

In Search of Fractional Statistics: Anyon There?

Eun-Ah Kim

Stanford University

Thanks to...

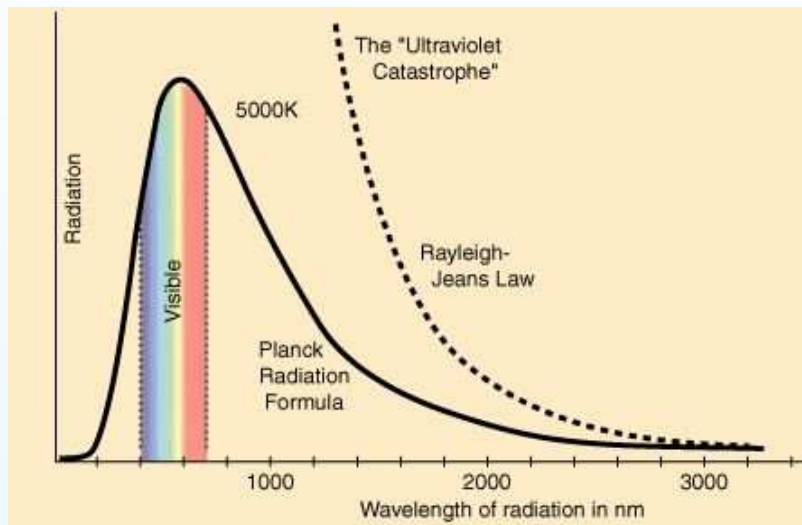
Michael Lawler, Smitha Vishveshwara, Eduardo Fradkin (UIUC),
Moty Heilblum (Weizmann), Claudio Chamon (BU),
Duncan Haldane (Princeton), Eddy Ardonne (KITP)
Steven Kivelson (Stanford), Vladimir Goldman (Stony Brook)

Contents

- ▶ Getting ready
 - Statistics: boson v.s. fermion
 - Fractional quasiparticles: anyons
 - The fractional quantum Hall effect
 - Edge states
- Cross current noise in T-junction
- Quantum Hall interferometer
- Summary

Statistics: boson v.s. fermion

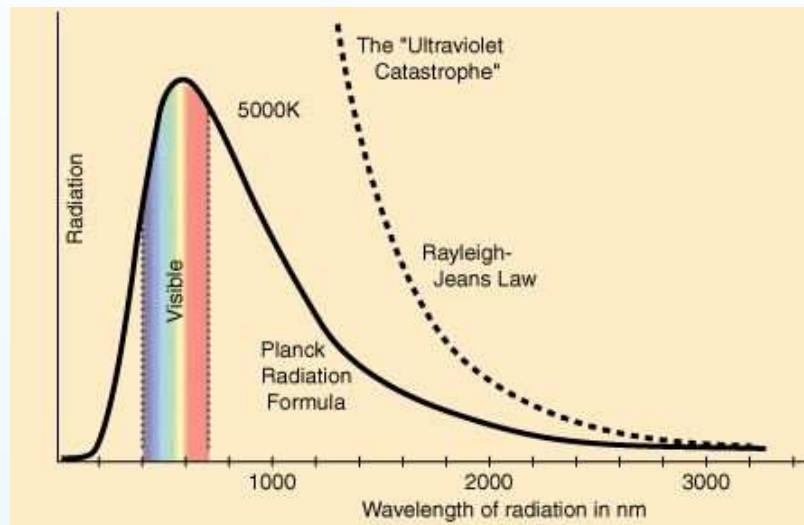
- Black body radiation



Bose-Einstein statistics of
photon

Statistics: boson v.s. fermion

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Bose-Einstein statistics of photon

- Periodic table

1																	18		
H 1.01	He 4.00																	He 4.00	
3	4																	9	10
Li 6.94	Be 9.01																	F 19.00	Ne 20.18
11	12																	17	18
Na 22.99	Mg 24.31																	Cl 35.45	Ar 39.95
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36		
K 39.10	Ca 40.08	Sc 44.96	Ti 47.87	V 50.94	Cr 52.00	Mn 54.94	Fe 55.85	Co 58.93	Ni 58.69	Cu 63.55	Zn 65.41	Ga 69.72	Ge 72.64	As 74.92	Se 78.96	Br 79.90	Kr 83.80		
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54		
Rb 85.47	Sr 87.62	Y 88.91	Zr 91.22	Nb 92.91	Mo 95.94	Tc (98)	Ru 101.07	Rh 102.91	Pd 106.42	Ag 107.87	Cd 112.41	In 114.82	Sn 118.71	Sb 121.76	Te 127.60	I 126.90	Xe 131.29		
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86		
Cs 132.91	Ba 137.33	La 138.91	Hf 178.49	Ta 180.95	W 183.84	Re 186.21	Os 190.23	Ir 192.22	Pt 195.08	Au 196.97	Hg 200.59	Tl 204.38	Pb 207.2	Bi 208.98	Po (209)	At (210)	Rn (222)		
87	88	89	104	105	106	107	108	109	110	111									
Fr (223)	Ra (226)	Ac (227)	Rf (261)	Db (262)	Sg (266)	Bh (264)	Hs (270)	Mt (268)	Ds (281)	Rg (272)									

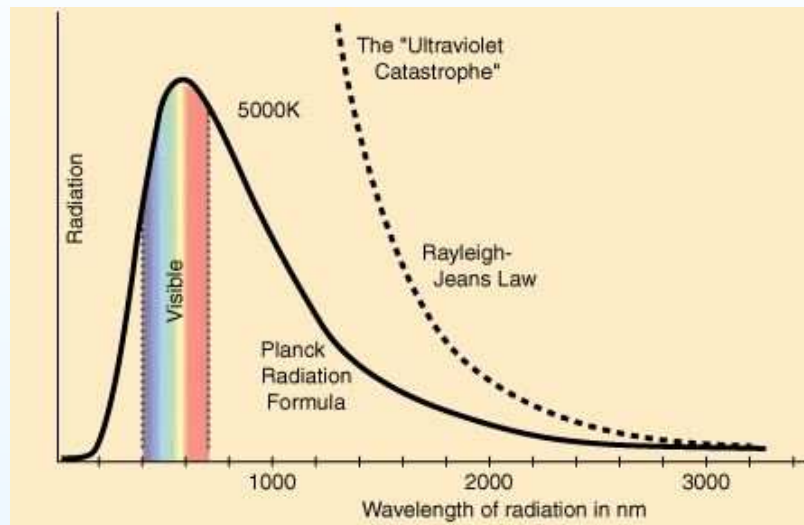
See "It's Elemental: The Periodic Table"
<http://pubs.acs.org/cen/80th/elements.html>

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce 140.12	Pr 140.91	Nd 144.24	Pm (145)	Sm 150.36	Eu 151.97	Gd 157.25	Tb 158.93	Dy 162.50	Ho 164.93	Er 167.26	Tm 168.93	Yb 173.04	Lu 174.97
90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th 232.04	Pa 231.04	U 238.03	Np (237)	Pu (244)	Am (243)	Cm (247)	Bk (247)	Cf (251)	Es (252)	Fm (257)	Md (258)	No (259)	Lr (262)

Fermi-Dirac statistics of electrons

Statistics: boson v.s. fermion

- Black body radiation



Bose-Einstein statistics of photon

- Superfluid He^4

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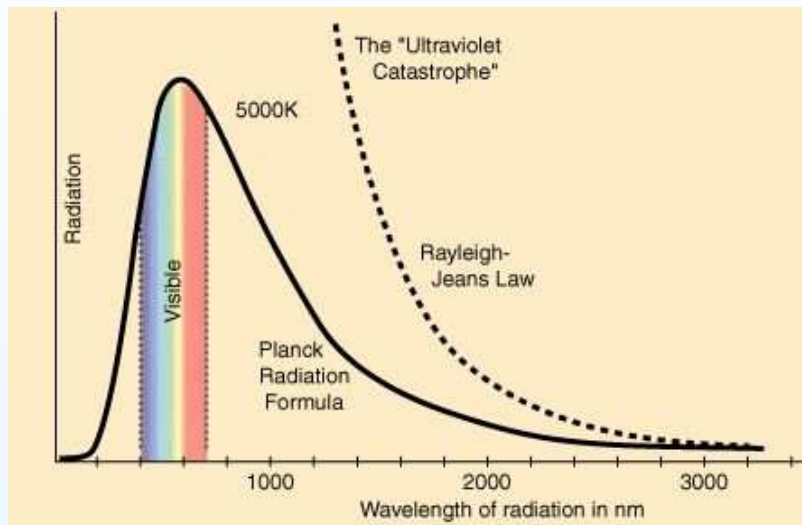
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Fractional quasiparticles: Anyons

- Fractional charge e^*

Fractional quasiparticles: Anyons

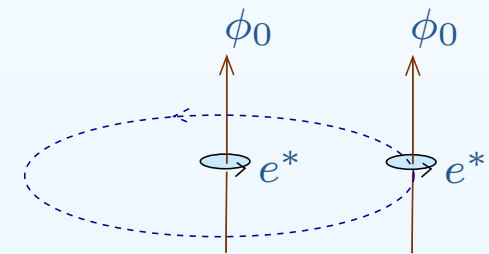
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Fractional quasiparticles: Anyons

- Fractional charge e^*
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 - Braiding statistics (Wilczek, 1982):

$$\Psi(r_1, r_2) = e^{i\theta} \Psi(r_2, r_1)$$

$$0 \leq \theta \leq \pi$$



An adiabatic process.

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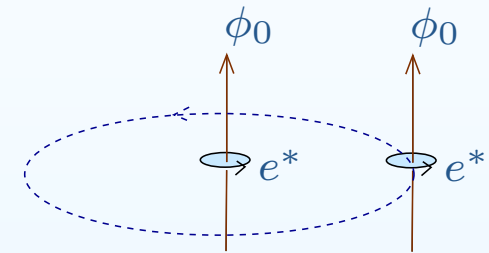
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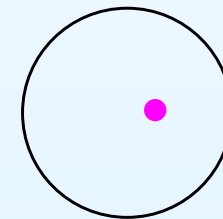
- Generalized exclusion statistics (Haldane, 1991):

$$\Delta d_\alpha = - \sum_{\beta} g_{\alpha\beta} \Delta N_\beta$$

$g_{\alpha\beta} = 0$ for boson, $g_{\alpha\beta} = \delta_{\alpha\beta}$ for fermion



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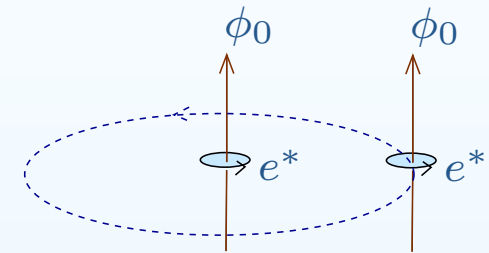
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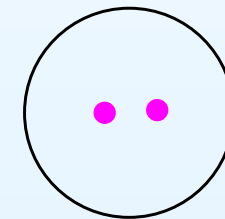
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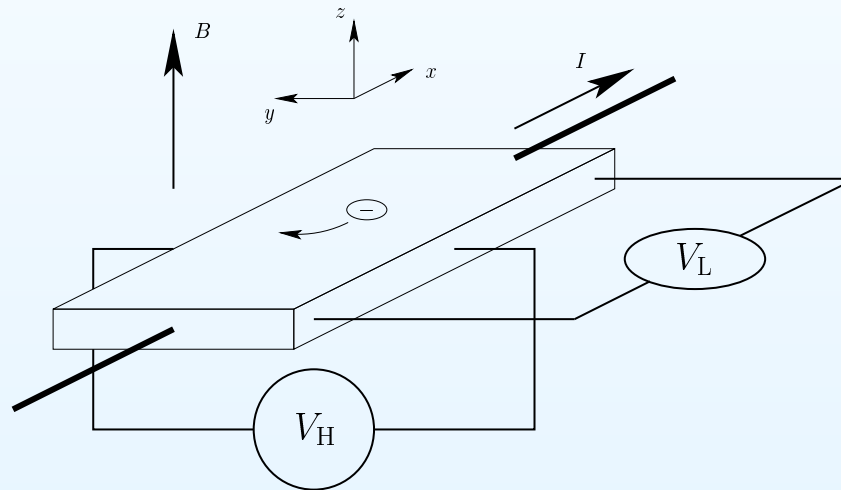


An adiabatic process.



The Fractional Quantum Hall Effect (Tsui et. al.,1982):

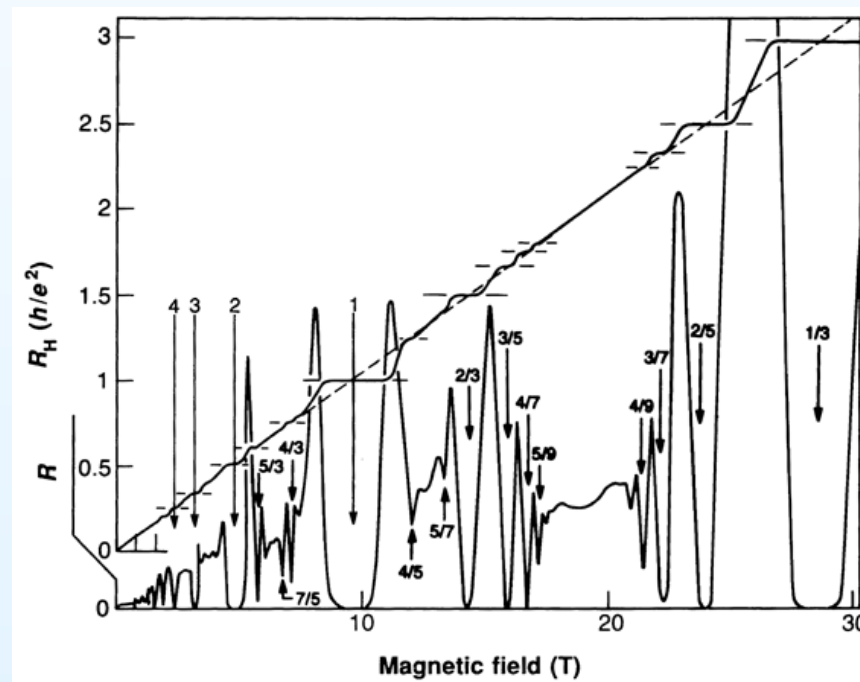
a quantization of Hall conductance ($\sigma_H = \nu e^2/h$) when the filling factor $\nu \equiv \rho h/eB$ (ρ is the surface density) is in the vicinity of a hierarchy of rational fractions.



A quantum Hall bar

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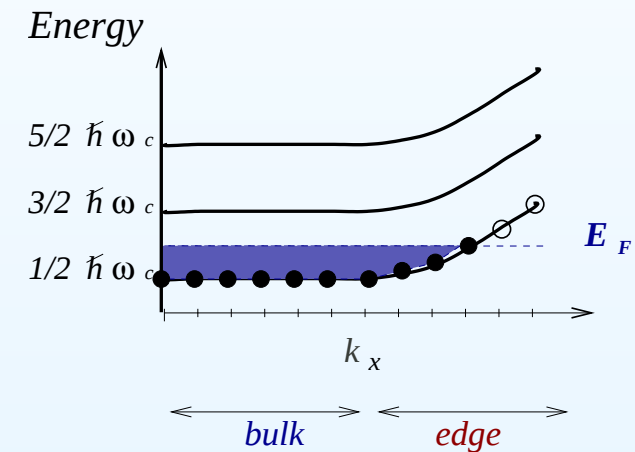
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Quantum Hall plateau (taken from Eisenstein and Stormer, 1990)

Edge States

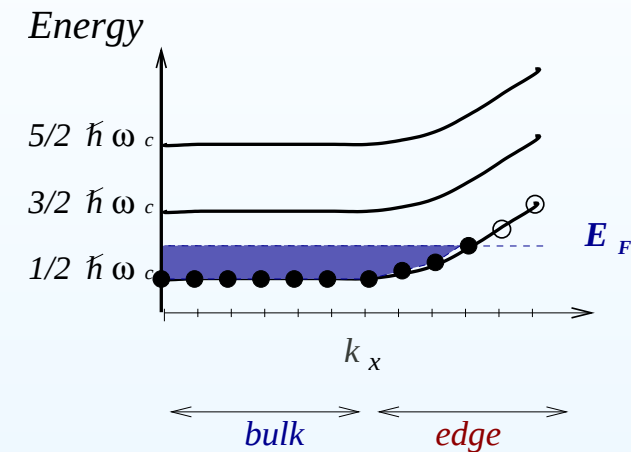
- The surface wave of edge distortions is the only **gapless** excitation.



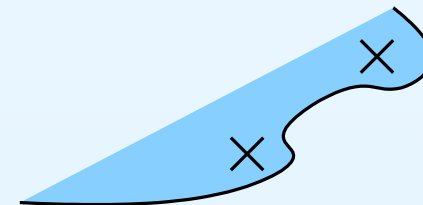
The Landau levels of confined 2DEG.

Edge States

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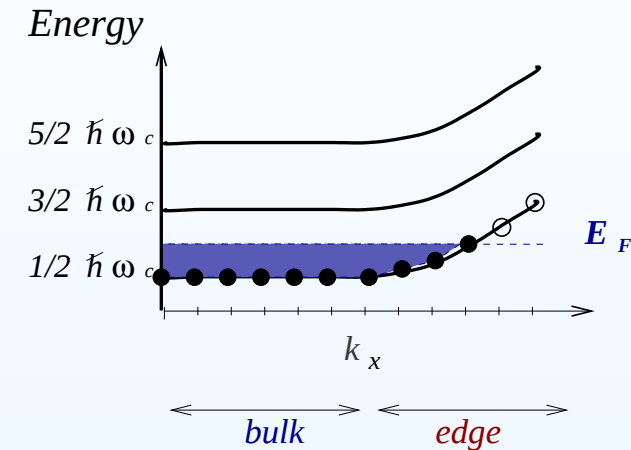
The edge mode is dissipationless.

Edge States

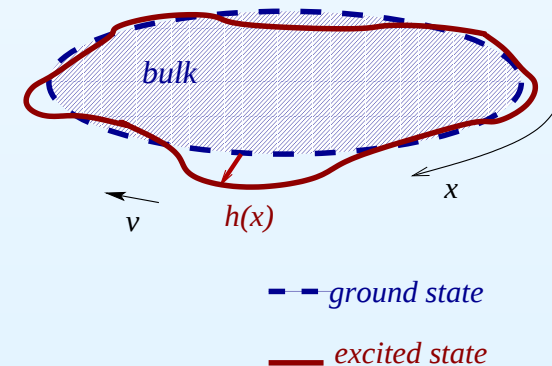
- The surface wave of edge distortions is the only **gapless excitation**.
- Hydrodynamic picture: dissipationless chiral Luttinger liquid. (Wen, 1990; Stone 1991)
- 1D density ripple $J(x) = \rho h(x)$ is related to **chiral boson** ϕ_+ through bosonization

$$J_+(x) \equiv -\frac{\sqrt{\nu}}{2\pi} \partial_x \phi_+, \quad \psi_+^\dagger = \frac{1}{\sqrt{2\pi}} e^{\frac{i}{\nu} \phi_+}$$

$$\mathcal{L} = \frac{1}{4\pi} \partial_x \phi_+ (\partial_t - \nu \partial_x) \phi_+$$



The Landau levels of confined 2DEG.



A displacement $h(x)$ at the point x .

Contents

- Getting ready
- ▶ Cross current noise in T-junction
 - T-junction in Jain states
 - $S = \mathcal{A} + \cos \theta \mathcal{B}$
 - Anatomy of the phase factor
 - Frequency spectrum
- Quantum Hall interferometer
- Summary

Fractional statistics in QH Jain States

- Pantry of Particles (since '84)

	Boson	$\nu = \frac{1}{\text{odd}}$ (Laughlin)	$\nu = \frac{p}{2np+1}$ (Jain)	Fermion
phase	$1 = e^0$	$e^{i\nu\pi}$	$e^{i\theta} \left(\frac{\theta}{\pi} = \frac{2n}{2np+1} + 1 \right)$	$-1 = e^{i\pi}$
charge		νe	$Q = \frac{-e}{2np+1}$	$-e$

$2n = \#$ of attached flux quanta

$p =$ effective filling factor

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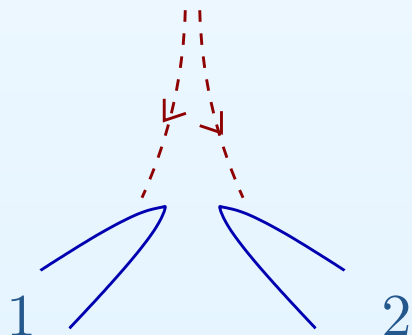
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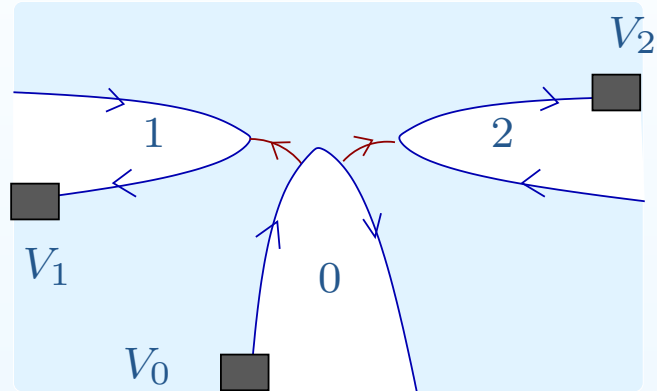
- Hanbury-Brown & Twiss (1956): Photon

0★



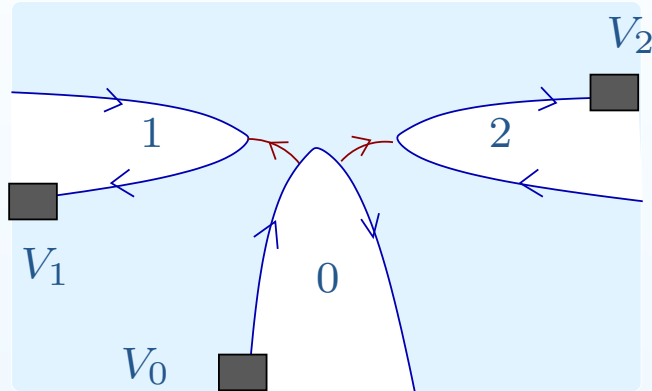
Intensity-intensity correlation
 $\Rightarrow$ Bunching

T-junction in Jain states, E.-A. Kim et al (PRL 05)



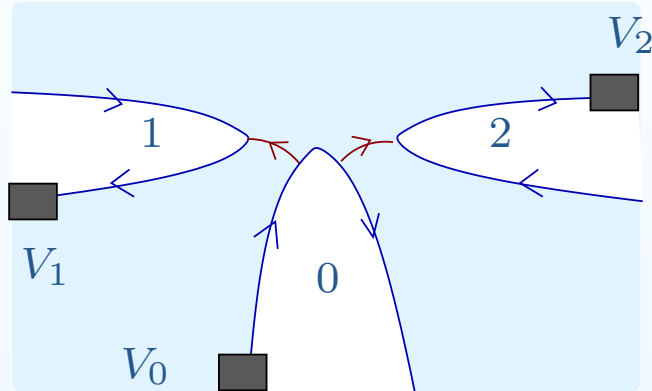
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- Cross correlations



$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

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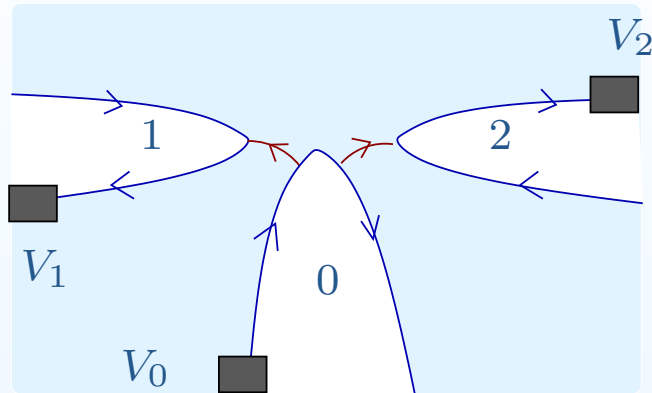


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$\Rightarrow$ **Non-equilibrium** ($V_1 - V_0 = V_2 - V_0 = V$),
finite T , perturbative calculation (Related
works on Laughlin states at $T=0$
by I. Safi et. al., S. Vishveshwara)

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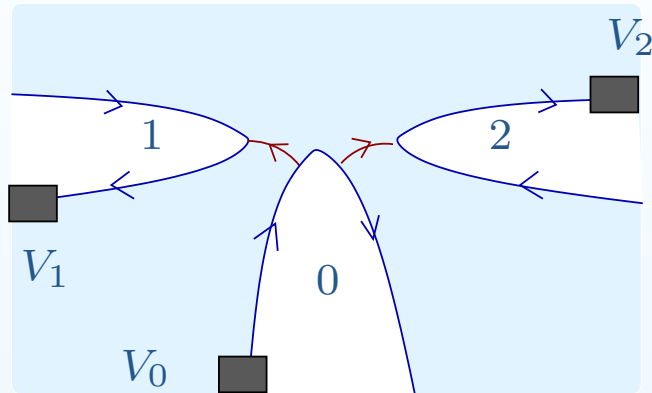
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- Tunneling Hamiltonian

$$\mathcal{L}_{int,l}(t) = \sum_{\epsilon=\pm} -\Gamma_l e^{i\epsilon\omega_0 t} V_l^{(\epsilon)}(t),$$

$$V_l^{(\epsilon)}(t) = (F_0 F_l^{-1})^\epsilon e^{i\epsilon\varphi_0(t)} e^{-i\epsilon\varphi_l(t)}$$

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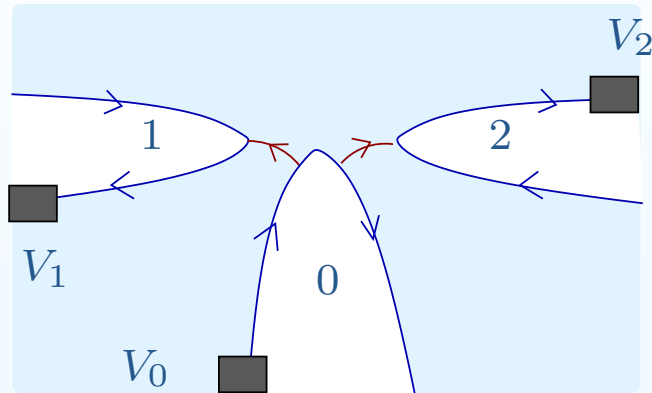
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- $\omega_0 = e^* V / \hbar$: **Josephson frequency**

- q.p. for edge l with unitary Klein factors F_l

$$\psi_l^\dagger \propto F_l e^{i\varphi_l},$$

$$F_l F_m = e^{-i\alpha_{lm}} F_m F_l$$

$$\alpha_{02} = \alpha_{21} = \alpha_{01} = \theta, \quad \alpha_{lm} = -\alpha_{ml}$$

Edge states for primary Jain sequence

- Chiral boson Lagrangian (charge mode ϕ_c , topological modes ϕ_N) (Lopez and Fradkin, 99)

$$\mathcal{L}_0 = \frac{1}{4\pi\nu} \partial_x \phi_c (-\partial_t \phi_c - \partial_x \phi_c) + \frac{1}{4\pi} (\partial_x \phi_N \partial_t \phi_N)$$

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- Quasi particle at $x = 0$:

$$\psi^\dagger(t) \propto e^{i(\frac{1}{p}\phi_c + \sqrt{1+\frac{1}{p}}\phi_N)} \equiv e^{i\varphi(t)}$$

$$\langle \psi(t) \psi^\dagger(0) \rangle = e^{\langle \varphi(t) \varphi(0) \rangle} = C(t) e^{-i\frac{\theta}{2} \text{sgn}(t)}, \quad C(t) \equiv \left| \frac{\frac{\pi\tau_0}{\beta}}{\sinh(\frac{\pi}{\beta}t)} \right|^K$$

$$\frac{K}{2} = \frac{1}{2p(2np+1)} : \text{scaling dimension, } \beta = 1/k_B T$$

Perturbative calculation

- $S^{\tilde{\epsilon}}(t)$ to lowest nontrivial order

$$\propto \tilde{\epsilon} \int dt_i^2 \cos[\omega_0(t-t_1-\tilde{\epsilon}t_2)] (C(t-t_1)C(t_2))^2 \left\{ \left(\frac{C(t-t_2)C(t_1)}{C(t)C(t_1-t_2)} \right)^{\tilde{\epsilon}} \sum_{\eta_1, \eta_2} \chi(\theta) - 1 \right\}$$

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$\tilde{\epsilon} = +/-$: relative tunneling orientation

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 - 1) comes from contour ordering
 - 2) carries the information of statistics

Perturbative calculation

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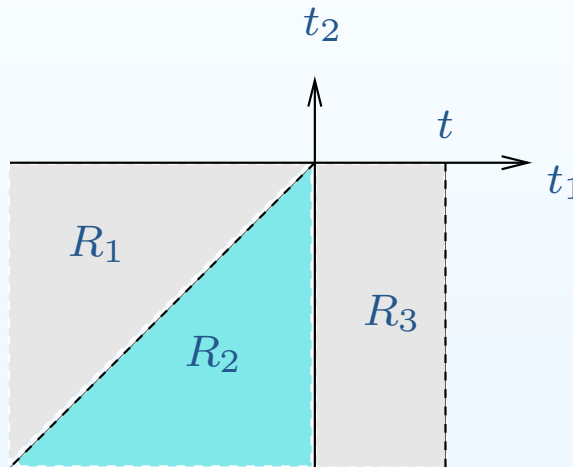
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$$\Rightarrow S(t) = \mathcal{A}(\omega_0 t; T/T_0, K) + \cos \theta \mathcal{B}(\omega_0 t; T/T_0, K)$$

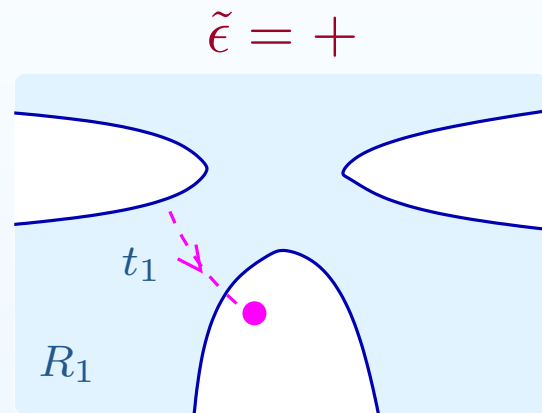
Anatomy of the phase factor

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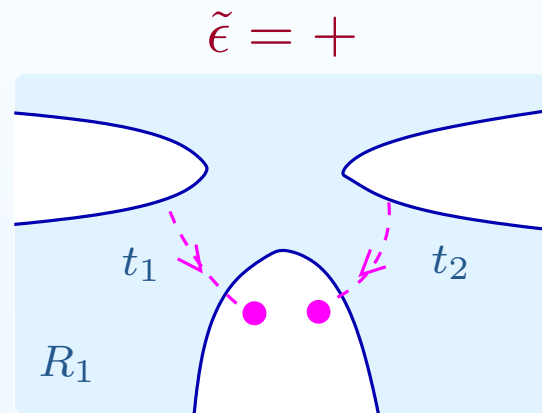
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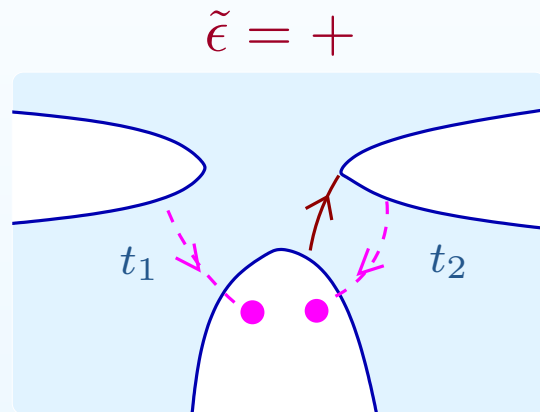
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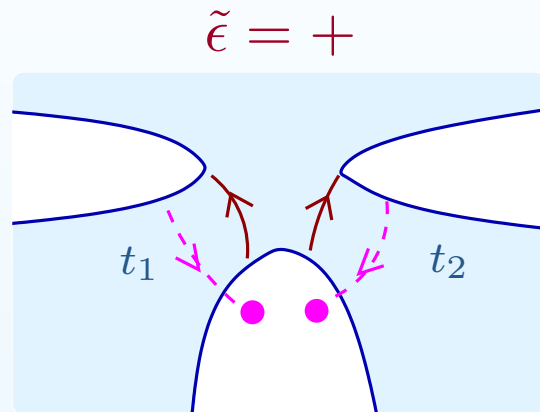
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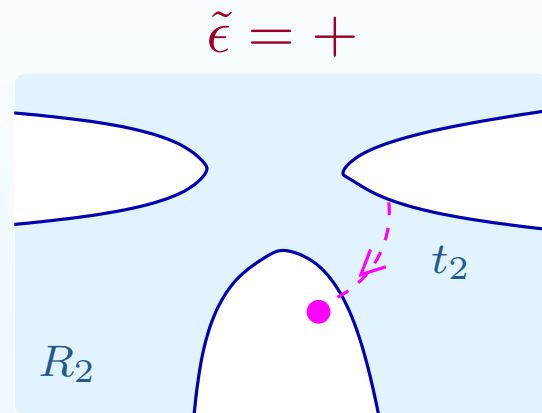
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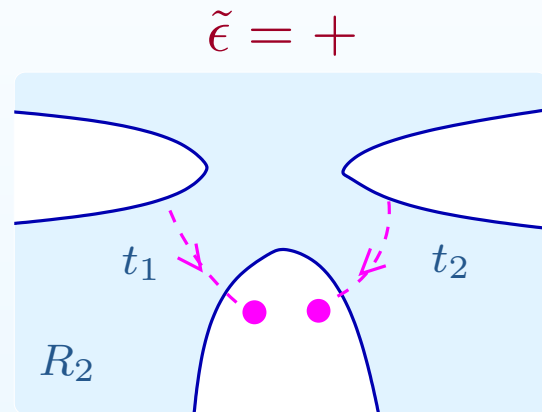
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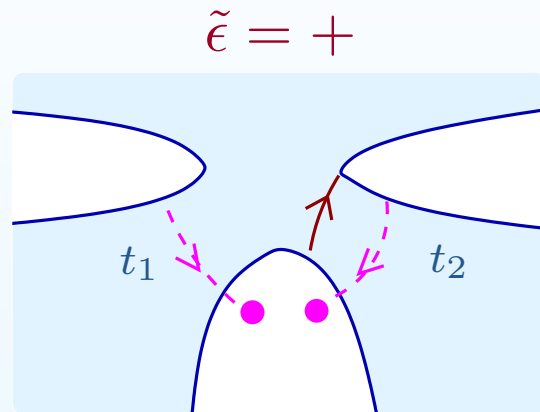
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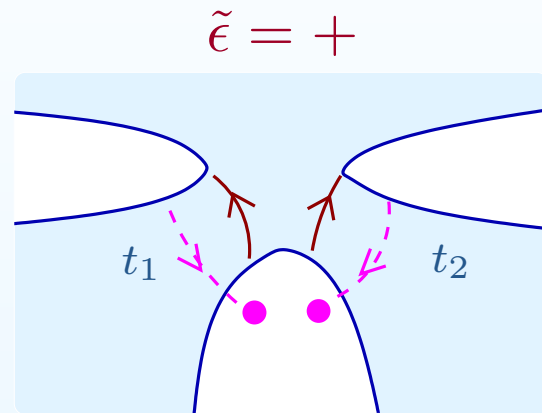
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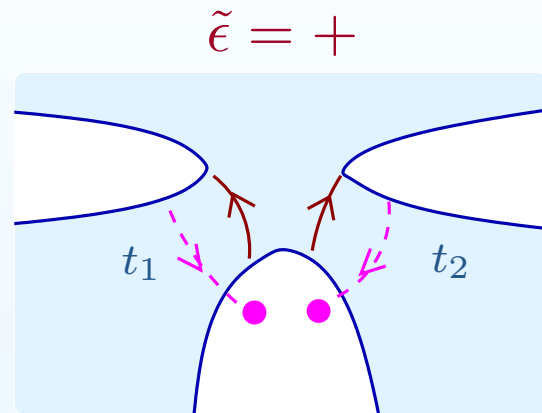


virtual exchange of qp's

$$\Rightarrow \chi[R_2; \eta] = e^{i\theta\eta} \chi[R_1; \eta]$$

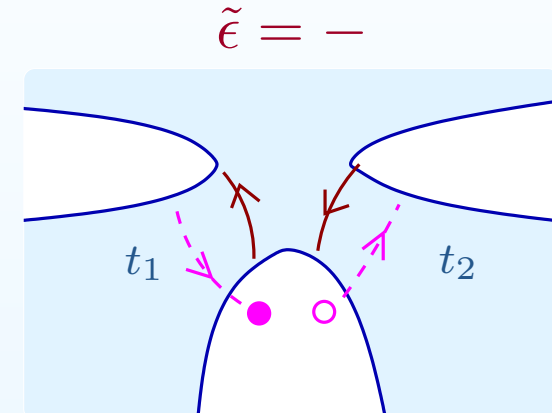
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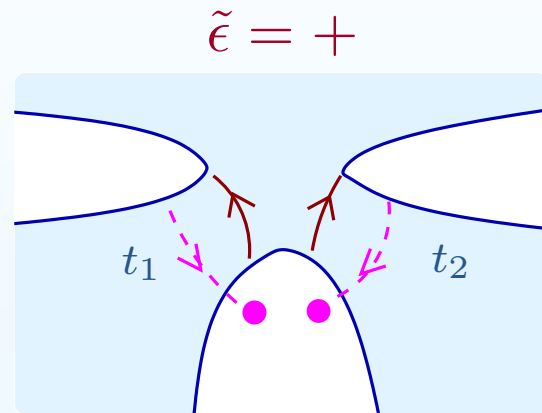


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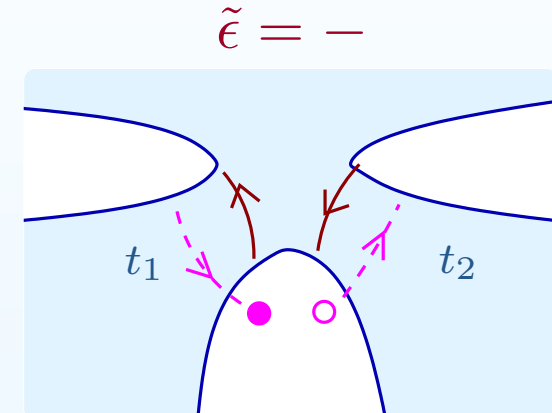
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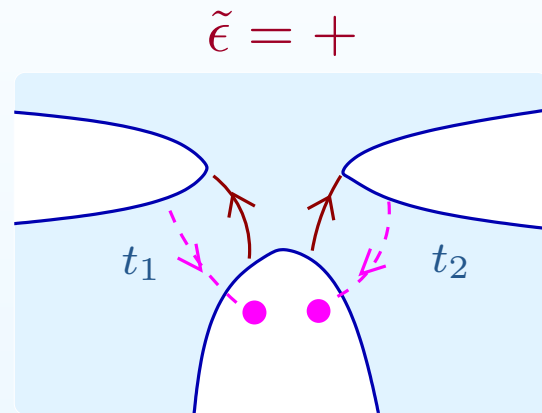
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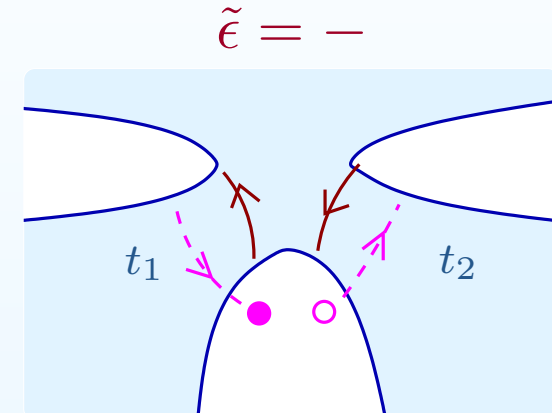
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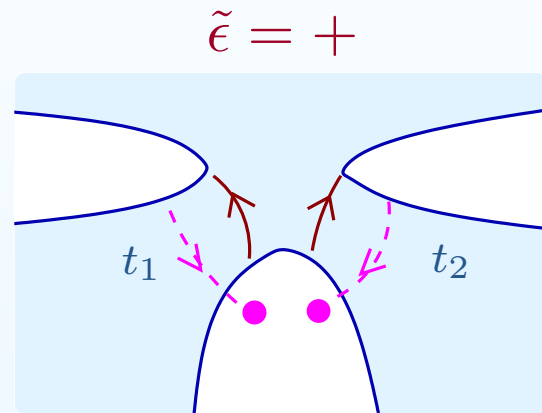
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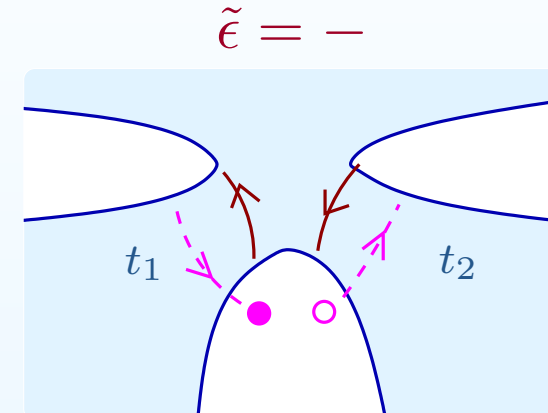
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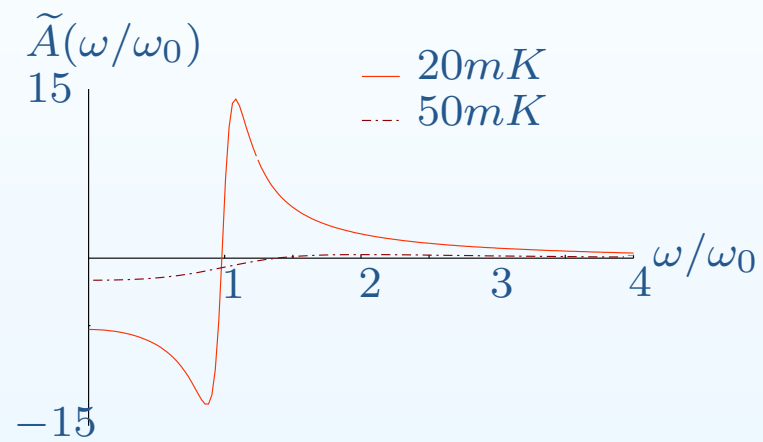
$$\sum_{\eta=\pm} \chi[R_2; \eta] (= \eta e^{i(\theta-\theta)\eta}) = 0$$

Frequency spectrum

- Direct term $\tilde{A}$ v.s. exchange term $\tilde{B}$

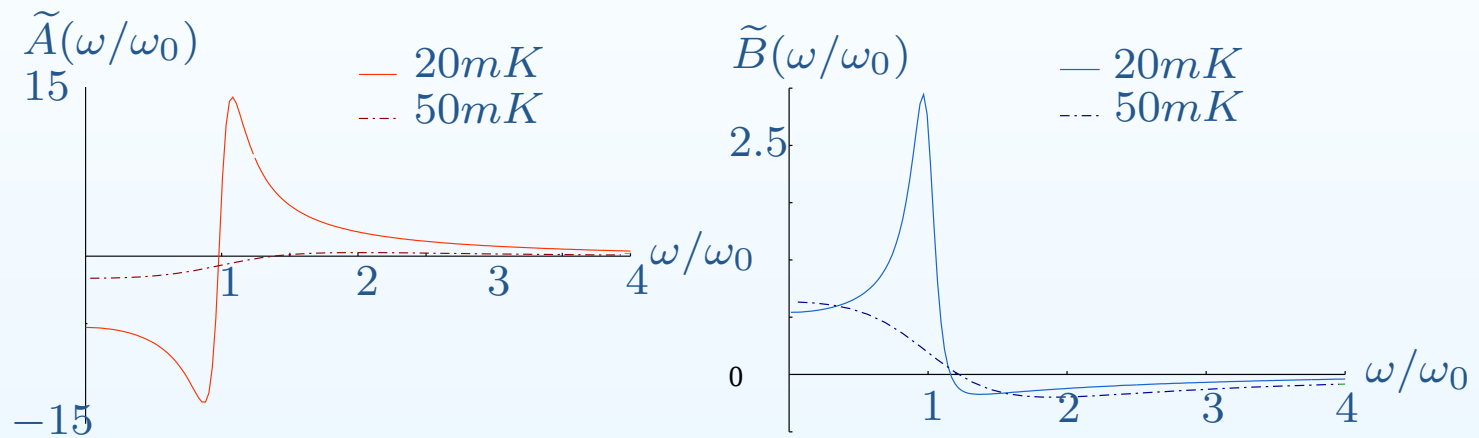
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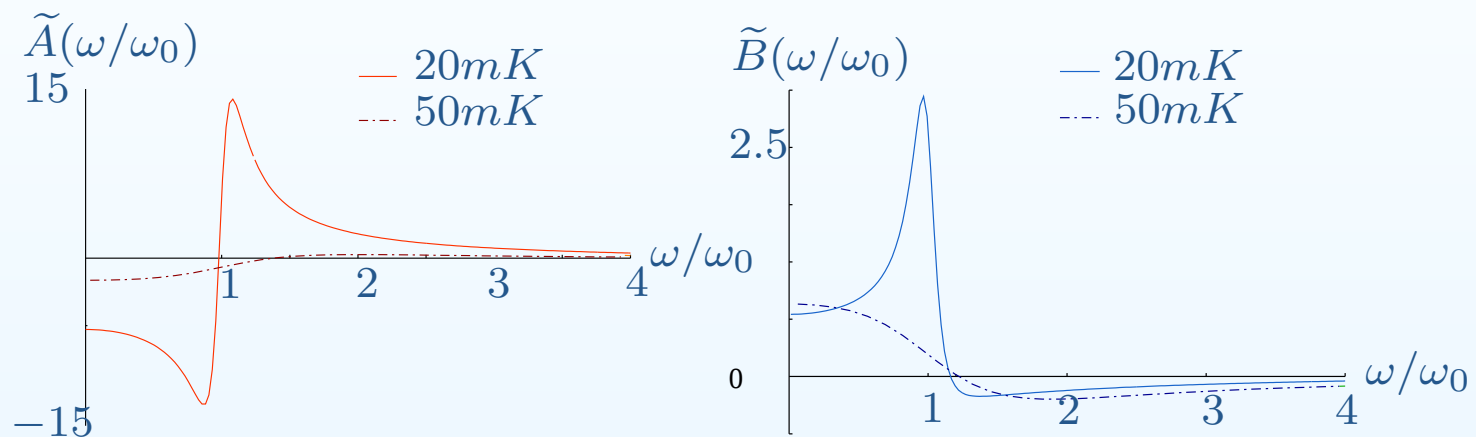
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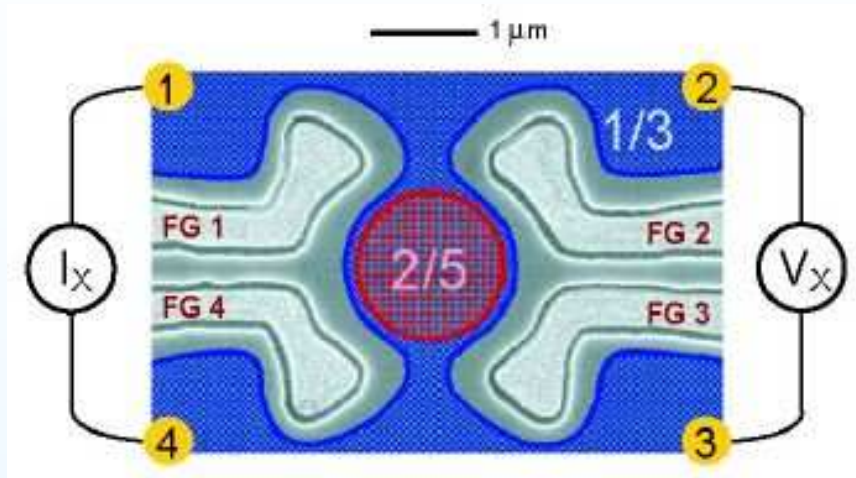


- $\tilde{S}(\omega/\omega_0; T) = \tilde{A} + \cos \theta \tilde{B}$.
- "Bunching" Laughlin qp ($\theta < \pi/2$) v.s. "anti-bunching" non-Laughlin qp ($\theta > \pi/2$).

Contents

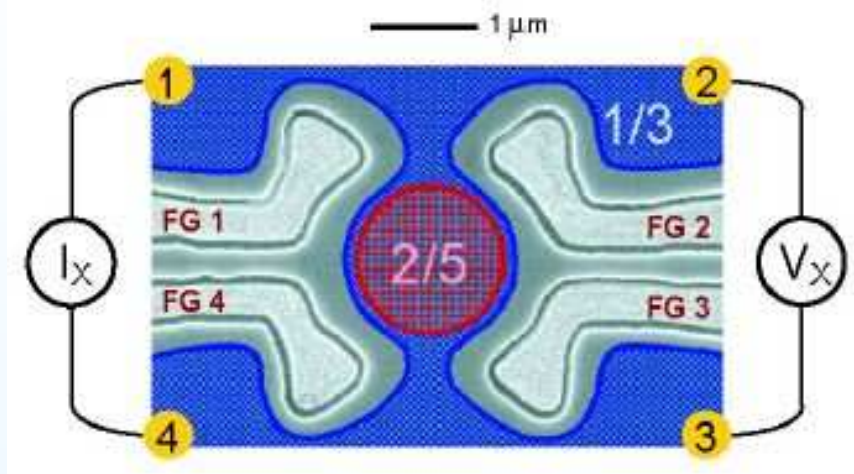
- Getting ready
- Cross current Noise in T-junction
- ▶ Quantum Hall Interferometer
 - Superperiodic Aharonov-Bohm effect
 - Interference conditions
 - Temperature dependence
- Summary

Superperiod Aharonov-Bohm effect (Goldman, 05)

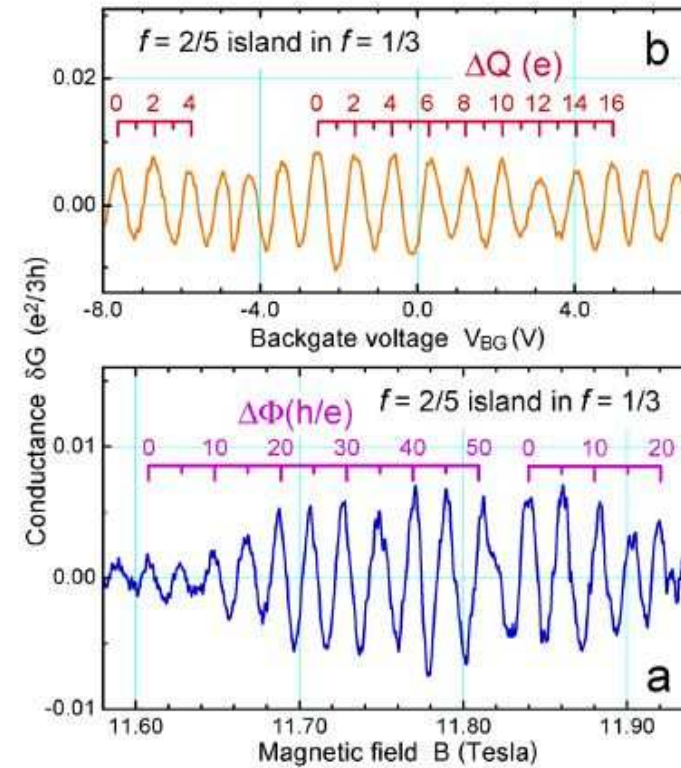


Four terminal measurements.

Superperiod Aharonov-Bohm effect (Goldman, 05)

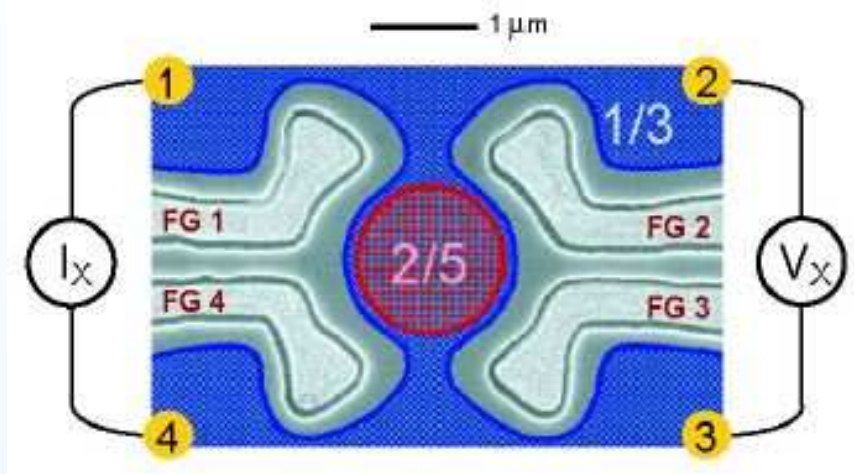


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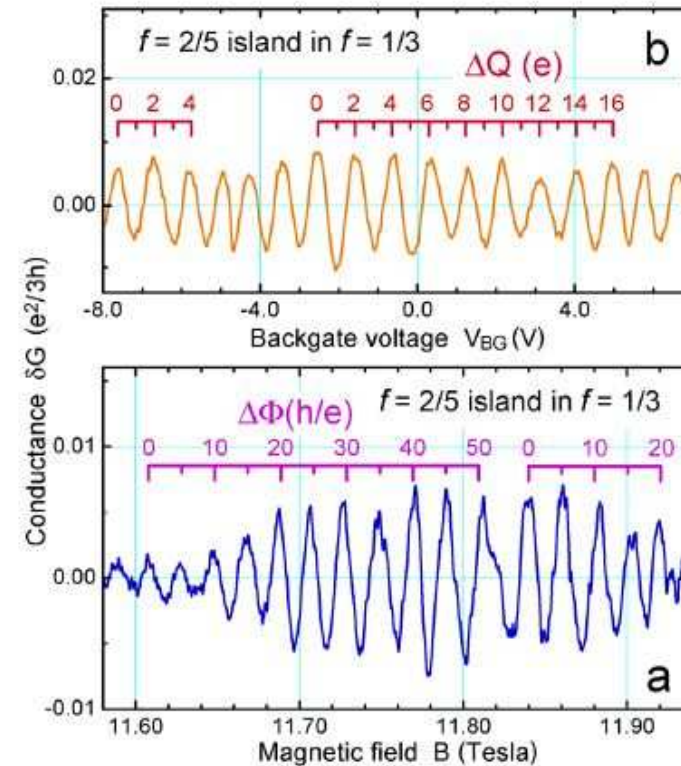


Superperiod oscillation with $\Delta\Phi = 5\phi_0$.

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Four terminal measurements.



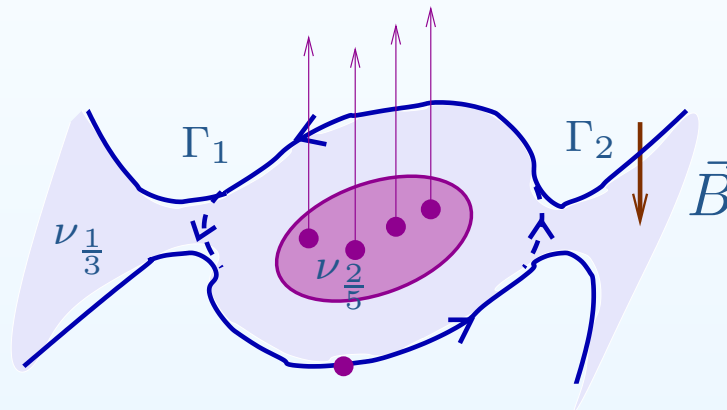
Superperiod oscillation with
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Is this REALLY a measure of fractional statistics?

Our model

- Double point contact interferometer

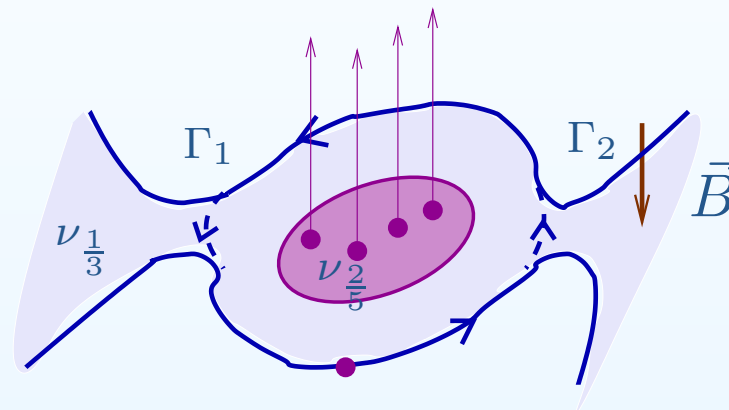
(Related works E.-A. Kim et al, PRL 03; Chamon et al, PRB 97)



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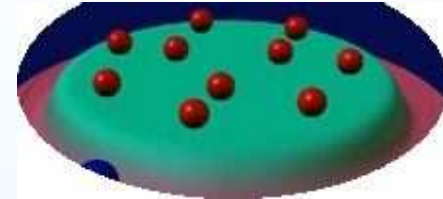
- Assumptions:

- No direct tunneling between outer edge and the inner puddle.
- Coherent propagation of $1/3$ qp along outer edge.
- Incompressibility of $2/5$ puddle.
- Fractional statistics between $1/3$ qp's with $\theta = \pi/3$.

Incompressibility of $2/5$ FQH liquid

- Hierarchical picture:

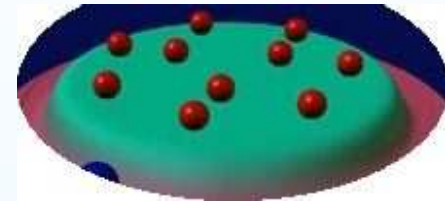
$1/3$ qp's condense to form a puddle of $2/5$ state



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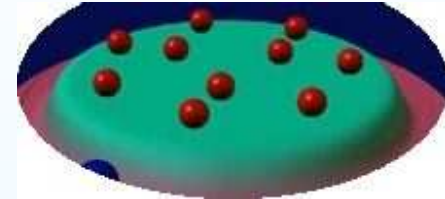


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- Incompressibility of $2/5$ state $\Rightarrow$ flux superquantization
 - $\frac{(\text{total charge of puddle } Q)/e}{Bs/\phi_0} = \nu_{2/5}$
 - $B \uparrow$ require $Q \uparrow$: N extra $1/3$ qp condense to the puddle of area s

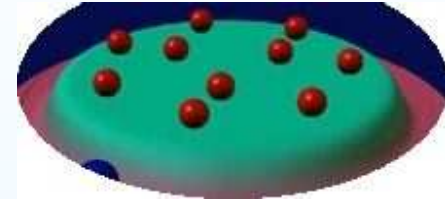
$$\nu_{1/3} \frac{Bs}{\phi_0} + \frac{1}{3}N = \nu_{2/5} \frac{Bs}{\phi_0}$$

(Jain, Kivelson, Thouless, 1993)

Incompressibility of $2/5$ FQH liquid

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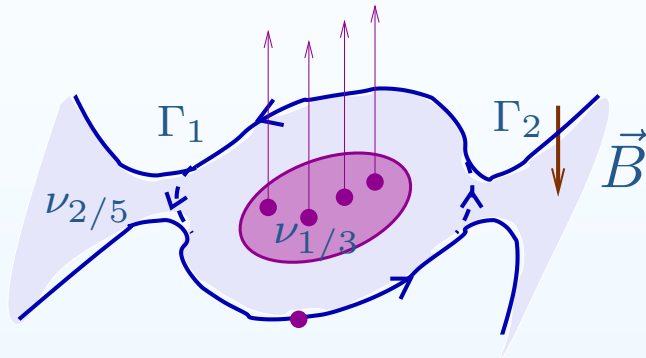


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$$N = \left[\frac{|B|s}{5\phi_0} \right]$$

(E.-A. Kim, in preparation)

Interference conditions

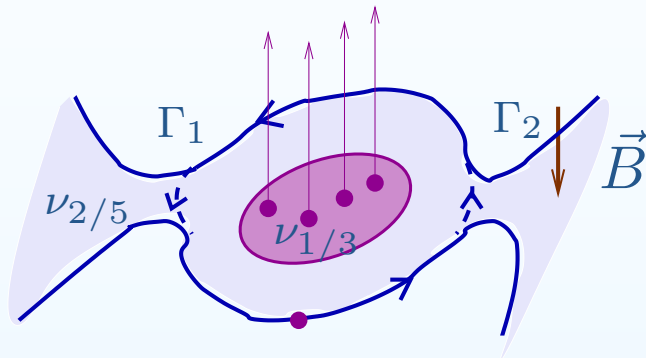


- Two independent periods:
 - Aharonov-Bohm phase due to flux through the area S

$$2\pi \frac{e^*}{e} \frac{|B|S}{\phi_0} = 2\pi \frac{1}{3} \frac{|B|S}{\phi_0}$$
 - Statistical phase due to N qp's in the puddle of area s

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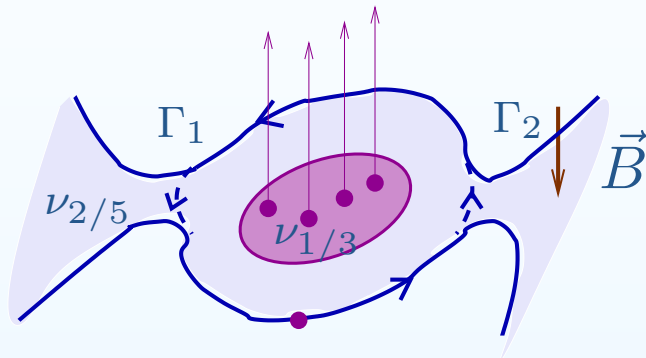
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$$\Delta \frac{\gamma}{2\pi} = \left(5 \frac{e^*}{e} \frac{S}{s} - \frac{\theta}{2\pi} \right) \Delta \left[\frac{|B|s}{5\phi_0} \right] = \left(\frac{5}{3} \frac{S}{s} - \frac{1}{3} \right) \Delta \left[\frac{|B|s}{5\phi_0} \right] = \text{integer}$$

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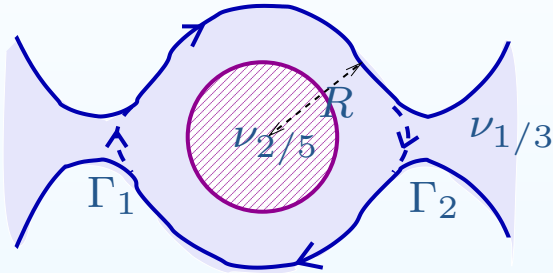
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- $$S/s = 1.43 \sim 7/5 \Rightarrow \Delta |B|s = 5\phi_0 :$$

consistent with the experiment

Temperature dependence of oscillation amplitude

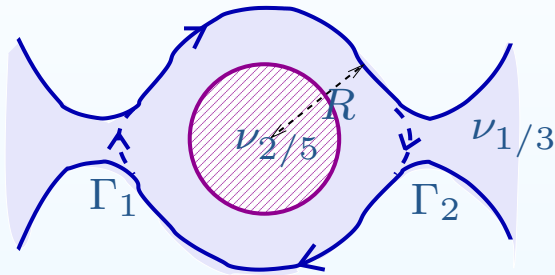
- Perturbative calculation to leading order in Γ



$$H_t = \frac{\Gamma_1}{2} e^{-i\omega_J t} \psi_{R,1}^\dagger \psi_{L,1} + e^{i\gamma} \frac{\Gamma_2^*}{2} e^{i\omega_J t} \psi_{L,2}^\dagger \psi_{R,2} + h.c.$$

Temperature dependence of oscillation amplitude

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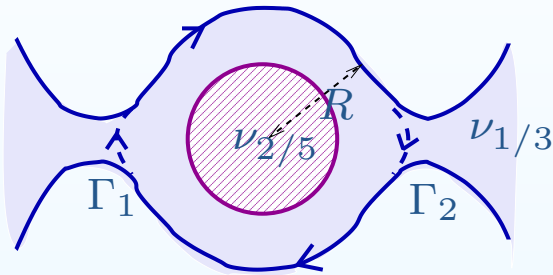
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- The conductance; data v.s. theory

$$G(\omega_0, v/R, T) = \bar{G}(\omega_0/T) + \cos \gamma \, \delta G(\omega_0, v/R, T), \quad \omega_0 = e^* V / \hbar$$

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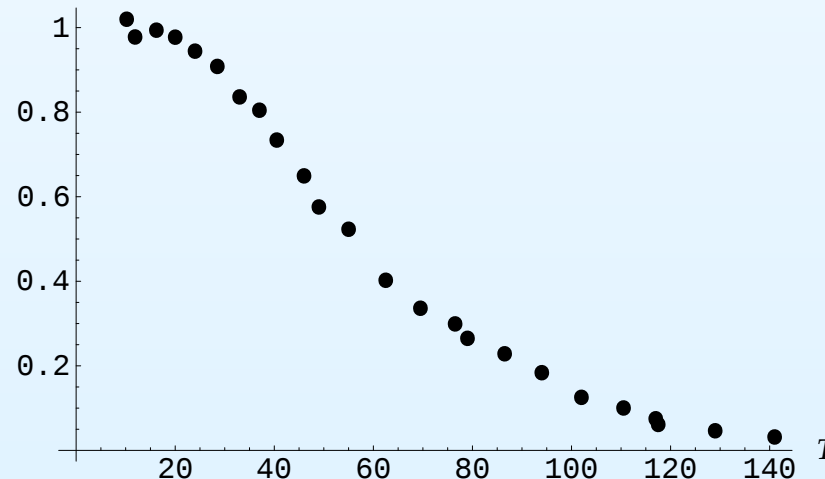


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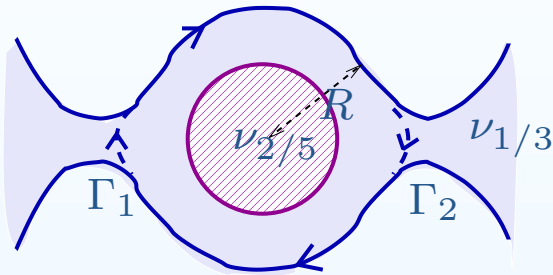
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$$\delta G(T) / \delta G(T = 11 \text{ mK})$$



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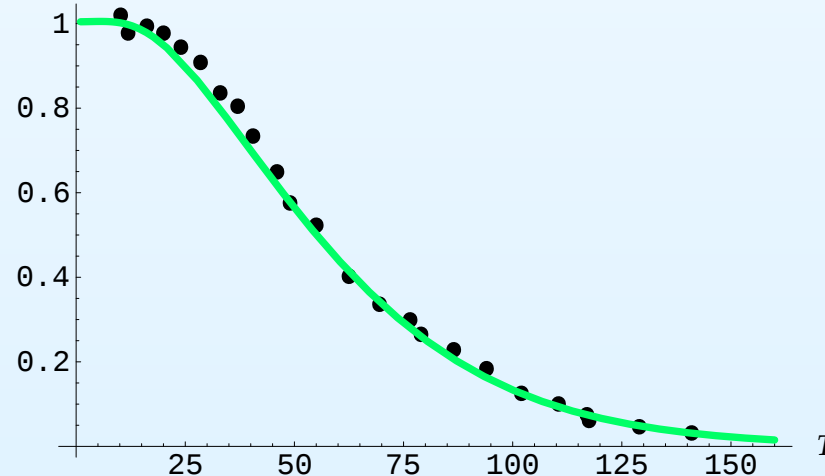


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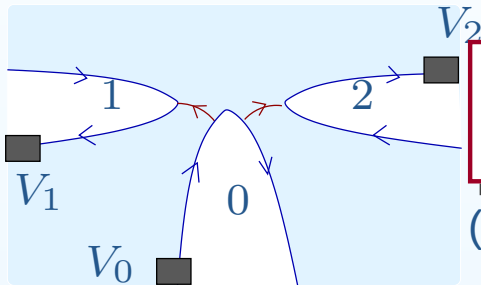
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Summary: Anyon there?

- T-junction proposal: the cross current noise $S(\omega)$

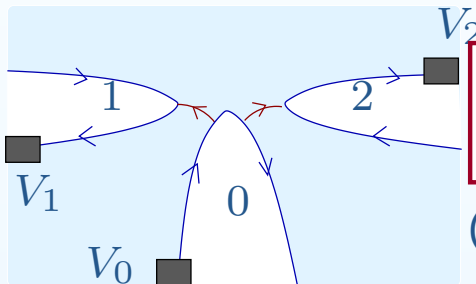


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(E.-A. Kim et al, PRL 05)

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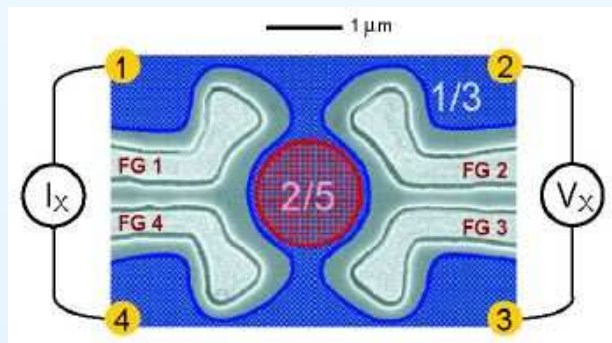
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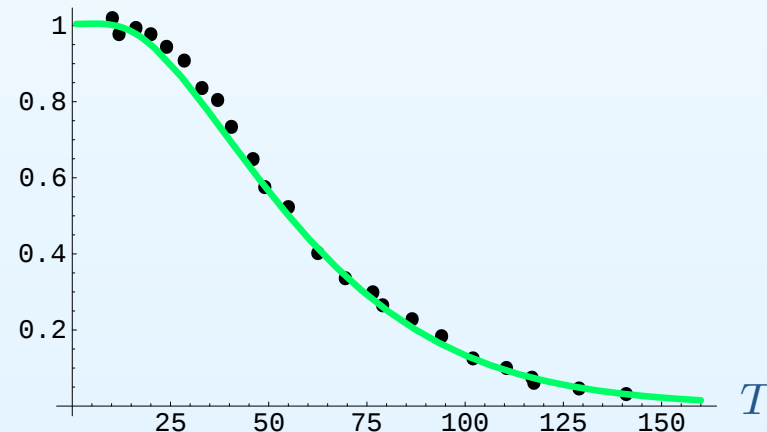
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The set up.

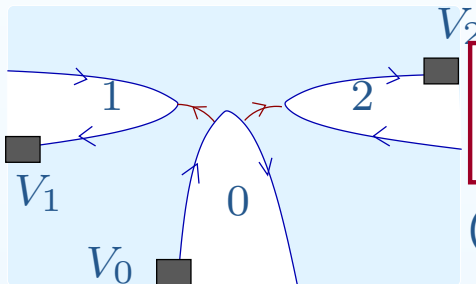
$\delta G(T)/\delta G(T = 11mK)$



The fit

Summary: Anyon there?

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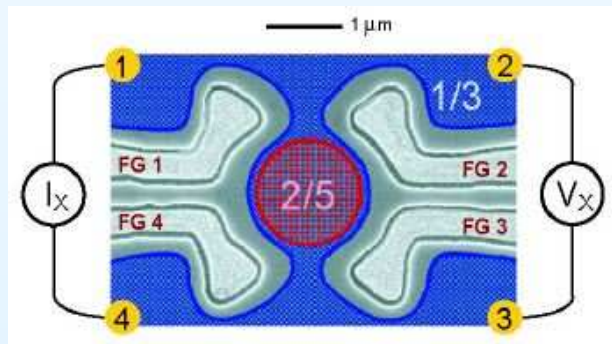


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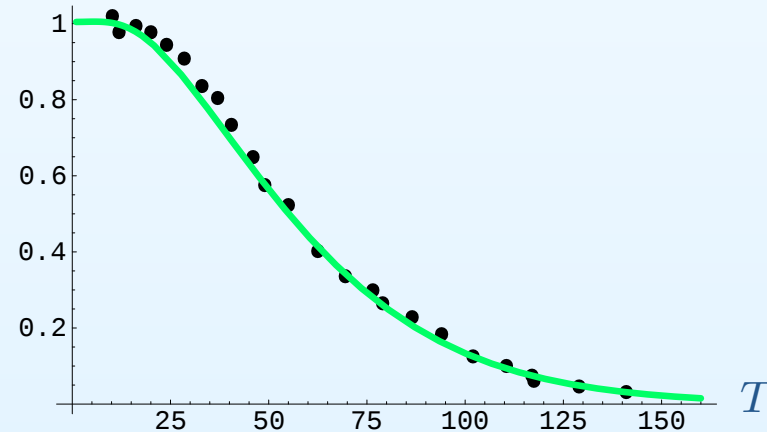
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$\delta G(T)/\delta G(T = 11mK)$



The set up.



The fit

- Future: non-Abelian statistics

Summary: Anyon there?

If they are there, we can now manipulate them!

References

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