Electron-Vortex Binding and Inter-Composite-Fermion Interaction in the Fractional Quantum Hall States

Gun Sang Jeon

Seoul National University

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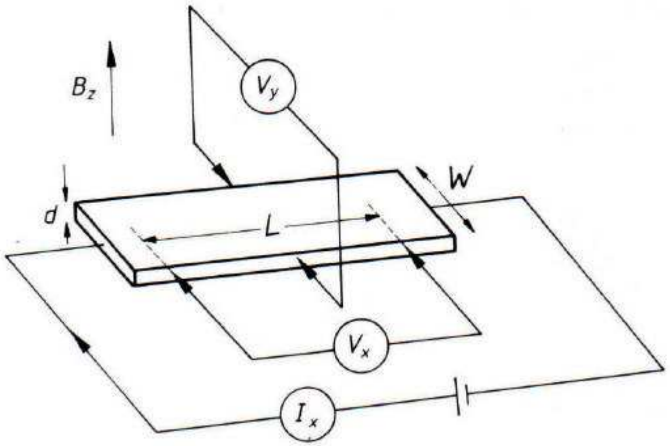
in collaboration with J.K. Jain(PSU), M.R. Peterson(UC Santa Cruz)

Jeon,Peterson,Jain, PRB (2005); Jain,Jeon, PRB(R) (in press) ; Chang,Jeon,Jain, PRL (2005)

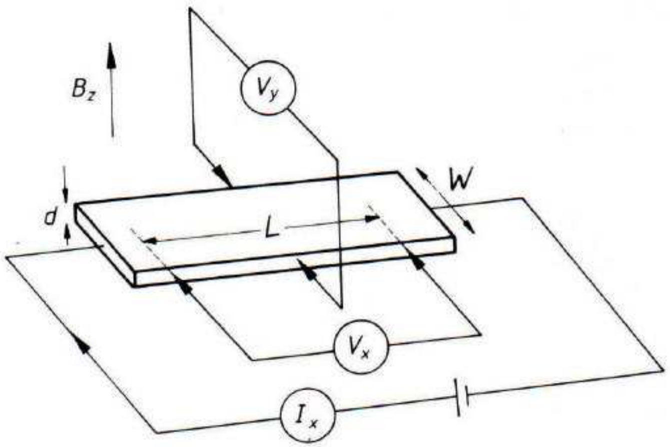
Fractional Quantum Hall Effect



longitudinal resistance $R_L \equiv \frac{V_x}{I_x}$

Hall resistance $R_H = \frac{V_y}{I_x} = \frac{B}{\rho e c}$ classically

Fractional Quantum Hall Effect



longitudinal resistance $R_L \equiv \frac{V_x}{I_x}$

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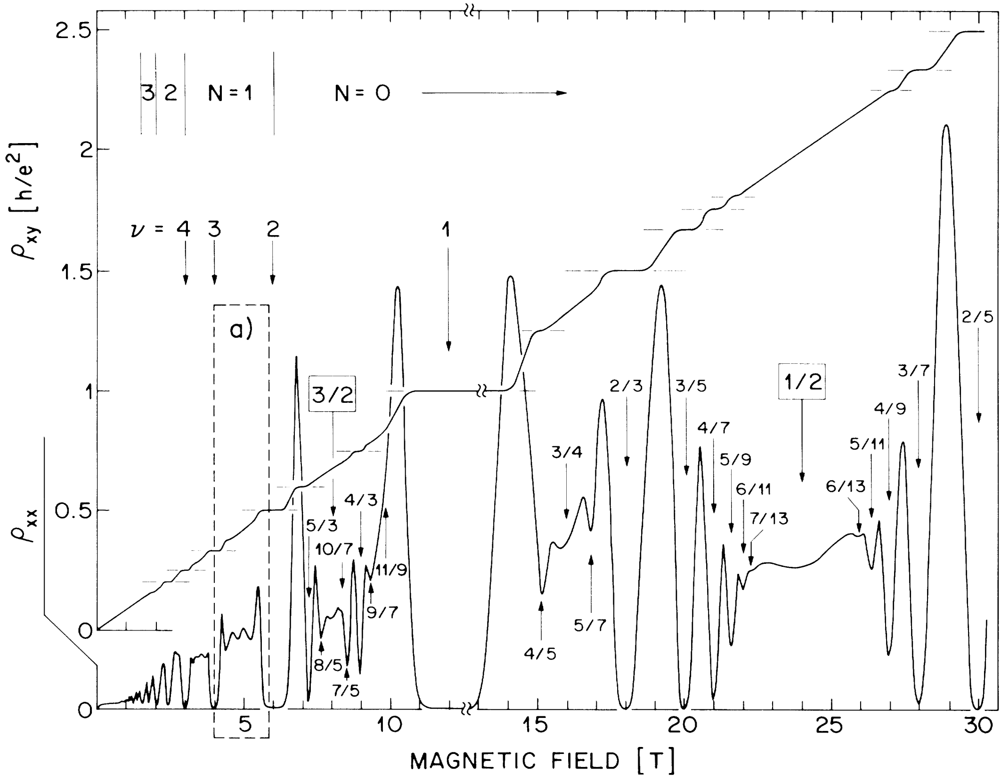
At integral and fractional $\nu = \frac{\rho \phi_0}{B}$

- vanishing longitudinal resistance
- quantized Hall resistance

$\phi_0 \equiv hc/e$: flux quantum

ρ : two-dimensional electron density

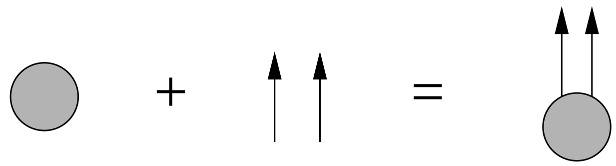


Willett, Eisenstein, Stormer, Tsui, Gossard, English (1987)

Composite Fermions [Jain (1989)]

bound states of electrons and an even number ($2p$) of quantized vortices

^{2}CF :

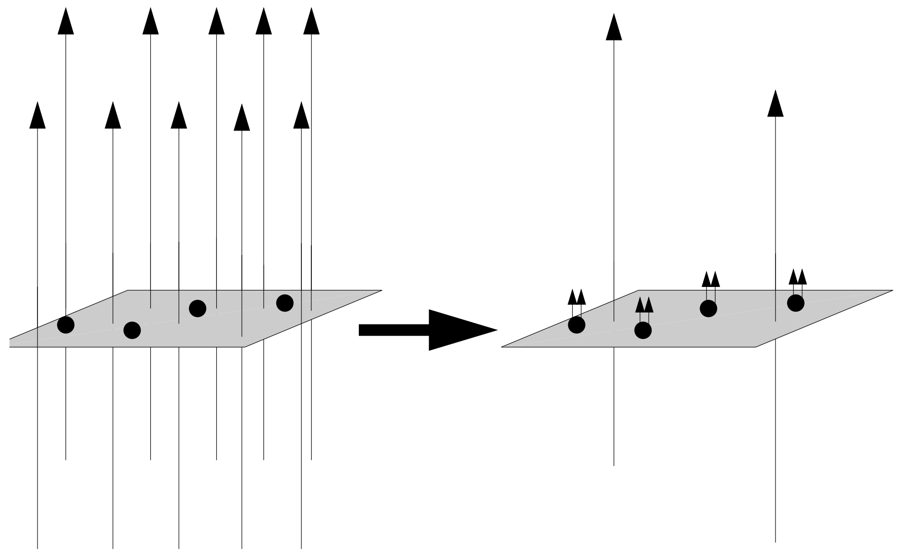


$$B^* = B - 2p\rho\phi_0$$

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

Recall

$$\nu = \frac{\rho\phi_0}{B}, \nu^* = \frac{\rho\phi_0}{B^*}$$



Composite-Fermion Wave Function

$$\Psi_\nu^J = \mathcal{P} \prod_{j<k} (z_j - z_k)^{2p} \cdot \Phi_{\nu^*}$$

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electrons at filling factor ν^*

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- attaches $2p$ vortices to each electron
- expands the system thereby reducing the filling factor

Composite-Fermion Wave Function

project into lowest Landau level

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CFs at filling factor $\nu = \dfrac{\nu^*}{2p\nu^* + 1}$

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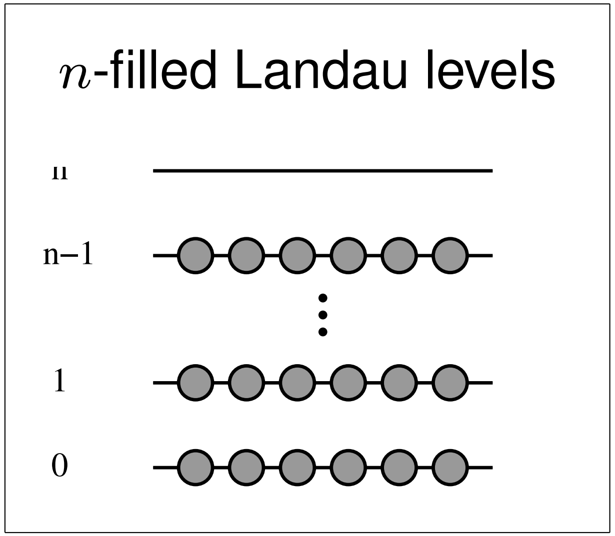
strongly interacting electrons → weakly interacting CFs

Integral quantum Hall effect at $\nu = n$

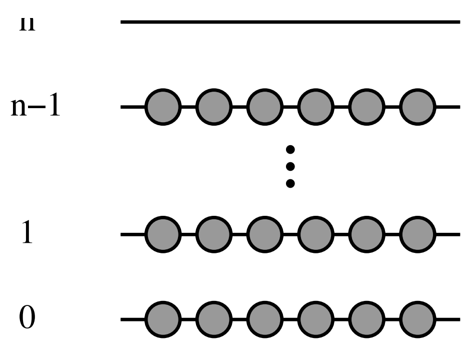
Φ_n:



Integral quantum Hall effect at $\nu = n$

n-filled Landau levels

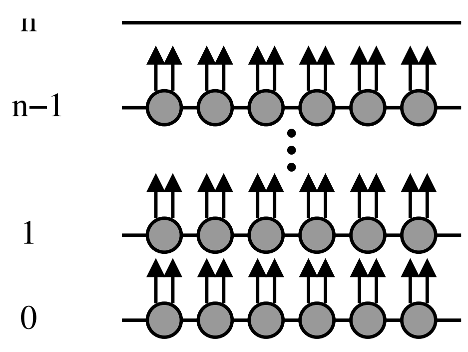
Φ_n:



Fractional quantum Hall effect at $\nu = \dfrac{n}{2pn+1}$

n-filled CF quasi-levels

$\Psi^J_{n/(2pn+1)}$:



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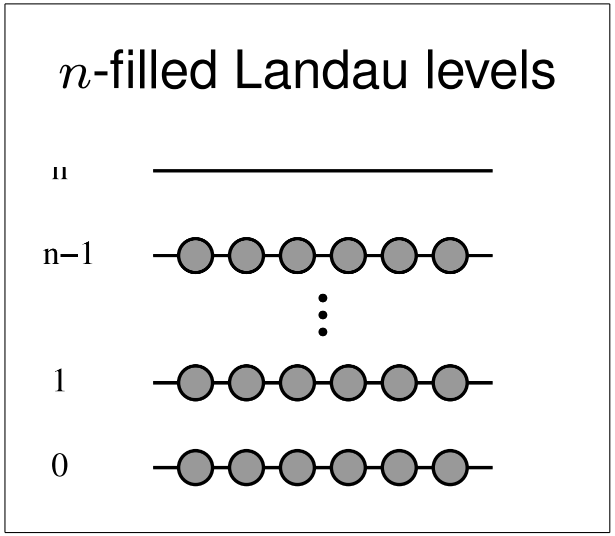

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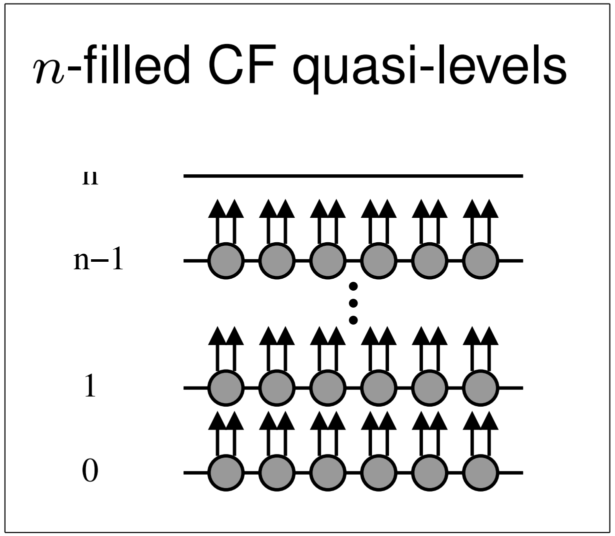


CF theory explains

the FQHE as

the IQHE of

composite fermions

$$\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots \text{ for } p = 1$$

Accuracy of Composite-Fermion Wave Function

relative errors are

smaller than 0.05%

Ground-state energy

ν	N	CF	exact
$\frac{2}{5}$	6	-0.5003	-0.5004
	8	-0.4802	-0.4802
	10	-0.4693	-0.4695
$\frac{3}{7}$	9	-0.4991	-0.4992
	12	-0.4825	-0.4826

Jain, Kamilla (1997)

Topological binding of electrons and vortices

Composite-fermion wave functions at $\nu = n/(2pn+1)$

$$\Psi_{n/(2pn+1)} = \mathcal{P} \prod_{j<k} (z_j - z_k)^{2p} \Phi_n(\{z_i\})$$

binds $2p$ vortices   fills LLs

complex vortex structures

Simple situation at $\nu = 1/m$ because ϕ_1 also has a simple vortex structure

$$\Psi^{(0)} = \prod_{j<k} (z_j - z_k)^m e^{-\sum_j |z_j|^2/4} \qquad \text{(Laughlin)}$$

: m vortices tied to each electron $(m = 2p + 1)$

Algebraic off-diagonal long-range order in a related bosonic wave function

[Girvin,MacDonald (1987)]

- gauge-transformed bosonic wave function Ψ_B

$$\Psi_B = \prod_{j<k} \left(\frac{|z_j - z_k|}{z_j - z_k} \right)^m \Psi$$

- one-particle reduced density matrix

$$G(\boldsymbol{r}, \boldsymbol{r}') \equiv \langle \Psi_B | c^\dagger(\boldsymbol{r}) c(\boldsymbol{r}') | \Psi_B \rangle$$

- algebraic ODLRO for $\Psi_B^{(0)}$

$$G^{(0)}(\boldsymbol{r}, \boldsymbol{r}') \propto |\boldsymbol{r} - \boldsymbol{r}'|^{-m/2} \quad \text{for } |\boldsymbol{r} - \boldsymbol{r}'| \gg \ell \qquad \ell \equiv \sqrt{\hbar c / eB}$$

cf.) no ODLRO for the fermionic FQHE wave function

$$\langle \Psi_{\mathrm{Fermion}} | c^\dagger(\boldsymbol{r}) c(\boldsymbol{r}') | \Psi_{\mathrm{Fermion}} \rangle \propto \exp(-|\boldsymbol{r} - \boldsymbol{r}'|^2 / 4)$$

- True ground state

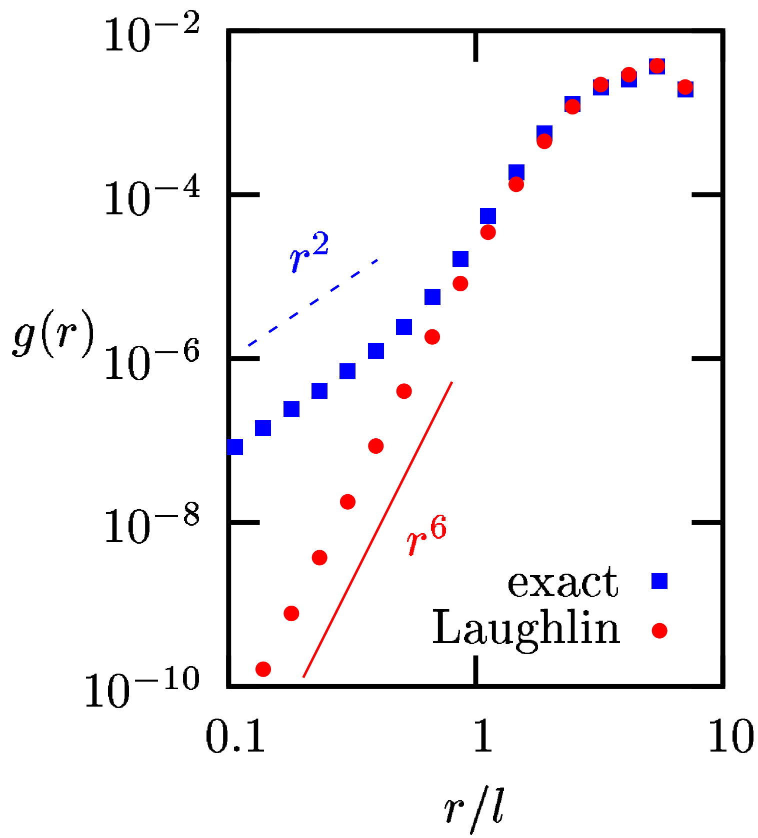
$$\Psi = \prod_{j<k} (z_j - z_k) F_S[\{z_i\}] e^{-\sum_j |z_j|^2/4}$$

$F_S[\{z_i\}]$: symmetric and analytic

Strictly speaking,

only one Pauli vortex is tied to each electron
i.e. bound vortex-antivortex pairs are produced
relative to Laughlin's wave function

pair correlation function



$N = 10, \nu = 1/3$

- Questions

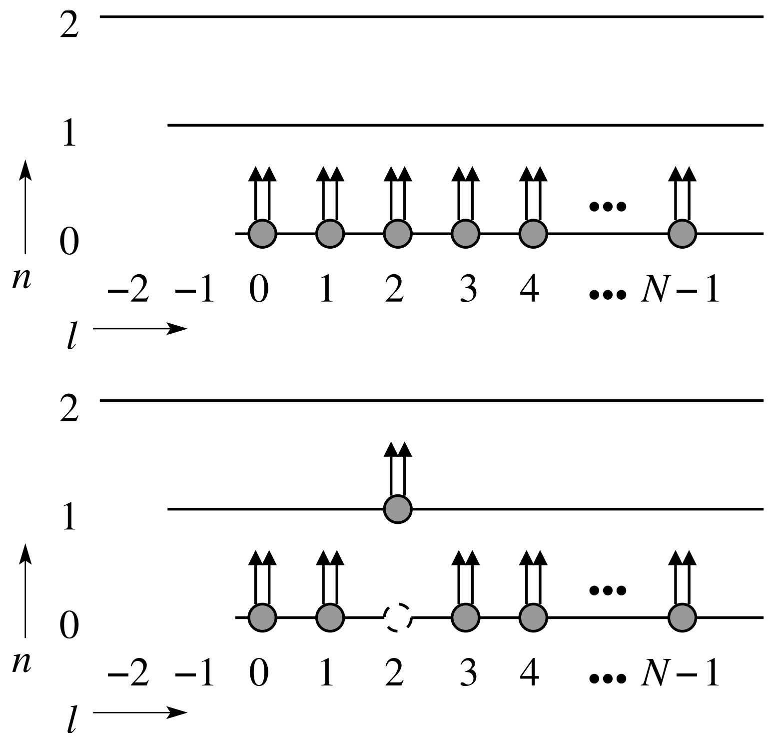
1. Does algebraic ODLRO persist for Ψ?
2. If it does, what is the exponent?

   Analogy to the KT transition might suggest a renormalization
   [cf. Girvin and MacDonald (1987)]

Improved Wave Functions at $\nu = 1/m$

$\Psi^{(0)}$: noninteracting composite fermions

Ψ' : better wave functions
obtained by CF diagonalization

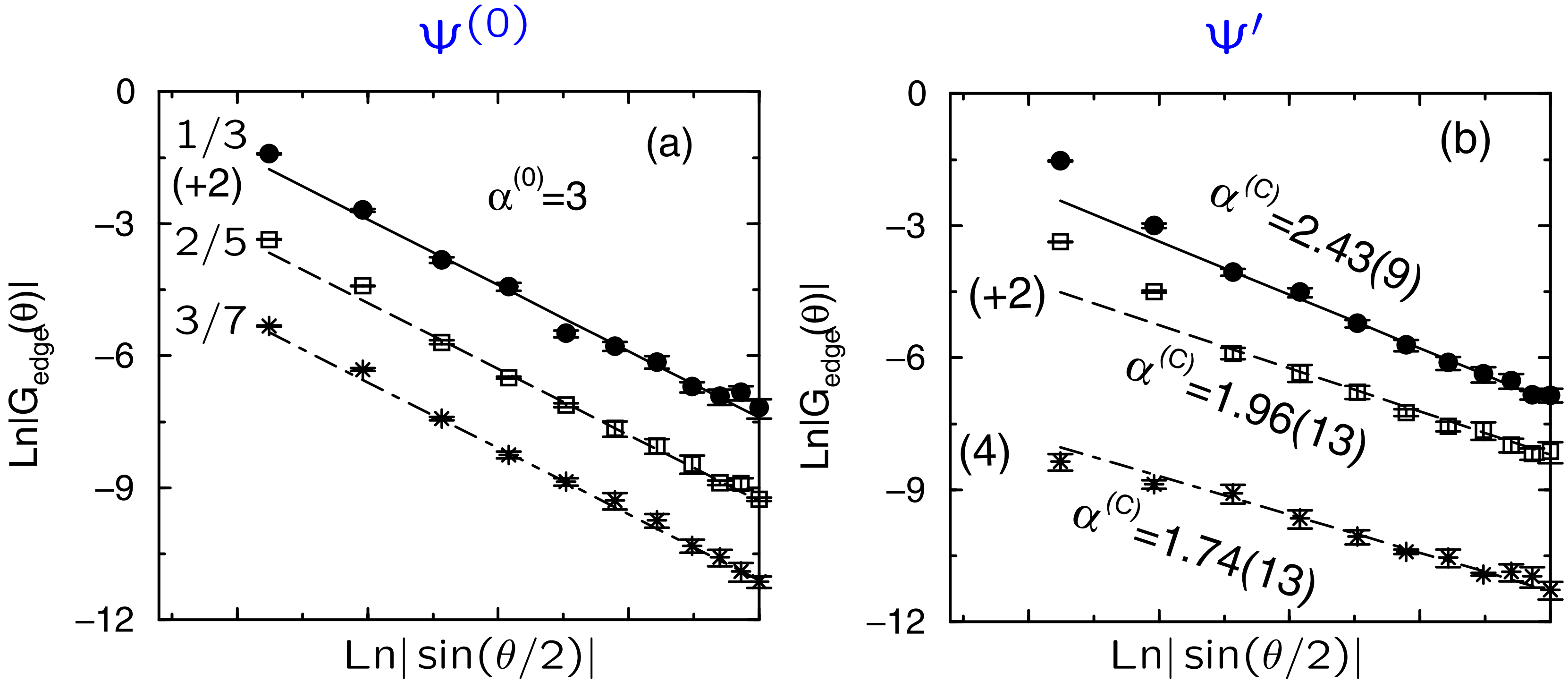


$\nu = 1/5$

N	$\|\langle\Psi^{(0)}\|\Psi'\rangle\|^2$	$E^{(0)}$	E'
17	0.73	13.693	13.683
18	0.72	15.066	15.055
19	0.68	16.487	16.475
20	0.69	17.952	17.940
21	0.71	19.465	19.452

Edge Exponent

$$G_{\text{edge}}(\boldsymbol{r},\boldsymbol{r}') = \langle\Psi|c^{\dagger}(\boldsymbol{r})c(\boldsymbol{r}')|\Psi\rangle \sim \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|^{\alpha_{\text{edge}}}}$$



Mandal,Jain(2002)

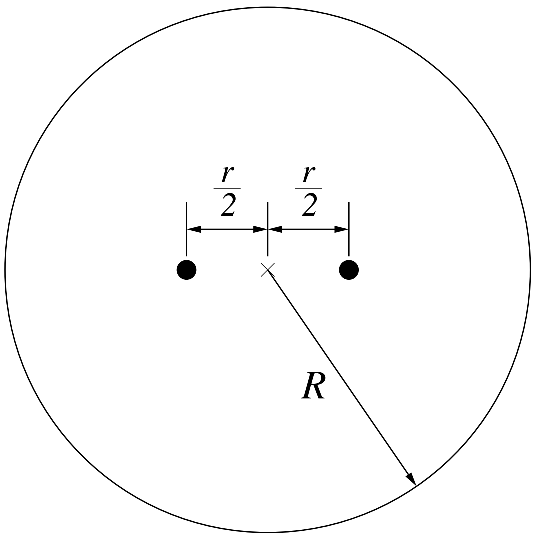
- $\alpha_{\text{edge}} = 3$ for $\Psi^{(0)}$.
- α_{edge} changes for Ψ'.

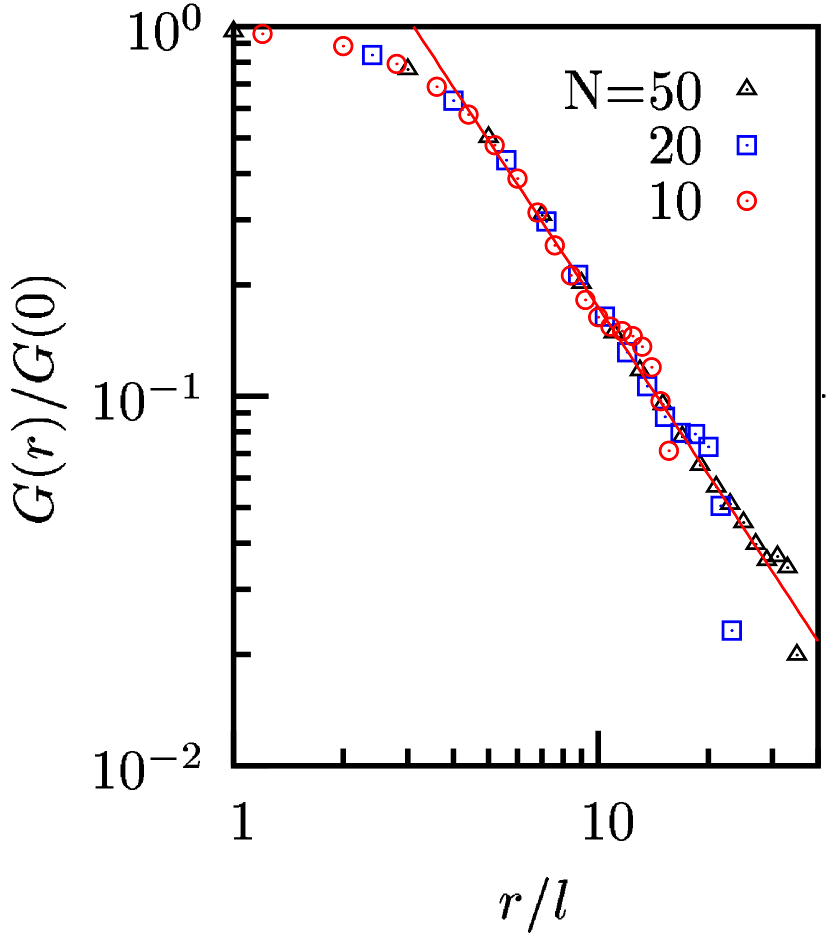
$\Psi^{(0)}$ at $\nu = 1/3$ in the disk geometry

- for $r \gg \ell$ (but $r < R$)

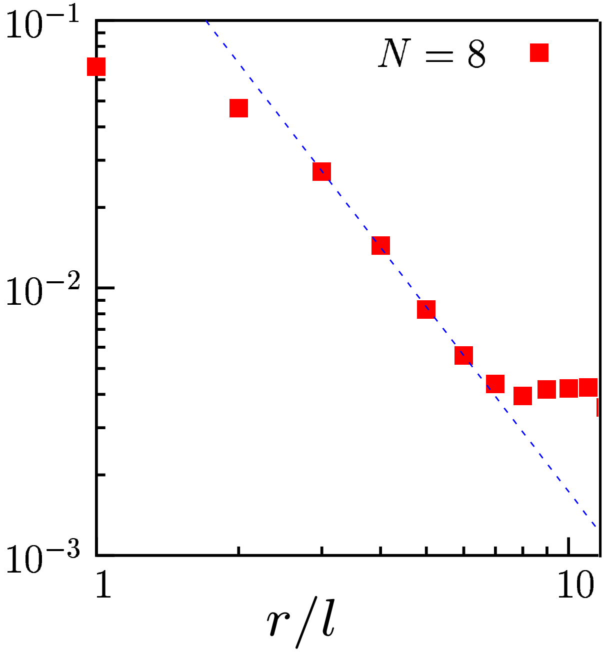
$$G(r) \equiv G\left(-\frac{r}{2}\hat{x}, \frac{r}{2}\hat{x}\right) \propto r^{-\alpha}$$

results consistent with $\alpha = 3/2$



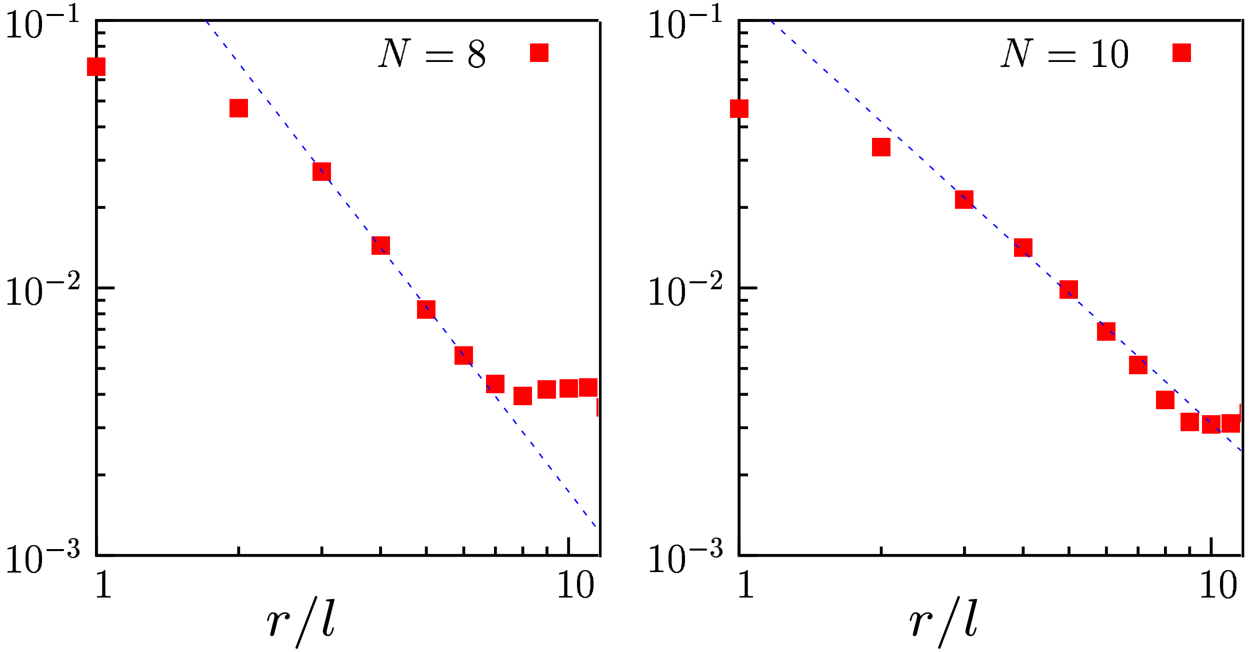


Ψ' at $\nu = 1/3$ in the disk geometry



$\alpha = 2.29$

Ψ' at $\nu = 1/3$ in the disk geometry



$\alpha = 2.29$ $\alpha = 1.61$

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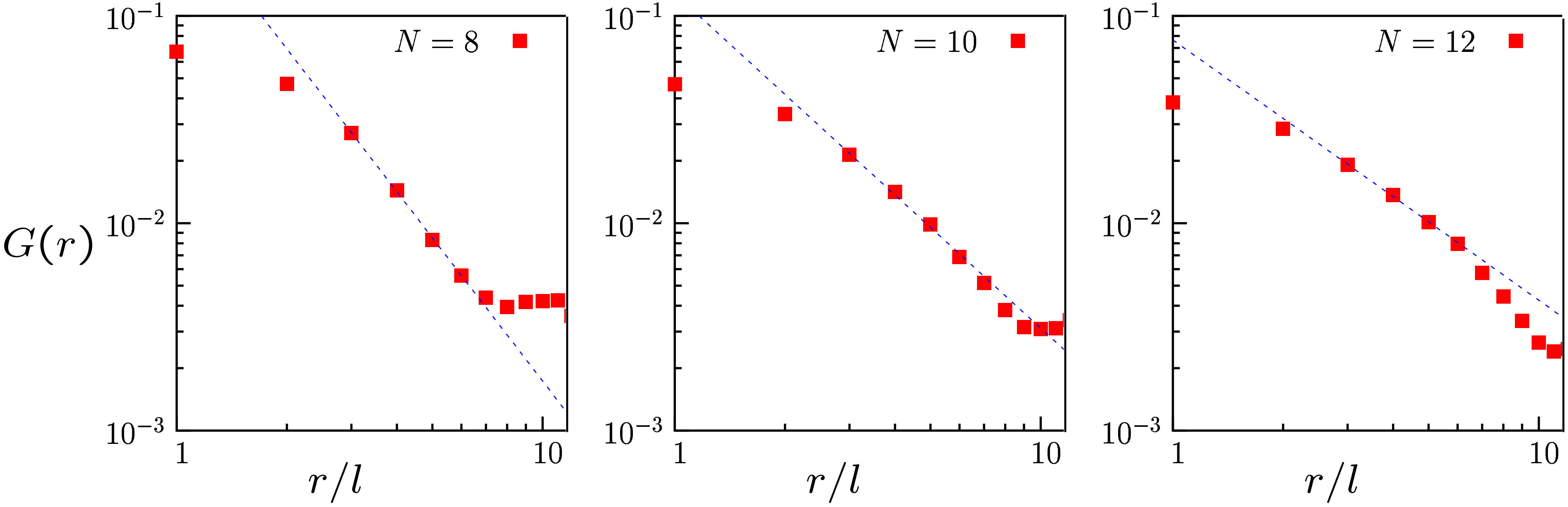
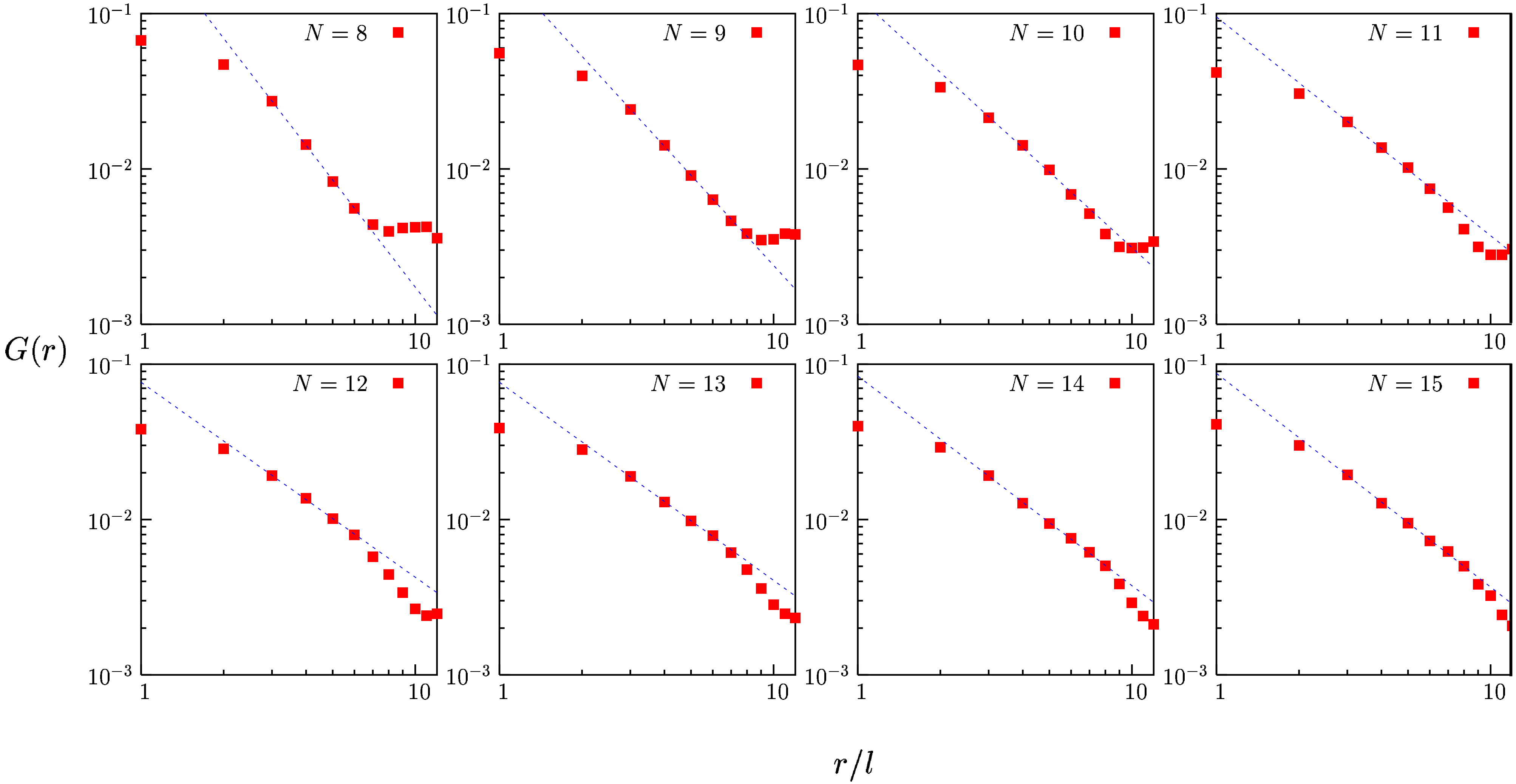


$\alpha = 2.29$ $\alpha = 1.61$ $\alpha = 1.25$

?

$\alpha > \; = \; < 3/2$

Ψ' at $\nu = 1/3$ in the disk geometry



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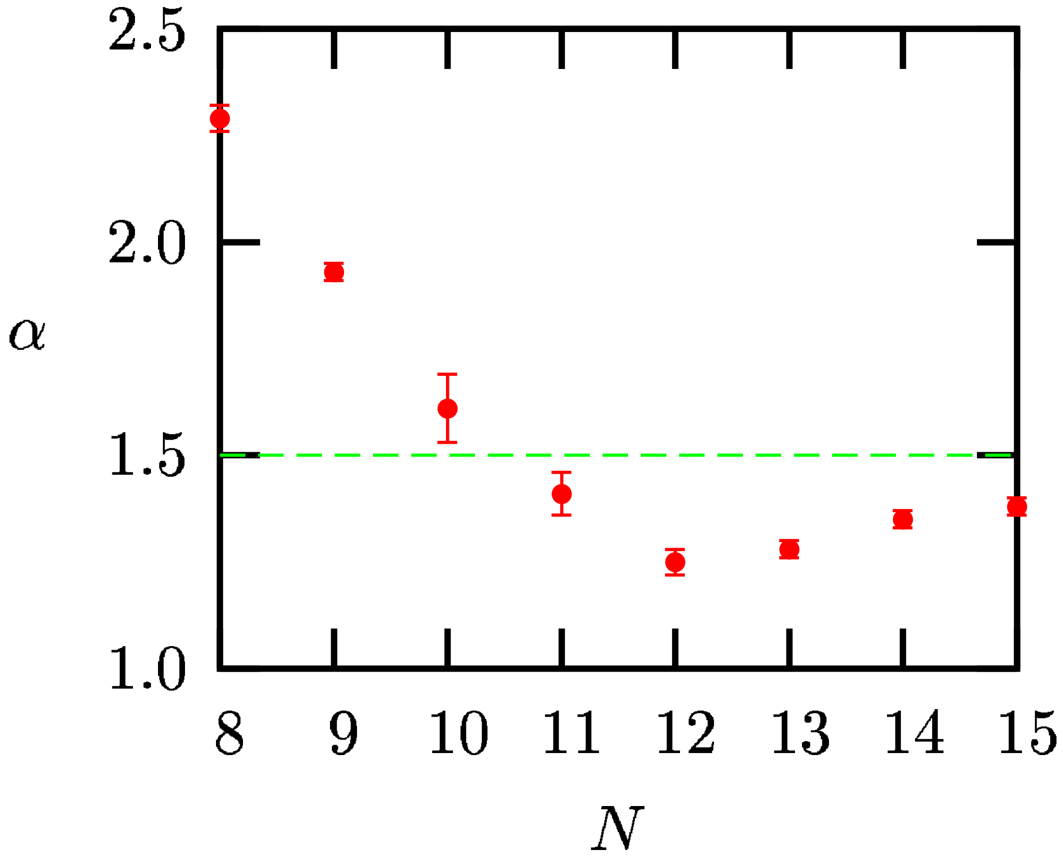
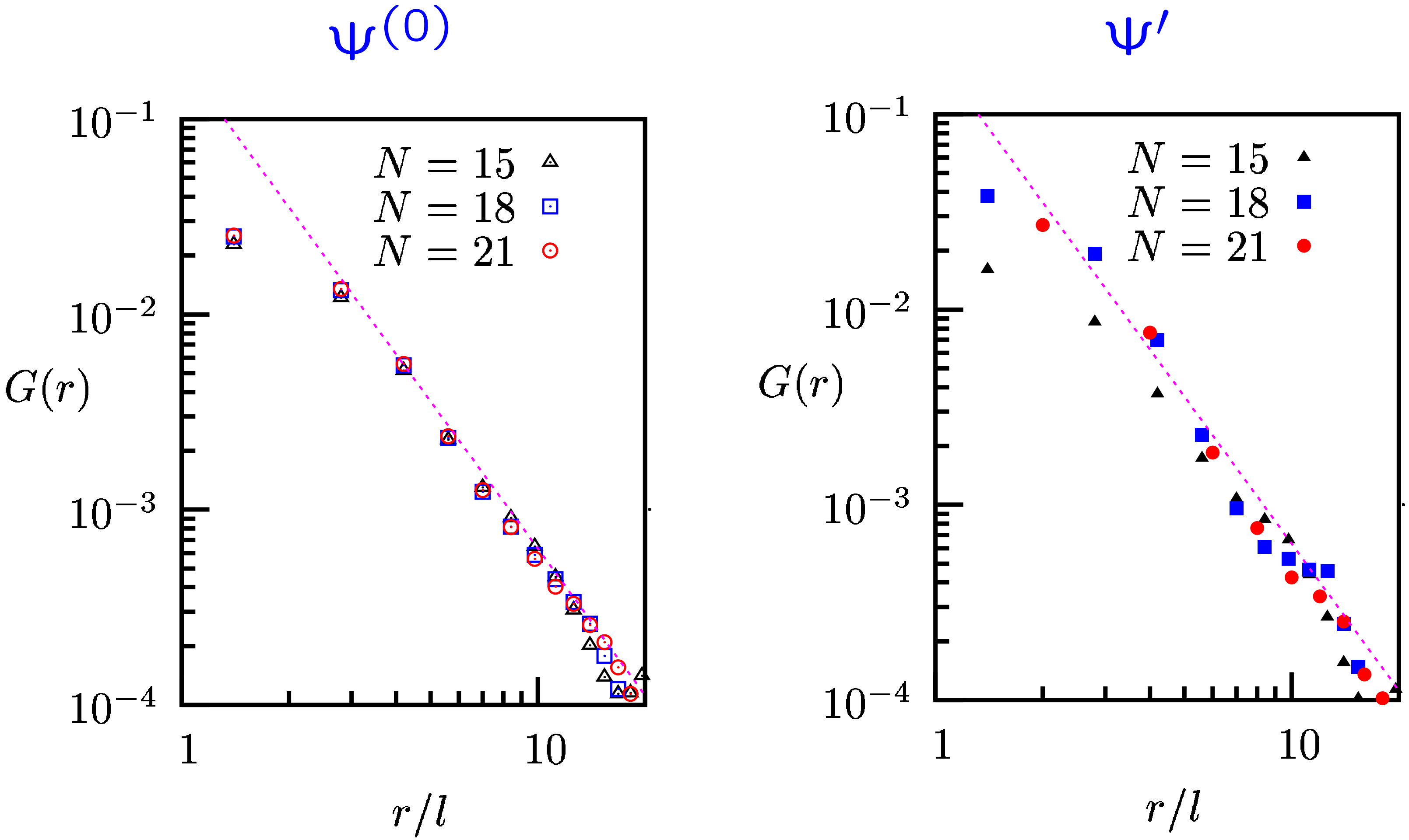


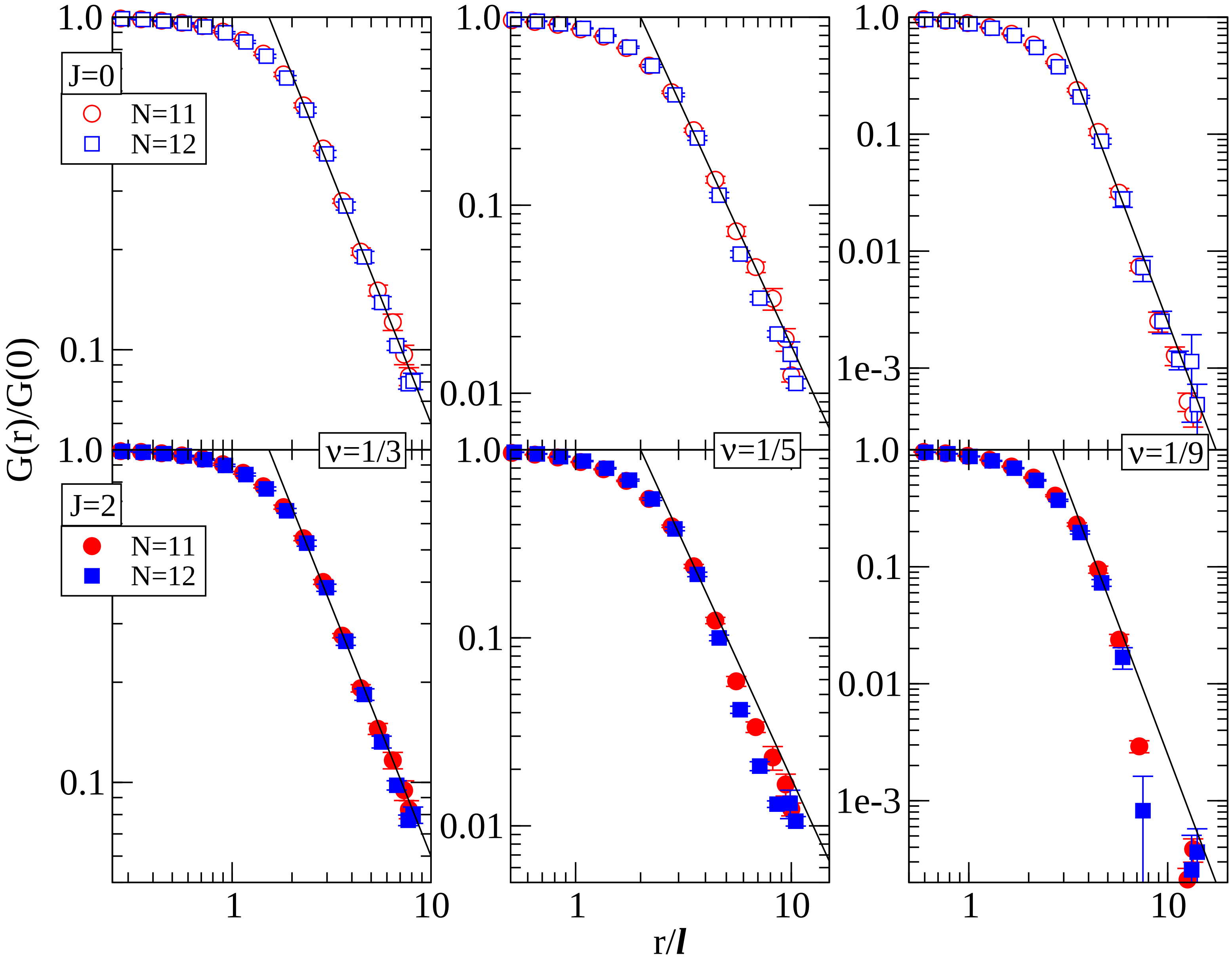
α appears to approach $3/2$

At $\nu = 1/5$ in the disk geometry



- $\Psi^{(0)}$ good power-law with $\alpha = 5/2$
- Ψ' large finite-size effects
  but consistent with $\alpha = 5/2$

In the spherical geometry



Summary of Part I

- Our calculation confirms
  an algebraic off-diagonal long-range order in bosonic wave functions
  is robust for a wide class of FQHE wave functions at $\nu = 1/m$.

- The exponents seem to be universal:

$$\alpha = m/2 \qquad (\nu = 1/m)$$

- As far as the long-distance behavior is concerned,
  the $1/m$ FQHE states behave as if m vortices were bound to each electron.

Other formulation of ODLRO at $\nu = 1/m$

Destruction of an electron at point η from $|\Psi^{(N+1)}\rangle$

$$|\Psi_1^{(N)}\rangle \equiv \hat{\psi}(\eta)|\Psi^{(N+1)}\rangle$$

Creation of m vortices at η from $|\Psi^{(N)}\rangle$

$$|\Psi_2^{(N)}\rangle \equiv \prod_{j=1}^{N}(z_j - \eta)^m|\Psi^{(N)}\rangle$$

"local electron-vortex binding amplitude" $\mathcal{B}$

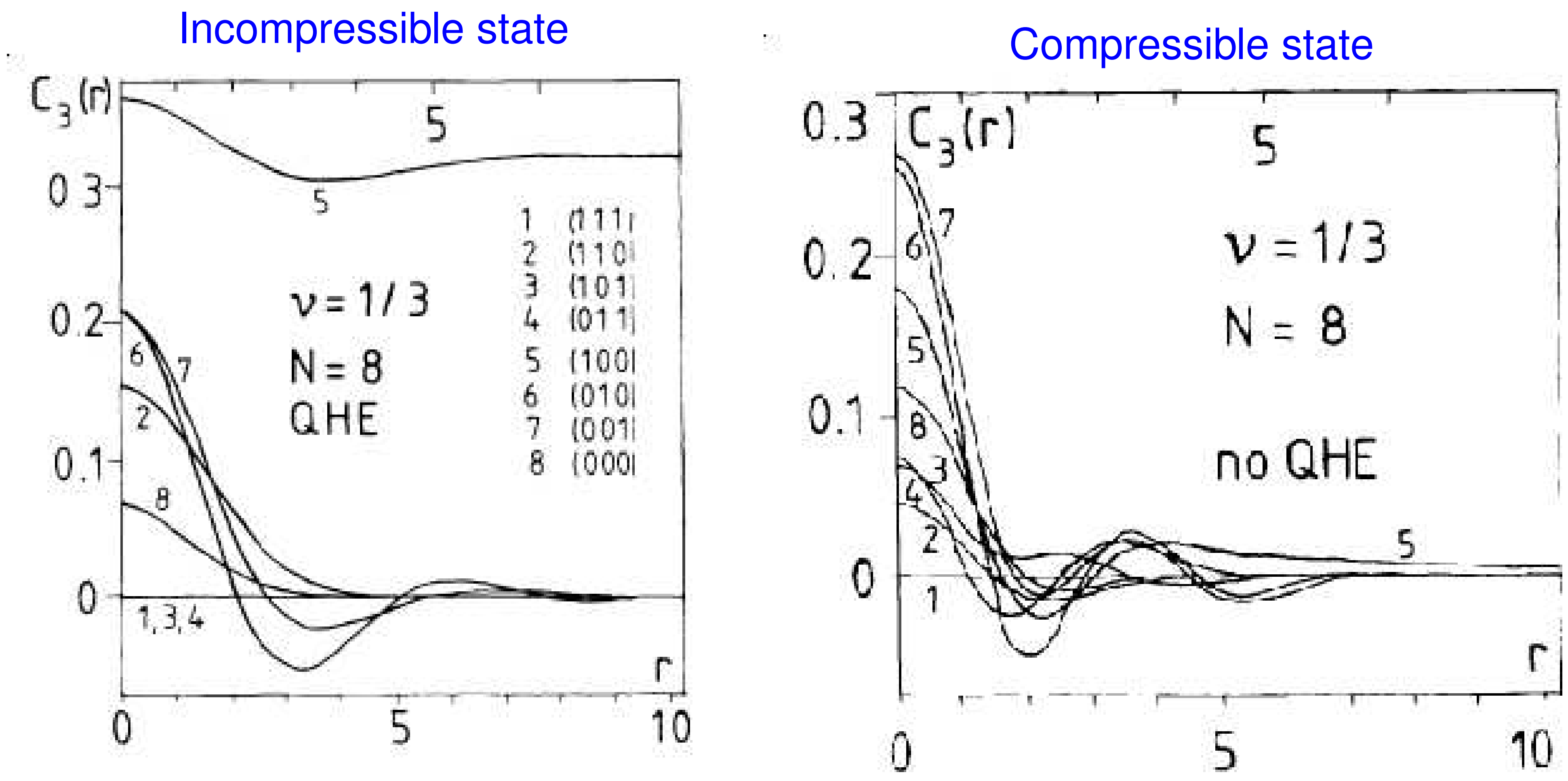
$$\mathcal{B}_\eta^{(N)} = \frac{\langle\Psi_1^{(N)}|\Psi_2^{(N)}\rangle}{\sqrt{\langle\Psi_1^{(N)}|\Psi_1^{(N)}\rangle\langle\Psi_2^{(N)}|\Psi_2^{(N)}\rangle}}$$

cf.) For Laughlin wave function

m vortices are strictly bound to each electron

$\mathcal{B}_\eta = 1$ for any η

In spherical geometry *Rezayi,Haldane(1988)*



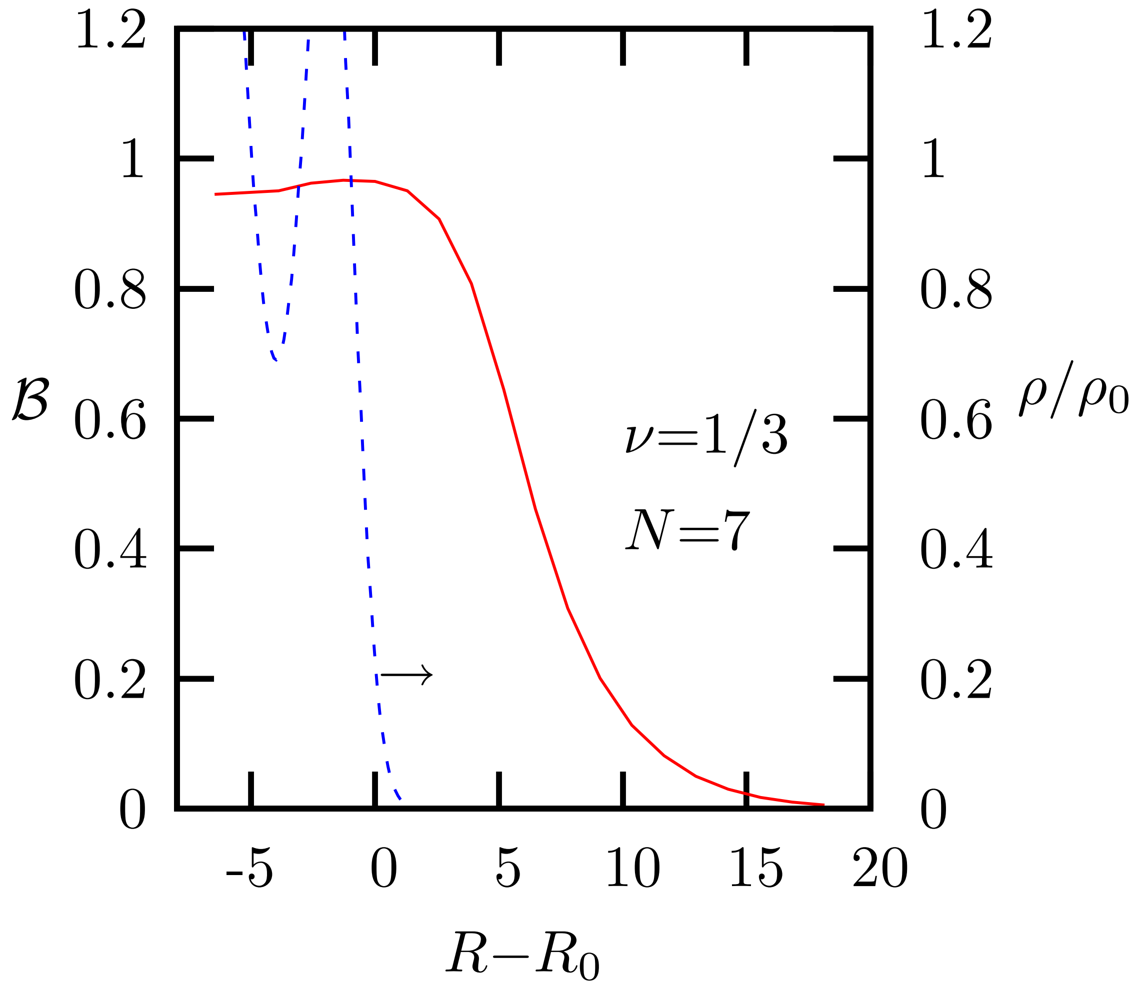
Similar quantity is finite only for incompressible state in bulk.

Electron-vortex binding at the edge ($\nu=1/3$, exact study)

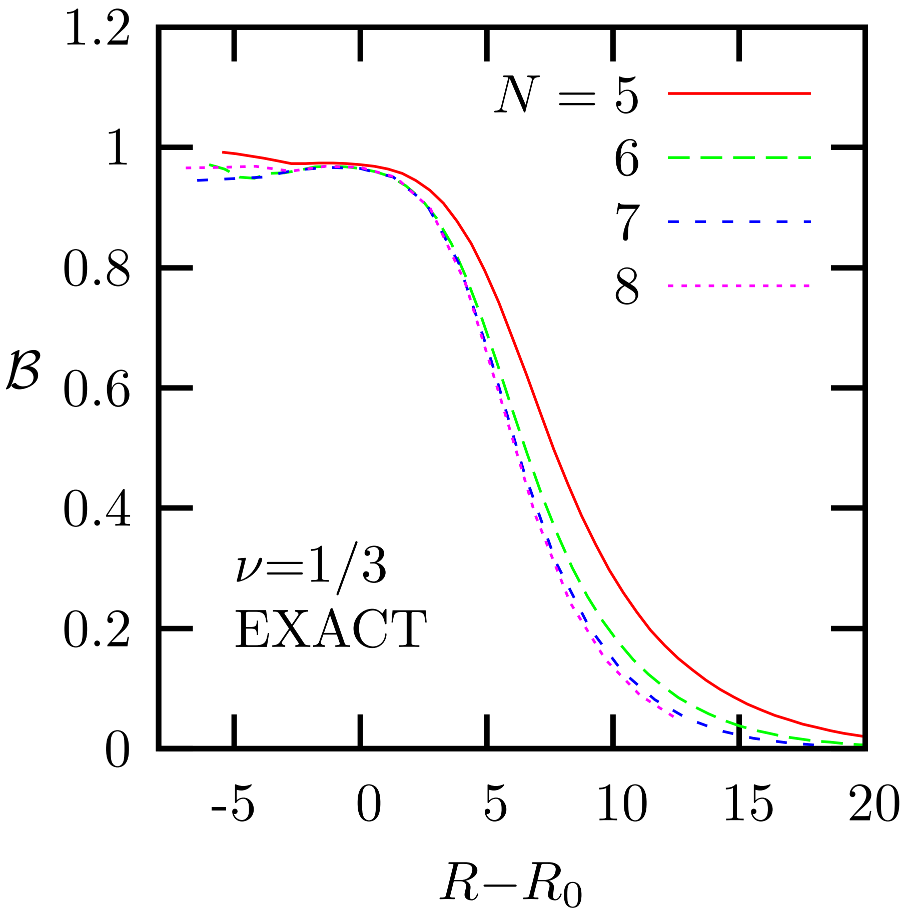
- Disk geometry is useful for the edge study.

- $\mathcal{B}$ decreases outside the edge. $(R_0 \equiv \sqrt{2N/\nu}$: standard edge)
  Note that $\mathcal{B} = 1$ for Laughlin wave function

- $\mathcal{B}$ shows rather slower decay compared with the density



Size-dependence (ν=1/3, exact study)



$\mathcal{B}$ decreases gradually

with N outside the edge.

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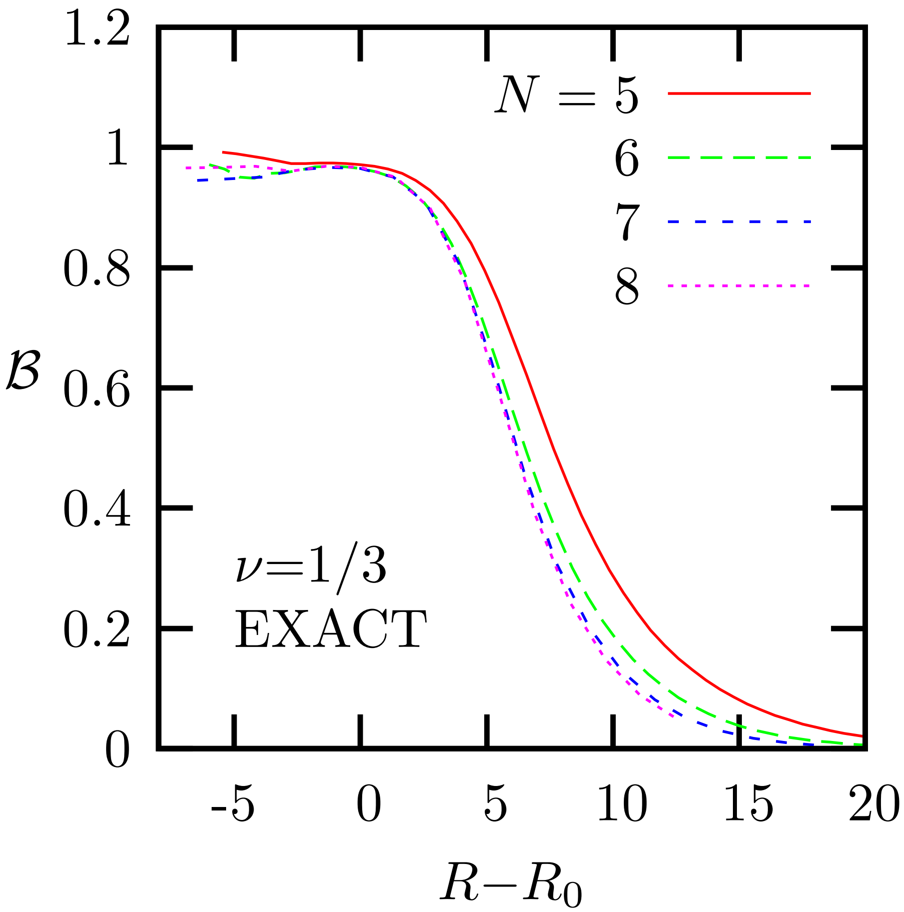

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Question

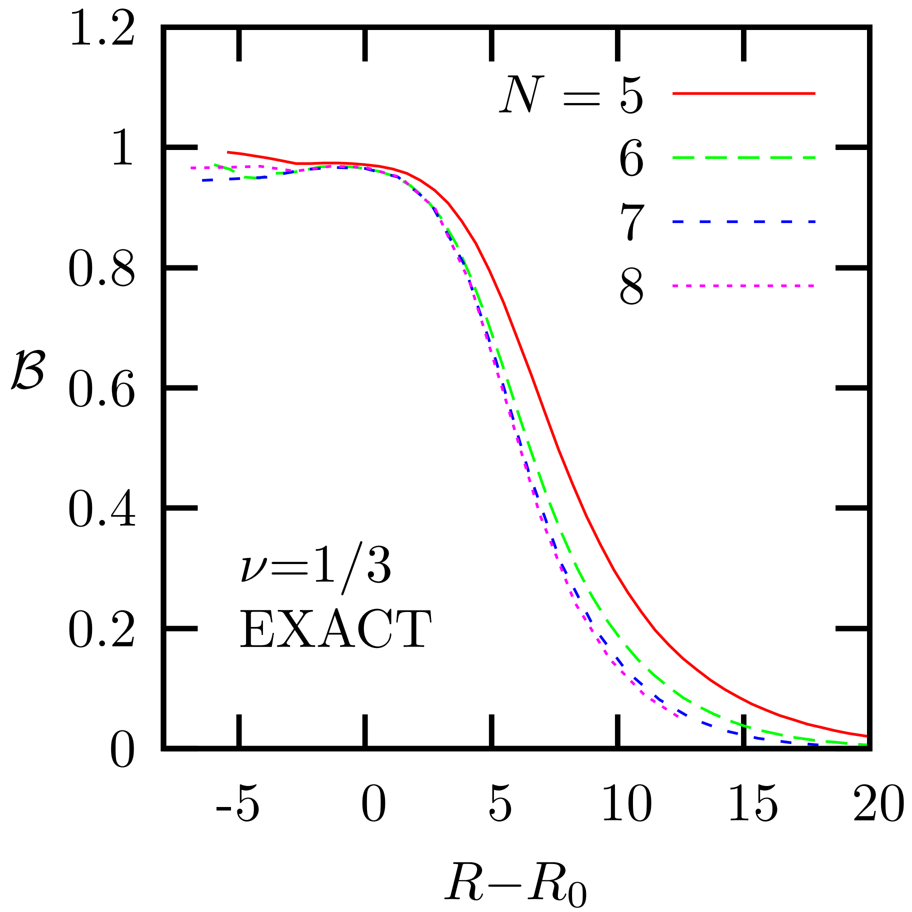
Does $\mathcal{B}$ vanish,

in the thermodynamic limit,

beyond a certain crtical distance

outside the edge?

Size-dependence (ν=1/3, exact study)



$\mathcal{B}$ decreases gradually with N outside the edge.

Question

Does $\mathcal{B}$ vanish, in the thermodynamic limit, beyond a certain crtical distance outside the edge?

: A reliable estimate of the themodynamic behavior requires systems larger than those accessible in exact studies.

CF$^{(1)}$ wave functions at $\nu = 1/m$ by CF diagonalization

Use basis functions with

up to one unit of “kinetic energy”

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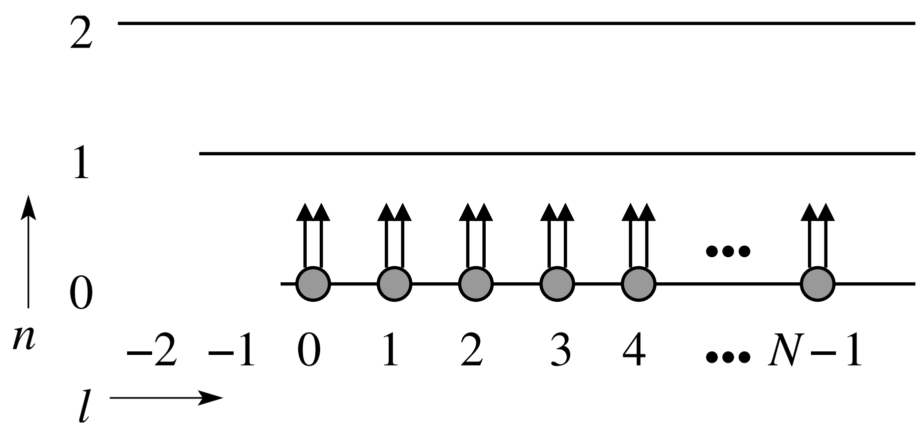
Use basis functions with

up to one unit of "kinetic energy"



CF$^{(1)}$ wave functions at $\nu = 1/m$ by CF diagonalization

Use basis functions with

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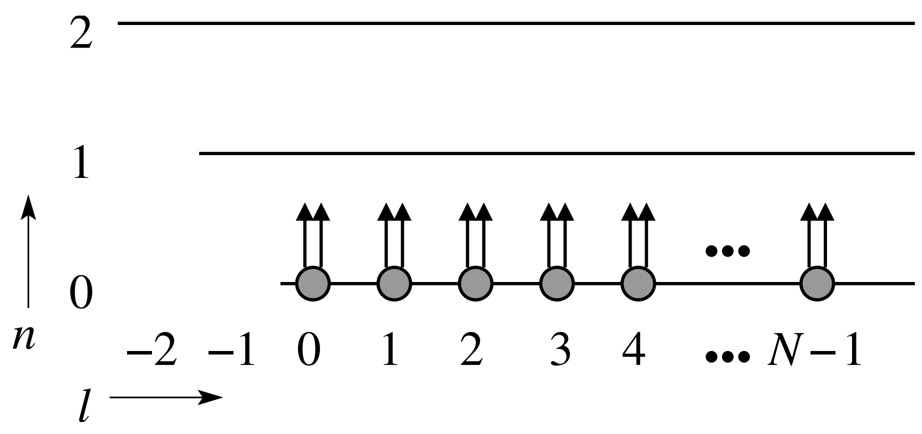

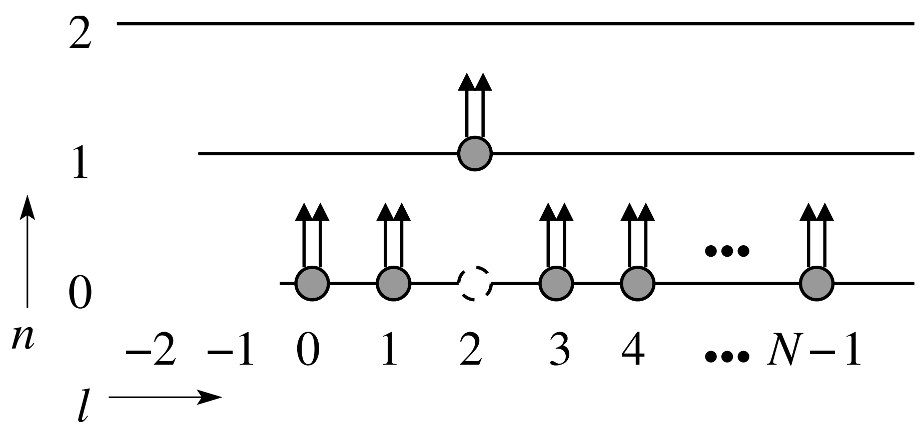

CF$^{(1)}$ wave functions at $\nu = 1/m$ by CF diagonalization

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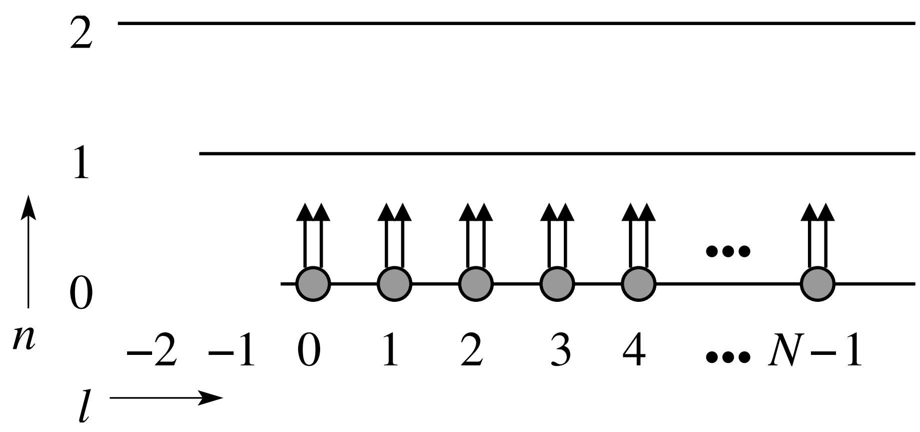
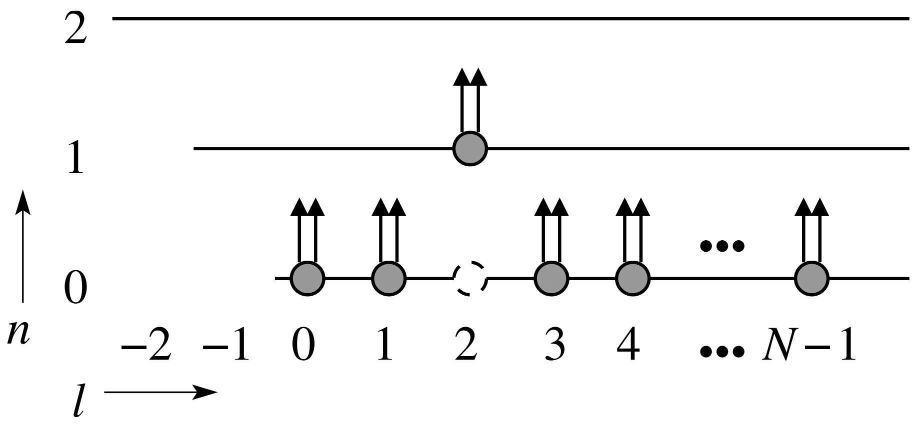
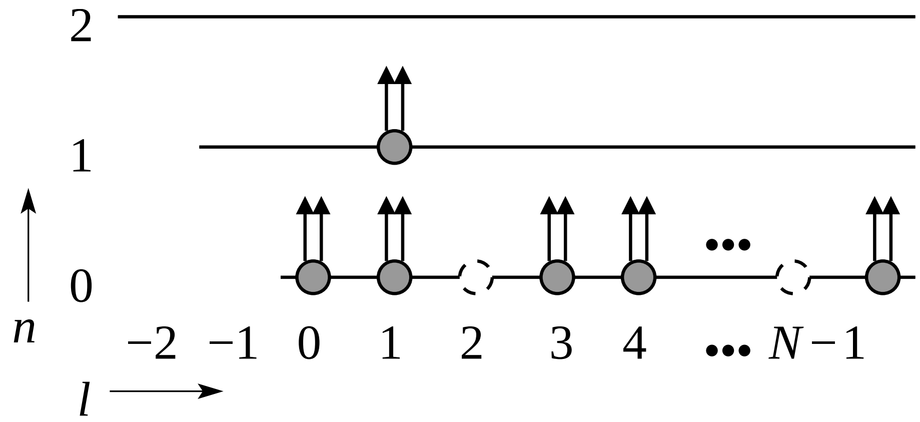




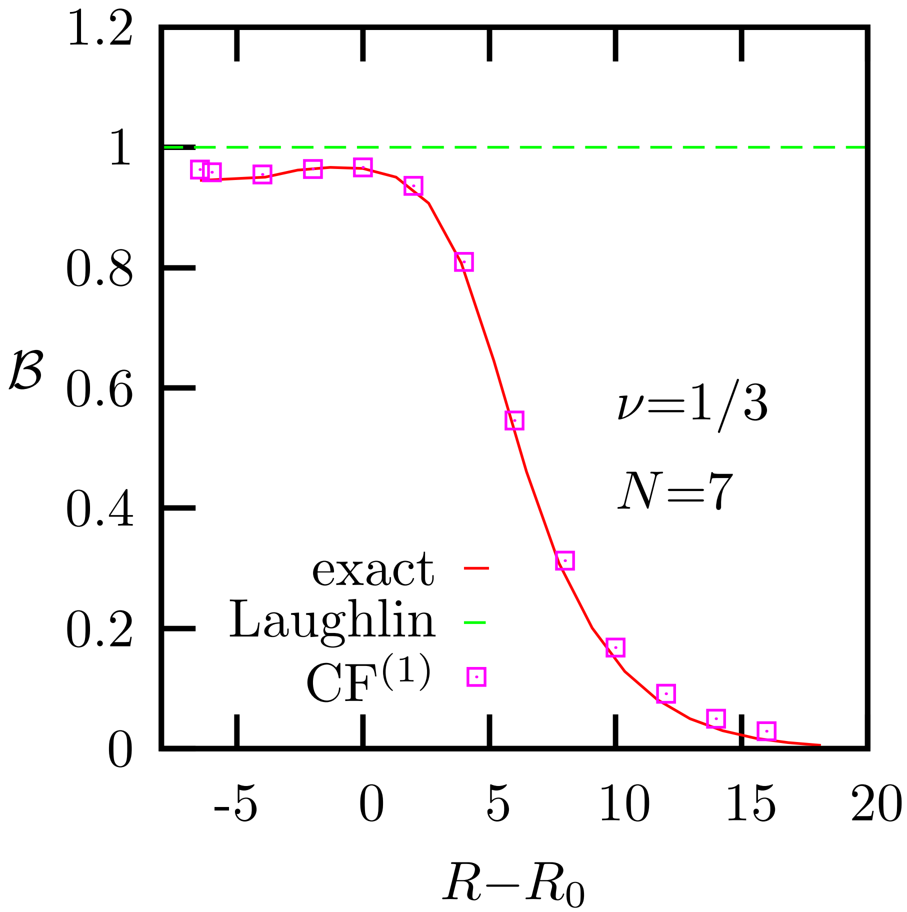


CF$^{(1)}$ wave functions at $\nu = 1/3$

N	D_{ex}	$D_{\text{CF}}^{(1)}$	E_{ex}	$E_{\text{CF}}^{(1)}$	$\langle\Psi_{\text{ex}}\|\Psi_{\text{CF}}^{(1)}\rangle$	$\langle\Psi_{\text{L}}\|\Psi_{\text{CF}}^{(1)}\rangle$
5	192	17	2.0273	2.0273(05)	0.9998(1)	0.9842(1)
6	1206	28	2.8602	2.8606(02)	0.9992(3)	0.9830(1)
7	8033	43	3.7949	3.7953(06)	0.9978(4)	0.9603(2)
8	55974	65	4.8299	4.8310(09)	0.9976(3)	0.9659(2)
9	403016	95	5.9559	5.9575(06)	0.9965(11)	0.9732(2)
10	2977866	137	7.1671	7.1679(29)		0.9692(2)
11	22464381	193		8.4610(13)		0.9665(2)
12	172388026	270		9.8318(20)		0.9635(2)

Electron-vortex binding at the edge ($\nu = 1/3$, CF$^{(1)}$ study)

CF$^{(1)}$ results reproduce exact behavior of $\mathcal{B}$ both inside and outside the edge.



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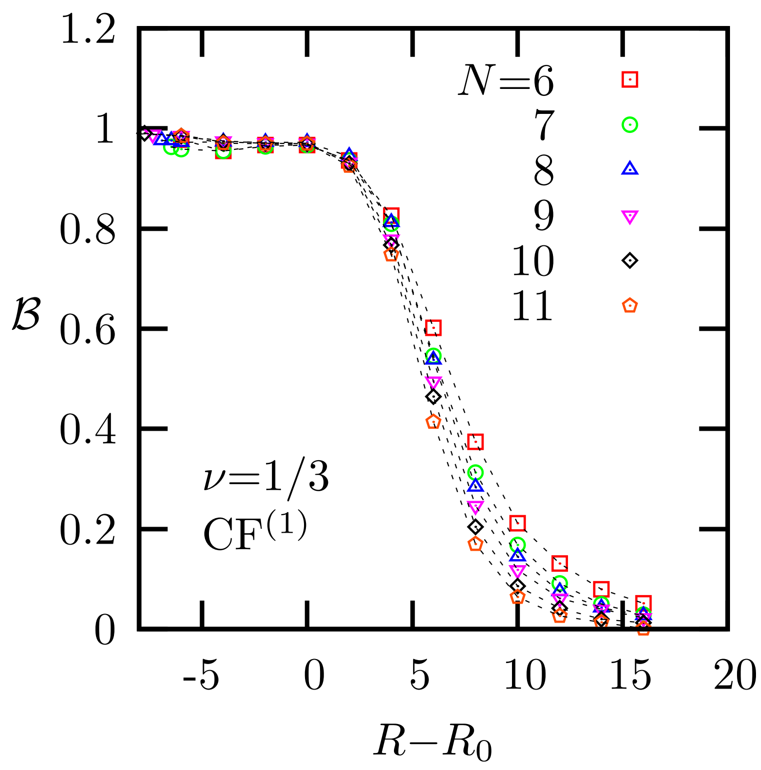
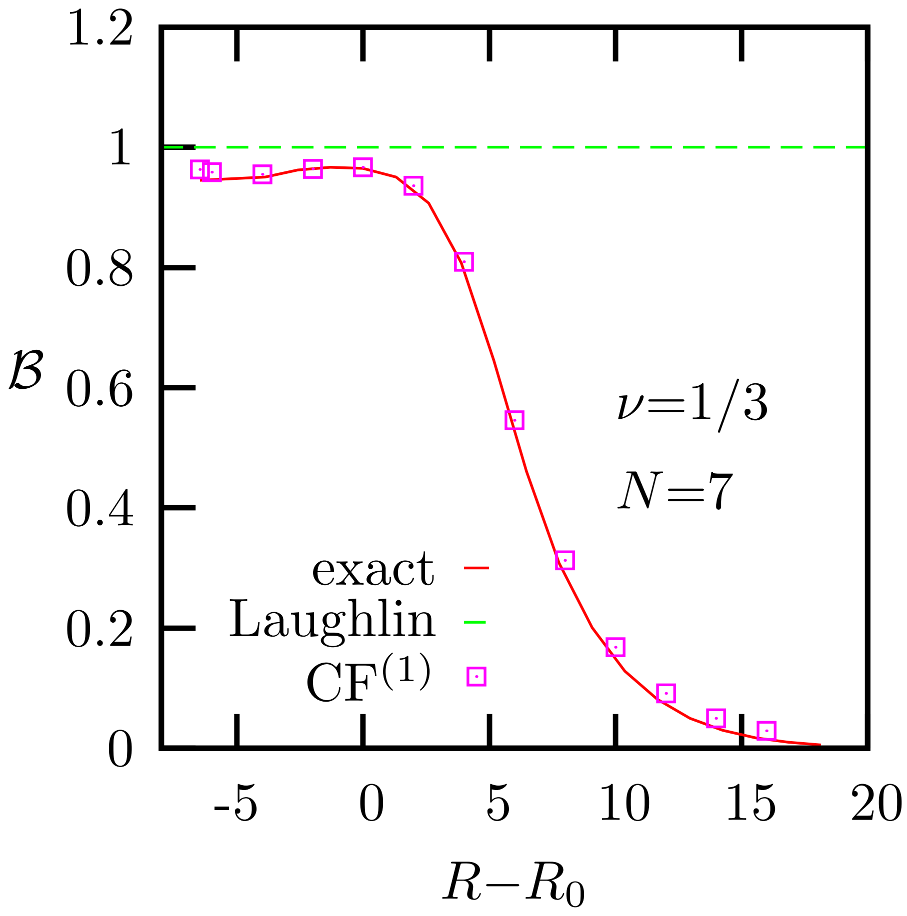
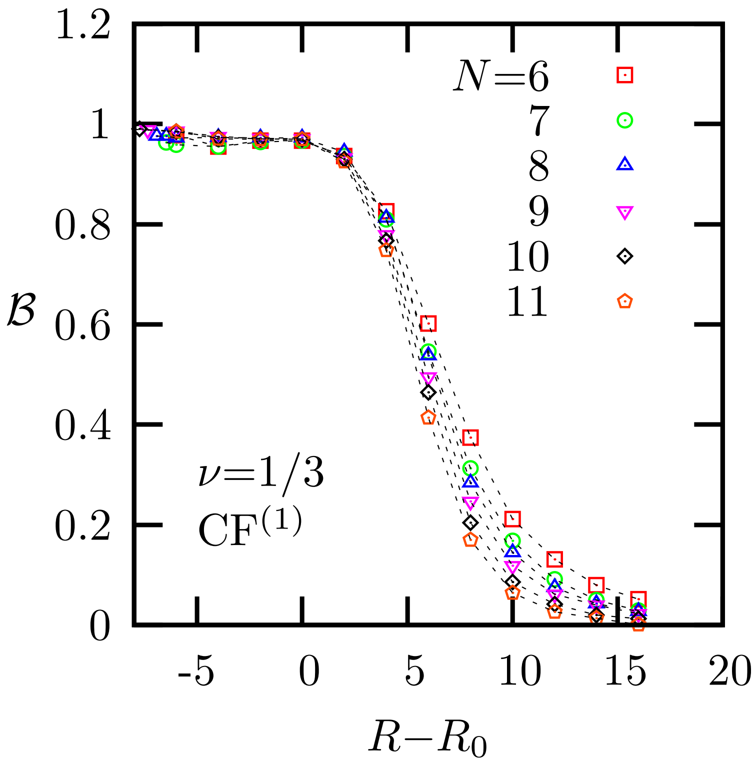


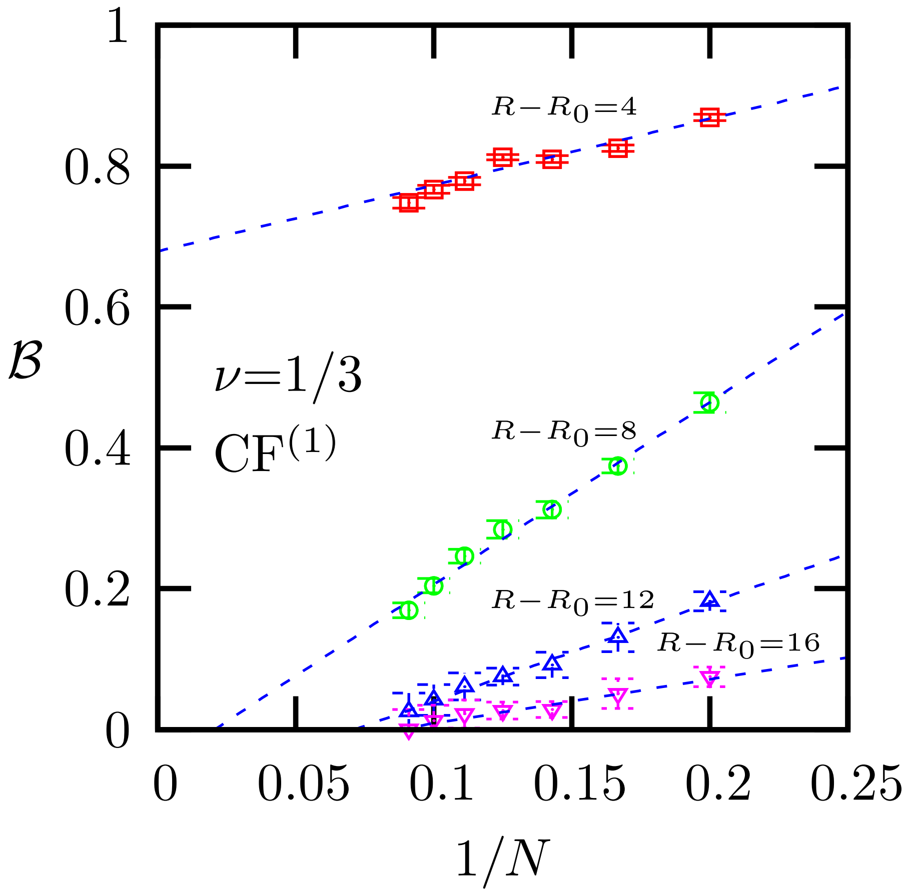


Thermodynamic limit of binding amplitude($\nu = 1/3$, CF$^{(1)}$ study)

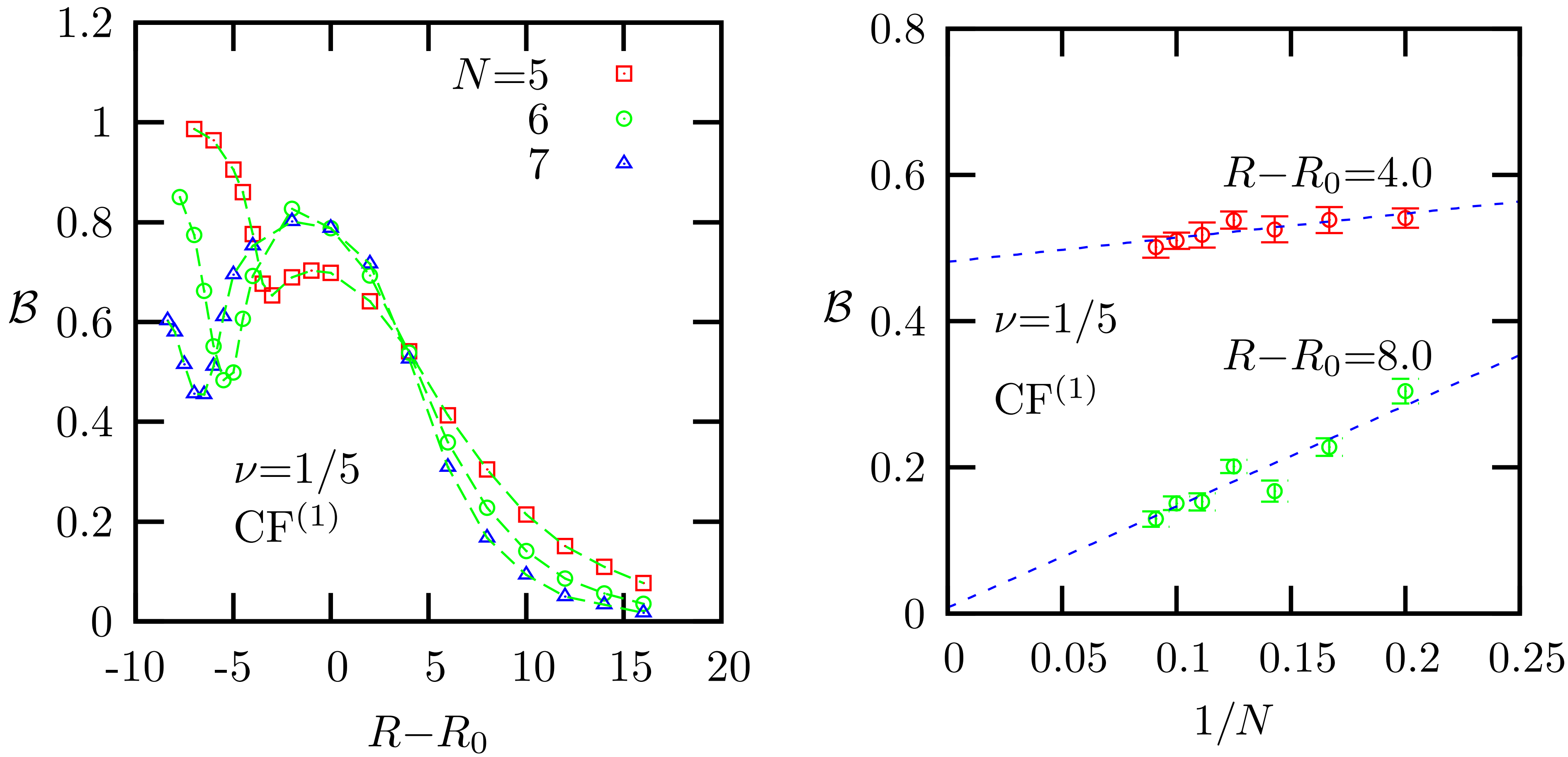
Existence of a critical distance

$\approx 7\ell$





Electron-vortex binding ($\nu = 1/5$, CF$^{(1)}$ study)



Larger fluctuations in $\mathcal{B}$ with N.

Stronger finite-size effect

Unbinding at a critical distance from the edge

Summary of Part II

- Computation of local electron-vortex binding amplitude indicates that electron and vortices are not bound beyond a certain critical distance from the edge in the thermodynamic limit.

- A rough estimate of the critical distance at $\nu = 1/3$ is 7 magnetic legnths. It is notable that electron density is extremely small at that distance.

Composite fermion crystal

Yi and Fertig (1998); Narevich, Murthy, and Fertig (2001)

Q. At very small ν (very large total angular momentum L), particles are far from one another. Do we get a crystal of electrons? (The overlap between neighboring wave packets is $\exp(-3.627/\nu)$. For $\nu = 1/9$, the overlap is $\sim 10^{-15}$.)

A. No. The ground state is an inherently quantum mechanical crystal of composite fermions.

Try the following wave functions:

- Hartree-Fock electron crystal : ψ_L^{EC}
- composite-fermion crystal : $\psi_L^{\text{CF}} = \prod_{j<k} (z_j - z_k)^{2p} \psi_{L^*}^{\text{EC}}$

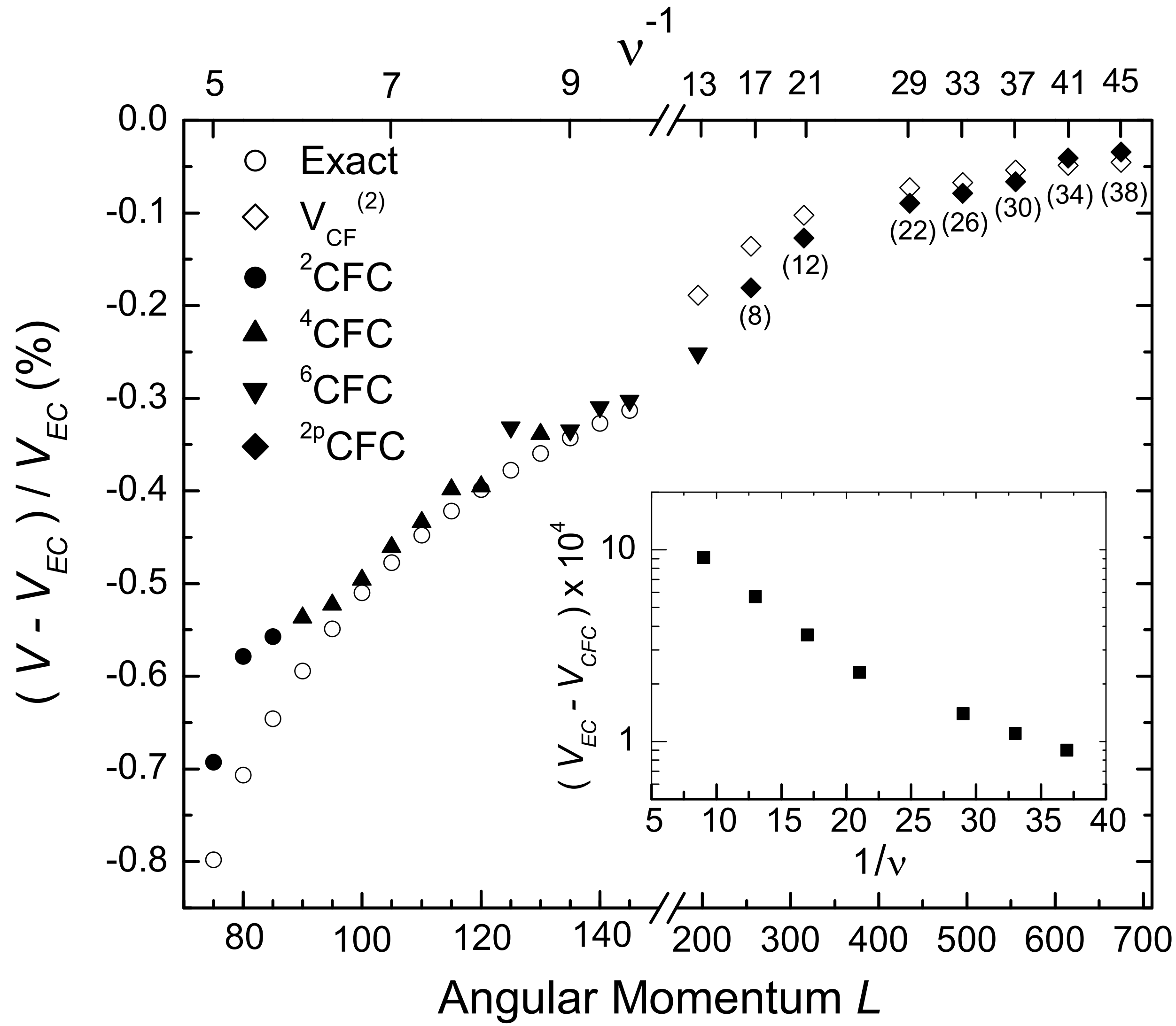
$$L^* = L - pN(N-1)$$

L : total angular momentum

Determine the optimal CF crystal by minimizing the energy

with respect to the flavor $(2p)$

Energy of crystals ($N = 6$)



Overlap

$|\langle\Psi^{\text{trial}}|\Psi^{\text{exact}}\rangle|^2/\langle\Psi^{\text{trial}}|\Psi^{\text{trial}}\rangle\langle\Psi^{\text{exact}}|\Psi^{\text{exact}}\rangle$

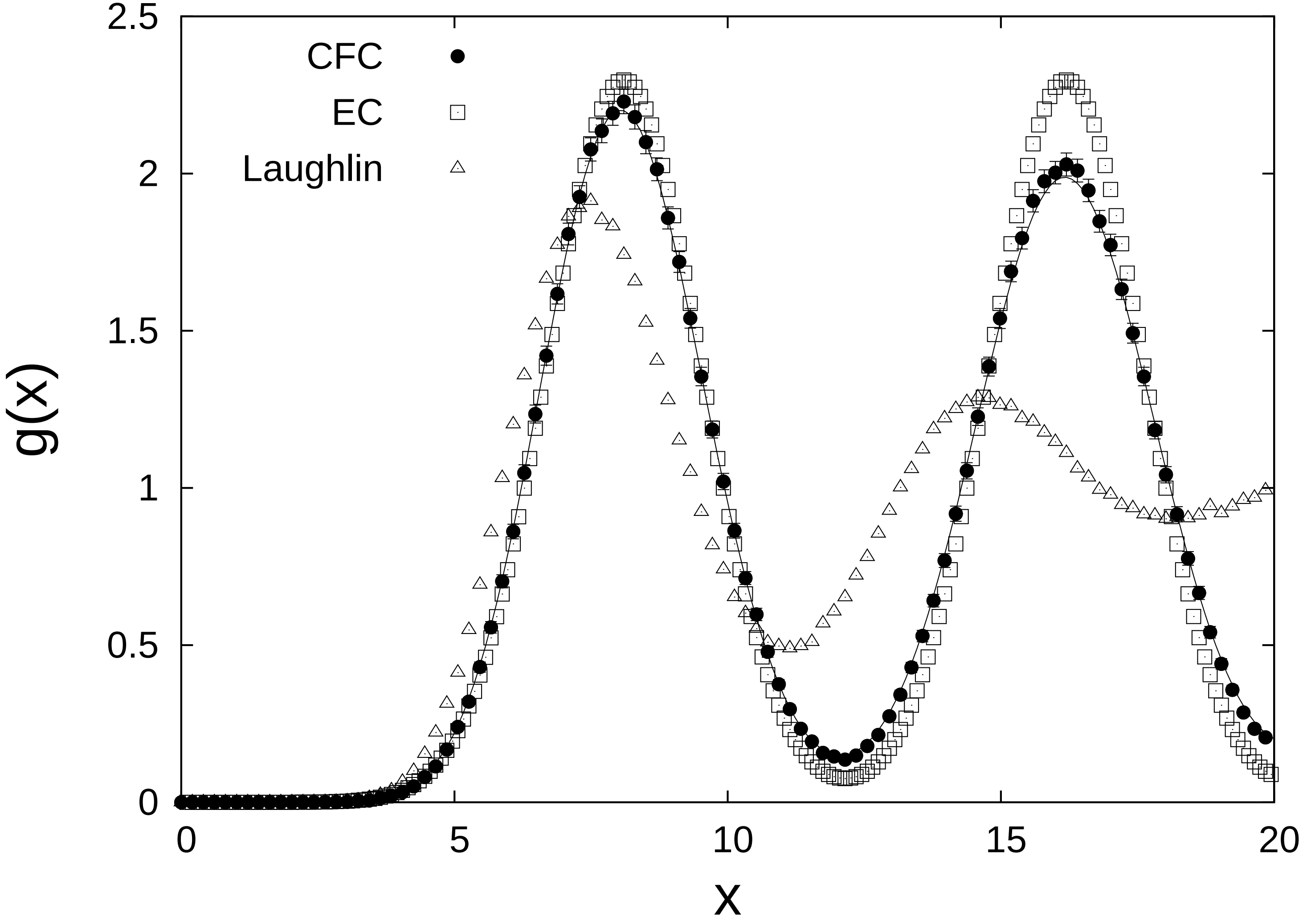
$L(\nu)$	D	CF crystal	electron crystal	Laughlin
75(1/5)	19858	0.891	0.645	0.701
105(1/7)	117788	0.994	0.723	0.504
135(1/9)	436140	0.988	0.740	0.442

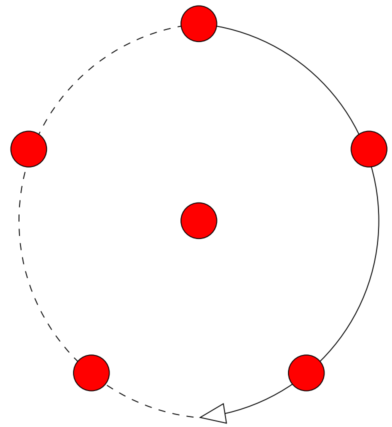
Energy

$L(\nu)$	exact	CF crystal	electron crystal	Laughlin
75(1/5)	2.2019	2.2042(5)	2.2196	2.2093(2)
105(1/7)	1.8533	1.8536(2)	1.8622	1.8617(2)
135(1/9)	1.6305	1.6306(1)	1.6361	1.6388(1)

The energy of the CF crystal at ν=1/7 and 1/9 is off by 0.016% and 0.006%. (For N=6 at ν=1/3, the energy of Laughlin's wave function is off by 0.15% and its overlap with the exact state is 0.964.)

Pair correlation functions ($N=6$, $\nu=1/7$)





Summary of Part III

- From our finite N study we cannot say when a transition into crystal takes place.

- However, the crystal is a CF crystal, even deep inside the crystal phase (very small ν).

- As ν decreases, the vorticity $2p$ goes on increasing.

- The CF crystal is expected to have qualitatively different properties than the electron crystal.

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