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OPTICAL TESTS OF QUANTUM MECHANICS

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crystal length. These conservation laws imply that conjugate (i.e., "twin") signal and idler photons are tightly correlated not only in their times of emission, but also in their energies and in their momenta. A rainbow of colored cones is produced around an axis defined by the direction of the UV beam, with photons always emitted in correlated pairs on opposite sides of the axis (Fig. 2).

Due to the fact that there are many ways to partition the energy of the parent photon, each daughter photon has a broad spectrum, and hence a narrow wave packet in time. However, the *sum* of the two daughter photons' energies is extremely well defined, since they must add up to the energy of the extremely monochromatic parent laser photon. Thus the *difference* in their arrival times and the *sum* of their energies can be simultaneously known to high precision. These characteristics prove important in several of the experiments described later.

C. Squeezed States of Light

The creation of conjugate photon pairs in spontaneous down-conversion is closely related to the process of squeezed light production (Rarity *et al.*, 1992). This becomes clear from the inspection of the quadrature-squeezed vacuum state (Kimble and Walls, 1987; Walls and Reid, 1986):

$$|\xi\rangle = \exp\left(\frac{\xi^*}{2}aa - \frac{\xi}{2}a^\dagger a^\dagger\right)|0\rangle \tag{4}$$

which corresponds to a vacuum state transformed by the creation ($a^\dagger a^\dagger$) and destruction (aa) of photons two at a time. When the gain arising from parametric amplification becomes large in the down-conversion crystal, a transition from spontaneous to stimulated emission occurs, and squeezed states of light are produced.This gain is dependent on the phase of amplified light relative to the phase of the pump light beam. Hence, vacuum fluctuations in one quadrature are squeezed, but stretched in the other quadrature, in such a way as to preserve the minimum uncertainty principle product of these fluctuations. That is, the fluctuations develop a periodicity at twice the optical frequency; this is a direct consequence of the fact that the photons are produced in pairs. In this way, the fluctuations in the squeezed quadrature can be reduced below the standard quantum limit observed for a vacuum state or a coherent state. For a review of quadrature-squeezed states and their applications, see Kimble and Walls (1987).

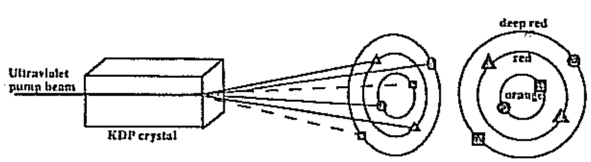

FIG. 2. Conical emissions of down-conversion from a nonlinear crystal. Photon energy depends on the cone opening angle, and conjugate photons lie on opposite sides of the axis, e.g., the inner "circle" orange photon is conjugate to the outer "circle" deep-red photon, etc. Energy and momentum are conserved in this two-photon decay.

A simpler type of squeezing (Yamamoto *et al.*, 1986) involves preparation of states with a well-defined photon number, that is, lacking the Poisson fluctuations of the coherent state. The possibility of producing such states demonstrates that "shot noise" in photodetection should not be thought of as merely the result of randomness in a semiclassical model, but as representing real properties of the electromagnetic field, accessible to experimental control.

D. Quantum Nondemolition

The uncertainty principle between number of quanta N and phase ϕ of a beam of light

$$\Delta N\ \Delta\phi \geq 1/2 \tag{5}$$

implies that to know the number of photons in the beam of light exactly, one must give up all knowledge of the phase of the wave. This is a manifestation of the particle wave complementarity principle. In theory a *quantum nondemolition* (QND) process is possible (Braginsky, 1989; Yamamoto *et al.*, 1986); Without annihilating any of the light quanta, one can count them. It might seem that this would make possible successive measurements on noncommuting observables of a single photon, in violation of the uncertainty principle; it is the unavoidable introduction of phase uncertainty by any number measurement that prevents this.

A *gedanken* example of QND is as follows: The Aharonov-Bohm effect can cause a phase shift of an electron wave function that arises from the time-varying vector potential associated with a nearby light beam; this phase shift can, in principle, be used to count photons in this beam (Chiao, 1970; Lee *et al.*, 1992). Because the photons in such a light beam are not annihilated, this is a QND process. Another QND scheme uses the intensity-dependent index of refraction (the Kerr nonlinearity) to detect the optical phase shift on a probe beam arising from the index change caused by the passage of a signal beam in a number state (Imoto *et al.*, 1985; Kitagawa and Yamamoto, 1986). Still other QND schemes use Rydberg atoms to give indirect information concerning the number of photons in a microwave cavity through which they have passed (Haroche *et al.*, 1992; Walther, 1992). A review of recent work is given in Roch *et al.* (1992).

III. Nonclassical Interference and "Collapse"

A. Single-Photon Interference and Berry's Phase

We turn now to several interference experiments performed at the one- and two-photon level. The simplest of these was performed by Grangier *et al.* (1986), using the cascade source discussed in Section I. One of the photons was directed to a "trigger" detector, while the other, thus prepared in an $n = 1$ Fock state (Clauser, 1974), was sent through a Mach-Zehnder interferometer. The detections at the output ports of the interferometer were registered in coincidence with the trigger photon, and the visibility of the fringes measured to be greater than 98%. Dirac's statement that a single photon interferes with itself is thus verified.

As an extension of these considerations, we have performed an experiment that demonstrates the existence of a Berry's phase at the single-photon level (Kwiat and Chiao, 1991). Berry's phase is a geometrical phase shift, which a quantum system acquires after a round-trip in the state space of the system, for example, by its adiabatic following of slow, circuital parameter changes in the system Hamiltonian. This phase shift has also been observed in classical optics (Chiao, 1989), and a controversy has resulted as to whether one should interpret it as classical or quantum [see references in Kwiat and Chiao (1991)]. Using the correlated photons from parametric down-conversion, we have done an experiment that excludes any classical explanation of the observed Berry's phase (Fig. 3). As in the Grangier experiment, one of our photons is directed to a trigger detector, and the other is directed into an interferometer. In one arm of the interferometer, two quarter-wave plates (one fixed, the other rotatable) cycle the polarization of the light, generating a specific type of Berry's phase known as *Pancharatnam's phase*. The state space in this case is the Poincaré sphere describing all polarization states. The phase acquired after a round-trip through the wave plates is equal to minus one-half of the solid angle subtended by the state-space trajectory with respect to the center of the sphere.

We have ruled out all classical wave theory explanations of these effects by using a beam splitter after the output port of the Michelson interferometer to measure the anticorrelation parameter of Grangier *et al.* (Section I). We found $\alpha = 0.08 \pm 0.04$, which clearly violates any classical wave prediction. Therefore, although this same Berry's phase shift also

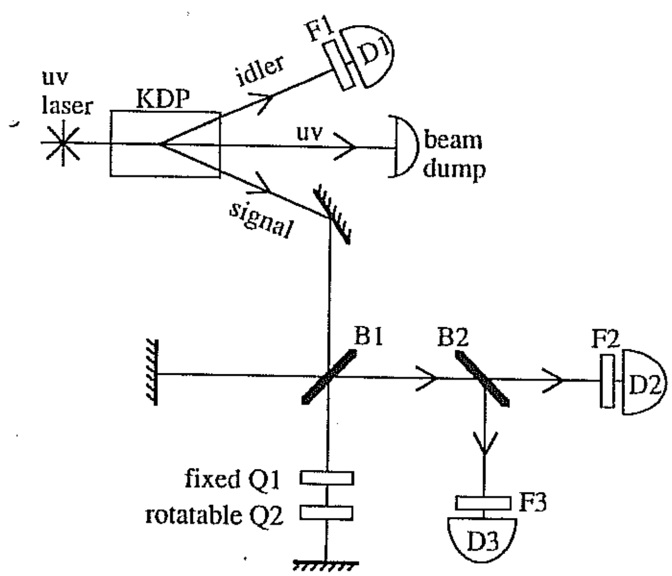

Fig. 3. Apparatus to measure Berry's phase for single photons, where Q1 and Q2 are quarter-wave plates. The energy-time uncertainty relation and wave function collapse were also studied by investigating the effect of various filters before the detectors (Kwiat and Chiao, 1991).

appears at the classical level, it is necessary to view it, and all other Berry's phases in optics, as a geometric effect that originates at the quantum level, but survives the correspondence principle limit into the classical level.

B. The Energy-Time Uncertainty Principle

The energy-time uncertainty principle

$$\Delta E\, \Delta t \geq \hbar/2 \tag{6}$$

is one of the fundamental aspects of quantum mechanics that we have tested in recent two-photon interference experiments at Berkeley. The motivation for this study arises from the fact that the energy-time uncertainty principle arises differently from the momentum-position uncertainty principle in the standard formulation of quantum mechanics, because time is not an operator. Unlike the case of momentum and position, where the fundamental commutator $[x,p_x] = i\hbar$ exists, from which one can derive in the usual way the momentum-position uncertainty principle, the corresponding commutator for time and energy $[t,H] = i\hbar$ does not exist, since the time operator t does not exist. This difficulty arises because the energy H is a positive definite operator, whereas the momentum p_x can range from negative to positive infinity. Since the t operator, if it existed, would be the infinitesimal generator of translations of either sign in energy (just as H is the generator of translations in time), it could generate arbitrarily large negative energies. Thus in the time-dependent Schrödinger equation, instead of being an operator, time is a c-number parameter, as is the case for position as well in relativistic quantum field theory (Aharonov and Bohm, 1961). We studied the energy-time relation using the correlations in down- conversion.

To account for the correlated, nonlocal effects in quatum theory, and in response to the Einstein-Podolsky-Rosen paper, Schrödinger (1980) introduced the concept of *entangled states*. An entangled state is a nonfactorizable sum of product states, for example, the Bohm singlet state,

$$|S=0\rangle = \frac{1}{\sqrt{2}}\{|\uparrow_1\rangle\,|\downarrow_2\rangle - |\uparrow_2\rangle\,|\downarrow_1\rangle\} \tag{7}$$

which represents the fact that two spin-1/2 daughter particles decaying from a spin-zero parent will always be correlated with each other, no matter how far apart they are from each other at the moment of detection. The mathematical fact that such a sum is nonfactorizable corresponds to the physical fact that the probabilites of detection of these particles are never independent of each other and, hence, that nonlocal correlations are intrinsic to this system. The energy correlations in down-conversion of a parent photon of sharp energy E_0 can be similarly described:

$$|2\text{ photons}\,\rangle = \int_0^{E_0} \mathrm{d}E_1\,\mathrm{d}E_2\,\delta(E_0 - E_1 - E_2)\,A(E_1,E_2)\,|E_1\rangle|E_2\rangle \tag{8}$$

where $A(E_1,E_2)$ is the probability amplitude for the production of two photons of energies E_1 and E_2. The meaning of this entangled state of energy is that after the measurement of the energy of one of the photons, which results in a definite value E_1, there is an instantaneous collapse of the state of the system to the state $|E_1\rangle|E_0 - E_1\rangle$. This implies an instantaneous increase of the width of the wave packet of photon 2.

We have investigated these issues in a modification of the single-photon Berry's phase experiment described earlier (Fig. 3). An interference filter F1 was used to measure sharply the frequency (and hence the energy) of trigger photon 1, whereas the Michelson interferometer was used to measure the width of the wave packet of conjugate photon 2. After the trigger photon passed through the filter of narrow width ΔE and was detected, we observed that photon 2 of the pair collapsed into a broad wave packet of a duration $\Delta t \approx h/\Delta E$, upon coincidence detection. This is a nonlocal effect in that the photons can be arbitrarily far away from each other when the collapse occurs. Again, the anticorrelation parameter α was measured and found to

violate any classical wave description by 9 standard deviations, thus ruling out all classical wave theory explanations of these collapse effects.

In our experiment, *two* collapses are actually occurring: The first occurs inside the Michelson interferometer, where the wave packet size of photon 2 discontinuously changes upon the detection of the "remote" photon 1; the second collapse occurs after the last beam splitter, where, due to its indivisibility, photon 2 is either transmitted or reflected, but not both. The first collapse is *temporal* in nature, in that the uncertainty in the position of photon 2 in *time* is changed after the collapse occurs; whereas the second collapse is a *spatial* one, in that the uncertainty in the position of photon 2 in *space* is changed.

C. Collapse Effects in Fluorescence

Two other studies of collapse have been provided by examining fluorescence in three-level atomic systems. One of them (Itano *et al.*, 1990) is generally known as the *quantum Zeno effect*, although it is sometimes described as the *watched pot effect*, which comes closer to the true meaning. The precursor to this effect was the observation of *quantum jumps* (Nagourney *et al.*, 1986). In the quantum jumps experiment, a strong laser field is used to check for a particle's presence in the ground state (via resonance fluorescence), while a second field couples the ground state to a metastable state. Because the atom spends some time in the ground state and some time "shelved" in the metastable one, the fluorescence abruptly and stochastically turns on and off, seeming to indicate observable quantum jumps of the atom from state to state. In the related watched pot experiment, a transition is *suppressed* by the action of a laser field, which is periodically used to check whether the transition has occurred yet. If this check occurs frequently enough, it is most likely to collapse the atom back into its original state, preventing a sufficient dipole from building up to allow the transition to progress. The more frequently one turns on the probe laser, the less likely a transition is to take place. Both experiments tend to be interpreted in terms of collapse of the wave function, although this interpretation has been the source of much controversy (Fearn and Lamb, 1992). It has been shown (Frerichs and Schenzle, 1991) that no collapse postulate is *necessary* in order to understand them. [Recall Mott's demonstration (1983) that quantum mechanics leads to the prediction of particle tracks in cloud chambers as joint-probability distributions, without the invocation of collapse.] Their interest lies in the fact that they relate the collapse picture directly to observable time evolution. While this evolution can be formally described in other ways (and should thus not be thought of as a proof of collapse), the ease of their interpretation in terms of collapse is a strong argument for the *usefulness* of the Copenhagen viewpoint.

D. Two-Photon Interference

We have so far discussed two-photon experiments in which one of the photons is used solely as a trigger for the other one, which is then prepared very nearly in an ideal $n = 1$ Fock state (Hong and Mandel, 1986). An early experiment to demonstrate *two*-photon interference using the down-conversion light source was performed by Ghosh and Mandel (1987). They looked at the counting rate of a detector illuminated by both of the twin beams. No interference of the two beams was observed, because although the sum of the phases of the two photons emitted in parametric fluorescence is well defined (by the phase of the pump), their *difference* is not. However, when Ghosh and Mandel looked at the rate of coincidence detections between two such detectors whose separation was varied, they observed high-visibility interference fringes. Whereas in the standard two-slit experiment, interference occurs between the two paths a single photon could have taken to reach a given point on a screen, in this case it occurs between the possibility that the signal photon reached detector 1 and the idler photon detector 2, and the possibility that the reverse happened. This experiment provides a manifestation of quantum nonlocality, since at a null of these coincidence fringes, the detection of one photon at the first detector excludes the possibility of the detection of the other photon at the second detector.

Such interference becomes clearer in the related interferometer of Hong *et al.*, (1987), which will play an integral part in several of the experiments discussed later. A simplified schematic of this interferometer is shown in Fig. 4. The identically polarized conjugate photons from a down-conversion crystal are directed to opposite sides of a 50-50 beam splitter, such that the

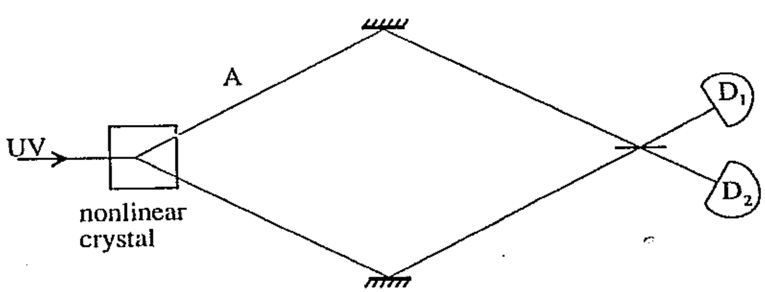

FIG. 4. Simplified setup for a Hong-Ou-Mandel interferometer (Hong *et al.*, 1987). Coincidences may result from both photons being reflected or both being transmitted. In a modification of this scheme, a half-wave plate in one arm of the interferometer (at point A) serves to distinguish these otherwise interfering processes, so that no null in coincidences is observed. Using polarizers before the detectors, one can "erase" the distinguishability, thereby restoring interference (Section IV.A).

transmitted and reflected modes overlap. If the difference in the path lengths prior to the beam splitter is larger than the two-photon correlation length (of the order of the coherence length of the down-converted light), the photons behave independently at the beam splitter, and coincidence counts between detectors in the two output ports are observed half of the time—the other half of the time both photons travel to the same detector. However, when the two path lengths from the crystal to the beam splitter are nearly equal, such that the photons' wave packets overlap at the beam splitter, the probability of coincidences is reduced, in principle to zero if the path lengths are identical. One can explain the coincidence null at zero path-length difference using the Feynman rules for calculating probabilities: Add the probability amplitudes of *indistinguishable* processes that lead to the same final outcome, and then take the absolute square of the resulting amplitude to find the probability of this outcome. The two indistinguishable processes that lead to coincidence detection in the preceding setup are both photons being reflected at the beam splitter (with Feynman amplitude $r \cdot r$) and both photons being transmitted (with Feynman amplitude $t \cdot t$). The probability of a coincidence detection is then

$$P_c = |r \cdot r + t \cdot t|^2 = \left| \frac{i}{\sqrt{2}} \cdot \frac{i}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right|^2 = 0 \tag{9}$$

where we have assumed a real transmission amplitude, and the factors of i come from the phase shift upon reflection at a beam splitter.

The possibility of a perfect null at the center of the dip is indicative of a nonclassical effect. Indeed, Mandel (1983) has shown that 50% coincidence-fringe visibility is the maximum possible from classical field predictions. The tendency of the photons to travel off together at the beam splitter can be thought of as a manifestation of the Bose-Einstein statistics for the photons (Fearn, 1990). In practice the bandwidth of the photons, and hence the width of the null, is determined by filters and/or irises before the detectors—widths as small as 5 μm have been observed, corresponding to time delays of only 15 fs (Steinberg *et al.*, 1992b). Consequently, one application of this phenomenon is the determination of various single-photon propagation times, with extremely high time resolution (see Section VIII).

IV. Complementarity

A. Quantum Eraser

The complementary nature of wave-like and particle-like behavior is commonly interpreted as follows: Due to the uncertainty principle, any attempt to measure the position (particle aspect) of a quantum will lead to an uncontrollable, irreversible disturbance in its momentum, thereby washing out any interference pattern (wave aspect) (Bohr, 1983). The measurement provokes an irreversible reduction of the state vector, irrevocably introducing an uncertainty in the phase. This picture, however, is incomplete; no state reduction, or collapse, is necessary to destroy wave-like behavior, and measurements that do not involve reduction can be reversible in a certain sense. To understand this phenomenon fully, one must view the loss of coherence as arising from an *entanglement* of the system wave function with that of the measuring apparatus (MA); this is identical to the first step in von Neumann's measurement theory (von Neumann, 1983), but lacks the step in which the off-diagonal elements of the expanded density matrix are postulated to vanish. Through this entanglement, previously interfering paths can become *distinguishable* (assuming that the final MA states are orthogonal), such that no interference is observed. This is true even though one may not actually make subsequent measurements on the MA to determine which path actually occurred, i.e., even if one does not look at the result of the MA. Whenever *welcher Weg* ("which way") information is available in principle about which possible path occurred, the paths are distinguishable, and no interference is possible. Interference may be regained, however, if one somehow manages to "erase" the distinguishing information, by correlating the results of measurements on the interfering particle with the results of particular measurements on the MA. This is the physical content of quantum erasure (Hillery and Scully, 1983; Scully *et al.*, 1991).

We have reported one demonstration of a quantum eraser, based on the Hong-Ou-Mandel interferometer described earlier (Fig. 4) (Kwiat *et al.*, 1992). A half-wave plate inserted into one of the paths before the beam splitter serves to rotate the polarization of light in that path. In the extreme case, the polarization is made orthogonal to that in the other arm, and the reflection-reflection and transmission-transmission processes become completely distinguishable (because one could use this "label" to tell after the beam splitter which photon had taken which path); hence, the destructive interference that led to a coincidence null does not occur. The distinguishability can be erased, however, by using polarizers before the detectors *after* the photons have left the beam splitter. In particular, if the initial polarization of the down-converted photons is horizontal, and the wave plate rotates one of photon's polarizations to vertical, then a polarizer at 45° before each detector will restore the original interference dip (use of a polarizer in front of only one detector is not sufficient, because the photon traveling to the other detector still carries which-path information). If one polarizer is at 45° and the other at −45°, interference is once again seen, but now in the form of a peak instead of a dip (Fig. 5). It is characteristic of quantum erasers that there are four basic measurements possible on the MA (here the polarization)—two of which yield which-path information,

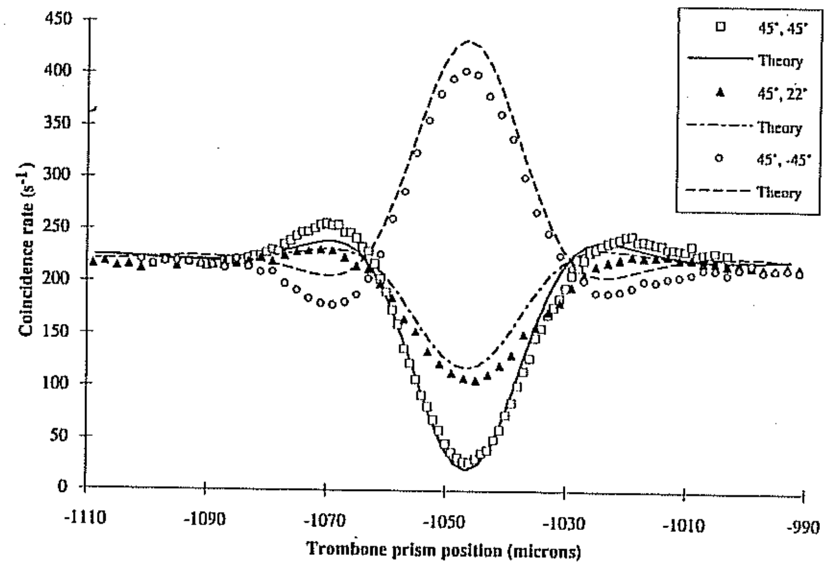

FIG. 5. Experimental data and scaled theoretical curves (adjusted to fit observed visibility of 91%) for a quantum eraser with polarizer 1 at 45° and polarizer 2 at various angles. Far from the dip, there is no interference and the angle is irrelevant (Kwiat *et al.*, 1992).

one of which recovers the initial interference fringes (here the coincidence dip), and one which yields interference *antifringes* (the peak instead of the dip). In all cases, one must correlate the results of measurements on the MA with the detection of the originally interfering system. The primary lesson of the quantum eraser is that one must consider the *total* physical state, which in addition to photon spatial wave functions may include the state of the photon polarization or even the state of distant photons or atoms. If the coherence of the measuring apparatus is maintained, then interference may be recovered by correlating results of measurements on the original system with results of particular measurements on the MA.

B. Vacuum-Induced Coherence

We now present a somewhat different demonstration of complementarity, involving two down-conversion crystals. In the experiment of Zou *et al.* (Greenberger *et al.*, 1993; Zou *et al.*, 1991), two nonlinear crystals, NL1 and NL2, are aligned such that the trajectories of the idler photons from each crystal overlap (Fig. 6). A beam splitter acts to superpose the trajectories of the signal photons. The basic interference effect arises between these signal

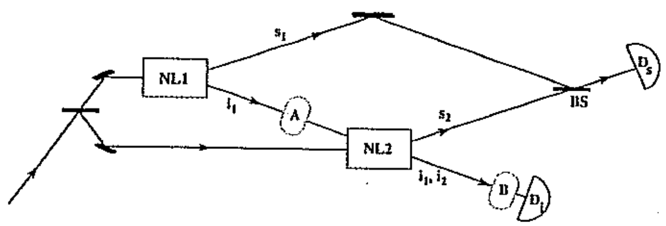

FIG. 6. Schematic of setup used in the Zou *et al.* experiment (1991), with the possible inclusion of additional elements to make it suitable for a quantum eraser. In the absence of these elements, the identically polarized idler photons from either crystal are indistinguishable. Consequently, interference fringes may be observed in the signal singles rate (at detector D_s) if any of the path lengths in the interferometer is varied.

photons as any of the path lengths in the interferometer is varied. If the path lengths are adjusted correctly and the idler beams overlap precisely, there is no way to tell, even in principle, from which crystal a photon detected at D_s originated—interference results in the signal singles rate at D_s (and also trivially in the coincidence rate between D_s and D_i). If the idler beam from crystal NL1 is prevented from entering crystal NL2 (or even if the two idler beams are only slightly misaligned), then the interference vanishes, because the presence or absence of an idler photon at D_i then labels the parent crystal. It is important to stress that the interference observable in the singles rate at D_s is *not* due to a stimulated emission in NL2 by idler photons from NL1; by using a weak pump beam, the experimenters have effectively ensured that the probability of more than one pair of photons in the interferometer at one time is very small.

Note that one could also make the two processes leading to a signal photon being detected at D_s distinguishable by inserting a halfwave plate between the two crystals (at point A in Fig. 6), thereby rotating the polarization of idlers from NL1 to be orthogonal to the polarization of idlers from NL2. As in the previous example of a quantum eraser, one could recover interference by using a polarizer before D_i (it is not necessary here to use a polarizer before D_s) and correlating counts between the two detectors. If one employed fast detectors and a rapidly switchable polarizer, one could perform a delayed-choice version, in which the orientation of the polarizer was not chosen until *after* the signal photon had already been detected. That is, the decision to measure which-path information (particle-like behavior of the photons) or to regain interference (wave-like behavior) could be made after the originally interfering particles were *detected*[1]. Of course, only by correlating the measurements at D_s and D_i could one determine the results of the decision, so there is no possibility of superluminal signaling. This and other improved quantum eraser schemes are discussed by Kwiat *et al.* (1994a).

[1]This is thus an extension of the original delayed-choice discussion by Wheeler (1979) and the experiments by Hellmuth *et al.* (1987) and Alley *et al.* (1986), in which the decision to display wave-like or particle-like aspects in a light beam may be delayed until after the beam has been split by the appropriate optics.

C. Suppression of Spontaneous Parametric Down-Conversion

A clever modification of the two-crystal experiment has recently been reported by Herzog *et al.* (Greenberger *et al.*, 1993; Herzog *et al.*, 1994); see Fig. 7. In this case only a *single* nonlinear crystal is used, but it is used in two directions. A given pump photon may down-convert in its initial right-ward passage through the crystal or in its left-going return trip (or not at all—the most likely outcome). As in the previous experiment, the idler modes from these two processes are made to overlap; moreover, the signal modes are also aligned to overlap. Thus, the left-going and right-going production processes are indistinguishable and interfere. The result is that fringes are observed in all of the counting rates (i.e., the coincidence rate and both singles rates) as any of the mirrors is translated. One interpretation of these results is as a variable enhancement (or suppression) of the spontaneous emission of the down-converted photons. In contrast to the cavity QED demonstrations discussed in Section II.A (Hinds, 1991), the distances to the mirrors can be much longer than the coherence lengths of the spontaneously emitted photons.

V. The EPR "Paradox" and Bell's Inequalities

A. Generalities

Nowhere is the nonlocal character of the quantum-mechanical entangled state as evident as in experiments involving the "paradox" of Einstein, Podolsky, and Rosen (EPR) (1935) and the related inequality by Bell (1964). Following the modified EPR-argument by Bohm (1983), we consider two photons traveling off back to back, described by the entangled polarization singlet-like state [cf. Eq. (7)]:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H_1, V_2\rangle - |V_1, H_2\rangle) \tag{10}$$

where the letters denote horizontal (H) or vertical (V) polarization, and the

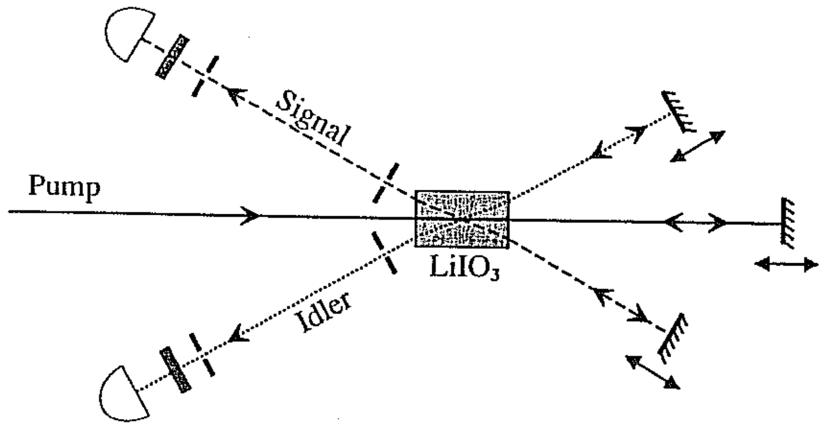

FIG. 7. Schematic of the Innsbruck experiment to demonstrate enhancement and suppression of down-conversion (Herzog *et al.*, 1994).

subscripts denote the photon propagation direction. This state is isotropic, in that measurement of any polarization component for one of the particles will yield a count with 50% probability; individually, each particle is unpolarized. Nevertheless, if we measure the polarization component of particle 1 in some basis, we can *predict with certainty* the value of particle 2's polarization in the same basis, seemingly without disturbing it, since it may be arbitrarily remote. Therefore, according to EPR, one must ascribe an "element of reality" to that component of polarization. The same logic applies for all polarization bases, implying that the polarization is well defined along all directions simultaneously. Of course, a quantum mechanical state cannot specify that much information, and is consequently an incomplete description, according to the EPR argument. An intuitive explanation that was implied by EPR (though not explicitly stated) is that the particles leave the source with definite, correlated properties; these properties are presumably determined by some local "hidden variables" not present in QM.

At the level of two entangled particles, a local hidden variable (LHV) theory can be advanced that correctly describes situations of perfect correlations or anti correlations (i.e., measurements made in the same polarization basis). Under these conditions, the choice of an LHV theory versus QM is a philosophical decision, not a physical one. It was only in 1964 that John Bell discovered that QM gives different *statistical* predictions than does any theory based on local realism, for situations of nonperfect correlations (i.e., analyzers at intermediate angles). In particular, he proved that any LHV theory's predictions for certain coincidence probabilities must satisfy an inequality that certain quantum mechanical predictions violate.

B. Polarization-Based Tests

Since Bell's inequalities were presented in 1964, they have been experimentally tested many times, and the vast majority of experiments have violated these inequalities (when several supplementary assumptions are made—see discussion later), in support of quantum mechanics. The initial tests were performed with pairs of photons produced via atomic cascades (Freedman and Clauser, 1972; Aspect *et al.*, 1982; Clauser and Shimony, 1978). Unfortunately, the angular correlation of the cascade photons is not very strong, and this weakens the correlations in polarization (due to transversality). In contrast, the strong correlations of the down-converted photons make them ideal for such tests. The first such tests were performed by Shih and Alley (1988), and Ou and Mandel (1988), using setups essentially identical to that already discussed in connection with quantum erasure (Fig. 4). At the condition of equal path lengths, and with the half-wave plate oriented to rotate the polarization from horizontal to vertical, the state of light after the 50-50 beam splitter is:

$$|\psi\rangle \approx \frac{1}{2}[|H_1\, V_2\rangle + i\cdot i\,|\,V_1 H_2\rangle + i\,|V_1 H_1\rangle + i\,|V_2 H_2\rangle] \qquad (11)$$

which reduces to (10) if one considers only terms that can lead to coincidence detections. A simple calculation reveals that the probability of coincidence detection depends only on the difference of the orientation angles of the two polarizers: $P_C \propto \sin^2(\theta_2 - \theta_1)$.

C. Nonpolarization Tests

The advent of parametric down-conversion has also led to the appearance of several *nonpolarization*-based tests of Bell's inequalities, similar to those first proposed by Horne *et al.* (1989). In the experiment of Rarity and Tapster (1990), an entanglement of the *momenta* of down-converted photons is employed (Fig. 8). By use of small irises (labeled A in the figure), only four down-conversion modes are examined: 1s, 1i, 2s, and 2i. Beams 1s and 1i correspond to one pair of conjugate photons; beams 2s and 2i correspond to a different pair. However, photons in beams 1s and 2s have the same wavelength, as do photons in beams 1i and 2i. With proper alignment, therefore, after the beam splitters there is no way, even in principle, to tell whether a pair of photons was previously in the 1s–1i or the 2s–2i paths. Consequently, although the singles rates at the four detectors indicated in Fig. 8 remain constant, the rates of coincidences between detectors display interference. This interference depends on the difference of phase shifts induced by rotatable glass plates P_i and P_s in paths 1i and 2s, respectively, and is formally

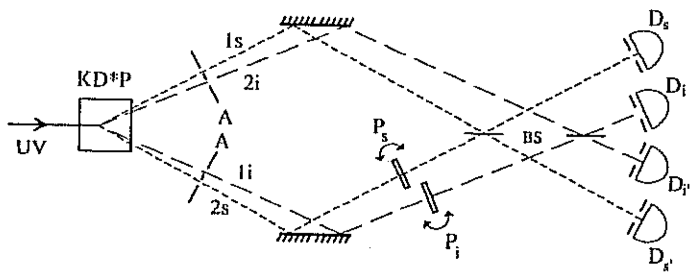

Fig. 8. Outline of apparatus used to demonstrate a violation of a Bell's inequality based on momentum entanglement (Rarity and Tapster, 1990).

equivalent to the polarization case considered earlier, in which it is the difference of polarization-analyzer angles that is relevant. By measuring the coincidence rates for two values for each of the phase shifters—a total of four combinations—the experimenters were able to violate an appropriate Bell's inequality, after making the standard supplementary assumptions. One interpretation is that the paths taken by a given pair of photons are not elements of reality.

A different Bell's inequality proposal, based on energy-time entanglement of the photons, was made in 1989 by Franson (1989), and implemented by several groups (Brendel *et al.*, 1992; Kwiat *et al.*, 1993a; Shih *et al.*, 1993). A schematic of the basic setup is shown in Fig. 9. Each member of a down-converted pair is directed into an unbalanced Mach-Zehnder interferometer, allowing both a short and long path to the final beam splitter. Correlations between detectors at two of the output ports are examined. For sufficiently large interferometer imbalances (i.e., larger than the 60-μm coherence length of the down-converted photons), no interference fringes are present in the detector singles rates. Nevertheless, fringes in the coincidence rate are predicted (as long as the difference in the path-length differences of the interferometers is less than 60 μm), arising from interference of the two indistiguishable processes that could lead to coincidence detection—"short-short" and "long-long." If the detectors are fast enough, it becomes possible to exclude the background of noninterfering processes in which one photon

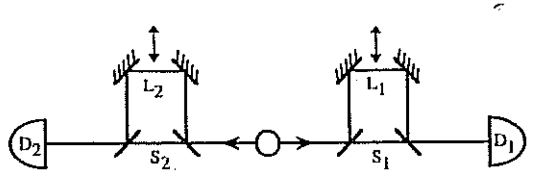

Fig. 9. Simplified schematic of a Bell's inequality experiment based on energy and time correlations as proposed by Franson (1989).

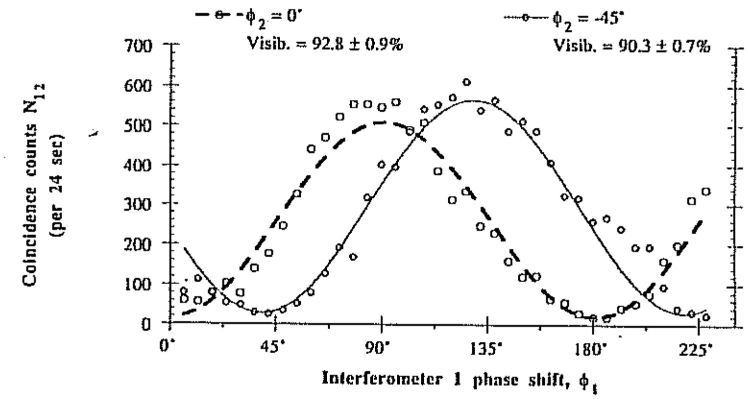

Fig. 10. High-visibility coincidence fringes in a Franson dual-interferometer experiment, for two values of the phase in interferometer 2, as the phase in interferometer 1 is slowly varied. The curves are sinusoidal fits.

takes its short path and the conjugate photon takes its long path; the resulting reduced quantum mechanical state [to be compared with (10)] is

$$|\psi\rangle = \frac{1}{2}(|S_1,S_2\rangle - e^{i\phi}|L_1,L_2\rangle) \tag{12}$$

where the letters indicate the short or long path, and the phase is proportional to the *sum* of the relative phases in each interferometer. The high-visibility fringes arising from (12) lead to a violation of an appropriate Bell's inequality (Brendel *et al.*, 1992; Kwiat *et al.*, 1993a). In particular, sinusoidal fringe visibilities in excess of 90% were observed (Fig. 10), whereas the maximum possible for a local hidden variable model is only 71%. One conclusion of such a violation is that it is incorrect to ascribe to the photons a definite time of emission from the crystal, or even a definite energy, until these observables are explicitly measured. A more general interpretation, applicable to *all* violations of Bell's inequalities, is that the predictions of quantum theory, which seem to be supported by nature, cannot be reproduced by any completely local theory. As discussed in slightly different ways by Jarrett (1984) and Shimony (1990), it must be that the results of measurements on one of the photons depend on the results for the other, and these correlations are not due to a common cause at their creation.

D. Loopholes and Loophole-Free Tests

No test of Bell's inequalities to date has been incontrovertible, due to several loopholes that reduce the true impact such an experiment might yield. One of these, the angular correlation problem, has essentially been solved by turning to down-conversion light sources over cascade sources, leaving the fast-switching loophole and the detection loophole. Only one Bell-type experiment, that of Aspect *et al.* (1982), has made any attempt at all to close the former, but even in that experiment the loophole remains. Although the experiment used rapidly varying analyzers, the variation was not random, and it has been argued that the speed of the polarization switching was not sufficient to disprove a causal connection between the analyzer and the source (Franson, 1985; Zeilinger, 1986).

The detection loophole arises from the nonunit detection efficiency in any real experiment. If the efficiency is sufficiently low, then it is possible for the subensemble of detected pairs to give results in agreement with quantum mechanics, even though the *entire* ensemble satisfies Bell's inequalities. Due to the inadequacy of existing detectors, experiments have so far employed an additional assumption, roughly equivalent to the "fair-sampling" assumption that the fraction of detected pairs is representative of the entire ensemble (Clauser *et al.*, 1969; Santos, 1992). Formerly, it was believed that $\sim 83\%$ was the lower efficiency limit needed to close this loophole experimentally. However, Eberhard (1993) has shown that by using a nonmaximally entangled state (i.e., one where the magnitudes of the probability amplitudes of the contributing terms are not equal), one may reduce the detector requirement to $\sim 67\%$, in the limit of no background.

At present, we know of only two efforts under way to attempt a loophole-free test of Bell's inequalities. One relies on the recent development of high-efficiency single-photon detectors, with measured efficiencies of 75%, and the possibility of even further improvements (Kwiat *et al.*, 1993b). Also central to this approach is a novel two-crystal down-conversion arrangement (Kwiat *et al.*, 1994b), in which it is not necessary to discard half of the counts to produce a polarization singlet-like state, unlike most of the previous down-conversion schemes [cf. Eq. (11)]. A very different proposal has been made by Fry and Li (1991), who for the first time proposed dissociated atoms (mercury dimers) as the correlated particles. The advantage is that detection efficiencies of 95% are possible by photoionizing the atoms and detecting the photoelectrons.

VI. Related Issues

A. Information Content of a Quantum

It is now reasonably well known that it is impossible to "clone" photons. That is to say, given an unknown polarization state of a single photon, it is impossible to create a second photon in the same state, without destroying the first one. If cloning *were* possible, one could communicate instantaneously over arbitrarily large distances using EPR correlations. If two separated scientists each receive a particle from an EPR pair, one of them (the "sender") can choose to measure the spin in any basis. This collapses the "receiver's" particle into a corresponding spin state, depending on the choice and *random outcome* of the measurement. Tracing over the state of the sender's particle to find the effective density matrix for the receiver's, we always find the density matrix of an *unpolarized* particle, with no information about which basis the sender used. The message of which measurement the sender made cannot be decoded by the receiver. However, if one or more "clones" of the receiver's photon existed, a set of polarization measurements could in fact determine the precise polarization of this photon, to an accuracy in bits equal to the number of copies available. Thus to preserve causality (the impossibility of such an instantaneous transfer of information), photon-cloning must be impossible (Glauber, 1986; Wootters and Zurek, 1982). Even though a photon may be in a definite polarization state (described by an angle drawn from a continuous distribution, and thus containing, in principle, an unlimited number of bits of information), no experiment can extract more than 1 bit of information about the polarization of a single photon: In the EPR example, this bit represents not the *choice* of measurement made by the sender, but that measurement's random outcome.

This makes the recent work by Bennett *et al.* (1993) on *quantum teleportation* all the more remarkable. They found that an unknown polarization state (with its, in principle, infinite amount of information) can be "teleported" to a distant lab by means of transmitting a mere 2 bits of classical information, as long as the distant lab ("B") and the one with the particle it wishes to teleport ("A") each possesses one of a pair of EPR particles. Thus, although one may only *extract* 1 bit of (normally useless) information from an EPR particle, the perfect correlations may be used to *transfer* an *infinite* amount of information, i.e., precise specification of a point in the state space of the particle (the Poincaré sphere for a photon or the Bloch sphere for an electron).

The basic idea of the scheme is that the joint system in A's possession, consisting of one particle of the EPR pair and a second particle in the unknown state $|\phi>$, has a total of four basis states. (We consider spin-1/2 particles for simplicity.) These basis states can be described in terms of the two possible spin projections of each of the two particles, or alternatively in terms of the total angular momentum and spin projection of the joint system. Envision A performing a measurement in the latter basis and finding the system either in the singlet state or in one of the three triplet states. The process works in all four cases, but is easiest to understand for the singlet, which (as alluded to in Section V) corresponds to perfect anticorrelation of the spins of the two particles, regardless of the choice of quantization axis. This anticorrelation implies that upon A's detection of a singlet state, his member of the EPR pair collapses into a state with spin exactly opposite that of $|\phi>$. But the two EPR particles themselves are (by definition) in a singlet state:

1	↺	↻	↔	↕	↕	↔	↔	↻	↺	↕	↻	↺
2	↕	↻	↔	↕	↺	↺	↔	↻	↕	↻	↻	↺
3		√	√	√			√	√			√	√
4		1	0	1			0	1			1	0

FIG. 11. A simple chart, demonstrating the use of EPR-correlated photon pairs for cryptography. Each collaborator measures the polarization of one of each EPR pair in a linear or circular basis, chosen randomly. Lines 1 and 2 depict their results. The collaborators determine (via *public* communication) for which pairs they used the same measurement basis, and keep only these results (line 3). The correlations ensure that they both now possess the same random binary key (line 4) (Collins, 1992).

then B's particle has spin exactly opposite to A's, which is to say identical to $|\phi>$. Experimenter A need only send B the 2-bit message that of the four possible outcomes, a singlet state was found, and B knows that she has a perfect copy of the particle A was studying. This "teleportation" scheme works even if neither experimenter had any prior information about the state $|\phi>$. The "no cloning" theorem is not violated, since the state of A's particle is irrevocably altered by the measurement he performs, leaving only one particle in the original state.

B. Cryptography

As discussed earlier, the EPR schemes cannot be used to send any signals superluminally; nevertheless, the proposal has been made that these be utilized for a different kind of communication, namely, cryptography. In the "one-time pad" of classical cryptography (Brassard, 1988), two collaborators need to share a secret "key" (a random string of binary digits) in order to encode and decode a message. Such a key may provide an absolutely unbreakable code, provided that it is unknown to any potential eavesdropper. The problem arises in key distribution: Any classical distribution scheme is subject to noninvasive eavesdropping, for example, using a fiber-coupler to tap the line, without disturbing the transmitted classical signal. In the quantum cryptography proposals and demonstrations made to date, the security is guaranteed by using single-photon states (Bennett *et al.*, 1992; Ekert, 1991); some of the schemes employ particles prepared in an EPR-entangled state. [Alternatively, the sender can send photons in a state of random, but *definite*, polarization (Bennett, 1992); equivalently, some schemes use phase (Ekert *et al.*, 1992).] Each collaborator receives one member of each correlated pair, and measures the polarization in a random basis. After repeating the process many times, the two then discuss publicly which bases were used for each measurement, but

not the actual measurement results. The cases where different bases were chosen are not used for conveying the key, and may be discarded, along with instances in which one party detected no photon. In cases where the same bases were used, however, the participants will now have correlated information. From this, a random, shared key can be generated (Fig. 11). As long as single photons are used, any attempt at eavesdropping, even one relying on QND, will necessarily introduce errors due to the uncertainty principle. For if the eavesdropper uses the wrong basis to measure the polarization (phase) of a photon before sending it on to the real recipient, the very act of measuring will disturb the original state.

C. Nonlocality without Inequalities

In the preceding experiments for testing nonlocality, the prediction of local realistic theories and those of standard quantum theory need not be contradictory in situations of perfect correlation. (Here we mean situations in which one can predict *with certainty* the results of a measurement on one particle, given the outcomes of measurements on other particles of the system; this is the EPR criterion for an "element of reality.") Indeed, one might even say it was the very fact that LHV theories *could* intuitively explain such cases that led EPR to conclude that quantum mechanics was incomplete. As Bell showed, the nonlocality of a singlet-like entangled state becomes manifest when *statistical* tests are made in cases of nonperfect correlation.

Recently, Greenberger, Horne, and Zeilinger (GHZ) proved that no local theory is consistent with even the perfect correlations predicted for some systems involving three or more entangled particles (Greenberger *et al.*, 1990; Greenberger *et al.*, 1989; Mermin, 1990). One can thus prove the incompatibility of local realistic theories with quantum mechanics without the need for inequalities. (In practice, their incompatibility with *nature* can only be tested with inequalities.) A schematic of one version of the GHZ apparatus is shown in Fig. 12. It is essentially a three-particle generalization of the Rarity-Tapster experiment discussed earlier (cf. Fig. 8). The source at the center emits trios of correlated particles, just as a down-conversion crystal produces pairs. Just as the Rarity-Tapster experiment selected two pairs of photons, the GHZ source selects two trios of photons; these are denoted by abc and $a'b'c'$. Hence, the state coming from the source may be written

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|abc\rangle + |a'b'c'\rangle) \qquad (13)$$

After passing through a variable phase shifter (e.g., ϕ_a), each primed beam is recombined with the corresponding unprimed beam at a 50-50 beam splitter. Detectors (denoted by Greek letters) at the output ports of all three

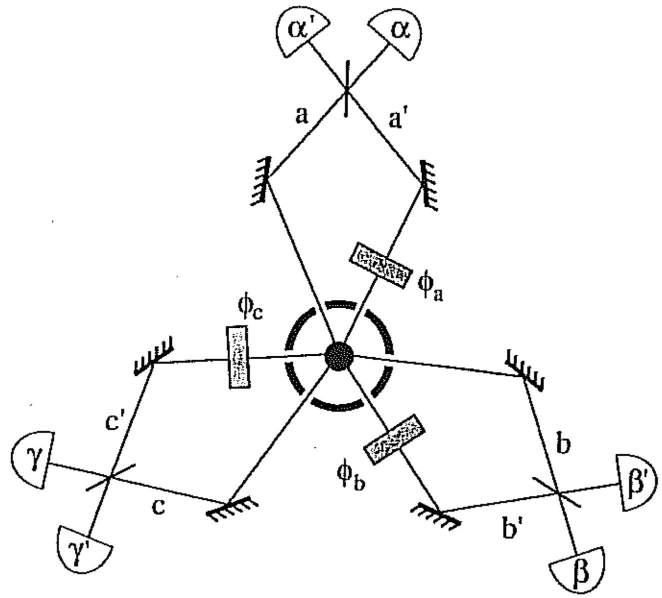

FIG. 12. A three-particle *gedanken* experiment to demonstrate the inconsistency of quantum mechanics and any local realistic theory. All beam splitters are 50-50 (Greenberger *et al.*, 1993).

beam splitters allow one to record the occurrence of triple coincidences, involving one detector for each beam splitter. A detailed discussion of this experiment is beyond the scope of this review and is given in Greenberger *et al.*, (1990); here we present a simplified argument to convey the spirit of the GHZ result. Given the state (13), one can calculate from standard quantum mechanics the probability of a triple coincidence as a function of the three phase shifts, assuming ideal optical elements and perfect detectors:

$$P(\phi_1,\phi_2,\phi_3) = \frac{1}{8}[1 \pm \sin(\phi_a + \phi_b + \phi_c)] \tag{14}$$

where the plus sign applies for coincidences between all unprimed detectors, and the minus sign for coincidences between all primed detectors. Consider first the case in which all phases are equal to 0; then occasionally (one-eighth of the time) there will be a triple coincidence of all primed detectors. Using what is known as a *counterfactual* approach, we can ask what *would* have happened if ϕ_a had been $\pi/2$ instead. We assume that this would not have changed the state from the source, and by the locality assumption, it certainly would not have changed the fact that detectors β' and γ' went off. But from (14) probability of a triple coincidence for primed detectors is zero in this case; therefore, we can conclude that if ϕ_a had been $\pi/2$, then detector α would have "clicked." The exact same logic implies that if ϕ_b had been $\pi/2$, detector β would have clicked, and if ϕ_c had been $\pi/2$, detector γ would have clicked. Consequently, if all the phases had been equal to $\pi/2$, we would have seen a triple coincidence between unprimed detectors. But according to (14) this is impossible: The probability of triple coincidences between unprimed detectors when all three phases are equal to $\pi/2$ is strictly zero! Hence, if one believes the quantum mechanical predictions for these cases of perfect correlations, it is not possible to have a consistent local realistic model. Lucien Hardy was the first to show that one could, at least in a *gedanken* experiment, demonstrate nonlocality without inequalities using only *two* particles. His first proposal relied on electron-positron annihilation (Hardy, 1992a); later he proposed a quantum optical version of the experiment (Hardy, 1992b), relying on down-converted photon pairs. The argument is similar to that for the GHZ case.

Central to the arguments used to infer nonlocality in the GHZ and Hardy experiments is the ideality of all components—to satisfy EPR's criterion of reality, one must be able to predict with *certainty* the results of particular measurements. This is never possible in any real experiment, so the arguments must be modified somewhat, as GHZ (Greenberger *et al.*, 1990), Hardy (1992b), and Eberhard and Rosselet (1994) have discussed. This done, one is left once again with an inequality that can only be violated with high-efficiency detectors (or by making supplementary assumptions). Specifically, detection efficiencies of 91% are needed for the GHZ scheme, and greater than 98% for the Hardy scheme! Moreover, one must still contend with the rapid-switching loophole in *any* test of nonlocality. Given this, it seems more likely that a loophole-free test of nonlocality would be based on Bell's inequalities, where the requirements are less stringent.

Although demonstrations of nonlocality without inequalities exist only at the *gedanken* level (and generalizations to real experiments seem to have even more difficult experimental constraints than loophole-free tests of Bell's inequalities), they are important because of their pedagogical value. In the original EPR argument, it is assumed that quantum mechanical calculations yield correct predictions for the case of perfect correlations. A more intuitive, local model is then espoused, which explains the correlations as being due to elements of reality, essentially arising at the source of the particles. The meaning of the GHZ result, and also the Hardy result, is that one is not even able to explain the *perfect* correlations of quantum mechanics with a local realistic model.

VII. The Reality of the Wave Function

A. Test of the de Broglie Guided-Wave Theory

The interpretation of the quantum-mechanical wave function ψ was a subject of no small import to the founders of quantum theory. Although it is now common to interpret ψ as simply a mathematical tool for calculating probabilities of measurement outcomes, other approaches have been suggested. The guiding wave (sometimes called a *pilot wave*) model of de Broglie is one such alternative, according to which the wave function describes a real physical wave (de Broglie, 1969). In the case of light, this approach describes both photons and electromagnetic waves guiding them, much like ocean waves guide a surfer. According to one version of the de Broglie model (Croca *et al.*, 1990), these waves are assumed to exist in ordinary three-dimensional space, so that waves associated with *different* particles may interfere. It is then possible to test experimentally the validity of the picture [at least this version (Holland and Vigier, 1991)] as it leads to predictions different from those of standard quantum theory. An experiment of this type, using down-conversion photons, was performed by Wang *et al.* (Fig. 13). First consider the singles rate at detector D_2: Assuming 50-50 beam splitters and ideal detectors, the signal photons (*s*) will be detected at D_2 one-fourth of the time, whereas idler photons (*i*) never make it to this detector. Idler photons can only be detected by D_1, and this occurs one-fourth of the time. To arrive at D_1, the signal photons may bounce off either BS_2 or BS_3, which act like the mirrors of a Michelson interferometer. Hence, the rate of counts at D_1 from *signal* photons depends on the interferometer path imbalance Δx, and the total rate at D_1 displays fringes with visibility 50% (again, for 50-50 beam splitters).

These considerations apply equally to both conventional quantum theory and the de Broglie approach. Differences between the theories are manifested in the rate of coincidences between D_1 and D_2. In the conventional theory the coincidence rate is a constant (equal to one-sixteenth the total rate of down-converted pairs), because there is only one possible way for a coincidence event to occur—idler photon detected at D_1; signal photon detected at D_2, after passing through beam splitters BS_1 and BS_2. According to the version of the de Broglie picture tested here, however, even if a signal photon is detected at D_2, the resulting empty signal wave propagating to D_1 interferes with the idler wave. Because the signal wave depends on Δx, so too does the total guiding wave acting on the idler photon, and coincidence fringes of 50% visibility are predicted. The results of Wang *et al.* (1991) strongly confirm the quantum calculation, disproving this particular version of the de Broglie theory.

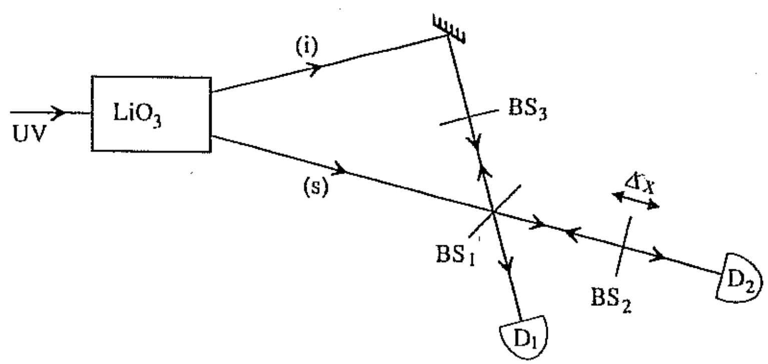

FIG. 13. Simplified schematic of experiment to test one version of a de Broglie guiding-wave theory. All beam splitters are 50-50 (Wang *et al.*, 1991).

B. Bohm's Deterministic Model of Quantum Mechanics

While de Broglie's early picture of real waves carrying particles along like surfers fails to explain some of the deeper surprises in quantum mechanics, Bohm's natural extension of his work fares much better. This extension differs from the simpler de Broglie model in that it sees the wave function as existing in configuration space, not in real space, as it must in order to replicate the quantum mechanics of more than one particle. While it is a realistic model, it is not local. Bohm demonstrated (Bohm, 1952; Bohm and Hiley, 1993; Holland, 1993b) that quantum mechanics is exactly equivalent to a theory of classical particles that undergo deterministic evolution and interact with a "quantum potential" determined by the wave function $\Psi \equiv Re^{iS/\hbar}$ (which obeys the Schrödinger equation). In particular, if we assume that the initial distribution of these particles is described by $|\Psi|^2 = R^2$ as in the Born interpretation of quantum mechanics and a similar relation for their initial momenta, and we assume that they interact with a *physical field* Ψ according to the equation of motion

$$m\ddot{x} = -\nabla[V(x) - (\hbar^2/2m)\nabla^2 R/R] \tag{15}$$

then their distribution at later times will agree exactly with the quantum mechanical prediction. This is equivalent to saying that the ensemble of particles, described by the probability distribution $R^2(x)$, follows a velocity field

$$v(x) = \nabla S(x)/m \tag{16}$$

The difference between this picture and the Born interpretation is that we need only assume the *initial* distribution of the particles. As time passes, deterministic evolution of the state of each particle preserves the equivalence between $|\Psi|^2$ and the probability density, not as a hypothesis but as a mathematical consequence. The Ψ-field is not directly detectable, but serves merely to "guide" the particle. Each particle has a well-defined position at all times,[2] and for this reason the picture is philosophically more palatable to many people, although it is sufficiently unwieldy that most physicists disregard it. Perhaps its most serious problem is that the real trajectories it predicts seem completely unphysical, unless it can be shown that these trajectories lead to some conclusion that can be tested experimentally and are more difficult to extract from standard quantum mechanics.

The origin of the model's idiosyncracy is the fact that (16) has the mathematical form of a fluid-flow equation. The various particle trajectories describing the members of an ensemble may never cross, because the velocity field is a single-valued function of position. How, then, is a two-slit experiment described? If all the trajectories are calculated, it will be seen that none of them crosses the symmetry plane. Quite contrary to our mechanistic intuitions, all the particles that pass through the upper slit "bounce" off the central plane, and contribute to the *upper* half of the interference pattern. Does this not contradict experiment, in that if we insert a measuring device to record which slit is traversed, the screen records the diffraction pattern of this slit, extending on both sides of the symmetry plane? No, not at all. For the trajectories of Bohm particles are (nonlocal) hidden variables, entirely inaccessible to measurement. If we introduce some measuring device, this contributes another degree of freedom to the configuration space in which Bohm's "quantum potential" and thus the velocity field resides; the trajectories may now cross in real space without doing so in configuration space. The Bohm theory is in agreement with our intuitions about the particle trajectories under these conditions. Unfortunately, the conclusion seems to be that the information added by Bohm to our understanding of quantum mechanics is only valid as long as we do not attempt to make any use of it! Perhaps there is some indirect effect of these trajectories that will afford the model some predictive power. As an analogy, we no longer believe in Bohr's planetary description of the hydrogen atom, yet his model correctly describes features that are less evident in modern quantum mechanics.

C. Application of Bohm's Theory to Tunneling Times

One attempt to apply Bohm's theory has been introduced by Leavens and Aers in the context of the tunneling-time controversy (Leavens, 1990a, 1990b; Leavens and Aers, 1993). It has long been known (Büttiker and Landauer, 1982; Hauge and Støvneng, 1989; Wigner, 1955) that there are difficulties associated with calculating the duration of the tunneling process, due to the fact that evanescent waves do not accumulate any phase. First of all, the momentum in the barrier region is imaginary, so the most naive semiclassical traversal time is not a real quantity. Second, if one applies the stationary phase approximation to calculate the group delay—which we define as the time of arrival of a wave packet peak at the far side of the barrier, relative to the time at which it would have arrived at the same point had no barrier been present—the insensitivity of the phase to barrier thickness leads to a delay that tends to a constant as the thickness diverges, in seeming violation of relativistic causality (Bolda *et al.*, 1993). As we explain later, we have recently confirmed this striking prediction experimentally, finding effective tunneling velocities as high as $1.7c$. It is not trivial to overcome this latter problem in the same way that one deals with superluminal group velocities in classical optics—as pulse reshaping effects (Siegman, 1986). Since the standard interpretation of quantum mechanics does not associate definite trajectories with individual particles, it is tempting to try to define some more meaningful "interaction time" for tunneling, which might accord better with our relativistic intuitions and perhaps even have implications for the ultimate speed of devices that rely on tunneling (Büttiker and Landauer, 1985). Various proposals have been made along these lines, and some experiments have even been done (Landauer, 1989), but by necessity they all consider modified experimental setups. In general, some additional interaction must be introduced to serve as a "clock," and as we have learned from Bohr (1983), it is only when we "refer only to observations obtained under circumstances whose description includes an account of the whole experimental arrangement" that quantum mechanics can provide an unambiguous description of a phenomenon.

One scheme that might describe tunneling times without introducing experimental modifications is the Bohm-de Broglie pilot wave model. As explained earlier, this model *does* define trajectories for individual particles. Thus one can calculate a distribution of traversal times for the tunneling particles using this model, and in general they do not share the apparent superluminality of the group delay. Like the trajectories for the particles in the two-slit experiment, however, these trajectories display some very odd features. In particular, it is found that *all* the transmitted particles correspond to those particles that were nearer to the leading edge of the incident pulse; this explains how they can arrive faster than had the incident peak traversed the barrier at c, though they themselves never travel superluminally. This behavior is due to the absence of crossing trajectories in the theory: If a particle that originates at a time t_0 is transmitted, *all* particles that originate earlier than t_0 must likewise be transmitted; otherwise, their trajectories on reflection would intersect that of the first particle considered. This all-or-nothing behavior has a very odd consequence. Consider a

[2]At present, the relativistic generalization of the Bohm model does not ascribe real trajectories to *photons* in the same way as to electrons (Bohm and Hiley, 1991; Holland, 1993a). However, the phenomena discussed in this paper generally have electronic analogs and involve quasi-free propagation of a sort that should be interpretable in similar terms. We implicitly consider the Bohm theory of the electron throughout this chapter, even in contexts where the experiments have been performed with photons.

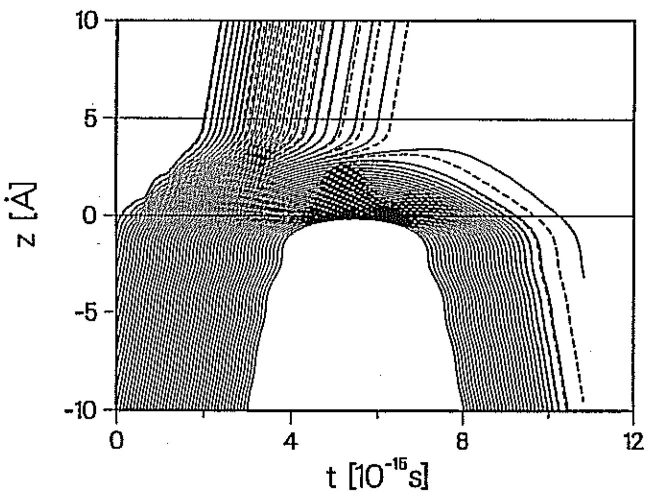

FIG. 14. Bohm trajectories for a Gaussian wave packet with $E_0 = 5$ eV, $\Delta k = 0.04$ Å^{-1} incident on a static barrier $V_0 = 10$ eV (solid curves) and an oscillating barrier (dashed curves). Only the earliest arriving trajectories make it across the barrier. (Reprinted from Leavens and Aers, 1991, 1015–1023, with kind permission from Pergamon Press Ltd, Headington Hill Hall, Oxford OX3 0BW, UK.)

narrow-bandwidth wave packet incident on a tunnel barrier with a 1% transmission. Quantum mechanics and the Bohm model both predict that the group delay does not depend on the precise bandwidth; as long as the bandwidth is small enough for the method of stationary phase to apply, the location of the transmitted peak is essentially constant. But the location of the *leading* 1% of the incident wave packet (where all of the particles predicted by the Bohm theory to be transmitted are said to originate) is *not* bandwidth independent. In fact, its initial location (relative to the peak) diverges as the bandwidth of the wave packet tends toward zero. Clearly, for small incident bandwidths, the Bohm trajectories for the transmitted particles must be delayed enormously by the tunnel barrier, because they arrive at the barrier long before the incident peak, but leave shortly after the peak's arrival. This is confirmed by direct calculation (see Fig. 14). Although this behavior does not actually violate any principle of physics, it is difficult for many physicists to believe that such oddly behaved trajectories could possess physical significance.

The fact that only the particles in the early part of the wave packet are transmitted accords well with intuitions one might have from considerations of classical wave propagation. As discussed in Chu and Wong (1982) and Garrett and McCumber (1970), superluminal group velocities of a classical wave can be understood as a "reshaping" of the leading edge into an early, scaled-down version of the incident pulse. In the context of tunneling, we understand this as a sort of coherent transient effect whose times scale depends only on energy differences, not barrier thicknesses. At a physical level, the reflection from a multilayer dielectric, for example, occurs because of coherent multiple reflections between the different layers. Over time, a steady-state field builds up inside the structure, which interferes destructively with the incident field at the exit face and constructively at the entrance face. At early times, before this field reaches its steady-state value (which is critically dependent, of course, on the frequency of the incident field), there is little destructive interference, and a significant part of the wave is transmitted. At later times, there is little transmission. This preferential treatment of the leading edge engenders a sort of optical illusion, shifting the transmitted peak earlier in time.

D. Status of the Wave Function in Measurement

The odd behavior of these tunneling wave packets pushes us to reexamine our interpretation of wave packets. Is it true that a particle's position should never be thought of as more precisely determined than the wave function suggests? Or does the tunneling phenomenon—coupled with our knowledge of causality—imply that location really is an "element of reality," in that once we see a transmitted photon we can say with certainty that it originated toward the front of its wave packet? These questions raise difficult issues of quantum philosophy. We are accustomed to using the wave function as a calculational tool without pondering such questions, but we have also learned that the wave function is in some sense physical, and should not be regarded merely as some distribution from classical statistics. A recent proposal by Aharonov *et al.* (1993; Aharonov and Vaidman, 1993) demonstrates that under certain carefully controlled conditions, a single-particle wave function may be measured. When a state is "protected" from change, for example, by an energy gap, and measurements are performed sufficiently "gently" (i.e., slowly enough that no excitations across the energy gap are likely), one should be able to determine not just the expectation value of position but the wave function at many different positions, without altering the state of the particle. No violation of the uncertainty principle or of the no-cloning theorem (see Section VI.A) arises from this, because the ability to "protect" a state relies on some preexisting knowledge about the state, but it assigns a deeper significance to the wave function, one Aharonov terms *ontological*, as opposed to merely epistemological. For instance, it would certainly be illustrative to measure a particle's wave function, allow it to tunnel, and then measure its new wave function, thus removing all questions about how to interpret the superluminal peak motion.

E. Observable Effects of "Empty Waves"

The real (but undetectable) fields of the Bohm model are similar to the "vacuum fluctuations" of quantum field theory. This can be seen particularly well by considering a *gedanken* experiment recently proposed by Elitzur and Vaidman (1993), although the experiment can be described perfectly well by standard quantum mechanics as well. This experiment consists of a simple single-particle interferometer, a Mach-Zehnder, for example. Particles are injected one at a time into one input port. The path lengths are adjusted so that all the particles leave a given output port (A), and never the other (B). Now we suppose that a "bomb" is inserted into one of the interferometer's two arms. The hypothesis is that this is an infinitely sensitive bomb, such that interaction with even a single photon will cause it to explode. However, there is some chance that the bomb is a dud, that is, that it lacks the sensitive trigger; in this case, incident photons sail through the bomb with no consequences. Unfortunately, classical intuition is quite clear: Any attempt to check for the presence of the trigger involves interacting with it in some way, and by hypothesis this will inevitably set it off, rendering it useless. Elitzur and Vaidman's insight is that quantum mechanics gives us another option, allowing us some portion of the time to be certain that the bomb is functioning, *without* setting it off. This point is in some sense complementary to the usual picture of quantum mechanics as denying the reality of any phenomena not directly observed: In contrast to classical mechanics, we can learn something about reality *without* (in a technical sense) "observing" it. If the trigger is missing, the interferometer functions as before. All incident particles exit output port A. After a sufficient number of attempts, we can be fairly certain that the bomb is a dud. The same would occur classically. But now consider what happens when a functioning bomb is inserted and a photon enters the interferometer. A Mach-Zehnder interferometer begins with a beam splitter, after which the photon has a 50% chance of heading toward the bomb. If this occurs, the bomb explodes, and we are no better off than the classical physicists down the hall. On the other hand, if the photon takes the path without the bomb, there is no more interference, since the fact of the bomb *not* exploding provides *welcher Weg* information. Thus the photon reaches the final beam splitter and chooses randomly between the two exit ports. Some of the time, we see a photon at output port B, something which never happened before the bomb was inserted. This immediately tells us that a trigger is in place—even though (since the bomb is unexploded) the trigger has not interacted with any photon. It is the possibility that the trigger *could have* interacted with a photon that destroys the interference.

How can the final state at the detectors depend on what is in a given arm of the interferometer, when we simultaneously know that no particle has gone through that arm? According to the Bohm theory, of course, the answer is straightforward: The real Ψ-field enters both arms, but is incapable of setting off the bomb. At the final detector, Ψ (which now depends on whether or not the bomb is a dud) "pilots" the particle accordingly. This interpretation is surprisingly similar to one orthodox description of the two-crystal *welcher Weg* experiment performed by Mandel and described in Section IV.B. In such a picture, space is permeated by electromagnetic *vacuum fluctuations*, real (but random) fields that have measurable effects, but do not consist of any photons. These fields are part of the electromagnetic ground state, so no energy can be extracted from them and they will thus never trigger photodetectors such as those considered by Elitzur and Vaidman, according to the treatment originally due to Glauber (1963). At a beam splitter, however, these vacuum fields can interfere with other incident fields (Caves, 1981, 1987) and influence the port of the beam splitter from which a photon will exit. The placement of a functioning bomb in the path of the vacuum fields destroys the coherence at the final beam splitter, on which interference relies.[3] The choice the photons make at the final beam splitter has thus been affected not by direct action of the bomb on any photons, but by its influence on the random fields permeating the vacuum.

VIII. The Single-Photon Tunneling Time

A. An Application of EPR States: Dispersion Cancellation

The crucial element in the Einstein-Podolsky-Rosen argument is the observation that while position and momentum are noncommuting operators, the difference of two positions and the sum of two momenta commute (Einstein *et al.*, 1935). In their idealized example, the Wigner distribution

$$W(q_1,q_2,p_1,p_2) = 2\pi\,\delta(q_1 - q_2 + q_0)\,\delta(p_1 + p_2) \tag{17}$$

represents a pair of particles, each with completely uncertain position and momentum, the sum of whose momenta is precisely 0 and the difference of whose positions is precisely q_0 (Bell, 1986). As discussed in Section V.C, this is analogous to the state of the photon pairs emitted in parametric

[3]Even if the bomb's trigger is of the QND sort, the number-phase uncertainty relation guarantees this.

fluorescence, which exhibit simultaneous correlations in time-of-arrival (the analog of position) and in wave vector (the analog of momentum); these correlations exceed by many orders of magnitude what would be allowed by the uncertainty relation for the position and momentum of a single photon. This suggests that apart from its importance in elucidating philosophical questions tied to quantum theory, the EPR state has practical importance in high-resolution timing experiments. Due to the uncertainty principle, in order to perform such experiments, it is necessary to introduce a large energy uncertainty, or bandwidth. Such a large bandwidth means that even normally small dispersive effects, such as those found in glass or even air, can be of great importance, rapidly broadening a pulse. Until recently, it seemed that this trade-off between pulsewidth and bandwidth was unavoidable, leading to a fundamental limit on the resolution of pulse propagation experiments in dispersive media (Franson, 1992). However, one can capitalize on the EPR correlations of down-converted photons to make high-resolution time measurements essentially immune to dispersive broadening (Steinberg *et al.*, 1992ab). The Hong-Ou-Mandel interferometer (see Section III.D) can be used to measure delay times and we shall see that in this setup the dispersion is sensitive only to the *sum* of the two photons' energies, while the time measurement is sensitive to the *difference* of the two emission times! Because these are compatible observables, they may be simultaneously specified arbitrarily well, thus preserving measurement accuracy.

The effect can be understood intuitively by applying Feynman's rules for interference more carefully, taking into account once more the fact that outcomes that are in principle distinguishable do not interfere. First of all, recall the discussion of Section III.D, which showed that no coincidences are observed in a perfectly aligned Hong-Ou-Mandel interferometer. The interference effect only occurs as long as the reflection-reflection and transmission-transmission processes are fundamentally indistinguishable, which is only the case if the two photons reach the beam splitter at the same time. To be indistinguishable, the two processes must also "agree" as to what frequency photon is seen by each detector. As can be seen in Fig. 15, this means that in some sense, each interferometer arm samples both of the anticorrelated frequencies, leading to an automatic cancellation of any first-order dispersive broadening. This is why the interferometer can be used for precise time-delay measurements; if a delay greater than the coherence time of about 20 fs is inserted in one arm, the coincidence null vanishes until this delay is compensated, for example, by piezoelectrically translating a mirror by some fraction of a micron. Such measurements can be more than 5 orders of magnitude more precise than would be possible via electronic timing of direct detection events, and in principle better than those performed with nonlinear autocorrelators (which rely, after all, on the same fundamental physics as down-conversion, but do not benefit from a cancellation of dispersion).

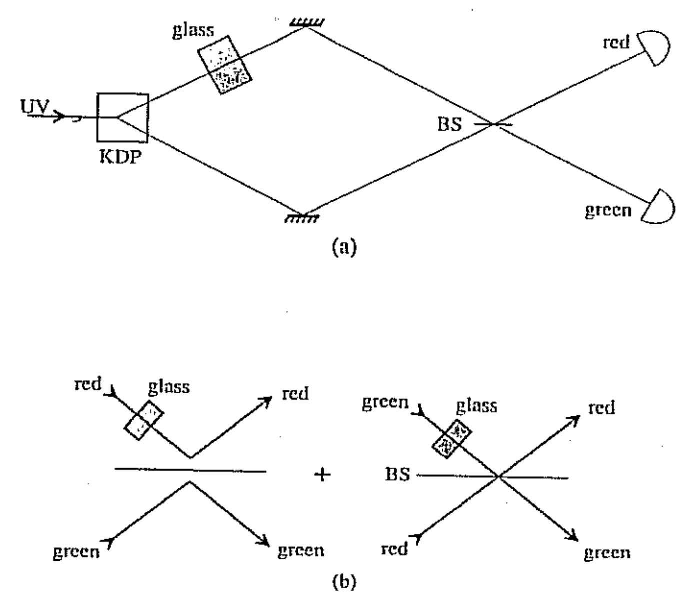

FIG. 15. (a) Simplified setup for a Hong-Ou-Mandel interferometer with a dispersive medium inserted in one arm. (b) Interfering Feynman paths for coincidence detection involve anticorrelated colors traversing the glass.

B. Single-Photon Propagation Time Measurements

This interference technique has certain unique features that immediately recommend it for a variety of investigations. As explained in the early sections of this chapter, coincidence counting allows one to study light at the quantum (i.e., single-photon) level, as opposed to most of modern optics, which can generally be described self-consistently using only classical field theory. The Hong-Ou-Mandel interferometer is thus an ideal apparatus for studying the quantum propagation of light, a problem that has been of great theoretical interest; at least until recently, the only quantum theory of light in dispersive media was an *ad hoc* one (Glauber and Lewenstein, 1991). Our first application of the interferometer was therefore to confirm that single photons in glass travel at the group velocity (Steinberg *et al.*, 1992b). While not unexpected, this result is an interesting one at the interpretational level. It suggests that when looking for a microscopic description of dielectrics, it is inappropriate to consider the medium as being polarized by an essentially classical electric field due to the collective action of all photons present, and reradiating accordingly. Linear dielectric response is not a collective effect in this sense, and really involves each photon interfering only with itself (as per Dirac's dictum) as it is partially scattered from the atoms in the medium. The fact that the particle's propagation is described by a group velocity is an example of wave-particle duality observed in a single experiment. This is a relatively trivial application of the interferometer. More interesting questions arise when we go beyond simple dielectrics to the consideration of the tunneling problem discussed earlier. Tunneling, even more strikingly than the group velocity, is a wave phenomenon. It is also manifestly nonlocal (although not in the strong sense of EPR or of Aharonov-Bohm) in that the final state involves a coherent superposition of two far-separated wave packets. Yet at the quantum level, a particle is detected only at a single position (recall Fig. 14). At the completion of a tunneling "event," the particle has either been transmitted or reflected, but its position is described by the solution to a wave equation, renormalized to account for this extra bit of information. This would not be so surprising were it not for the superluminality of tunneling. As alluded to earlier, such superluminality is explicable at the classical-field level. In particular, the area of the transmitted pulse is so small that it can be seen as originating entirely in the leading edge of the incident pulse [although this need not be the case in planned experiments (Chiao, 1993; Steinberg and Chiao, 1994)]. See Fig. 14.

If we accept, however, the collapse hypothesis of the Copenhagen interpretation, what happens when a given particle is detected on the far side of the barrier (or, for that matter, if a perfectly efficient detector on the *near* side of the barrier *fails* to detect it)? Suddenly, this small wave packet is renormalized according to the projection postulate (von Neumann, 1983). At this point, the entire quantum ($\hbar\omega$ if the particle in question is a photon) appears as a single unit in one wave packet. We can no longer get around the question of superluminality as easily as in the classical case. According to standard quantum theory, the incident wave function is seen as a *complete* description of the physical state of the photon, not merely an expression of our own personal uncertainty as to the photon's position. We are led to say that whenever the photon is transmitted, it arrives (on average) earlier than it would have had it travelled always at c. Although this effect depends critically on analytic wave packets with arbitrarily long tails and therefore cannot be used for superluminal signaling [as a signal must correspond to an *abrupt* change, which can never propagate faster than c (Brillouin, 1960)], it goes against deeply held intuitions about the meaning of causality, which need to be reexamined in this context. [Since the anomaly occurs not for a complete ensemble, but for a *subensemble* determined by joint preselection (preparation of a Gaussian input state) and postselection (discarding reflected photons), it is an example of what Aharonov and Vaidman (1990) have termed weak measurement (Steinberg, submitted).]

C. Tunneling Time in a Multilayer Dielectric Mirror

While some tunneling time experiments have been performed in the past (Landauer, 1989), optical tests offer certain unique advantages (Martin and Landauer, 1992). In our experiment, the main advantages include the ease of construction of a barrier, the high resolution of the timing possible, the absence of dissipation, the fact that the photon is intrinsically relativistic, and that the particular barrier we were able to use has very little energy dependence. At a theoretical level, the fact that photons are described by Maxwell's (fully relativistic) equations is an important argument against interpreting superluminal tunneling predictions as a mere artifact of the nonrelativistic Schrödinger equation. At an experimental level, it puts us much closer to a regime in which an apparent violation of causality might be observed (Deutch and Low, in press; Low and Mende, 1991). The fact that our barrier (described later) has little energy dependence means that the transmitted wave packets suffer little distortion, and are essentially indistinguishable from the incident wave packets. It also means that one is denied the recourse suggested by some workers (Dumont and Marchioro, 1993) of interpreting the superluminal appearance of transmitted peaks to mean that only the high-energy components [which traveled faster even before reaching the barrier (Hauge *et al.*, 1987)] were transmitted.

In our setup (Fig. 16), the tunnel barrier was a multilayer dielectric mirror with 11 alternating layers of low and high index material, each one quarter wave thick at the design frequency of the mirror, which corresponded to a wavelength of 700 nm in air (Steinberg *et al.*, 1993). Such a periodic structure leads to a photonic bandgap (Yablonovitch, 1993) analogous to that in the Kronig-Penney model of solid-state physics. The gap represents a forbidden range of energies, in which the multiple Bragg reflections will interfere constructively so as to exponentially damp any incident wave. We

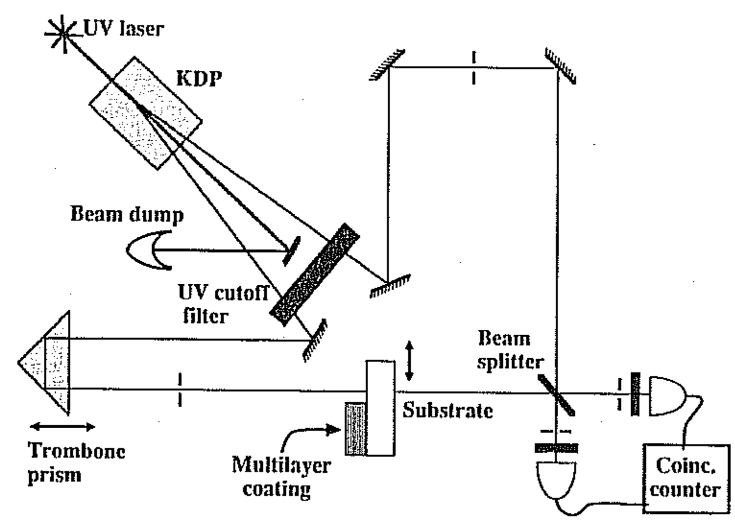

FIG. 16. Apparatus for measuring the single-photon tunneling time (Steinberg *et al.*, 1993).

worked at 700nm, the center of the gap, where this 1.1-μm thick structure reflected 99% of all incident light. Throughout the bandgap (extending from 600 to 800 nm in wavelength), the transmissivity is relatively flat; outside the gap, it reaches a series of resonant transmission peaks, easily understood as Fabry-Perot resonances, but also analogous to resonant transmission for electrons incident on a square barrier.

The analogy with tunneling in nonrelativistic quantum mechanics arises because of the exponential decay of the field envelope within the periodic structure, that is, the imaginary value of what in solid-state physics would be termed the *quasimomentum*. The analogy is sufficiently accurate that the same qualitative features arise for the transmission time: For thick barriers (including ours, where $kd \approx 10$), it saturates at a constant value of approximately 1.7 fs. This means that 700-nm light transmitted through a mirror should arrive on average 1.9 fs *sooner* than a similar wavepacket traveling through a micron of air instead of the micron-thick mirror! To measure such a small time difference, we had the multilayer structure coated on one-half of one surface of a high-quality optical flat. This way, as we scanned across the Hong-Ou-Mandel dip, we could slide the coated half in and out of place without affecting any of the alignment or changing the optical path length of the rest of the apparatus. This also enabled us to avoid problems associated with hysteresis when translating the prism that was used to adjust the path-length difference; the reflective coating was slid in and out of place periodically while the prism made a single slow scan, thus forming a pair of coincidence dips. Each dip was subsequently fitted to a Gaussian, and the difference between their centers was calculated. Typical data are shown in Fig. 17. When several such runs were combined, we found that the tunneling peak arrived 1.47 ± 0.21 fs *earlier* than the one traveling through air.

In this way, we confirmed that the tunneling delay time was superluminal. Our results excluded the semiclassical time, which corresponds to treating the *magnitude* of the (imaginary) crystal momentum as a real momentum. This time is of interest mainly because it also arises in a calculation by Büttiker and Landauer (1982) of a critical time scale in problems involving oscillating barriers, which they interpret to mean that this time is a better measure of the duration of the *interaction* than is the group delay. Our data are consistent with the group delay, and also with the Larmor time as defined by Büttiker (1983). The Larmor time is one of the early efforts to attach a "clock" to a tunneling particle, in the form of a spin aligned perpendicular to a small magnetic field confined to the barrier region. The basic idea is that the amount of Larmor rotation (in the plane perpendicular to the applied field) experienced by a transmitted particle is a measure of the time spent by that particle in the barrier.

At midgap, this time agrees with the group delay; both predict superluminal delay times. In other regimes, the two times differ. Most of this difference reflects the fact that the aligned spin component is more easily

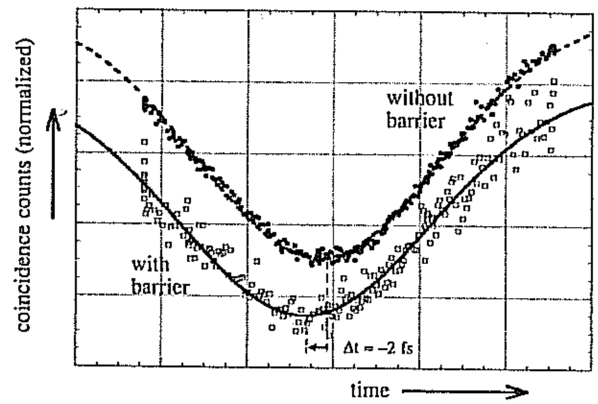

FIG. 17. Coincidence profiles with and without the tunnel barrier, taken by scanning the trombone prism of [Fig. 16], map out the single-photon wave packets. The lower profile shows the coincidences with the barrier; this profile is shifted by ≈ 2 fs to *negative* times relative to the one with no barrier (upper curve). The wave packet that tunnels through the barrier arrives *earlier* than the one that travels the same distance in air.

transmitted than the anti-aligned component, leading to a rotation not perpendicular to the B-field but *toward* the B-field. It is debatable to what extent this sort of an alignment should be considered as contributing to an interaction time. Recently, we have studied the energy dependence of the tunneling time, not by tuning the frequency directly, but by angling the dielectric mirror, thus shifting its bandgap. Preliminary results are qualitatively in better agreement with the group delay than with the Larmor time; a more accurate test is currently under way.

D. Interpretation of the Tunneling Time

Although no conclusion about interaction times can be drawn from our experiments, they provide a very clear verification of superluminal tunneling delays. The simplicity of the geometry we used and the absence of extraneous degrees of freedom strongly suggests (within the framework of standard quantum mechanics) that there are no further time scales "hidden" in this particular experiment. At the same time, we would like to understand these superluminal effects in the context of a relativistic theory. We do not believe that any particle travels faster than c at any given time, or that any *signal* could be transmitted faster than c, and yet it is nontrivial to reconcile these statements with the data.

One attempt made recently (Deutch and Low, 1993) relied, like the Larmor time, on attaching an extra parameter such as spin to a tunneling wave packet. [This idea had appeared elsewhere in the literature as well

(Martin and Landauer, 1992).] The interesting observation was made that if the incident Gaussian wave packet consisted of two different polarizations—one for the first half of the wave packet and an orthogonal (hence distinguishable) one for the second half—then the superluminally transmitted peak would consist entirely of this first polarization, demonstrating the causal "disconnection" of the outgoing peak from the incident one. Further analysis of this example (Steinberg *et al.*, 1994) shows that it involves mixing particle and wave pictures of tunneling in a way that would not be experimentally testable. The addition of the polarization information in question suppresses the interference of multiply reflected waves (in analogy with the quantum eraser discussed earlier) and the transmitted beam is no longer a simple Gaussian; the discontinuity introduces rapid oscillations at later times. It is only the small peak which arrives earliest that has the character described by Deutch and Low. It is not possible in a single experiment to observe *both* the superluminal transmission *and* the polarization information. A modified version of this *gedanken* experiment, due to Rolf Landauer, involves *smoothly* varying the polarization across the incident pulse. In this case, the transmitted peak appears superluminally, as before, but reproduces the original polarization profile, providing no extra information about where the transmitted particles originate. Even if one believes that no particle can travel faster than light (certainly all serious researchers in this field agree that no *signal* can, at any rate), it appears that attempts within the framework of quantum mechanics to "label" particles and thus verify this claim are doomed to failure, thanks to the complementarity principle.

IX. Envoi

A. Phase Operators

Let us recall Section III.B and difficulty of interpreting the energy-time uncertainty principle. The number-phase uncertainty relation is related to this better known example; energy, after all, is number times $\hbar\omega$, while time is phase divided by ω. Like time, phase does not correspond to a Hermitian operator in quantum mechanics, and the usual nonrelativistic derivation of uncertainty relations via commutation relations cannot apply.[4] The first

[4]Just as going from nonrelativistic quantum theory to field theory reduces the position momentum uncertainty relation to the same status as the time-energy relation (see Section III.B), it reduces other uncertainty relations to that of number and phase. For example, polarization is no longer a property of a particle, but a label of different field modes. In a given basis, spin projection along the quantization axis is the difference of the occupation number for the two eigenstates; spin projection in a nonorthogonal basis depends on the phase difference *between* these two eigenstates.

attempt at a phase operator was made by Dirac, who reasoned that phase should be the conjugate variable to number, and therefore postulated the existence of a phase operator ϕ whose commutator with the number operator $\underset{\sim}{N}$ is $-i$, from which the number-phase uncertainty principle $\Delta N\,\Delta\phi \geq 1/2$ follows automatically (Dirac, 1927). It was later shown (Louisell, 1963) that no such Hermitian operator exists. This is connected with the fact that N has a positive-definite spectrum, and a translation operator for N (which role ϕ would play) cannot exist, in precise analogy to the question of a time operator. Nevertheless, there does seem to be some meaningful content to this relation: If we pick a reasonable definition of phase for simple test cases (such as the angular coordinate of the Wigner distribution in electromagnetic phase space), the phase of number states is indeed perfectly undefined, and the phase uncertainty of a coherent state does indeed fall as the reciprocal of its number uncertainty. Other attempts have been made at generalized phase operators, but these are of unclear physical significance and no universal preference has emerged (Barnett and Pegg, 1986; Noh *et al.*, 1993).

Mandel's group has therefore taken a practical approach to the problem (Noh *et al.*, 1991). They have set up a homodyne measurement that is typical of how one might measure phases in classical optics. Classically, the phase can be determined as a function of differences between the photocurrents at various detectors in such a scheme. Mandel's group defines the operator by taking the expression appropriate for their experiment, and replacing the *c*-number photocurrents with their operator counterparts. They have shown theoretically that this operator may differ in certain regimes from the other proposals we have discussed, although in the correspondence principle limit they all of course agree. They have experimentally confirmed the validity of this definition, and found results that do not match the other proposed operators.

Raymer's group has performed related experiments (Smithey *et al.*, 1993), in which they are able to measure the total quantum state of an electromagnetic field mode. This differs from the proposal of Aharonov's discussed earlier. For one thing, the measurement involves studying an entire ensemble, not a single system. For another, it examines the properties of a single-mode field whose occupation is quantized, as opposed to the (first-quantized) wave function of a single particle. It is novel in that it reconstructs the entire density matrix for the state, allowing a calculation of all properties from number statistics to quadrature amplitudes to phase fluctuations. This is done by integrating various "slices" of the Wigner distribution by performing appropriate homodyne measurements, and performing an inverse transformation of the type used in computer-aided tomography to reconstruct the whole function. We do not review these experiments here, but only mention that they can be used to measure directly the number-phase uncertainty product for various states of the field.

B. Nonlinear Quantum Mechanics

So many quantum mechanical predictions have already been verified that it becomes difficult to see most optics experiments as tests of quantum theory, as opposed to mere ongoing confirmations. The real problem is that, although people are dissatisfied with certain aspects of quantum mechanics (QM), there is very little in the way of convincing extensions or generalizations of QM. One noteworthy exception is Weinberg's nonlinear quantum mechanics. Linearity, in the sense of the superposition principle, is arguably the heart of quantum mechanics, as of any wave mechanics. Not only does it make calculation easier than in strongly nonlinear theories, but it is really the source of interference effects and particle-wave duality. On the other hand, the most striking problem of quantum mechanics is the discrepancy between linear, unitary (Schrödinger) evolution and the observation in real life of individual events or quantum jumps: the absence of Schrödinger cats in nature. Because information is gained whenever a collapse occurs from some pure linear superposition of states to one particular eigenstate of a given operator, such a process cannot be explained in terms of a linear theory. Perhaps if a small nonlinear component were present in a more complete formulation of quantum theory, this collapse would emerge naturally?

Weinberg (1989) proposed an example of such a nonlinear extension of quantum mechanics. In particular, he offered an example of what a nonlinear Hamiltonian might look like for a nuclear spin-3/2 system. The tests of this generalization have relied on optical spectroscopy. For example, Bollinger *et al.* (1989) have looked for a dependence of the transition frequencies between hyperfine states of $^9Be^+$ on the populations of these states. The hyperfine state involves the nuclear spin, and a nonlinear term in the nuclear Hamiltonian would lead to energy shifts based on these populations. A limit of less than $10\,\mu$Hz was placed on these frequency shifts, implying that less than about 10^{-26} of the binding energy per nucleon could be attributed to a Weinberg nonlinearity. As discussed by Polchinski (1991), if such a nonlinearity does nevertheless exist, it will necessarily lead to great changes in our understanding of QM. It appears that the breakdown of the superposition principle would either lead to the possibility of superluminal commnication or of communication "between different branches of the wave function," that is, dependence of experimental observables on contrary-to-fact conditions such as what the experimenter *would have done* in different experimental circumstances!

C. Issues in Causality

It might seem that while optics is a useful tool for investigating complex systems, optics itself is completely understood; this is not the case. In fact,

some of the most interesting causal paradoxes that have yet to be resolved occur in optics. The most well known of these is Barton's paradox (Barton and Scharnhorst, 1993), closely related to the Casimir effect [itself a QED phenomenon recently given indirect confirmation in a quantum optics experiment (Sukenik *et al.*, 1993)]. In this case the amplitude for light-light scattering, one of the radiative corrections in QED, is modified by the presence of closely spaced parallel plates (an extremely small, but in principle observable, effect). In certain arrangements, it appears that this can lead to propagation of light in vacuum (albeit a vacuum "colored" by the presence of the Casimir plates) faster than c. Similar paradoxes have been pointed out in connection with localization of any particle in a quantum field theory (Hegerfeldt, 1985) and with the Glauber theory of photodetection (Bykov and Tatarskii, 1989). It may be that the high momentum/energy components introduced by localizing a detector to within a wavelength lead to spurious "dark" detections (Ho, 1993), which can be mistaken for causality-violating events.

D. Conclusions

Optics experiments have allowed wide ranging and important tests of QM. In conjuction with atomic physics, they have been used to investigate such diverse questions as parity nonconservation and nonlinear extensions of quantum mechanics, issues whose proper place is many orders of magnitude above the 1-eV energy scale. Optics is also important in the study of gravitational phenomena, such as gravitational antennas and gravitational lensing by some of the most massive objects in the universe. Thanks to novel optical techniques, we have new abilities for manipulating single atoms and ions, which promise new devices for time keeping (and thus further tests of general relativity), as well as studies of such diverse phenomena as Bose-Einstein condensation, the Casimir effect, the Unruh effect, and atomic interference.

The two great revolutions in physics that occured at the beginning of the twentieth century, quantum mechanics and relativity, both arose from optical experiments. At the end of this century, we still find that optical experiments probe deeply our understanding of these two branches of physics. The reason for this is that the photon is at once both a quantal and a relativistic particle. The conceptual tension between the fundamentally nonlocal nature of the quantum on the one hand, and the intrinsically local nature of space-time on the other, is most sharply manifested in optical phenomena and provides a strong motivation for pushing optical tests to new frontiers.

Acknowledgments

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