phy 101 Dec 14 2008

CHAOS + NONLINEARITY,

chaos:

→ deterministic.

→ effectively random or unpredictable.

→ sensitive dep on initial conditions:

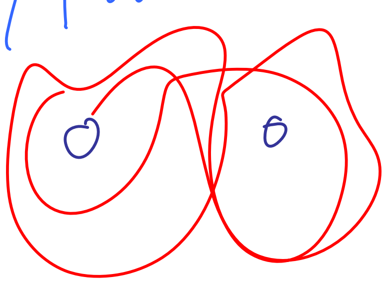
random fluctuations are always around (warm, quantum fluctuations).

not enough initial data (weather).

EXACT "analytic" solutions of Eqns do not generally exist.

2Body: Newton gravity + initial cond ⇒ orbits elliptical exactly

3Body problem: no single formula.



Solve by TIME STEPPING. approximate for Δt small.

→ computer.

perfect prediction → ∞ amount of memory etc.

Computational Complexity

CHAOTIC systems are NONLINEAR.

output is disproportionate to input.

S_1 S_2 are 2 solutions (trajectories).

LINEAR $S_1 + S_2$ is also a solution

NONLINEAR $S_1 + S_2 \neq$ solution

EXAMPLE of a chaotic MAP

population Dynamics. Robert May 1976.

x_n = population of some bugs in year n.

DIFFERENCE EQN (MAP) $x_{n+1} \leftarrow x_n$

stepping in time, next year this year

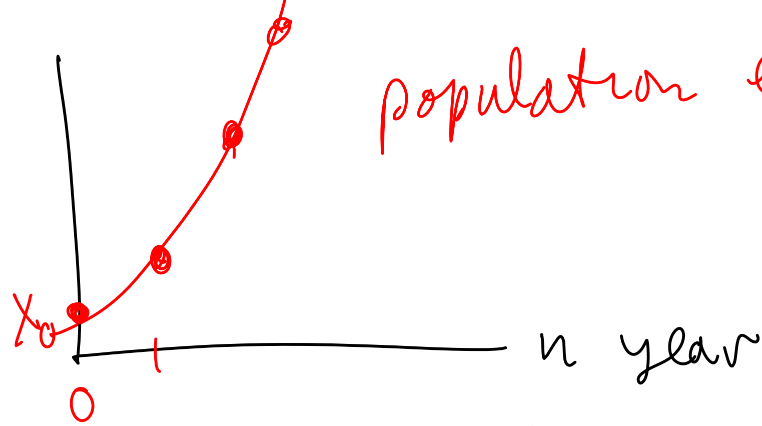
Simple Model

$$x_{n+1} = R\, x_n$$

$R > 1$

randiness

predicts:



population explosion $R > 1$

$x_n \to \infty$! not realistic.

Malthus' idea: population limited by resources.

let pop max $= 1$.

LOGISTIC EQ$\underline{\text{N}}$

$$x_{n+1} = R\, x_n (1 - x_n),$$

$$= R\,(x_n - x_n^2).$$

if x_n is small $x_n^2 \ll x_n \to$ same as before.

if x_n close to CAPACITY 1.

$1 - x_n$ is small and growth $\underline{\text{stops}}$.

Key fact is $x_n^2 \to$ NONLINEAR TERM

single state for small R as $n \to \infty$

period doubling → period 2

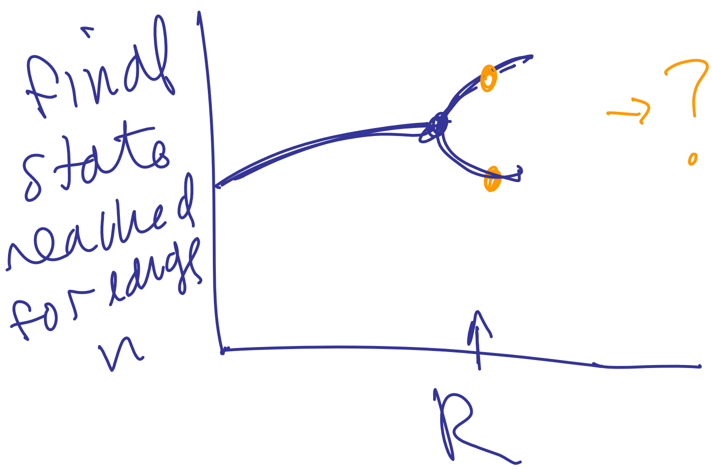
p 2 → p 4 → p 4 → p 8 → ...

period doublings faster in R

→ chaos.

Every possible period emerges within chaotic regime!

Now plot "bifurcation diagram"



→ ? fractal object!

embedded "windows" of every period.

FRACTAL: geometric object which is SELF-SIMILAR

blow up → contains whole thing!

even simple chaotic systems
contain infinite complexity —

Fractal objects

More Fractals
next
time.