Nonlinear optical interactions of wave packets in photonic crystals

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Abstract: We develop a new formalism for describing nonlinear interactions of beams and pulses in photonic crystals, providing a convenient method of solving optical frequency conversion and optical control problems.

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Photonic crystals provide new opportunities for enhancing and controlling nonlinear optical processes, such as the generation of optical harmonics and combination frequencies, self- and cross-phase-modulation, and the like. Different theoretical approaches have been developed for describing nonlinear processes in "low contrast" periodic materials. These types of media are usually one-dimensional, such as the index-modulated optical fibers. The traditional approaches are not appropriate for "high contrast" periodic structures with rapid spatial variations in the linear optical properties that are typical of two- and three-dimensional photonic crystals. We present a treatment of the problem based on an effective-fields formalism that was recently proposed for nonlinear self-action of a single wave-packet [1].

In our approach wave packets in a photonic crystal are comprised of Bloch functions in the same photonic band with close crystal wave vectors; the contributions of the different Bloch functions to a wave packet are described by a slowly varying function of the crystal wave vector. The Fourier transform of this wave-vector function is a slowly varying function of the spatial coordinates, which we call the effective field $g_a(\boldsymbol{r}, t)$ of the wave packet a. The effective fields give an "averaged" description for the electromagnetic field in a periodic medium; in terms of them expressions for the average energy density and Poynting vector can be given.

A simple example is the application of our formalism to the optical parametric amplification, where a strong pump wave packet with the central frequency ω_2 amplifies a weaker wave packet of frequency ω_1 in the nonlinear optical process $\omega_2 = \omega_1 + \omega_3$. Fig. 1 gives a graphical representation of this process on the photonic band diagram, where the energy flows from the effective field $g_2(\boldsymbol{r}, t)$ to the effective fields $g_1(\boldsymbol{r}, t)$ and $g_3(\boldsymbol{r}, t)$.

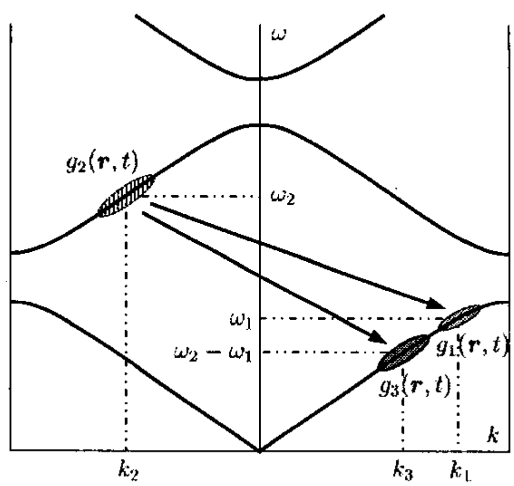

Fig. 1. A schematic for the nonlinear wave-packet interaction process $\omega_2 = \omega_1 + \omega_3$. Shaded ovals mark the parts of the photonic bands that correspond to the Bloch modes contributing to the wave packets $a = 1, 2, 3$. The effective fields $g_a(\boldsymbol{r}, t)$ are the Fourier transforms of the slowly varying amplitudes describing the contributions from different Bloch modes.

The effective field $g_1(\boldsymbol{r},t)$ of the amplified wave packet is governed by the dynamical equation

$$\begin{aligned}\frac{\partial g_1(\boldsymbol{r},t)}{\partial t} &= -\mathrm{i}\,\omega_1\, g_1(\boldsymbol{r},t) - \frac{\partial \omega_1}{\partial k^i}\frac{\partial g_1(\boldsymbol{r},t)}{\partial r^i} + \frac{\mathrm{i}}{2}\frac{\partial^2\omega_1}{\partial k^i \partial k^j}\frac{\partial^2 g_1(\boldsymbol{r},t)}{\partial r^i \partial r^j} \\ &\quad + 2\mathrm{i}\chi^{(2)}_{\text{eff}}\, g_2(\boldsymbol{r},t)\, g_3^*(\boldsymbol{r},t)\exp[\mathrm{i}(\boldsymbol{k}_2 - \boldsymbol{k}_1 - \boldsymbol{k}_3)\cdot \boldsymbol{r}]. \qquad (1)\end{aligned}$$

Eq. (1) is formally very similar to the equation for a slowly varying amplitude of the electric field in a homogeneous medium, so we can use well established methods to solve it. Yet the coefficients in Eq. (1) are determined by the photonic band structure, and can differ greatly from typical values in a uniform medium. $\chi^{(2)}_{\text{eff}}$ is a volume integral of the spatially periodic quadratic optical susceptibility $\overleftrightarrow{\chi}^{(2)}(\boldsymbol{r})$ multiplied by the three Bloch functions corresponding to the central crystal wave vectors $\boldsymbol{k}_a$, $a = 1, 2, 3$. With a proper choice of $\boldsymbol{k}_a$ we could compensate the material dispersion of the medium and achieve phase matching. Unlike in the case of a homogeneous medium, the dependence of $\chi^{(2)}_{\text{eff}}$ on $\boldsymbol{k}_a$ can be very strong, and we may have to take it into account *via* a series expansion that will lead to the appearance of new nonlinear terms in Eq. (1) featuring spatial derivatives of $g_a(\boldsymbol{r},t)$.

The effective-fields formalism describes nonlinear optical processes in photonic crystals in a convenient and intuitive way, so that the correct dynamical equations can be guessed in simple cases. However, we rigorously ground our theory on a Hamiltonian formalism that allows us to easily quantize the problem, find conserved quantities, and investigate their relation to symmetries. Our approach can easily be extended to more complex problems. We also treat other examples that highlight the new features of the dynamical equations for the effective fields, which are specific to photonic crystals. We demonstrate how a spatially localized dc electric field or a controlling (pump) optical beam can influence the propagation of a wave packet through a nonlinear phase shift and change of the group velocity. The effective field formalism allows us to describe and easily calculate the consequences of such effects, which alternatively can be viewed as resulting from a spatially localized shift in the photonic band structure due to the electro-optic or Kerr-effect.

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[1] J. E. Sipe, N. A. R. Bhat, P. Chak, and S. Pereira, "Effective field theory for the nonlinear optical properties of photonic crystals," Phys. Rev. E *(in press)*.