

# Quantization of the Electromagnetic Field in Dispersive and Absorptive Structured Dielectrics

Navin A. R. Bhat and J. E. Sipe

Department of Physics, University of Toronto  
60 St. George St., Toronto, ON, Canada, M5S 1A7  
nbhat@physics.utoronto.ca

**Abstract:** We present a canonical Hamiltonian formulation for electromagnetic fields in dispersive and absorptive inhomogeneous structured dielectrics obeying the Kramers-Kronig relations. We present an effective band description of the polariton modes in the absorption band.

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Dispersion and absorption have become increasingly important in the study of photonic metamaterials. Indeed, many proposed applications require delicately engineered absorption and/or dispersion characteristics. While these effects can be added “by hand” to envelope function equations, or even to the underlying Maxwell equations, the advantage of formulating a calculation in terms of photonic modes of the system is often thereby lost. Such a formulation is crucial, of course, for many applications in quantum optics. Here we present a canonical Hamiltonian formulation of the EM field in dispersive and absorptive photonic metamaterials with dispersive and absorption properties explicitly satisfying the Kramers-Kronig relations. It is based on an identification of the photonic modes of the entire system, and is applicable to problems in both classical and quantum optics.

Hamiltonian theories for dispersive media have of course been considered before. They necessarily require introduction of material degrees of freedom to account for the temporal nonlocality of the response. Ideally, medium degrees of freedom should be introduced based on an intuitive, physical model of the material. The role and expression for the dielectric response should be clear, and one should obtain canonical quantities that are just the photon modes augmented by medium excitation. Two concepts currently dominate for structured media, each with its own strengths: In the Langevin noise method [1-3], the basic operators are noise currents, from which the EM field modes are given only indirectly using the Helmholtz Green function; the other, the Auxiliary field method [4,5], involves *ad hoc* postulation of auxiliary fields coupled to the usual Maxwell fields *via ad hoc* dynamics, with the result that the physical intuition and physical role of the dielectric response are somewhat opaque. Huttner and Barnett [6] introduced a physically intuitive formulation for uniform media based on the Hopfield model for the dielectric, but others have argued that it would be cumbersome to apply their approach to inhomogeneous media[5]. However, here we present just such a scheme. It involves an intuitive model of the medium, the end result is both easy-to-use and elegant, and the final canonical quantities identified are indeed the most natural ones, for they are just the photon modes augmented by medium excitation. The approach allows us to move naturally from the canonical description of a nondispersive and nonabsorptive medium to one that is both dispersive and absorptive.

The medium response is introduced *via* a Hopfield-type model, as harmonic oscillator fields  $\psi_{\Omega}(\mathbf{r})$  of frequency  $\Omega$  coupled to the EM fields by a coupling constant  $\Lambda^{ij}(\mathbf{r}, \Omega)$ . This is well-justified physically in the linear regime. The full (and arbitrary) response of the actual medium,  $\chi^{(1)}(\mathbf{r}, \Omega)$ , is then generated implicitly by the coupled dynamics of the bare EM field with the medium fields. The dielectric function is easily understood, with dispersion being due to virtual coupling and absorption due to direct coupling. Our theory is applicable to inhomogeneous media in general, and we formulate it in terms of a ‘nominal’ nondispersive and nonabsorptive structure with index of refraction  $n(\mathbf{r})$ . It should thus be particularly useful in the design of structured dispersive and absorptive dielectrics for which the practitioner has good intuition for the mode structure of the nominal medium.

The dressed modes of the system, which are the polaritons, are obtained through a mixed real-space / mode-space multichannel Fano diagonalization. For a finite system or an infinite system subject to periodic boundary conditions, we identify two qualitatively different regimes in the polariton spectra. At frequencies below the onset of absorption, we obtain discrete polariton spectrum, while in the absorption band we obtain a continuous one. At frequencies  $\Omega$  in the absorption band (*e.g.*, above the electronic band

gap), the expression for the polariton operators  $c_{\Omega m}$  in terms of the bare EM mode operators  $a_{\mu}$  and their physical conjugates  $a_{\mu}^{\dagger}$ , and in terms of the bare medium operators  $\psi_{\Omega'}(\mathbf{r})$  and  $\psi_{\Omega'}^{\dagger}(\mathbf{r})$ , is

$$c_{\Omega m} = \sum_{\mu} \left( \alpha_{\mu}^{\Omega m} a_{\mu} + \beta_{\mu}^{\Omega m} a_{\mu}^{\dagger} \right) + \int d\Omega' \int d\mathbf{r} \left( \alpha_{\Omega'}^{\Omega m}(\mathbf{r}) \cdot \psi_{\Omega'}(\mathbf{r}) + \beta_{\Omega'}^{\Omega m}(\mathbf{r}) \cdot \psi_{\Omega'}^{\dagger}(\mathbf{r}) \right) \quad (1)$$

where we have implicitly defined the various coefficients of the canonical transformation, and the frequency and spatial integrals are over the absorption band and absorbing region respectively. Then for  $\gamma_{\mu}^{\Omega m}$ , which are proportional to the bare EM coefficients in (1) for the contribution of electromagnetic nominal mode  $\mu$  to the polaritons, we obtain the generalized Hermitian eigenvalue problem

$$\sum_{\mu} \left[ \frac{(\hbar\Omega)^2 - (\hbar\omega_{\mu})^2}{2\hbar\omega_{\mu}} \delta_{\mu'\mu} + (\Delta Re\Gamma(\Omega))_{\mu\mu'} \right] \gamma_{\mu}^{\Omega m} = \frac{z^{\Omega m}}{\pi} \sum_{\mu} (Im\Gamma(\Omega))_{\mu\mu'} \gamma_{\mu}^{\Omega m} \quad (2)$$

where  $\omega_{\mu}$  are the nominal mode frequencies, and we define the mode overlap integrals with the dielectric response  $\Gamma^{ij}(\mathbf{r}, \Omega)$ , as

$$\begin{aligned} (\Delta Re\Gamma(\Omega))_{\mu\mu'} &\equiv \frac{1}{\epsilon_0} \int d\mathbf{r} D_{\mu}^{i*}(\mathbf{r}) [Re\Gamma^{ij}(\mathbf{r}, \Omega) - \Gamma_{\text{nom}}^{ij}(\mathbf{r})] D_{\mu'}^j(\mathbf{r}) \\ (Im\Gamma(\Omega))_{\mu\mu'} &\equiv \frac{1}{\epsilon_0} \int d\mathbf{r} D_{\mu}^{i*}(\mathbf{r}) Im\Gamma^{ij}(\mathbf{r}, \Omega) D_{\mu'}^j(\mathbf{r}) \end{aligned} \quad (3)$$

where  $\Gamma_{\text{nom}}^{ij}(\mathbf{r})$  is the nominal dielectric response with which the nominal modes are determined, and  $D_{\mu}(\mathbf{r})$  are the displacement fields of the nominal modes. **We stress that although the derivation makes use of the underlying model of the dielectric, only the aggregated real and imaginary parts of the dielectric response appear in the eigenvalue problem, and each appears in a physically distinct role.**

Furthermore, the real part of the response appears as its difference from the nominal response, giving a continuity of intuition from the nominal mode structure to that of the polaritons. The eigenvalue  $z^{\Omega m}$  determines the correct normalization of the electromagnetic component of the polaritons, and also determines the resonant contribution of the medium at the polariton frequency. The contributions of the medium fields with natural frequencies detuned from the polariton frequency are determined straightforwardly, once the eigenvalue problem has been solved; see Fig. 1. We note that despite having a generalized Hermitian eigenvalue problem, which is associated with nonorthogonal bases, we have unitary evolution and orthogonal polaritons.

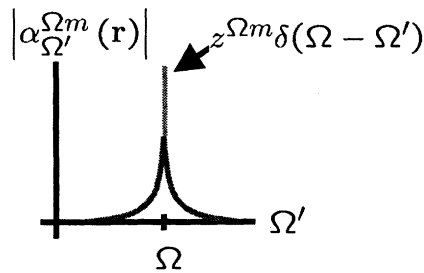


Fig. 1.: Schematic showing the contribution of the medium fields of natural frequency  $\Omega'$  to a polariton at frequency  $\Omega$ . The coefficient  $\beta_{\Omega'}^{\Omega m}(\mathbf{r})$  has similar behavior.

At frequencies below the onset of absorption, the right hand side of (2) vanishes, and there are no longer solutions at all frequencies  $\Omega$ . Instead, the equation gives a discrete spectrum, and in fact it is just the equation for the shifted electromagnetic modes of the dispersive system. This is convenient, since for problems of interest, codes are often freely available to calculate the dispersive modes of the system [7]. Then, as before, the medium contribution is determined straightforwardly from the calculated dispersive photon modes. Here we see the utility of this method relative to others: Given the dispersive electromagnetic mode structure, which is well-understood and well-calculated, the canonical polaritons follow. Furthermore, the polaritons have a natural interpretation. At frequencies below the onset of absorption, they are just the (dispersive) electromagnetic field modes augmented by the associated medium

excitation. This is a significant advantage over the Langevin noise method, in which the electromagnetic field modes are obtained only indirectly from the canonical noise current operators.

It has been pointed out in the past [6] that absorption causes the loss of a polaritonic dispersion relation in this approach. However, we are able to present an effective band structure concept. Formulated in terms of the expectation value of electromagnetic field energy in the polaritonic state, the nominally discrete bands are broadened in frequency (and shifted) due to absorption. An example is given in Fig. 2. In the limiting case of no absorption, we recover the usual photonic dispersion relation.

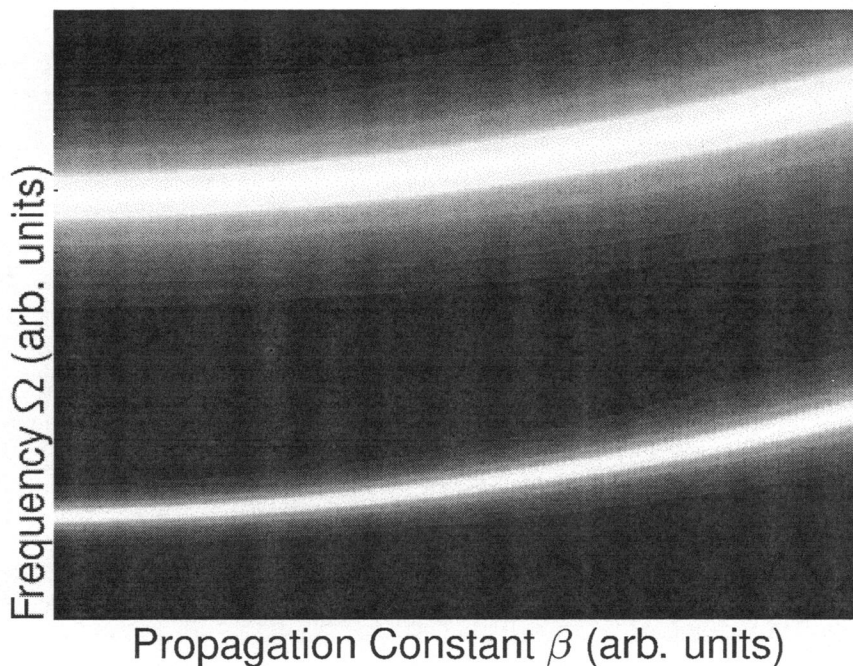


Fig. 2.: Two bands of the effective polaritonic band structure of a photonic waveguide of highly dispersive and absorptive dielectric obeying the Kramers-Kronig relations.

Finally, the model is prescriptive. That is, given the absorptive response specified over all space for realistic inhomogeneous structured dielectrics, the appropriate model coupling constant can be constructed straightforwardly, in the basis of the local principal axes of the imaginary part of the dielectric response, as

$$\Lambda(\mathbf{r}, \Omega) = \sqrt{\frac{2}{\pi} \Omega \text{Im} \Gamma(\mathbf{r}, \Omega)} \quad (4)$$

**Thus, for a specified position and frequency dependent imaginary part of an underlying dielectric tensor, a canonical description of the modes of the coupled medium and electromagnetic field can be immediately given.** We will present particular applications to photonic crystals that there is not room here to describe.

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