Singularites in Fermi liquids and the instability of a ferromagnetic quantum-critical point

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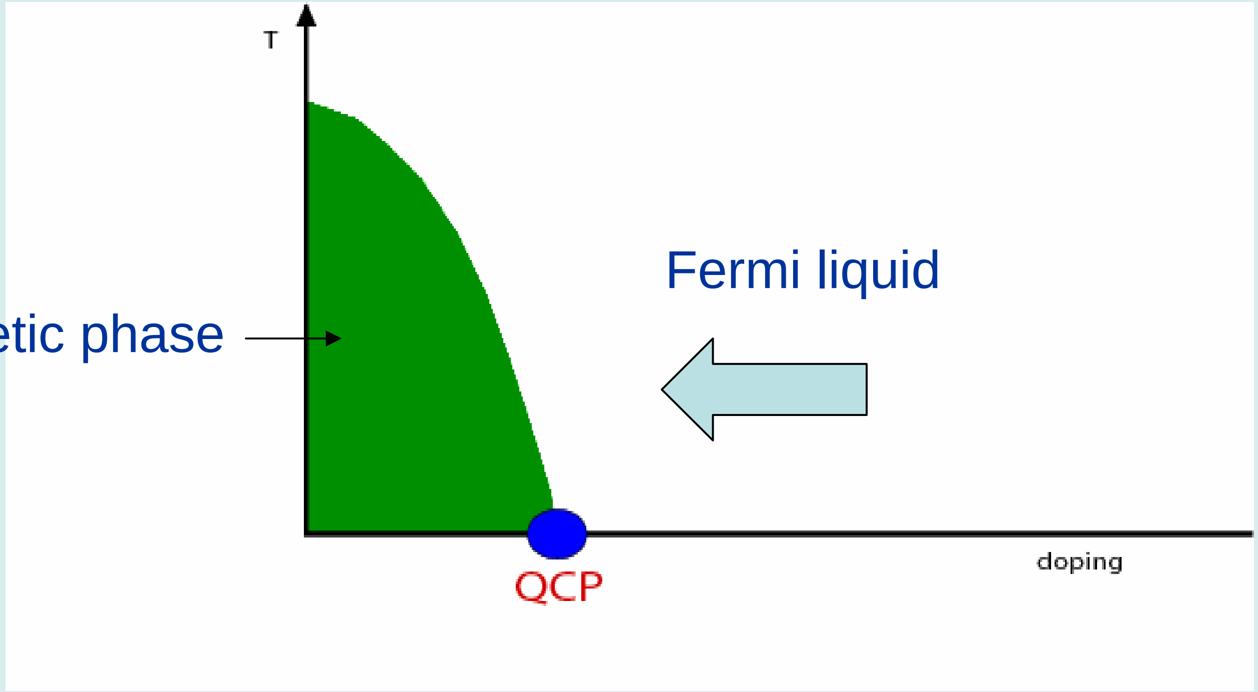
Interest to isotropic models of fermions interacting with low-energy, long-wavelength bosons

- Fermions with $\varepsilon(\mathrm{k}) = \mathrm{v}_F (\mathrm{k} - \mathrm{k}_F)$
- Landau-overdamped bosons with $\chi^{-1}(\mathrm{q}, \Omega) \propto \mathrm{q}^2 + \gamma \frac{\Omega}{\mathrm{q}}$
- Residual spin-fermion coupling g $(\mathrm{c}_\mathrm{k}^+ \mathrm{c}_{\mathrm{k+q}} \mathrm{b}_\mathrm{q} + \mathrm{h.c})$

The most known example –
a ferromagnetic quantum-critical point

The case of a ferromagnetic (Stoner) instability for itinerant electron system

Ferromagnetic phase



What is the critical theory?

Interest to isotropic models of fermions interacting with low-energy, long-wavelength bosons

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- Landau-overdamped bosons with $\chi^{-1}(\mathrm{q}, \Omega) \propto \mathrm{q}^2 + \gamma \frac{\Omega}{\mathrm{q}}$
- Residual spin-fermion coupling g $(c^{+}_{k} c_{k+q} b_q + h.c)$

Other examples: interaction with gauge fields, an instability towards nematic ordering, quarks interacting with gluons, ...

P.A. Lee, Nagaosa, Wen; Ioffe & Larkin, Kee & Kim, Lawler & Fradkin, A.C. & Schmalian...

The problem is particularly interesting in D =2

Three issues:

- What happens with fermions near criticality?
  (fluctuations are strong and destroy a Fermi liquid behavior. What replaces a Fermi liquid?)

- Whether the Landau-overdamped form of the bosonic propagator $\chi^{-1}(\mathrm{q},\Omega) \propto \mathrm{q}^2 + \gamma \frac{\Omega}{\mathrm{q}}$ survives?

(is a second order continuous transition protected against fluctuations?)

- Is there a superconductivity near a QCP?

- What happens with fermions near criticality?
  (fluctuations are strong and destroy a Fermi liquid behavior. What replaces a Fermi liquid?)

Not an academic issue:

Essential for the calculation of the specific heat near a quantum-critical point

$$C_{FL}(T) \propto m^{*}\, T$$

m* diverges at QCP

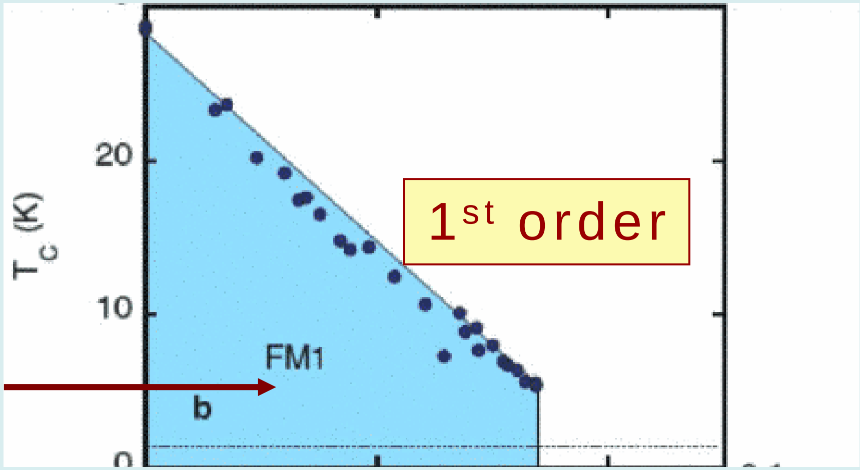
- Whether the Landau-overdamped form of the bosonic propagator $\chi^{-1}(q,\Omega) \propto q^2 + \gamma \frac{\Omega}{q}$ survives?

(is a second order continuous transition protected against fluctuations?)

Ferromagnetic transition is often 1st order

$ZrZn_2$

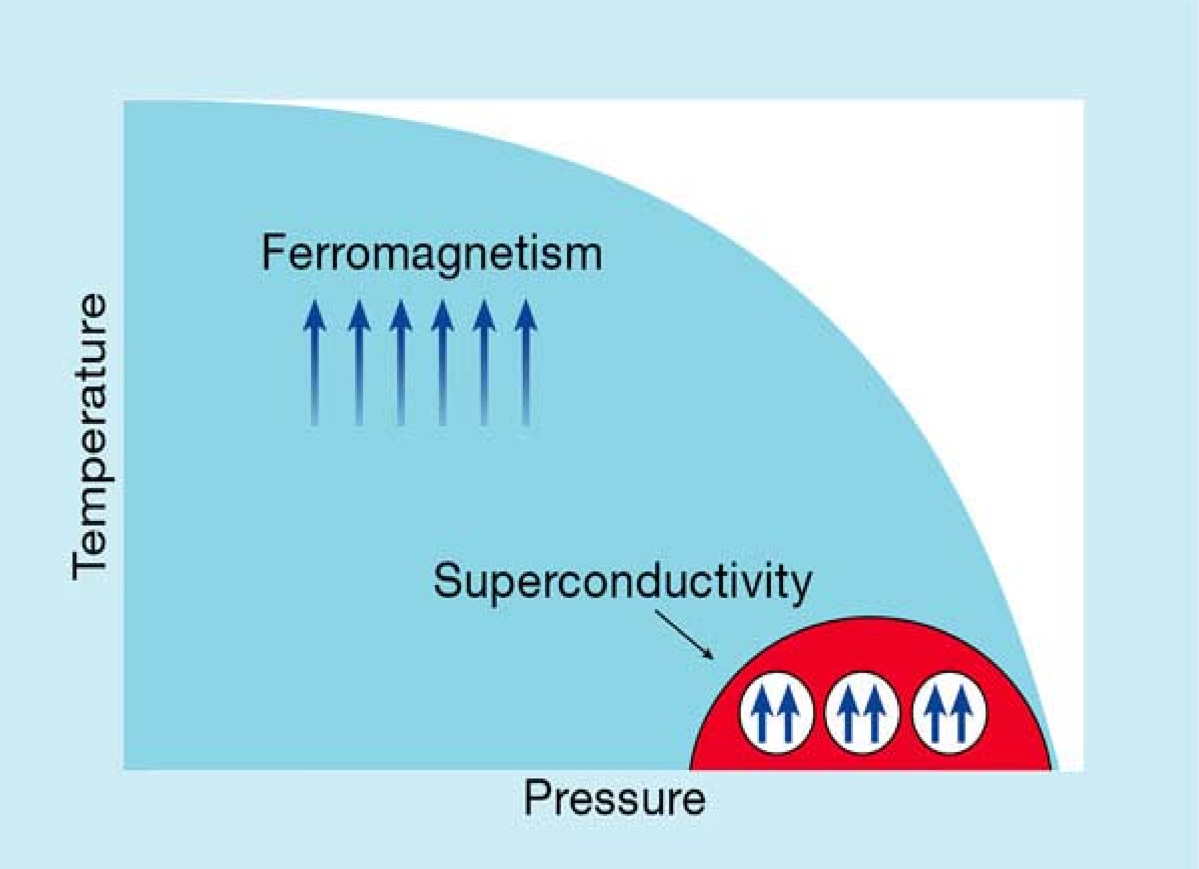
Ferromagnetic phase



pressure

- Is there a superconductivity near a QCP?

UGe_2



Fermions at QCP

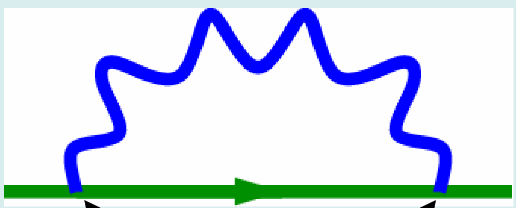
Eliashberg-type theory (D=2)

Altshuler, Ioffe, Millis, Kopietz; Metzner et al; Morr, A.C., ...

$$\Sigma(k,\omega) = \Sigma(\omega)$$

+ no vertex corrections

fermionic self-energy



g

$$\Sigma(\omega) = (i\,\omega)^{2/3}\,\omega_0^{1/3}$$

$$\omega_0 = \frac{3\sqrt{3}}{16\,\pi^2}\frac{g^2}{E_F}$$

non –Fermi liquid at QCP

$$G^{-1}(k,\omega) = i(\omega + \Sigma(\omega)) - v_F(k - k_F) \approx i(\omega^{2/3}\omega_0^{1/3} - v_F(k - k_F))$$

$$G(r, t=0) \propto G_0(r, t=0)/\sqrt{r} \sim 1/r^2$$

Issue of (still) current interest:

Is the Eliashberg theory with $\Sigma(\omega) \sim \omega^{2/3}$ stable?

Earlier perturbative calculations – yes

Altshuler, Ioffe, Millis, Metzner, A.C. ...

Recent calculations based on bosonization – no
(argued that Eliashberg theory is broken below some $\omega_{\min}$)

Fradkin, Lawler...

Our results:

- For a charge QCP, Eliashberg theory survives, but controllable calculations are only possible if one extends the model to a large number of fermionic flavors N

- For SU(2) symmetric spin case, Eliashberg theory of a ferromagnetic QCP is internally unstable

A conventional reasoning for the Eliashberg theory:

If $G^{-1}(k,\omega) \sim (i\,\omega)^{2/3} - v_F\,(k - k_F)$ and $\chi^{-1}(q,\omega) \propto q^2 + \gamma \dfrac{\omega}{q}$

Then, for the same frequency,

typical fermionic momenta $k - k_F \sim \omega^{2/3}$

fermions are much faster than bosons

typical bosonic momenta $q \sim \omega^{1/3}$ (much larger)

Quite generally, in this situation,
Migdal theorem must be valid

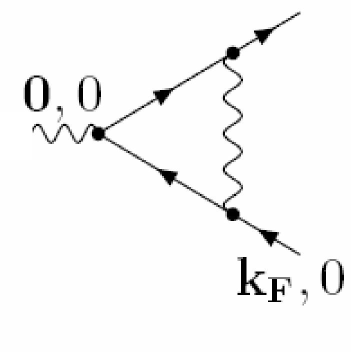
$$\Sigma(\omega) = (i\,\omega)^{2/3}\,\omega_0^{1/3}$$

$$\omega_0 = \frac{3\sqrt{3}}{16\,\pi^2}\frac{g^2}{E_F}$$

Fermions must remain fast up to $\omega \sim \omega_0$ $\Rightarrow$ $\alpha = \frac{g}{E_F} << 1$

If $\alpha << 1,$ vertex corrections and k-dependent self-energy are small

Static vertex in the limit of vanishing momentum



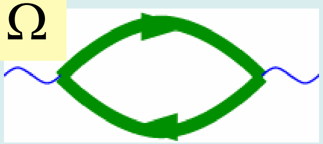
$= O(\alpha^{1/2})$

This is not enough.

The theory must satisfy Ward identities, associated with the conservation of the total number of particles, and the total spin

$$\chi_{\text{tot}}(q = 0, \Omega \neq 0) = 0$$

q, Ω



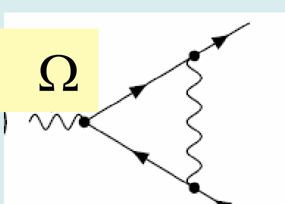
$$\chi_0(q, \Omega) = \frac{m}{2\pi}\left(1 - \frac{|\Omega|}{\sqrt{\Omega^2 + v_F^2 q^2}}\right) = 0 \ \text{ when } q = 0$$

However, “Eliashberg” susceptibility

$$\chi_0(q, \Omega) = \frac{m}{2\pi}\left(1 - \frac{|\Omega|}{\sqrt{(\Omega + \Sigma(\Omega))^2 + v_F^2 q^2}}\right) \neq 0 \text{ when } q = 0$$

Vertex corrections are NOT small when q=0, and Ω is nonzero

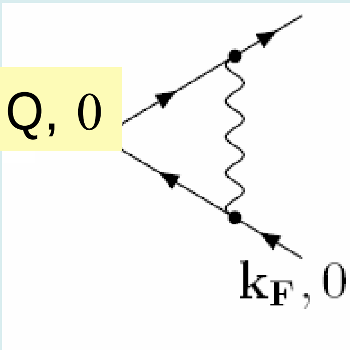
0, Ω



$$\frac{\delta g}{g} = \frac{\partial \Sigma(\omega)}{\partial \omega} \sim \left(\frac{\omega_0}{\omega}\right)^{1/3}$$

Finite momentum, zero frequency

Q, 0

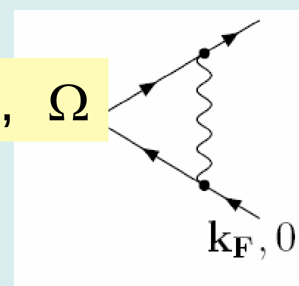


$$= O(\alpha^{1/2})$$

vertex corrections are small

Finite frequency, zero momentum

0, Ω



$$\frac{\delta g}{g} = \frac{\partial \Sigma(\omega)}{\partial \omega} \sim \left(\frac{\omega_0}{\omega}\right)^{1/3}$$

vertex corrections are large

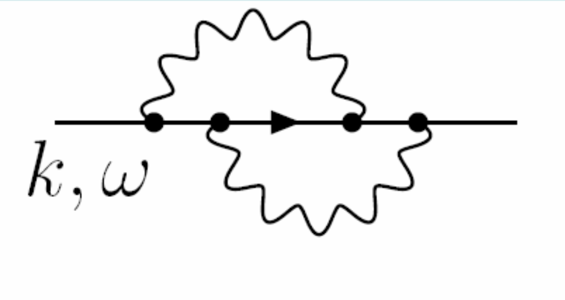
Finite frequency, finite momentum

$$v_F Q \sim \Sigma(\omega)$$

$$\frac{\delta g}{g} = O(1)$$

As a result, the theory is NOT under control:

$$\Sigma_2(\omega) =$$



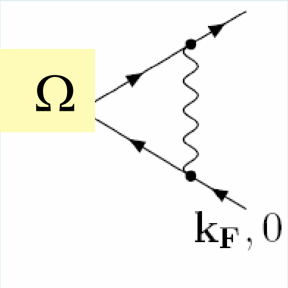
$$\Sigma_2(\omega) \sim \omega^{2/3}\,\omega_0^{1/3} \sim \Sigma_1$$

$$\Sigma_3(\omega) \sim \Sigma_2 \text{ and so on}$$

This is dangerous

Example:

$0, \ \Omega$



$$\frac{\delta\,\mathrm{g}}{\mathrm{g}} = \frac{\partial\,\Sigma(\omega)}{\partial\,\omega} \sim \left(\frac{\omega_0}{\omega}\right)^{1/3}$$

In order by order perturbation theory

$$\frac{\delta\,\mathrm{g}}{\mathrm{g}}\Big|_{\omega=0} = 1 + 0.7854 + 0.7578 + \ldots = \text{infinity}$$

Another powerful way to do calculations is bosonization.

Lawler & Fradkin

The bosonization result in 2D: $G(r, t=0) = G_0\,(r, t=0)\;\; e^{(-r/r_0)^{1/3}}$ is consistent

with divergent series for the self-energy (i.e., $\omega^{2/3}$ does not survive)

If $\Sigma(\omega) \sim \omega^{2/3}$ $\quad G(r, t=0) \propto G_0\,(r, t=0)\,/\sqrt{r} \sim 1/r^2$

One detail – bosonization assumes that the curvature of the Fermi surface is irrelevant

$$\epsilon_k = v_F k_\perp + \frac{k_\parallel^2}{2m_B}.$$

The curvature of the Fermi surface is crucial to the physics in 2D

- one can extend the theory to N fermionic flavors

Altshuler et al, Rech et al

$$\Sigma_2(\omega) \sim \omega^{2/3}\ \omega_0^{1/3} * ((\mathrm{Log\ N})/\mathrm{N})^2$$

The factor 1/N comes from the curvature of the Fermi surface

$$\epsilon_k = v_F k_\perp + \frac{k_\parallel^2}{2m_B}.$$

Without the curvature, we would still have $\Sigma_2(\omega) \sim \Sigma_1(\omega)$, even at N >>1

One can improve bosonization to include the curvature

Khveshchenko & A.C.

Result: $G(r) = G_0(r)\ e^{-Z(r)}$

Without curvature $Z(r) \sim r^{1/3}$

With curvature $Z(r \to \infty) \sim \log r$

$G(r, t=0) \propto 1/r^{\eta}$ at the largest r, => small frequency behavior of the Green's function is consistent with

$$\Sigma(\omega) \propto (\omega)^{2/3}$$

(perturbation theory does not diverge)

Intermediate conclusion

For 2D fermions, interacting with a given

$$\chi^{-1}(\mathrm{q},\Omega) \propto \mathrm{q}^2 + \gamma \frac{\Omega}{\mathrm{q}}$$

$\Sigma(\omega) \propto (\omega)^{2/3}$ is very likely the exact solution.

This was the effect from bosons on fermions.

What about the opposite effect?

Does the bosonic propagator preserve its form when one includes the corrections due to fermions?

A conventinal answer – fermions only account for Landau damping of bosons

Then, once the fermions are integrated out,

$$S = \int d^dQ\, d\omega\, [Q^2 + \frac{|\omega|}{Q}]\eta^2 + b\,\eta^4 + \ldots$$

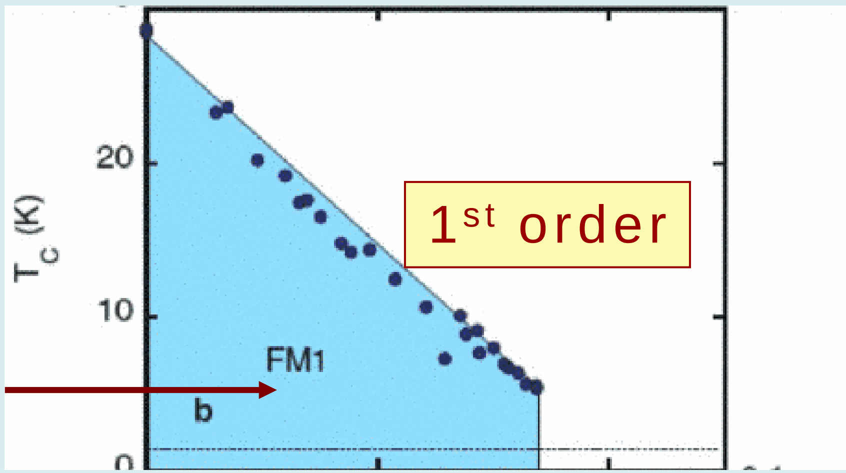
Z=3, Dcr =1 (Dcr + Z =4)

Bosonic propagator is not affected by fluctuations in D >1

QC properties	Specific Heat	Resistivity exp.
D=2	$T^{2/3}$	$T^{4/3}$
D=3	$-T \ln T$	$T^{5/3}$

In reality, ferromagnetic transition is 1st order

$ZrZn_2$



1st order

Ferromagnetic phase

pressure

p (kbar)

$$S = \int d^dQ\, d\omega\, [Q^2 + \frac{|\omega|}{Q}]\eta^2 + b\eta^4 + \dots$$

Verify this

Apply a magnetic field and use the exact formula for the thermodynamic potential in the Eliashberg theory

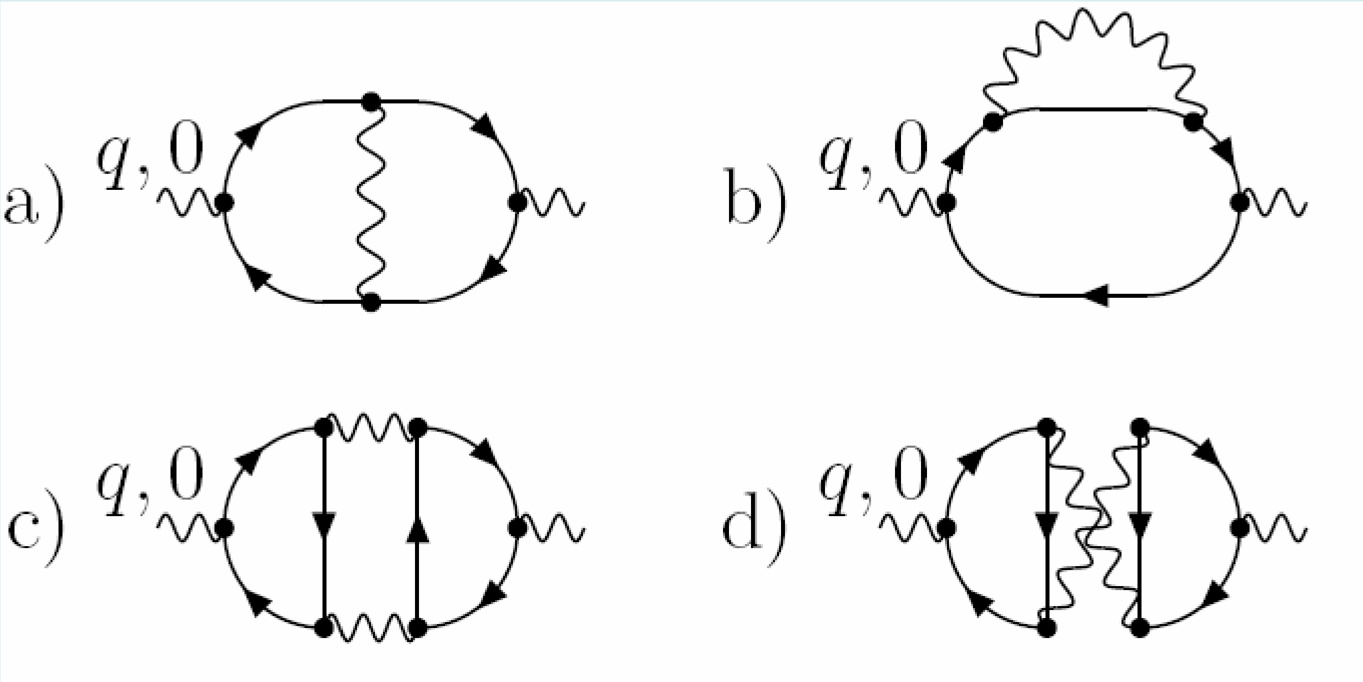
$$\Omega = \Omega_{free} + \frac{1}{2}\int \frac{d^2q\, d\Omega}{(2\pi)^3}\left(2\log(1+g\Pi_{+-}) + \log(1+g\Pi_{++})\right)$$

$$\Pi_{+-} = \int G_+ G_- ,\quad \Pi_{++} = \int G_+ G_+$$

Betouras, Efremov, A.C.
Maslov & A.C.

$$\chi = -\frac{\partial^2 \Omega}{\partial H^2}$$

Diagrams for $\chi(q)$ in the Eliashberg theory

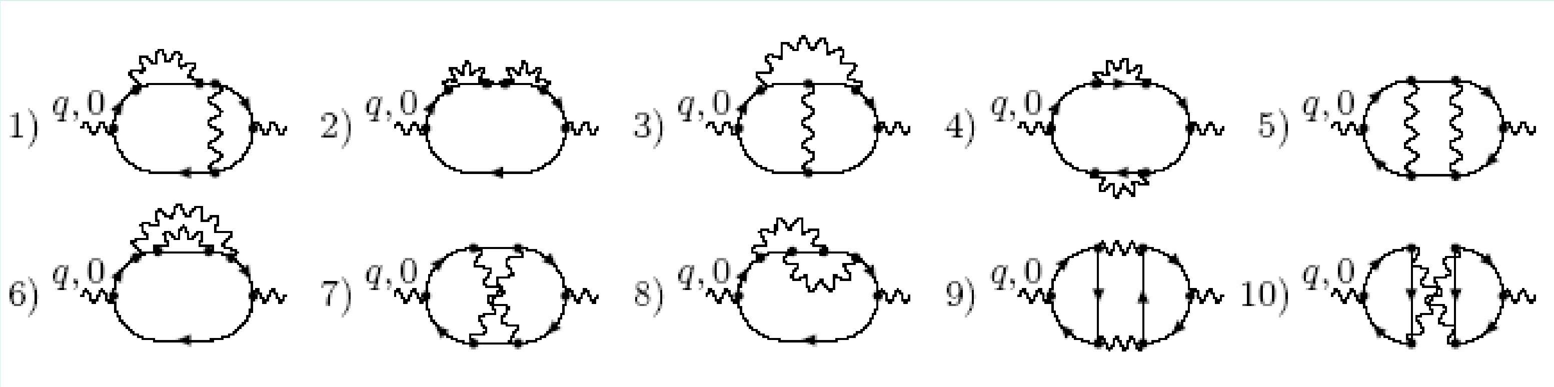


$$\delta\chi_{spin}(Q) = -0.25\, Q^{3/2}\, p_F^{1/2}$$

Comes from processes in which bosons are vibrating at fermionic frequencies

(opposite to Migdal processes)

Corrections: go outside Eliashberg theory



$$\delta\chi^{(2)}_{\text{spin}}(Q) \propto Q^{3/2}\, p_F^{1/2} * \left(\frac{\log N}{N}\right)^2$$

Same corrections as for the self-energy

$$\Sigma_2(\omega) \sim \Sigma_1(\omega) * \left(\frac{\log N}{N}\right)^2$$

Physics -- Landau damping

$$\chi^{-1}(q,\Omega) \propto \frac{\Omega}{|q|}$$

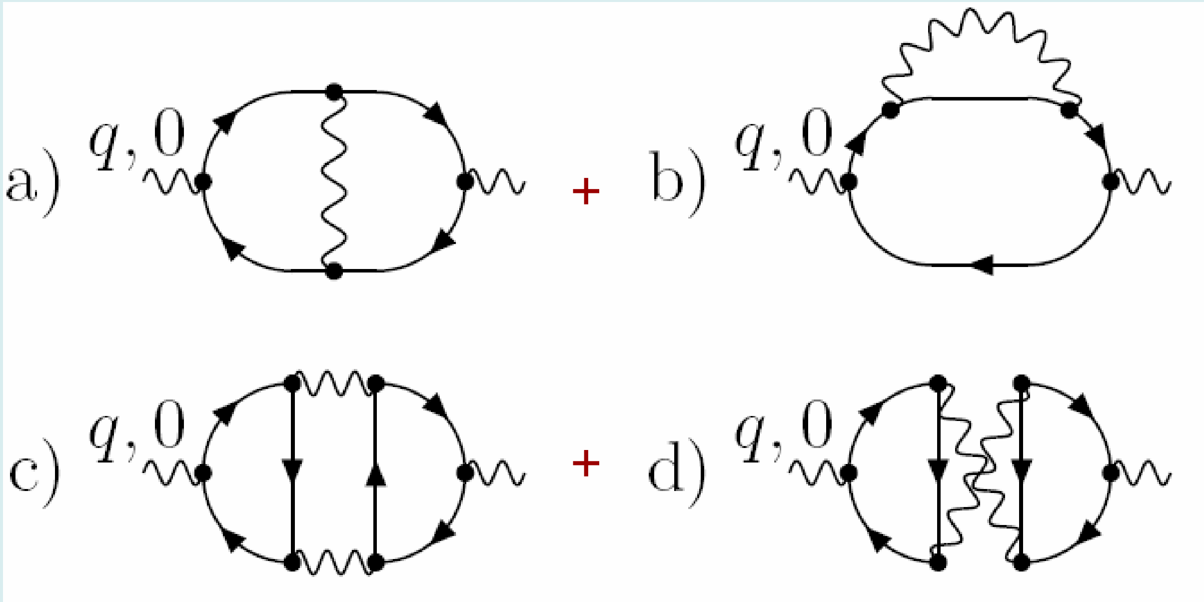
Long-range dynamic interaction Ω/r between fermions

A magnetic field changes $\frac{\Omega}{|q|}$ into $\frac{\Omega}{\sqrt{q^2+(iH)^2}}$ i.e. restores analyticity

As a result, the derivative with respect to H is singular

For the charge susceptibility, χ measures the response to the change of the chemical potential. Ω/q survives this change, hence the charge susceptibility should be regular.

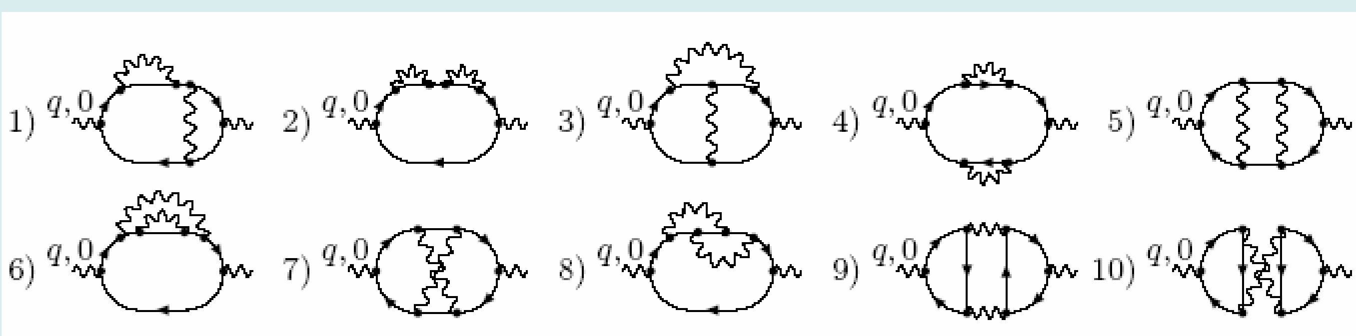
Charge susceptibility



= 0

= 0

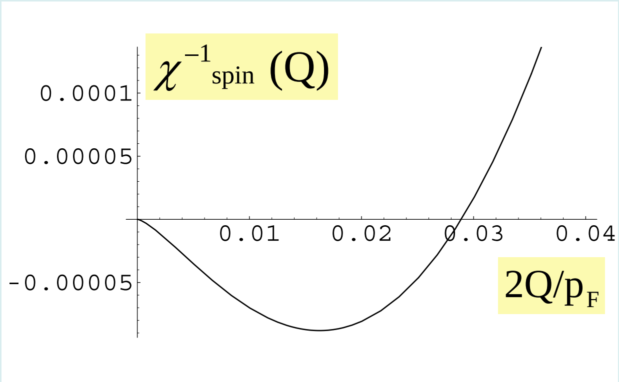
$$\delta\chi_{\text{charge}}(Q) \propto Q^2$$



$= 0(q^2)$

Back to the static spin susceptibility

$$\chi_{\text{spin}}(\mathrm{Q}) = (\mathrm{Q}^2 - 0.25\ \mathrm{Q}^{3/2}\ \mathrm{p}_F^{1/2})^{-1}$$



Internal instability of z=3 QC theory in D=2

Another way to see the instability of a ferromagnetic QCP is to calculate the q=0 susceptibility at a finite T

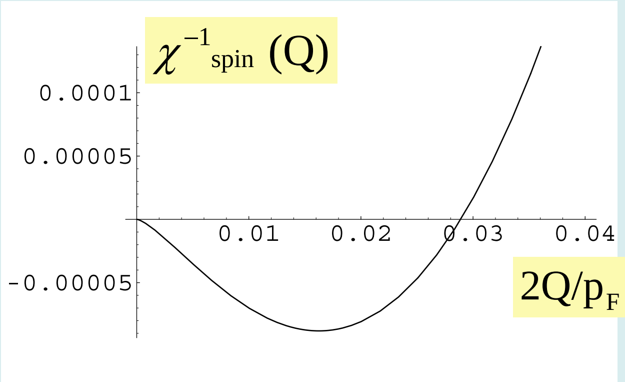
$$\chi^{-1}(q=0,T) = -\frac{T}{T_0}\log\frac{T_0}{T}$$

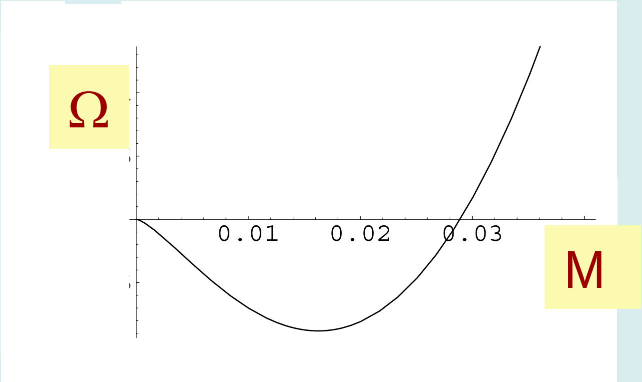
Or calculate the thermodynamic potential at a finite magnetization

$$\Xi(M) = b\,M^4 - a\,M^3$$

Maslov & A.C.,
Belitz, Kirkpatrick & Vojta

What can happen?





a transition into a spiral state

a first order transition to a FM

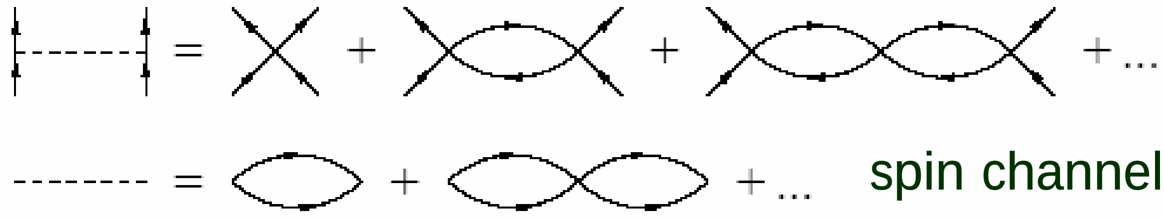
Belitz, Kirkpatrick, Vojta,
Maslov and A.C., Rech, Pepin, A.C.

Both scenarios are possible

The instability of the Ferromagnetic QCP can be also understood in the frameworks of the Hertz-Millis-Morya theory

How one obtains Hertz theory

Stage one: obtain the spin-fermion model
(fermions, collective spin fluctuations, and interacton)
(integrate out high energy fermions)



spin channel

$$\mathcal{H} = \sum_{\mathbf{k},\alpha} \mathbf{v}_k(\mathbf{k}-\mathbf{k}_F)c^\dagger_{\mathbf{k},\alpha}c_{\mathbf{k},\alpha} + \sum_q \chi_0^{-1}(\mathbf{q})\mathbf{S}_q\mathbf{S}_{-q}$$
$$+g\sum_{\mathbf{q},\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k}+\mathbf{q},\alpha}\,\sigma_{\alpha,\beta}\,c_{\mathbf{k},\beta}\cdot\mathbf{S}_{-\mathbf{q}}\,.$$

$$\chi_0\,(\mathrm{q}) \propto \frac{1}{\mathrm{q}^2+\xi^{-2}}$$

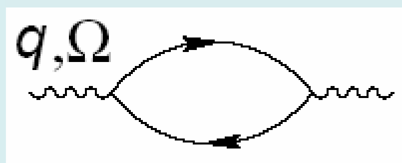
low-energy fermions

spin-fermion interaction

static collective spin fluctuations

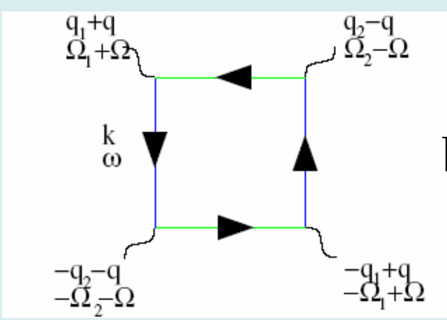
Stage two: obtain the theory for collective modes (integrate out low energy fermions)

Hertz theory $$S = \int d^dQ \, d\Omega \, [Q^2 + \zeta^{-2} + \frac{|\Omega|}{Q}] \eta^2 + b\,\eta^4 + \ldots$$



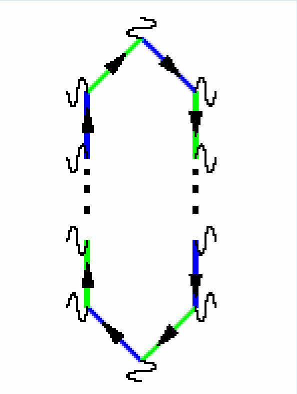
$$= \frac{\Omega}{q}$$

Landau damping



$$b = \text{const} + \frac{\Omega}{(\Omega + v_F\, q)^3}$$

vertex is non-analytic in momentum and frequency



$$= \frac{\Omega}{(\Omega + v_F\, q)^{2n-1}}$$

fermions produce long-range dynamic interaction between collective modes

This dynamic interactions fits back into $q^{3/2}$ term in the static susceptibility

The non-analyticity of the spin susceptibility is NOT specific to a ferromagnetic quantum criticality

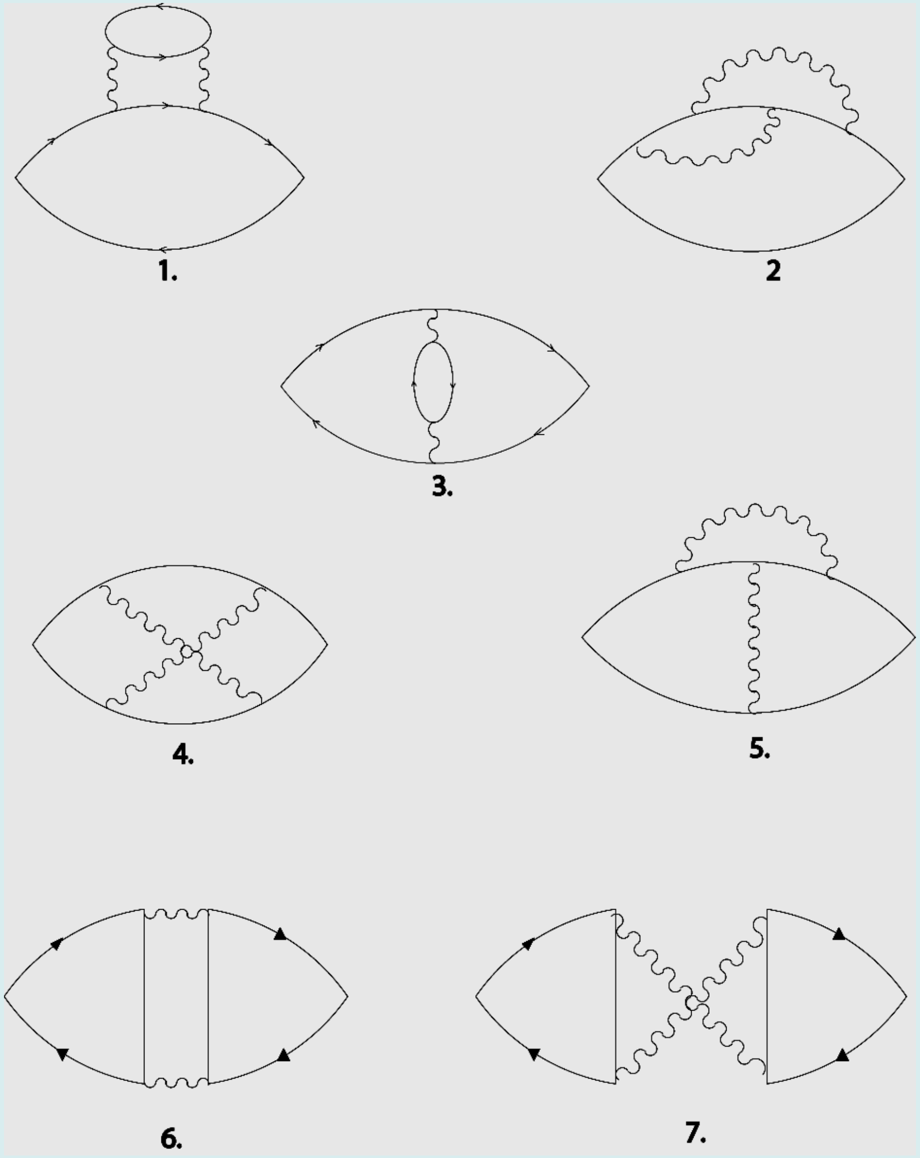
The same effect is already seen in weak coupling perturbation theory, the only difference is that $q^{3/2}$ is replaced by $|q|$ as the effect of fermionic self-energy is weak at small coupling.

T=0, finite Q

$$\delta\chi_{spin}(Q) = \frac{4m}{3\pi^2}\left(\frac{m\,U(2p_F)}{4\pi}\right)^2 \frac{|Q|}{p_F}$$

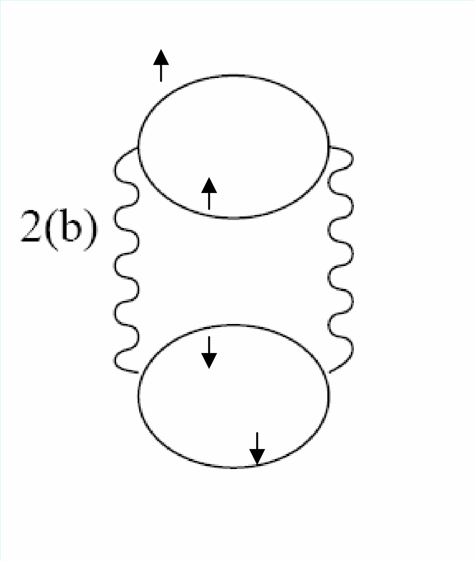
Q=0, finite T

$$\delta\chi_{spin}(T) = \frac{2\,m}{\pi}\left(\frac{m\,U(2p_F)}{4\,\pi}\right)^2 \frac{T}{E_F}$$



Why only U(2pF) matters?

In perturbation theory, only bubbles with the same spin projection are present. The 2kF bubble is singular and is affected by the magnetic field



$$\Pi\left(q \approx 2p_F, \omega\right) = \frac{m}{\pi}\left(1 - \sqrt{\frac{q-2p_F}{2\,p_F} + \left[\left(\frac{q-2p_F}{2\,p_F}\right)^2 - \left(\frac{\omega}{2\,v_F\,p_F}\right)^2\right]^{1/2}}\right)$$

$\omega/\sqrt{2\,p_F - q}$ singularity

In the presence of H, $p_F^{\uparrow} \neq p_F^{\downarrow}$

$$\Omega \sim T\sum_{\omega}\omega^2 \int d\,q \frac{1}{\sqrt{2p_F^{\uparrow} - q}\sqrt{2p_F^{\downarrow} - q}} \sim T\sum \omega^2 \log\left(H^2 + \omega^2\right)$$

$$\chi(T) = -\frac{\partial^2\Omega}{\partial H^2} \propto T$$

For a constant, but arbitrary interaction U

$$\delta\chi^{-1}(T) \propto T\left(\log(1+\Gamma) - \frac{\Gamma}{1+\Gamma}\right)$$

$$\Gamma = \frac{U\,N(0)}{1 - U\,N(0)} \rightarrow \infty \text{ at QCP}$$

$$\delta\chi^{-1}(T) \propto T\,\Gamma^2 \text{ at two-loop order}$$

$$\delta\chi^{-1}(T) \propto T \log \zeta$$

$$(T \log T \text{ at QCP})$$

For arbitrary U (q), near a ferromagnetic QCP

$$\delta\chi^{-1}(T) = -\chi^{-1}_{\text{Pauli}} \frac{T}{E_F}\left(\log(1+f_{o,s}) + O(f_{i,s})\right)$$

Instability of a ferro-magnetic QCP in 2D

Component of the Landau function

What would happen if $\delta\chi_s\,(\mathrm{q},\mathrm{T})$ was positive ?

Self-consistent calculations:

$$\chi\,(\mathrm{q},\Omega) \propto (\mathrm{q}^{\alpha} + |\,\Omega\,| \,/\, |\,\mathrm{q}\,|)^{-1}$$

$$\alpha = 1$$

$$\chi\,(\mathrm{q},\omega) \propto (|\,\mathrm{q}\log\mathrm{q}\,| + |\,\omega\,| \,/\, |\,\mathrm{q}\,|)^{-1}$$

$$\Sigma\,(\omega) \propto \omega\ \log(|\log\omega\,|)$$

Marginal Fermi liquid in D=2

Conclusions

The QCP towards charge (e.g., nematic) ordering is stable in 2D. The fermionic self-energy is $\Sigma(\omega) \sim \omega^{2/3}$, and the theory is controllable at N >>1

A ferromagnetic Hertz-Millis critical theory is internally unstable in D=2

(and also in D = 3)

- static spin propagator is negative at QCP up to Q~ p_F
- free energy has an M^3 term

Either an incommensurate ordering,
or 1st order transition to a ferromagnet

Collaborators

- Jerome Rech (Saclay)
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- Mitya Maslov (U. of Florida)
- Andy Millis (Columbia)
- Joseph Betouras (St. Andew)
- Dima Efremov (TU Dresden)

THANK YOU!