

What happens if I let go?

Moving a block against friction at constant velocity



Work done by me = F.d

Work done by friction = -f.d = -F.d

Total work done = **0**

What happens if I let go?	NOTHING!!

Gravity and spring forces are **Conservative**

Friction is NOT!!

Power

- **Average power**

$$P_{avg} = \frac{W}{t}$$

- **Instantaneous power** – the rate of doing work

$$P = \frac{dW}{dt}$$

- SI unit: J/s = kg*m²/s³ = W (Watt)

James Watt (1736-1819)

Power of a constant force

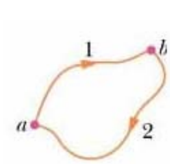
• In the case of a constant force

$$P = \frac{dW}{dt} = \frac{d(\vec{F}\cdot\Delta\vec{r})}{dt} = \vec{F}\cdot\frac{d\Delta\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

$$P = Fv\cos\phi$$

Conservative forces

• The net work done by a conservative force on a particle moving around any closed path is zero

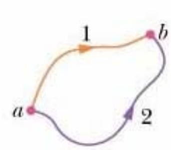


$$W_{ab,1} + W_{ba,2} = 0$$

$$W_{ab,1} = -W_{ba,2}$$

$$W_{ab,2} = -W_{ba,2}$$

$$W_{ab,1} = W_{ab,2}$$

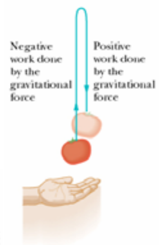


• The net work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle

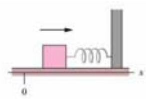
Conservative forces: examples

- Gravity force

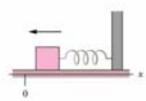
$$-mgh_{up} + mgh_{down} = 0$$



- Spring force

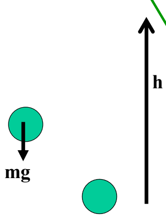


$$-\frac{kx_{right}^2}{2} + \frac{kx_{left}^2}{2} = 0$$



Potential Energy & Work

The change in potential energy is equal to minus the work done BY the conservative force ON the body.



Work done by gravity

= mg.(-h) = -mgh

Therefore change in PE is

$\Delta U = -\Delta W$

$\Delta U_{grav} = +mgh$

Lift mass m with constant velocity

Potential Energy

The change in potential energy is equal to minus the work done BY the conservative force ON the body.

Compress a spring by an amount x

Work done by spring is $\Delta w = \int -kx\, dx = \frac{1}{2} kx^2$

Therefore the change in PE is

$\Delta U = -\Delta W$

$\Delta U_{spring} = +\frac{1}{2} kx^2$

Potential Energy

The change in potential energy is equal to minus the work done BY the conservative force ON the body.

$$\Delta U = -\Delta W$$

but recall that

$$\Delta W = \Delta K$$

so that

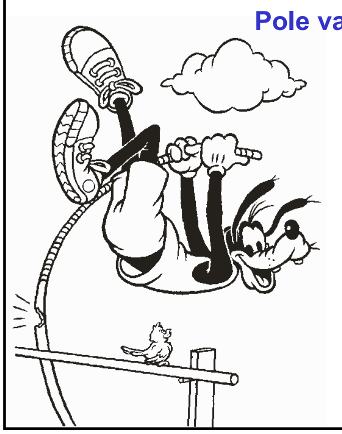
$$\Delta U = -\Delta K$$

or

$$\Delta U + \Delta K = 0$$

Any decrease in PE results from an increase in KE

Pole vault



Pole vault

- Energy stored in muscles

Pole vault

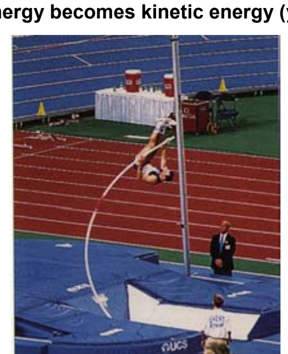
• Muscle energy becomes kinetic energy (x-direction)

Pole vault

• Kinetic energy becomes elastic potential energy

Pole vault

• Elastic energy becomes kinetic energy (y-direction)



Pole vault

- Kinetic energy becomes gravitational potential energy

Pole vault

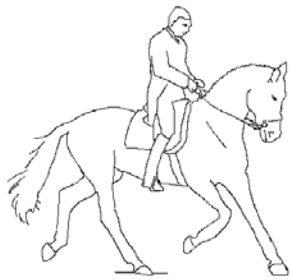
- Gravitational potential energy becomes kinetic energy (y-direction)

Pole vault

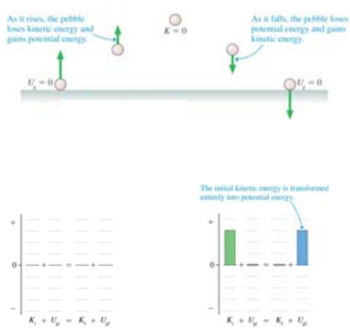
- Kinetic energy energy becomes part elastic potential energy and part internal energy

Pole vault

- The 'pole vault' phenomenon is ubiquitous…

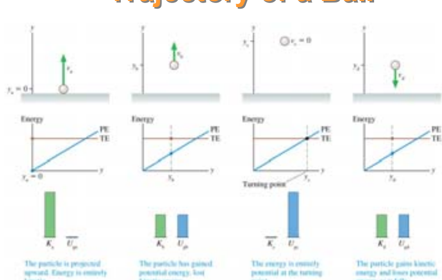


Energy Bar Charts

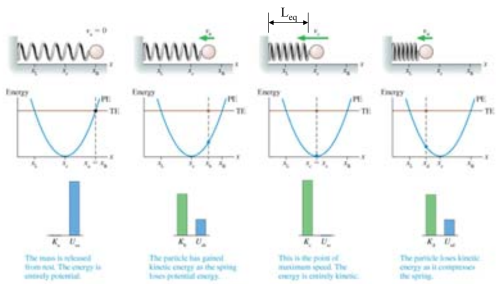


Energy Bar Charts

Trajectory of a Ball



A Mass and Spring



$$\Delta U + \Delta K = 0$$

In a system of conservative forces, any change in Potential energy is compensated for by an inverse change in Kinetic energy

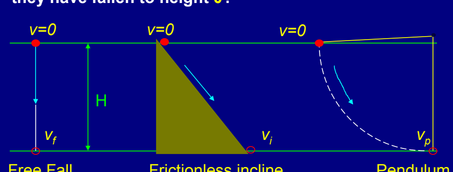
$$U + K = E$$

In a system of conservative forces, the mechanical energy remains constant

Quiz

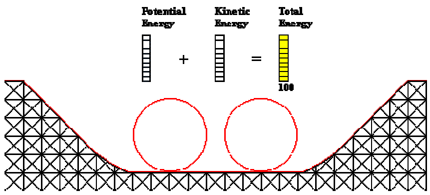
Falling objects

- Three objects of mass *m* begin at height *h* with velocity *0*. One falls straight down, one slides down a frictionless inclined plane, and one swings on the end of a pendulum. What is the relationship between their velocities when they have fallen to height 0?

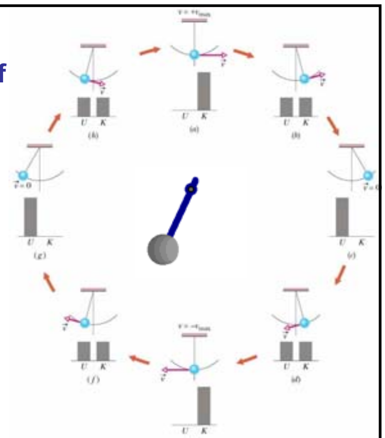


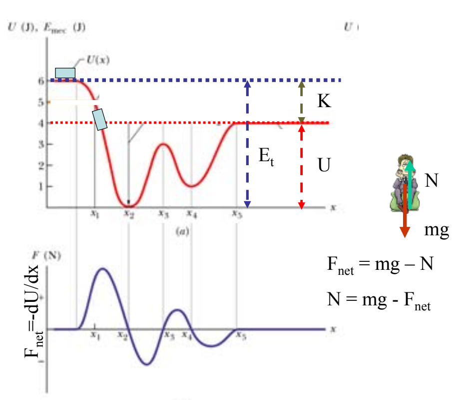
(a) $V_f > V_i > V_p$ (b) $V_f > V_p > V_i$ (c) $V_f = V_p = V_i$

Roller Coaster
Conservation of Energy



Conservation of mechanical energy: pendulum

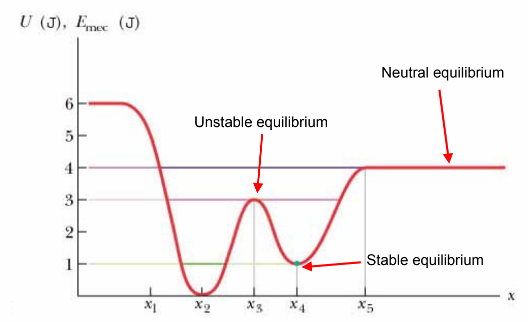




$F_{net} = mg – N$

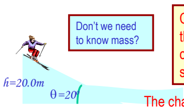
$N = mg - F_{net}$

Potential energy curve: equilibrium points



Example for Non-Conservative Force

A skier starts from rest at the top of a frictionless hill whose vertical height is 20.0*m* and the inclination angle is 20º. Determine how far the skier can get on the snow at the bottom of the hill where there is friction (coefficient of kinetic friction between the ski and the snow is 0.210).



Don't we need to know mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times 9.8\times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy.

$$\Delta K = K_f - K_i = -f_k d$$

Since $K_f = 0$ $\quad -K_i = -f_k d; \quad f_k d = K_i$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2\times 0.210\times 9.80} = 95.2m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% in the engine:
1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$m_{car} = 1450kg \quad Weight = mg = 14200N$

Coefficient of Rolling Friction; μ=0.016

$\mu n = \mu mg = 227N$

Air Drag $\quad f_a = \frac{1}{2}D\rho A v^2 = \frac{1}{2}\times 0.5\times 1.293\times 2v^2 = 0.647v^2$

Total Resistance $\quad f_t = f_r + f_a$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N)\cdot 26.8 = 18.5kW$$

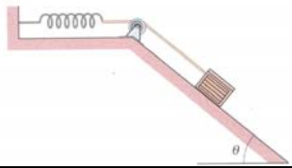
Power to overcome each component of resistance

$$P_r = f_r v = (227)\cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7)\cdot 26.8 = 12.5kW$$

Extra Problrm

••32 A 2.0 kg breadbox on a frictionless incline of angle $\theta = 40°$ is connected, by a cord that runs over a pulley, to a light spring of spring constant $k = 120$ N/m, as shown in Fig. 8-44. The box is released from rest when the spring is unstretched. Assume that the pulley is massless and frictionless. (a) What is the speed of the box when it has moved 10 cm down the incline? (b) How far down the incline from its point of release does the box slide before momentarily stopping, and what are the (c) magnitude and (d) direction (up or down the incline) of the box's acceleration at the instant the box momentarily stops?



Extra problem

77 In Fig. 8-61, a small block is sent through point A with a speed of 7.0 m/s. Its path is without friction until it reaches the section of length $L = 12$ m, where the coefficient of kinetic friction is 0.70. The indicated heights are $h_1 = 6.0$ m and $h_2 = 2.0$ m. What are the speeds of the block at (a) point B and (b) point C? (c) Does the block reach point D? If so, what is its speed there; if not, how far through the section of friction does it travel?

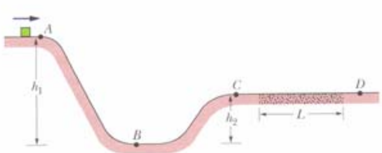


Fig. 8-61 Problem 77.