Increasing the delay-time–bandwidth product for microring resonator structures by varying the optical ring resonances

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We propose a scheme to increase the delay-time–bandwidth product for a periodic microring resonator structure in slowing or stopping light. The idea is based on the existence of a low group velocity and low dispersion region close to a band edge near a ring resonance. By putting different frequency components on resonance with different rings, one can drastically slow down the light without inducing large additional dispersive distortion. © 2007 Optical Society of America

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Resonant photonic bandgap structures (RPBGs) are promising canditates for slow light applications. An RPBG is an optical system in which optical resonances are periodically arranged. One important class of RPBGs is periodic microring resonator sequences, for which a broad variety of structures with different geometries has been proposed and studied.[1] Generally the maximum delay in such structures is limited by a delay-time–bandwidth product.[2] Stopping light inside a structure can increase the delay beyond the limit set by the delay-time–bandwidth product without inducing additional dispersive distortion[3–5]; however, in light-stopping processes, a fundamental parameter more relevant than the maximum delay is the maximum number of pulses that can be stopped simultaneously inside a structure of given length, or, equivalently, the minimum length of the structure needed to hold the whole pulse with a frequency spectrum restricted to the applicable band. For example, a recently proposed light-stopping scheme uses quantum-well Bragg structures,[4,6] in which the bandwidth of a special "intermediate band (IB)" between the excition resonance and the Bragg resonance is initially finite and then compressed to zero after the pulse is totally inside the structure. This requires a universal minimum length inversely proportional to the delay-time–bandwidth product, which is independent of the initial (i.e., before being compressed) bandwidth of the IB.

In this Letter we propose a scheme that increases the delay-time–bandwidth product for RPBGs far beyond the usual limit. We use the side-coupled integrated spaced sequence of resonators (SCISSOR)[1] microring resonator structure [see Fig. 1(a)] as an example, but the basic principle applies to any RPBG with similar features.[7] The idea is to gradually vary the resonances of the rings in an otherwise periodic structure, so that all frequency components of the pulse are on resonance or nearly on resonance with some of the rings. By putting different spectral components on resonance with different rings, one not only obtains a much larger group delay but also suppresses the dispersive distortion, since different frequency components are slowed down independently.[8] Although this interpretation of the slow light process is physically transparent, another approach based on the properties of the band structure is more suitable for a quantitative formalism; thus we mainly adopt this second approach in the rest of this Letter.

The schematic of a SCISSOR unit is illustrated in Fig. 1(a). We assume that light propagates linearly inside the structure and couples between the channel waveguides and the resonator only at the coupling points marked by the filled circles through the coupling equation

$$\begin{pmatrix} E_3^{\mathrm{up/low}} \\ E_2^{\mathrm{up/low}} \end{pmatrix} = \begin{pmatrix} \sigma & i\kappa \\ i\kappa & \sigma \end{pmatrix} \begin{pmatrix} E_4^{\mathrm{up/low}} \\ E_1^{\mathrm{up/low}} \end{pmatrix}, \tag{1}$$

where $E_m^{\mathrm{up/low}}$ are the electric fields and σ and κ are the self- and cross-ring-channel coupling coefficients, respectively. In our current model the coupling coefficients are assumed to be real and satisfy $\sigma^2+\kappa^2=1$ to conserve energy. Combining the linear propagation in the waveguides (both channels and resonator) with the coupling equation (1), one can obtain the

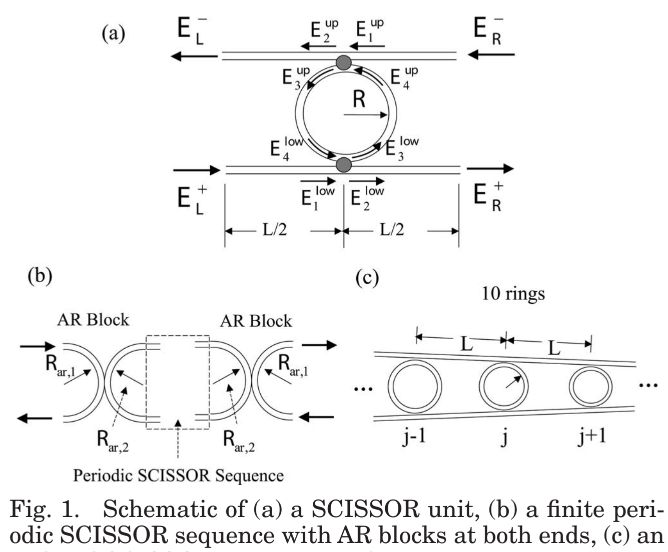

Fig. 1. Schematic of (a) a SCISSOR unit, (b) a finite periodic SCISSOR sequence with AR blocks at both ends, (c) an inclined SCISSOR structure with varying ring resonances.

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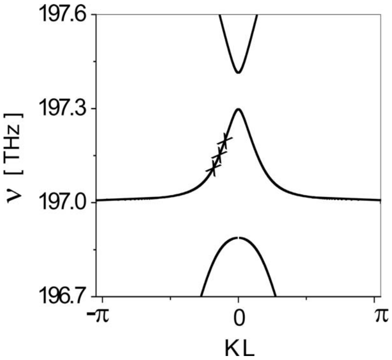

Fig. 2. Photonic band structure of the periodic SCISSOR structure around $100\nu_B$, with the crosses indicating the K components of the incident pulse.

unit cell transfer matrix in the frequency domain, $M(\nu)$, relating the fields on the left to the fields on the right[1]:

$$\begin{pmatrix} E_R^+ \\ E_R^- \end{pmatrix} = M \begin{pmatrix} E_L^+ \\ E_L^- \end{pmatrix}, \quad M = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix},$$

with

$$\alpha = \left\{ \exp\left[i\pi \left(\frac{\nu}{\nu_B} - \frac{\nu}{\nu_r} \right) \right] \right\} \times \left[\exp\left(2i\pi \frac{\nu}{\nu_r} \right) - \sigma^2 \right] \Big/ 2i\sigma \sin\left(\pi \frac{\nu}{\nu_r} \right)$$

and $\beta = \kappa^2 / [2i\sigma \sin(\pi \frac{\nu}{\nu_r})]$. Here $\nu_r = c/(2\pi n_r R)$ is the fundamental ring resonance frequency, with n_r and R being the effective refractive index and the radius of the ring, respectively; $\nu_B = c/(2n_B L)$ is the fundamental Bragg resonance frequency, with n_B and L being the effective refractive index and the length of the channel waveguides, respectively. For an infinite periodic sequence of such unit cells, the band structure, i.e., the quasi-wavenumber K as a function of the frequency ν, is given by $\cos(KL) = 1/2 \times \mathrm{Tr}\{M(\nu)\}$, where Tr is the trace of a matrix. In general, there exist bandgaps associated with, and spectrally close to, the ring resonances $m\nu_r$ and the Bragg resonances $m'\nu_B$, where m, m' are positive integers; between any two adjacent bandgaps, there is a photonic band. When a ring resonance and a Bragg resonance, $m\nu_r$ and $m'\nu_B$, are sufficiently close to each other, the photonic band between the ring resonance bandgap and the Bragg resonance bandgap is called the IB. This IB runs from $m'\nu_B$ to nearly $m\nu_r$, and the group velocity within the IB decreases as $m'\nu_B$ and $m\nu_r$ are closer to each other. In particular, when $m\nu_r = m'\nu_B$, the IB is completely flat, indicating a zero group velocity for all wavenumbers. This allows one to deliberately slow down, stop, and release light by manipulating the bandwidth of the IB, as has already been proposed by Yang *et al.*[4,6] for quantum-well Bragg structures. As mentioned above, the fundamental delay-time–bandwidth product limits the applicability of this process.

By numerically solving $\cos(KL) = 1/2 \times \mathrm{Tr}\{M(\nu)\}$, we illustrate in Fig. 2 a typical band structure of a periodic SCISSOR sequence with the parameters $\nu_B = 1.973$ THz, $\nu_r = 3.940$ THz, and $\sigma = 0.97$. This fundamental ring-resonance frequency ν_r corresponds to a ring radius of $R = 76.09\ \mu\mathrm{m}/(2\pi n_r)$. The band structures shown in Fig. 2 is around the adjacent 100th Bragg resonance ($100\nu_B = 197.3$ THz) and the 50th ring resonance ($50\nu_r = 197.0$ THz), where one can see the IB between the two bandgaps. Other than being quite sensitive to the change of ν_r or ν_B, the IB has another intriguing feature generally existing near the ring resonances: it is very flat at the band edge on the ring resonance side. Our scheme to increase the delay-time–bandwidth product is based on this flat region of the IB due to its low group velocity and low dispersion.

However, one cannot *directly* take advantage of this flat region for slowing light, since it is extremely narrow in the frequency domain compared with the IB bandwidth. The key to solving this problem is to vary the ring resonances of the unit cells in the otherwise periodic structure to provide a "quasi-force" f acting on light pulses *in K space*, as has been widely applied in optical Bloch oscillations in photonic crystals.[9] With this quasi-force, an incident pulse with a frequency spectrum of normal width, i.e., comparable with the IB bandwidth but still within the IB, will be pushed, in K space, into the flat region at the band edge on the ring resonance side while propagating inside the structure.

To further elaborate on this, we compare the group delay in a "uniformly periodic system," i.e., a finite periodic sequence without any variation in the parameters of the unit cells, and the delay in an "inclined system," i.e., a structure with gradually varying ring resonances but otherwise uniformly periodic propagation, by numerically solving the linear propagation equations[10] together with the coupling equation (1). The variation in the ring resonances can be realized by, e.g., varying the radii or the effective indices of the rings.

For the uniformly periodic system, we consider a 10-unit-cell structure with the parameter values as those used for Fig. 2. An antireflection (AR) block,[11] which is a coupled optical waveguide unit consisting of two coupled half-rings [see Fig. 1(b)], with radii $R_{ar,1}$ and $R_{ar,2}$ corresponding to fundamental ring resonance frequencies of (with the effective index being the same as that of the channel) $\nu_{r,1}^{ar} = \nu_r$, $50\nu_{r,1}^{ar} - 50\nu_{r,2}^{ar} = 3.66$ THz, and ring–ring self-coupling coefficient $\sigma_{ar} = 0.29$, is added at both ends of the structure to minimize the reflectivity at frequency $\nu_0 = 197.15$ THz in the middle of the IB. The incident Gaussian pulse is 8 ps FWHM, with the frequency spectrum centered at ν_0. The AR blocks allow the whole pulse to go through the structure. The output (transmitted) pulse is shown as the dashed curve in Fig. 3, which is delayed for about 5 ps and is broadened due to the group velocity dispersion. The transmitted pulse begins to emerge even before the incident pulse reaches its peak, indicating that at any

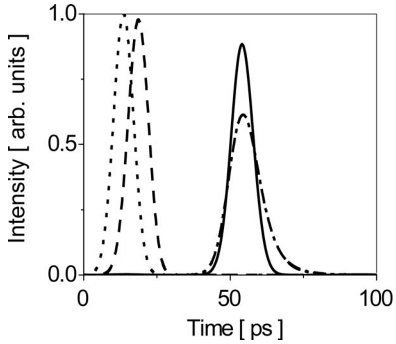

Fig. 3. Time dependence of pulse intensities: the input (dotted curve), transmitted in the 10-unit-cell uniformly periodic system (dashed curve), reflected in the 10-unit-cell inclined system (solid curve), and transmitted in the 90-unit-cell uniformly periodic system (dashed–dotted curve).

given time the pulse extends well outside the uniformly periodic system.

We now turn to the inclined system, as sketched in Fig. 1(c), with varying ring resonances but otherwise the same as the above uniformly periodic system (also with the same AR blocks). The fundamental ring resonance of the jth ring, $\nu_{r,j}(j=1,2,3\ldots10)$, is given by $50\nu_{r,j}=50\nu_r+(j-1)(100\nu_B-50\nu_r)/12=50\nu_r+(j-1)\times25$ GHz. As one can see, variation of the ring resonances $\nu_{r,j}$ with j is linear, with the 50th ring resonance of the first unit cell $(50\nu_{r,1})$ being the same as that in the uniformly periodic system, and the 50th ring resonance of the 7th ring $(50\nu_{r,7})$ is in the middle of the IB (Fig. 2) of the uniformly periodic system. We can analyze this scenario of slowing light inside this system in terms of the quasi-force f in K-space induced by the inclination of the band structure due to the variation of the ring resonances throughout the system. Here we only have space to mention that in this IB at each K this quasi-force is given by $f(K)=\Lambda(K)(\nu_{r,j+1}-\nu_{r,j})$, where $\Lambda(K)<0$. Since this quasi-force is negative, it will push negative K toward $-\pi$ and positive K toward 0 in the 1st Brillouin zone. For an incident pulse with the negative K (positive group velocity) components around the middle of the IB, as indicated by the crosses in Fig. 2, the quasi-force f will push its K spectrum toward $-\pi$ into the flat region; after the K passes the Brillouin zone boundary from $-\pi$ to π, it continues to move toward 0 into the positive K states around the middle of the IB, where the group velocity is negative, which means that in real space the pulse is moving backward. Thus the final output is a reflected rather than a transmitted pulse. Because in K space the flat region near the ring resonance is large, the light pulse stays in the low group velocity and low dispersion states for a long time, resulting in a much longer delay with only small additional dispersive distortion.

We can confirm this by numerical simulations of pulse propagation in the inclined structure, giving the result for the reflected pulse as the solid curve in Fig. 3. It is remarkable that the current output pulse is delayed for about 40 ps, 8 times the delay in the uniformly periodic system. The output pulse starts to come out long after the incident pulse disappears, indicating that the whole pulse is totally inside the structure for a long time and thus can be totally stopped inside the structure by shifting all the 50th resonances of the rings to coincide with the 100th Bragg resonance; this will be discussed in detail in a later paper. Moreover, the additional distortion in the output pulse is small compared with the distortion in a pulse traveling through a 90-cell uniformly periodic system that would give the same delay, which is plotted as the dashed–dotted curve in Fig. 3.

In summary, by varying the ring resonances in an otherwise periodic microring resonator sequence, thus making many frequency components of an incident pulse on resonance with rings independently, one can obtain a much larger delay-time–bandwidth product with little additional dispersive distortion.

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