1.03

$$u(\nu,T)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}\,d\nu$$

$$\nu = c/\lambda$$

$$\frac{d\nu}{d\lambda} = \frac{-c}{\lambda^2} \Rightarrow d\nu = \frac{-c}{\lambda^2}d\lambda$$

$$\therefore u(\lambda,T)\,d\lambda = \frac{8\pi h}{c^3}\frac{c^3}{\lambda^3}\left(e^{hc/kT\lambda}-1\right)^{-1}\frac{c}{\lambda^2}d\lambda$$

$$= \frac{8\pi hc}{\lambda^5}\left(e^{hc/kT\lambda}-1\right)^{-1}d\lambda$$

To find the max set

$$\frac{\partial u(\lambda,T)}{\partial\lambda} = 0$$ and solve for λ

(in derivative treat T as a const)

$$0 = \cancel{8\pi hc}\left(\frac{-5}{\lambda^6}\left(e^{hc/kT\lambda}-1\right)^{-1} - \frac{1}{\cancel{\lambda^5}}\left(e^{hc/kT\lambda}\left(\frac{-hc}{kT\lambda^2}\right)\right)\left(e^{hc/kT\lambda}-1\right)^{-2}\right)$$

let $x = hc/kT\lambda_{MAX}$

$$0 = -5 + \frac{e^x x}{e^x-1}$$

$$-5e^x+5+e^x x = 0$$

$$(x-5)+5e^{-x} = 0$$

$$(5-x) = 5e^{-x}$$ solve graphically or numerically

or Taylor expand RHS about $x=5$

$$(5-x) = 5(e^{-5}+\dots)$$

$$x = 5(1-0.00674)$$

$$= 4.966$$

with Maple $x = 4.9651$

$$\therefore \lambda_{MAX} = \frac{b}{T},\quad b = \frac{hc}{k_B x} = 0.289\ \mathrm{cm \cdot K^\circ}$$

$k_B = 1.38\times10^{-16}\ \mathrm{erg\cdot K^{-1}}$

$h = 2\pi\times1.054\times10^{-27}\ \mathrm{erg\cdot s}$

$c = 2.998\ \mathrm{cm/s}\times10^{10}$

$\therefore \lambda_{Max}(T_{SUN}=6000\,K^\circ) = 482.92\ \mathrm{nm}$ which is Visible!

1.04

$\lambda = 3500\,\text{Å}$

$E_{MAX} = 1.6\,eV$

$E = \frac{hc}{\lambda}$

$h = 2\times\pi\times 6.58\times 10^{-22}\,MeV\cdot s$

$c = 2.998\times 10^{8}\,m/s$

$$W = E_{photon} - E_{MAX}$$

$$= \frac{2\times\pi\times 6.58\times 10^{-22}\,MeV\cdot s\times 2.998\times 10^{8}\,m/s}{3500\times 10^{-10}\,m} - 1.6\,eV$$

$$= 3.54\,eV - 1.6\,eV = 1.9\,eV$$

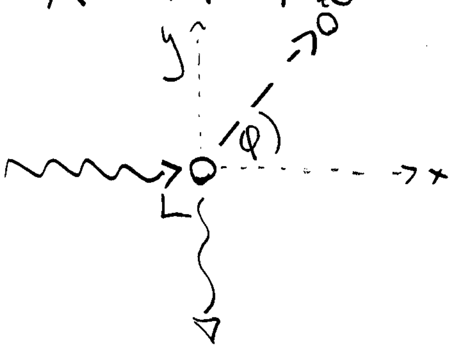
1.07

$E_{photon} = 100\text{ keV} = 0.1\text{ MeV}$

$\theta = 90°$

$$\lambda^F - \lambda^i = \frac{h}{m_e c}(1-\cos\theta)$$

$$\lambda^F - \lambda^i = \frac{h}{m_e c} \quad \text{For } \theta = 90°$$



$m_e = 0.511\text{ MeV}/c^2$

$c = 2.998 \times 10^8\text{ m/s}$

$h = 2\pi\hbar$

$= 2\pi \times 6.58 \times 10^{-22}\text{ MeV}\cdot\text{s}$

$$\lambda_i = \frac{hc}{E} = \frac{2\pi(6.58\times10^{-22})(2.998\times10^8)}{0.1}$$

$= 1.24 \times 10^{-11}\text{ m}$

$= 0.12\text{ Å}$

$$\lambda^f = \frac{h}{mc} + \lambda^i = 1.48\times10^{-11}\text{ m} = 0.148$$

conservation of momentum

$$\hat{x}: \quad p_x^{(photon)} = p_x^{(electron)}$$

$$\hat{y}: \quad \underbrace{0}_{initial} = \underbrace{p_y^{(photon)} + p_y^{(electron)}}_{final}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{p_y^{(electron)}}{p_x^{(electron)}}\right) = \tan^{-1}\left(\frac{-p_y^{(photon)}}{p_x^{(photon)}}\right)$$

$$p = \hbar k = \frac{h}{\lambda} \qquad \therefore \quad p_y^{(photon)} = \frac{h}{\lambda^F} \qquad p_x^{(photon)} = \frac{h}{\lambda^i}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{\lambda^i}{\lambda^F}\right) = \tan^{-1}\left(\frac{1.24}{1.48}\right)$$

$= 44°$

Conservation of Energy

$$E^{(photon)} = \hbar\omega = h\nu = \frac{hc}{\lambda}$$

$$E^{(electron)} = E^{(photon)}_{initial} - E^{(photon)}_{Final}$$

$$= hc\left(\frac{1}{\lambda^i} - \frac{1}{\lambda^F}\right)$$

$$= (1.24\times10^{-12})(1.31\times10^{10}\text{ m}^{-1})$$

$= 16\text{ keV}$

1.12

a) $\lambda = 150\,\text{Å}$

$$\lambda = \frac{h}{p}$$

$$E = \tfrac{1}{2}mv^2 \quad , \quad p = mv$$

$$= \tfrac{1}{2}m\left(\frac{p}{m}\right)^2$$

$$= \frac{1}{2}\frac{p^2}{m}$$

$$E = \frac{1}{2}\frac{h^2}{\lambda^2 m}$$

from Gasiorowicz p. 460

$\hbar = 6.58\times10^{-22}$ MeV·s

$h = \hbar 2\pi$

$M_e = 0.511\ \text{MeV}/c^2$

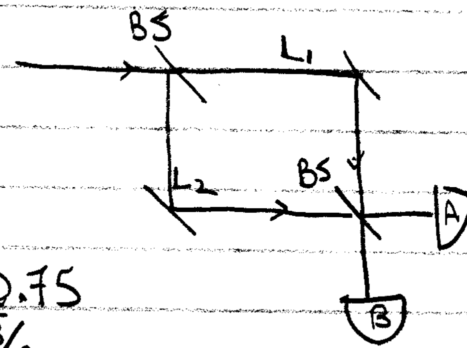
$c = 2.998\times10^{8}$ m/s

For $\lambda = 150\,\text{Å} = 150\times10^{-10}\,m$, $E = 6.68\times10^{-9}\,\text{MeV} = 6.68\times10^{-3}\,\text{eV}$

b) For $\lambda = 5\,\text{Å} = 5\times10^{-10}\,m$

$$E = 6.01\,\text{eV}$$

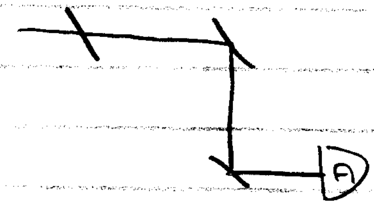
$r = |r| e^{i\theta_r}$ $\quad |r|^2 = 0.25$
$= \frac{1}{2} e^{i\theta_r}$ $\quad |r| = \frac{1}{2}$



$t = |t| e^{i\theta_t}$ $\quad \theta_t = 0 \quad |t|^2 = 0.75$
$|t| = \sqrt{3}/2$

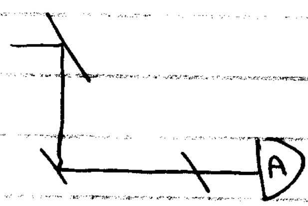
Two Amps which lead to a detection at A

A_1:



$A_1 = t e^{ikL_1} r$ $\quad$ PATH 1

A_2:



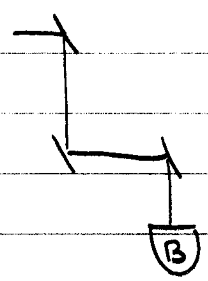
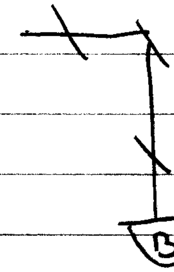
$A_2 = r e^{ikL_2} t$ $\quad$ PATH 2

$$P_A = |A_1 + A_2|^2 = |tre^{ikL_1} + tre^{ikL_2}|^2 = |tr(e^{ikL_1} + e^{ikL_2})|^2$$
$$= |tr|^2 |e^{ikL_1} + e^{ikL_2}|^2$$
$$= |t|^2|r|^2 (2 + 2\cos k(L_1 - L_2))$$
$$= 3/8 (1 + \cos(L_1 - L_2)k) \quad \text{with interference}$$

Without interference, $P_{A1} = |te^{ikL_1} r| = 3/16$
$P_{A2} = |te^{ikL_2} r| = 3/16$
so $P_A = P_{A1} + P_{A2} = 3/8$

With interference:
$Q = k(L_1 - L_2)$
$Q = 0 \Rightarrow P_A \max = 3/4$
$Q = \pi \Rightarrow P_A \min = 0$
Average $\Rightarrow P_A = 3/8$

ALWAYS EQUAL

$A_3:$  rre^{ikL_2} $A_4:$  tte^{ikL_1}

$$P_B = |A_3 + A_4|^2 = |r^2 e^{ikL_2} + t^2 e^{ikL_1}|^2$$

$$= \left| \tfrac{1}{4} e^{i2\phi_r} e^{ikL_2} + \tfrac{3}{4} e^{ikL_1} \right|^2$$

$$= \tfrac{1}{16}\left(e^{i2\phi_r} e^{ikL_2} + 3e^{ikL_1} \right)\left(e^{-i2\phi_r} e^{-ikL_2} + 3e^{-ikL_1} \right)$$

$$= \tfrac{1}{16}\left(1 + 3e^{i(2\phi_r + kL_2 - kL_1)} + 3e^{-i(2\phi_r + kL_2 - kL_1)} + 9 \right)$$

$$= \tfrac{1}{16}\left(10 + 6\cos\left(2\phi_r + \underbrace{k(L_1 - L_2)}_{Q}\right)\right)$$

$$= 1 \text{ max}, \ \tfrac{1}{4} \text{ min} \qquad \text{Average} = \tfrac{5}{8}$$

$$1 = P_A + P_B$$

$$= \tfrac{3}{8}(1 + \cos Q) + \tfrac{1}{16}\left(10 + 6\cos(Q + 2\phi_r)\right)$$

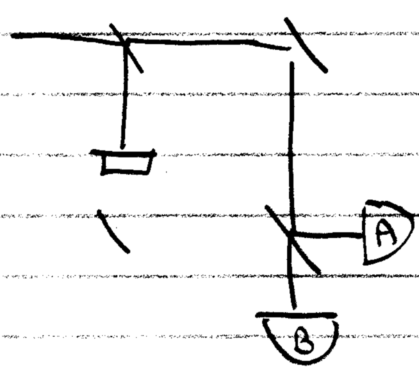
$$= 1 + \tfrac{3}{8}\left(\cos Q + \cos(Q + 2\phi_r)\right)$$

$$0 = \cos Q + \cos(Q + 2\phi_r)$$

$$\cos Q = -\cos(Q + 2\phi_r)$$

$$\therefore -\cos x = \cos(x + \pi) \implies \phi_r = \pi/2$$

$P_0 = \frac{1}{4}$ with interference ie $Q=0$

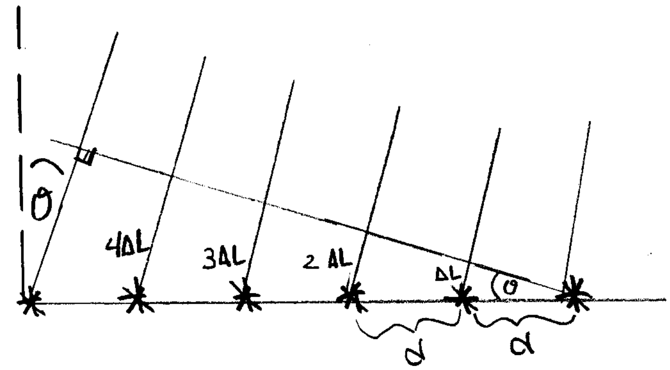


$P_{Block} = |r|^2$
$= 1/4$

$P_B = |t e^{ikL_1} t|^2$
$= |t^2|^2 = 9/16$

So blocking the lower path increases the probability of detecting a photon at B.

Additional Problem #2



when $\Delta L = n\lambda$ we have constructive interference

$d\sin\theta = n\lambda$, $\sin\theta \approx \theta$ for small θ

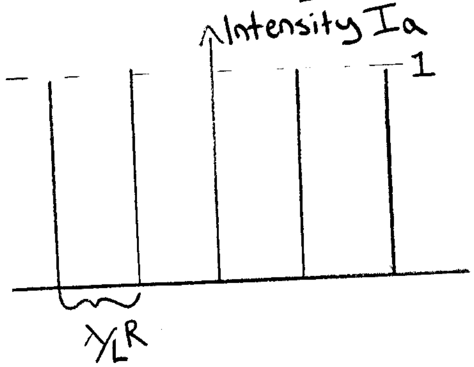
for a screen R distance from the slits

position on the screen $\theta = \frac{x}{R} \Rightarrow x = R\theta$

a) For $d = L$

$$Lx = Rn\lambda$$

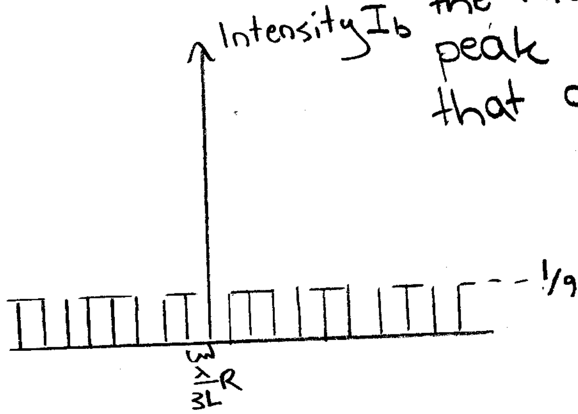
$$x = \frac{\lambda}{L}R$$



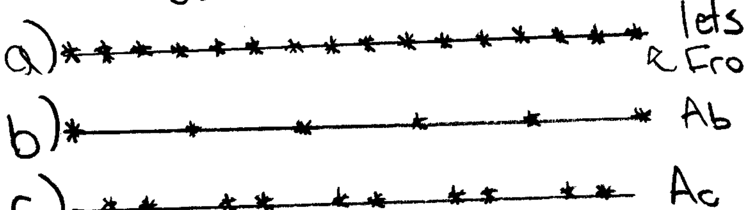
b) For $d = 3L$

$$x = \frac{\lambda}{3L}R$$

since there $\frac{1}{3}$ as many slits per length and 3 times as many peaks per unit length $\Rightarrow$ the intensity of each peak will be $1/9$ that of case a)



SLITS



lets call the amplitude for light at the screen from this slit pattern A_a

$$A_a = A_b + A_c$$

$$\therefore A_c = A_a - A_b$$

$$I_c = |A_c|^2 = |A_a - A_b|^2 = |\sqrt{I_a} - \sqrt{I_b}|^2 \Rightarrow$$