Emergence Lecture 15.

chaos + nonlinearity —

— sensitive dependence on initial conditions.

— deterministic, simple.

→ effectively random
complex.

(double pendulum).

dbl pendulum = "few modes" chaos.

weather = "many modes". "chaotic"

— eqNS are known. but useless w/o
a computer. — no analytic solutions.

— can only do numerical solutions
always approximate.

2 Body prob.

Newton solved this exactly.

formula

— sol^n of a differential eq^n

$$F = ma$$

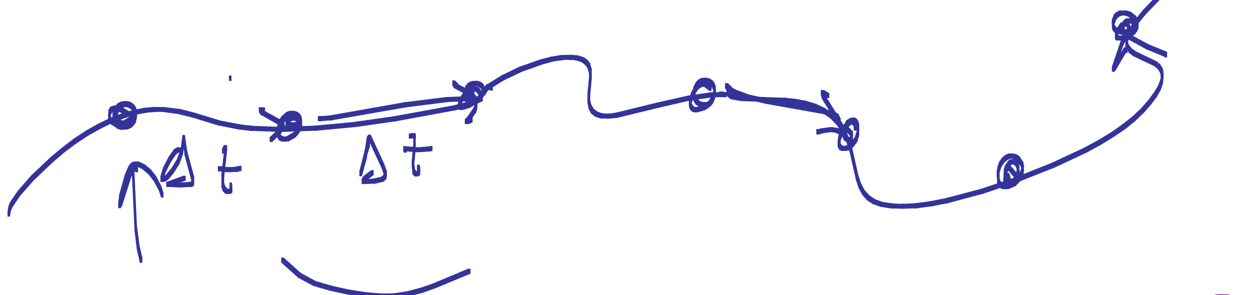
$$F = m\frac{d^2x}{dt^2}$$

no sol^n to 3 Body exactly.

$$\frac{dx}{dt} \approx \frac{\Delta x}{\Delta t}$$

↖ little steps in time

computer solves



— chaotic systems have NONLINEAR Eq^{ns}.

$\sim x^2 \sim x\frac{dx}{dt}$ etc.

→ POPULATION Model.

LOGISTIC Eq^{n}

example of a simple nonlinear model -

difference Eq$^{\underline{n}}$ — not differential.

X_n = population of bugs in year n.

x_0 = initial condition

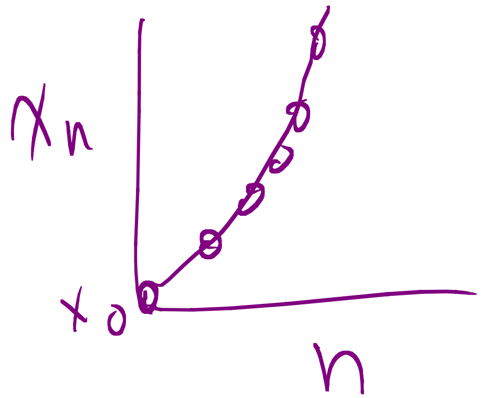
$$x_1 \xrightarrow{\text{rule}} x_2 \to x_3 \cdots \to X_n \to \cdots$$

1st Model — population explosion

$$X_n = R\, X_{n-1}$$

R = randiness

exponential growth. — bugs grow forever



$R > 1$.

— better model

reduce R as

$x_n \to$ max

"carrying capacity".

$x = 1$ = "full house".

Logistic Eq^n

$$x_{n+1} = R\, x_n \underbrace{(1 - x_n)}_{\text{new term.}}$$

– non linear model.

$\sim \quad -R\,x_n^2$ — creates all the complexity

the complex number version of this is THE MANDELBROT SET.

see links