PHY357 – Subatomic Physics – Midterm Exam

February 24, 2020

Duration - 50 minutes

PLEASE read carefully the following instructions.

Aids allowed: A non-programmable calculator without text storage.

There are three questions on this midterm exam. Each question is worth one-third of the total grade.

Partial credit will be given for partially correct answers, so show any intermediate calculations that you do and write down, **in a clear fashion** any relevant assumptions you are making along the way.

There are two pages of background material, not all of which you'll need to answer the questions, that are found on pages two and three of this midterm paper. The questions start on page four.

Do no separate the sheets of the question paper. Hand in the question sheets with your exam booklet at the end of the test.

Good luck!

PHYSICS 357S – Constants and Particle Properties

Possibly Useful Physical Constants:

Avogadro No:	$6\times10^{23}\,mole^{-1}$
pi	$\pi = 3.1416$
speed of light:	$c = 3.0\times10^{8}\,m/s$
Plank's constant:	$\hbar = 6.6\times10^{-22}\,MeV\cdot s$
	$\hbar c = 197\,MeV.fm$
	$(\hbar c)^2 = 0.4\,GeV^2\cdot mb$
	$1\,eV = 1.6\times10^{-19}\,Joules$
	$1\,eV/c^2 = 1.8\times10^{-36}\,kg$
	$1\,fm = 10^{-15}\,m$
	$1\,mb = 10^{-27}\,cm^2$
1 year	1 year $\approx \pi\times10^{7}s$
electron charge:	$e = 1.602\times10^{-19}\,C$
electron magnetic moment:	$\mu_e = 9.3\times10^{-24}\,Joules\cdot Tesla^{-1}$
fine structure constant:	$\alpha = e^2/(\hbar c) = 1/137.0360$
strong coupling constant:	$\alpha_s(M_Z) = 0.116\pm0.005$
Fermi coupling constant:	$G_F = 1.166\times10^{-5}\,GeV^{-2}$
Cabibbo angle:	$\sin\theta_C = 0.22$
Weak mixing angle:	$\sin^2\theta_W(M_Z) = 0.2319\pm0.0005$
Branching Ratios	$BR(Z\to e^+e^-) = 3.21\pm0.07\%$
	$BR(Z\to hadrons) = 71\pm1\%$
Electromagnetic constant	$1/(4\pi\varepsilon_0) = 8.99\times10^{9}\,Nm^2/C^2$

Particle Properties

Boson	Mass $\left(GeV/c^2\right)$
γ	$< 3\times10^{-36}$
$gluon$	~ 0
$W^{\pm}$	80.22
Z^0	91.187
H^0	125

Lepton	Mass $\left(MeV/c^2\right)$	Lifetime (s)
ν_e	$<10^{-5}$	-
e	0.510999	$>10^{33}$
ν_μ	<0.27	-
μ	105.658	2.197×10^{-6}
ν_τ	<10	-
τ	1777	2.906×10^{-13}

Hadron	Quark Content	Mass $\left(MeV/c^2\right)$	$\mathrm{I}\left(\mathrm{J}^{\mathrm{PC}}\right)$
π^+,π^0,π^-	$u\bar{d},\left(u\bar{u}-d\bar{d}\right)/\sqrt{2}, d\bar{u}$	139.57,134.97, 139.57	$1\left(0^{-+}\right)$
K^+,K^-	$u\bar{s}, s\bar{u}$	493.65	$\frac{1}{2}\left(0^-\right)$
$K^0,\bar{K}^0$	$d\bar{s}, s\bar{d}$	497.67	$\frac{1}{2}\left(0^-\right)$
ρ^+,ρ^0,ρ^-	$u\bar{d},\left(u\bar{u}+d\bar{d}\right)/\sqrt{2}, \bar{u}d$	775.7	$1\left(1^{--}\right)$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Delta^-,\Delta^0,\Delta^+,\Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
Λ^0	uds	1115.6	$0\left(\frac{1}{2}^+\right)$
$\bar{D}^0, D^0$	$u\bar{c}, c\bar{u}$	1863	$\frac{1}{2}\left(0^-\right)$
D^-, D^+	$d\bar{c}, c\bar{d}$	1869	$\frac{1}{2}\left(0^-\right)$
D_S^+, D_S^-	$c\bar{s}, \bar{c}s$	1968	$0\left(0^-\right)$
B^+, B^-	$u\bar{b}, \bar{u}b$	5279	$\frac{1}{2}\left(0^-\right)$
Λ_c^+	udc	2285	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+,\Sigma^0,\Sigma^-$	uus, uds, dds	1189	$1\left(\frac{1}{2}^+\right)$
Ξ^0,Ξ^-	uss, dss	1315	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Λ_b	udb	5624	$0\left(\frac{1}{2}^+\right)$

1. Particle 1, at rest, decays into particles 2 and 3 (ie. a reaction like $1 \rightarrow 2 + 3$ occurs).

   (a) Show that the energy of the outgoing particles can be obtained from the various masses by:

   $$E_2 = \frac{m_1^2 + m_2^2 - m_3^2}{2m_1} c^2$$

   (b) Consider the limit of this formula when $m_1 = m_2 + m_3$. Explain how the simplified expression describes the physics in that limit.

   (c) Use this formula to find the centre of mass energy of each decay product in the following reactions:

      i. $\pi^+ \rightarrow e^+ + \nu_e$;
      ii. $D^0 \rightarrow K^- + \pi^+$;
      iii. $\Xi^- \rightarrow \Lambda^0 + \pi^-$.

   For each of the reactions in part (c) you should provide two energies (one for each final state particle).

2. For each of the following reactions or decays state whether it can proceed by a strong, electromagnetic or weak interaction. For those that are possible through multiple interactions indicate which one is dominant (ie. most likely to occur). For interactions that involve neutrinos you should state clearly what kind of neutrino (both flavour and whether a neutrino or anti-neutrino) must be involved for the reaction to proceed. Draw a simple particle (quark or otherwise) flow diagram to show one way each reaction proceeds:

   (a) $\Omega^- \rightarrow \Lambda^0 + K^-$
   (b) $\Sigma^- \rightarrow \Lambda^0 + e^- + \nu$
   (c) $\bar{p} + n \rightarrow \pi^- + \pi^0$
   (d) $\Delta^+ \rightarrow n + \pi^+ + \pi^0$
   (e) $K^- \rightarrow \mu^- + \nu$

   Partial credit will be given for listing all the quantities that **are** conserved even if you end up getting the wrong interaction type or don't provide a particle flow diagram for the reaction. You are encouraged to provide some notes on your thought process as you work out how the reactions proceed.

3. This question consists of two, independent, questions about radioactive decay. Each is worth half of the credit for this problem:

   (a) A sample of 1 g of a radioactive isotope with atomic weight 208 decays via β emission. If 70 decays are recorded in a 24 hour period with a detector that has an efficiency of 15%, what is the mean lifetime of the isotope?

   (b) A 1 g sample take from a historical organic tapestry (a piece of fabric made from plant material) is found to have a β count rate of 2.1 counts per minute. These βs originate from the decay of ${}^{14}C$ contained in the fabric. ${}^{14}C$ has a mean lifetime of 8270 years. If the fraction of ${}^{14}C$ in living matter (ie. plants alive today, that were used to make the fabric) is 1.2×10^{-12}, what is the age of the tapestry? To estimate the age you should assume that when the plants were killed and the tapestry was originally produced the ${}^{14}C$ atoms it contained started to decay, and were not replenished, leaving a smaller fraction of ${}^{14}C$ in the historical tapestry, when it is measured in the present day.