# *In Search of Fractional Statistics: Anyon There?*

Eun-Ah Kim

Stanford University

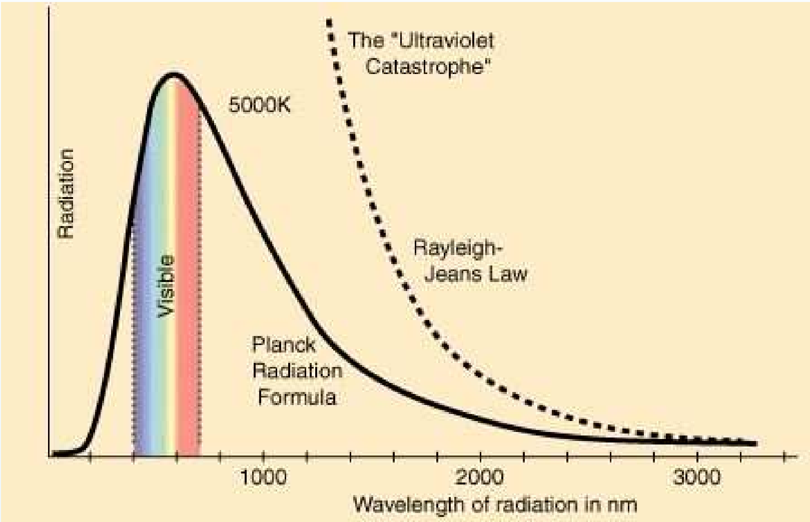
# Thanks to...

Michael Lawler, Smitha Vishveshwara, Eduardo Fradkin (UIUC),
Moty Heilblum (Weizmann), Claudio Chamon (BU),
Duncan Haldane (Princeton), Eddy Ardonne (KITP)
Steven Kivelson (Stanford), Vl adimir Goldman (Stony Brook)

# Contents

# Statistics: boson v.s. fermion

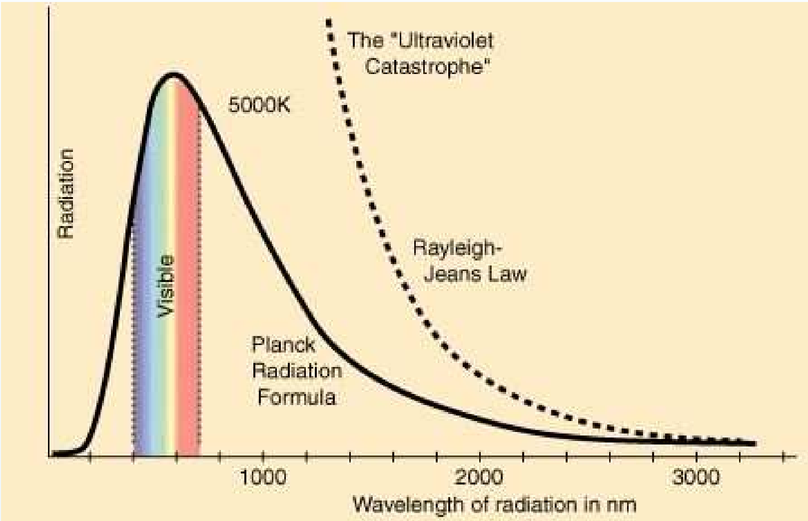
- Black body radiation




Bose-Einstein statistics of photon

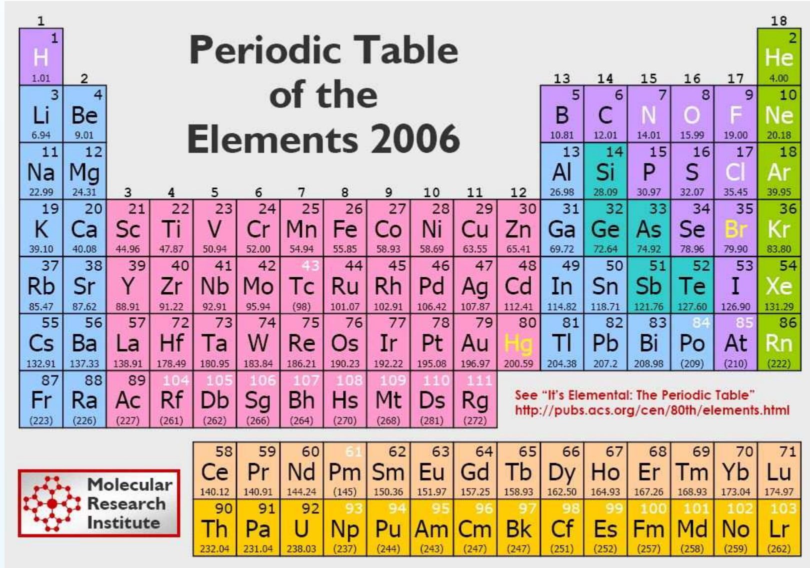
# Statistics: boson v.s. fermion

- Black body radiation




Bose-Einstein statistics of photon

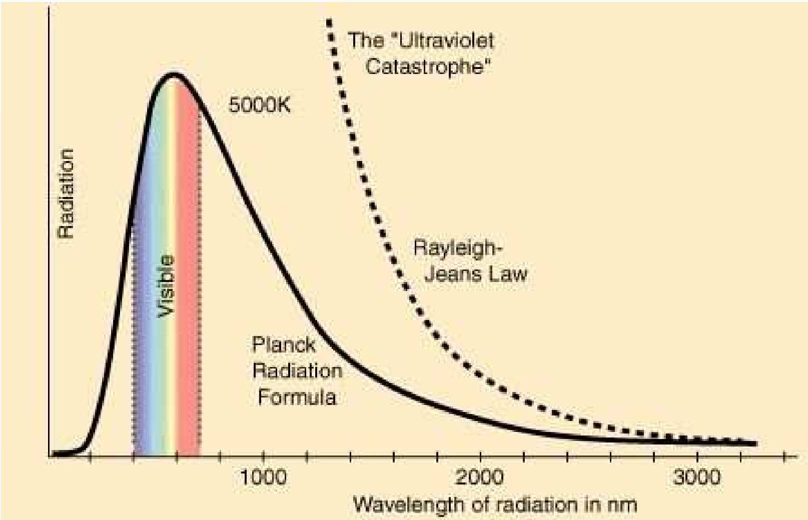
- Periodic table




Fermi-Dirac statistics of electrons

# Statistics: boson v.s. fermion

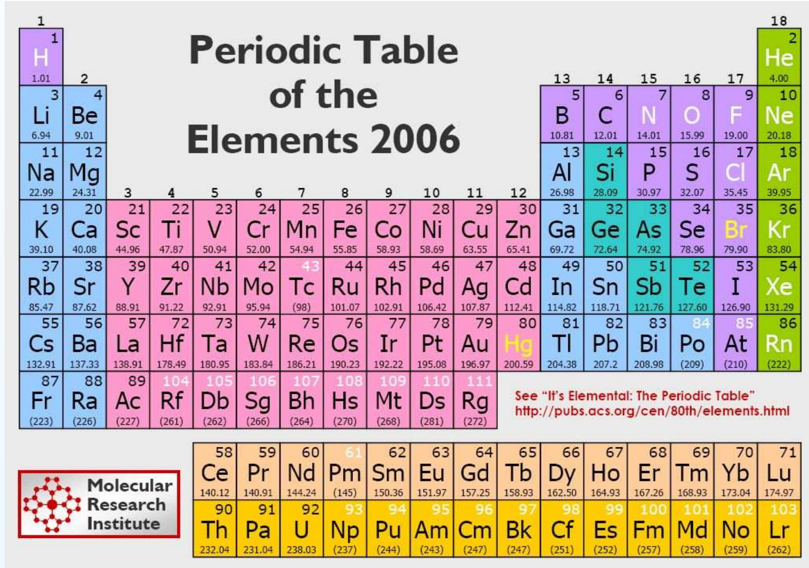
- Black body radiation




Bose-Einstein statistics of photon

- Superfluid $He^4$

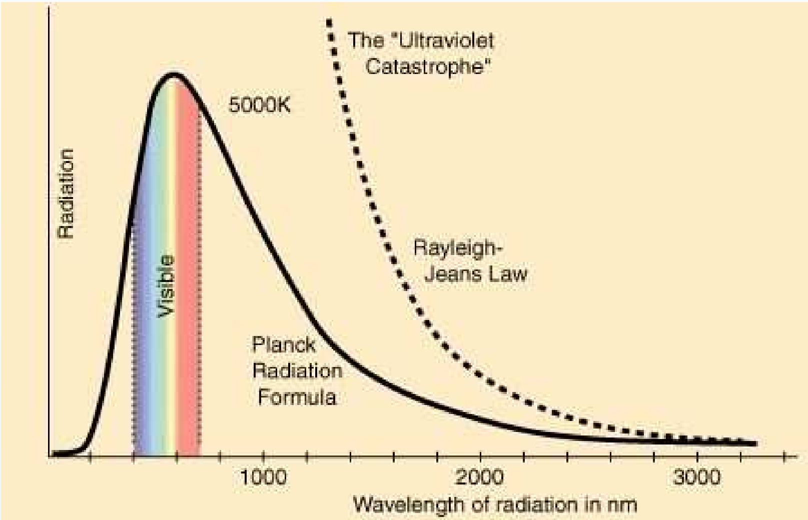
- Periodic table




Fermi-Dirac statistics of electrons

# Statistics: boson v.s. fermion

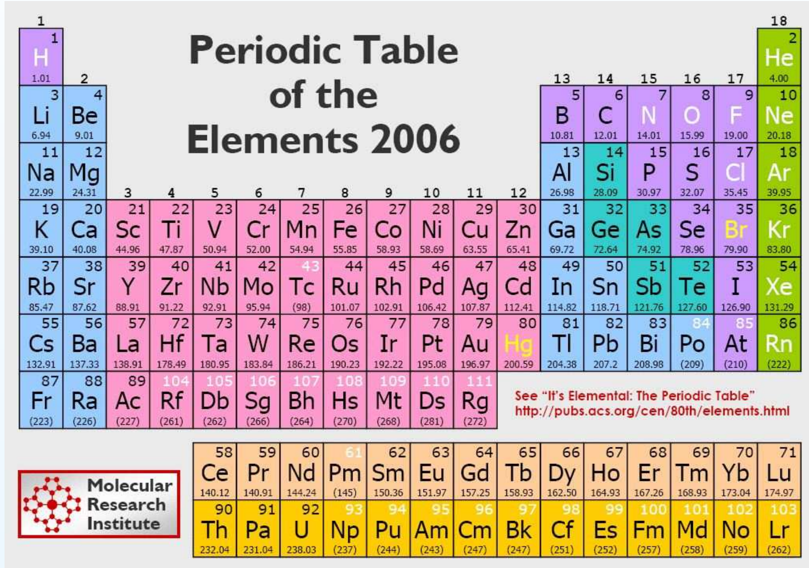
- Black body radiation




Bose-Einstein statistics of photon

- Superfluid $He^4$

- Periodic table




Fermi-Dirac statistics of electrons

- Normal fluid $He^3$

# Fractional quasiparticles: Anyons

- Fractional charge $e^*$

## Fractional quasiparticles: Anyons

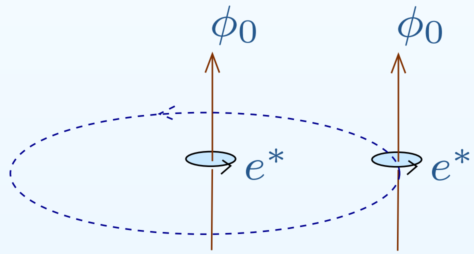
- Fractional charge $e^*$
- Fractional statistics

# Fractional quasiparticles: Anyons

- Fractional charge $e^*$
- Fractional statistics
  - Braiding statistics (Wilczek, 1982):

$$\Psi(\boldsymbol{r_1}, \boldsymbol{r_2}) = e^{i\theta}\Psi(\boldsymbol{r_2}, \boldsymbol{r_1})$$

$$0 \leq \theta \leq \pi$$



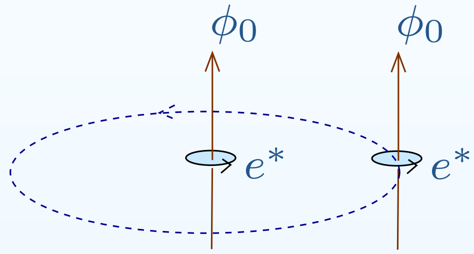

An adiabatic process.

# Fractional quasiparticles: Anyons

- Fractional charge $e^*$
- Fractional statistics
  - Braiding statistics (Wilczek, 1982):

$$\Psi(\boldsymbol{r_1}, \boldsymbol{r_2}) = e^{i\theta}\Psi(\boldsymbol{r_2}, \boldsymbol{r_1})$$

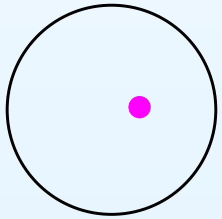
$0 \leq \theta \leq \pi$




An adiabatic process.

  - Generalized exclusion statistics (Haldane, 1991):

$$\Delta d_\alpha = -\sum_\beta g_{\alpha\beta}\Delta N_\beta$$



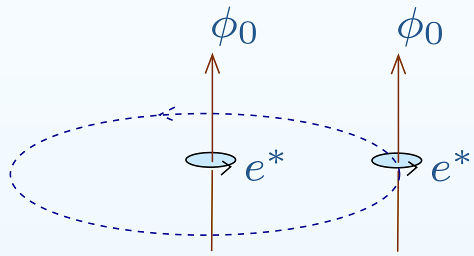
$g_{\alpha\beta} = 0$ for boson, $g_{\alpha\beta} = \delta_{\alpha\beta}$ for fermion

# Fractional quasiparticles: Anyons

- Fractional charge $e^*$
- Fractional statistics
  - Braiding statistics (Wilczek, 1982):

$$\Psi(\boldsymbol{r_1}, \boldsymbol{r_2}) = e^{i\theta}\Psi(\boldsymbol{r_2}, \boldsymbol{r_1})$$

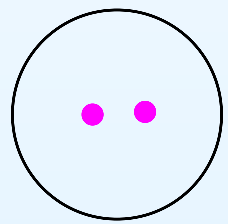
$0 \leq \theta \leq \pi$




An adiabatic process.

  - Generalized exclusion statistics (Haldane, 1991):

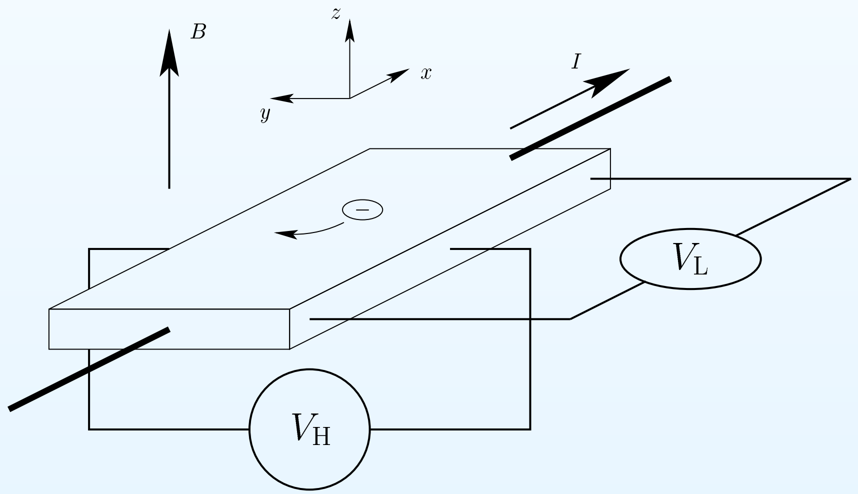
$$\Delta d_\alpha = -\sum_\beta g_{\alpha\beta}\Delta N_\beta$$



$g_{\alpha\beta} = 0$ for boson, $g_{\alpha\beta} = \delta_{\alpha\beta}$ for fermion

# The Fractional Quantum Hall Effect (Tsui et. al.,1982):

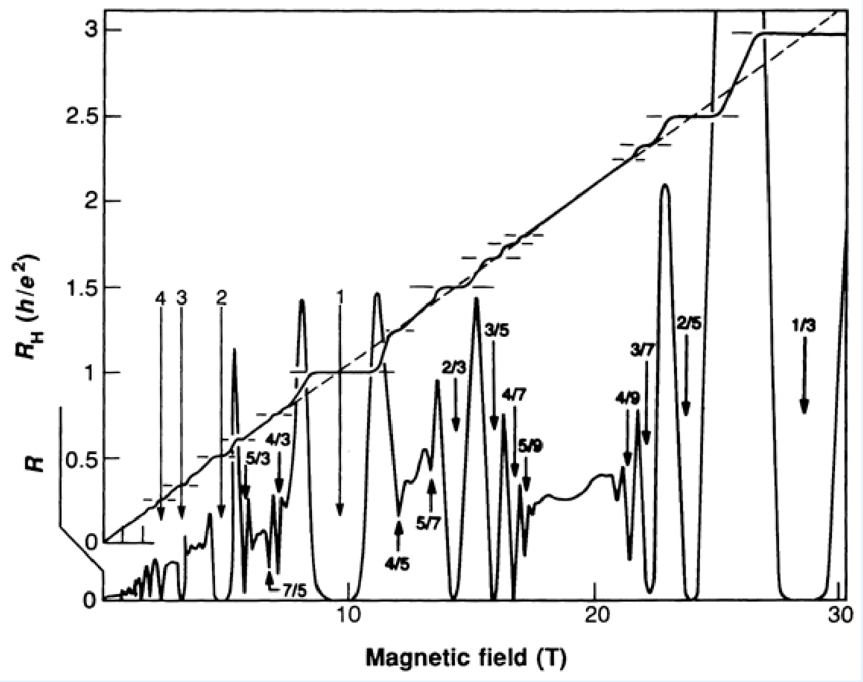
a quantization of Hall conductance ($\sigma_H = \nu e^2/h$) when the filling factor $\nu \equiv \rho h/eB$ ($\rho$ is the surface density) is in the vicinity of a hierarchy of rational fractions.




A quantum Hall bar

# The Fractional Quantum Hall Effect (Tsui et. al.,1982):

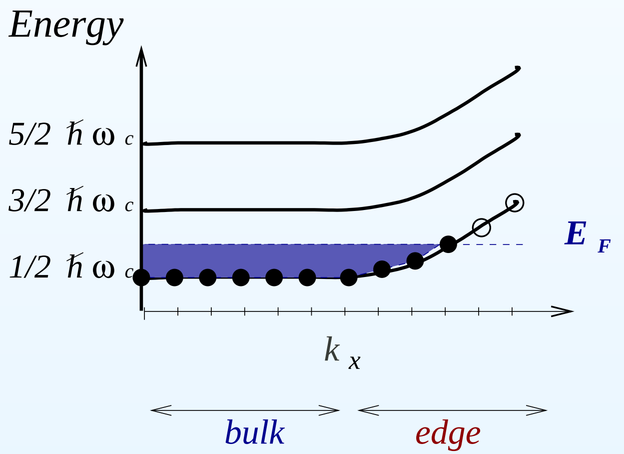
a quantization of Hall conductance ($\sigma_H = \nu e^2/h$) when the filling factor $\nu \equiv \rho h/eB$ ($\rho$ is the surface density) is in the vicinity of a hierarchy of rational fractions.




Quantum Hall plateau (taken from Eisenstein and Stormer, 1990)

# Edge States

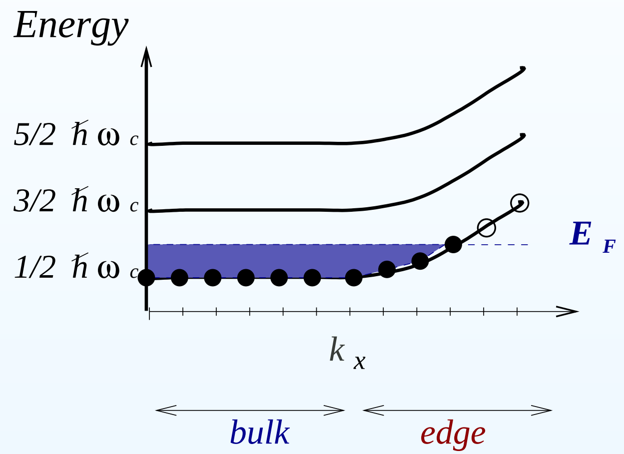
- The surface wave of edge distortions is the only gapless excitation.



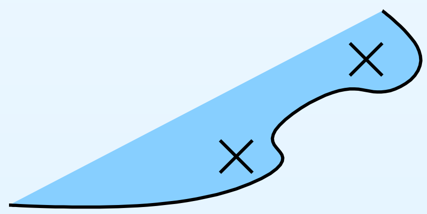

The Landau levels of confined 2DEG.

# Edge States

- The surface wave of edge distortions is the only gapless excitation.
- Hydrodynamic picture: dissipationless chiral Luttinger liquid. (Wen, 1990; Stone 1991)




The Landau levels of confined 2DEG.



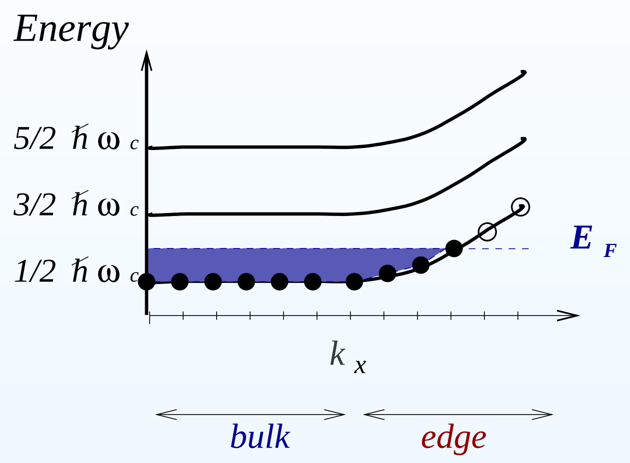
The edge mode is dissipationless.

# Edge States

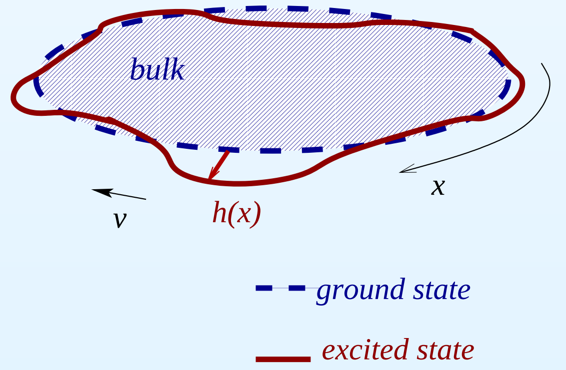
- The surface wave of edge distortions is the only gapless excitation.
- Hydrodynamic picture: dissipationless chiral Luttinger liquid. (Wen, 1990; Stone 1991)
- 1D density ripple $J(x) = \rho h(x)$ is related to chiral boson $\phi_+$ through bosonization

$$J_+(x) \equiv -\frac{\sqrt{\nu}}{2\pi}\partial_x\phi_+ \; , \; \psi_+^\dagger = \frac{1}{\sqrt{2\pi}} e^{\frac{i}{\nu}\phi_+}$$

$$\mathcal{L} = \frac{1}{4\pi}\partial_x\phi_+(\partial_t - \nu\partial_x)\phi_+$$




The Landau levels of confined 2DEG.




A displacement $h(x)$ at the point $x$.

## Contents

# Fractional statistics in QH Jain States

- Pantry of Particles (since '84)

| | Boson | $\nu = \frac{1}{\text{odd}}$ (Laughlin) | $\nu = \frac{p}{2np+1}$ (Jain) | Fermion |
|---|---|---|---|---|
| phase | $1 = e^0$ | $e^{i\nu\pi}$ | $e^{i\theta} (\frac{\theta}{\pi} = \frac{2n}{2np+1} + 1)$ | $-1 = e^{i\pi}$ |
| charge | | $\nu e$ | $Q = \frac{-e}{2np+1}$ | $-e$ |

$2n =$ # of attached flux quanta
$p =$ effective filling factor

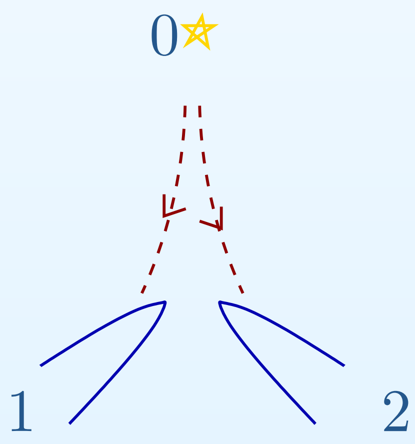
# Fractional statistics in QH Jain States

- Pantry of Particles (since ’84)

| | Boson | $\nu = \frac{1}{\text{odd}}$ (Laughlin) | $\nu = \frac{p}{2np+1}$ (Jain) | Fermion |
|---|---|---|---|---|
| phase | $1 = e^0$ | $e^{i\nu\pi}$ | $e^{i\theta}(\frac{\theta}{\pi} = \frac{2n}{2np+1} + 1)$ | $-1 = e^{i\pi}$ |
| charge | | $\nu e$ | $Q = \frac{-e}{2np+1}$ | $-e$ |

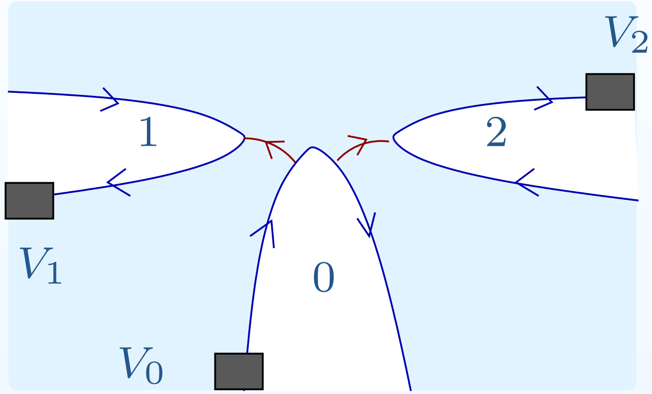
$2n = \#$ of attached flux quanta
$p =$ effective filling factor

- Hanbury-Brown & Twiss (1956): Photon

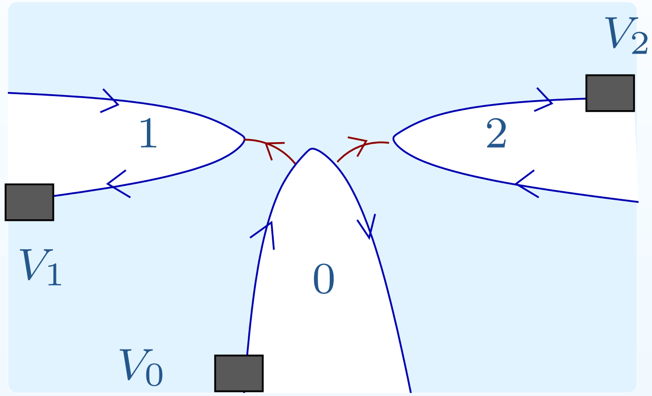



Intensity-intensity correlation
$\Longrightarrow$ Bunching

# T- junction in Jain states, E.-A. Kim et al (PRL 05)



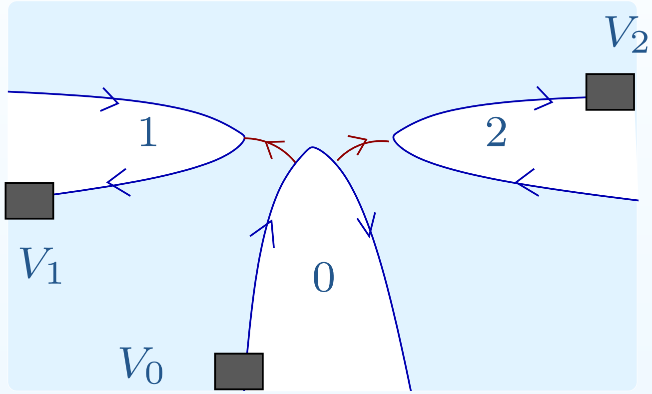
# T- junction in Jain states, E.-A. Kim et al (PRL 05)




- Cross correlations

$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

# T- junction in Jain states, E.-A. Kim et al (PRL 05)



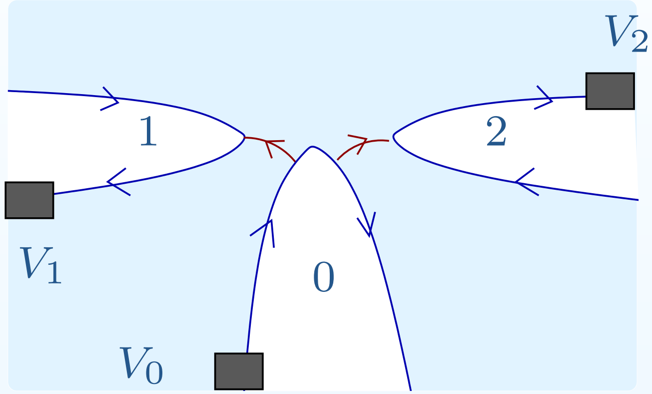

- Cross correlations

$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

$\Rightarrow$ Non-equilibrium ($V_1 - V_0 = V_2 - V_0 = V$), finite $T$, perturbative calculation (Related works on Laughlin states at $T = 0$

by I. Safi et. al., S. Vishveshwara)

# T- junction in Jain states, E.-A. Kim et al (PRL 05)




- Cross correlations

$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

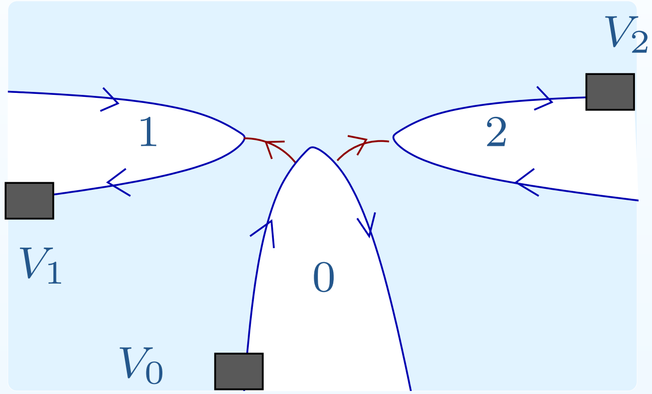
$\Rightarrow$ Non-equilibrium ($V_1 - V_0 = V_2 - V_0 = V$), finite $T$, perturbative calculation (Related works on Laughlin states at $T=0$ by I. Safi et. al., S. Vishveshwara)

- Tunneling Hamiltonian

$$\mathcal{L}_{int,l}(t) = \sum_{\epsilon=\pm} -\Gamma_l e^{i\epsilon\omega_0 t} V_l^{(\epsilon)}(t),$$

$$V_l^{(\epsilon)}(t) = (F_0 F_l^{-1})^{\epsilon} e^{i\epsilon\varphi_0(t)} e^{-i\epsilon\varphi_l(t)}$$

# T- junction in Jain states, E.-A. Kim et al (PRL 05)




- Cross correlations

$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

$\Rightarrow$ Non-equilibrium ($V_1 - V_0 = V_2 - V_0 = V$), finite $T$, perturbative calculation (Related works on Laughlin states at $T=0$ by I. Safi et. al., S. Vishveshwara)

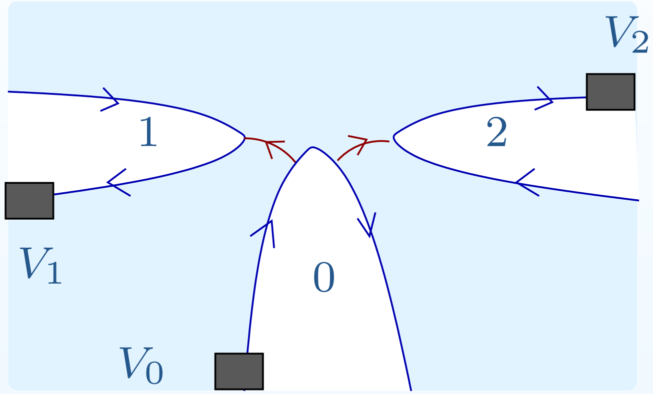
- Tunneling Hamiltonian

$$\mathcal{L}_{int,l}(t) = \sum_{\epsilon=\pm} -\Gamma_l e^{i\epsilon\omega_0 t} V_l^{(\epsilon)}(t),$$

$$V_l^{(\epsilon)}(t) = (F_0 F_l^{-1})^{\epsilon} e^{i\epsilon\varphi_0(t)} e^{-i\epsilon\varphi_l(t)}$$

  - $\omega_0 = e^* V/\hbar$: Josephson frequency

# T- junction in Jain states, E.-A. Kim et al (PRL 05)




- Cross correlations

$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

$\Rightarrow$ Non-equilibrium ($V_1 - V_0 = V_2 - V_0 = V$), finite $T$, perturbative calculation (Related works on Laughlin states at $T=0$ by I. Safi et. al., S. Vishveshwara)

- Tunneling Hamiltonian

$$\mathcal{L}_{int,l}(t) = \sum_{\epsilon=\pm} -\Gamma_l e^{i\epsilon\omega_0 t} V_l^{(\epsilon)}(t),$$

$$V_l^{(\epsilon)}(t) = (F_0 F_l^{-1})^{\epsilon} e^{i\epsilon\varphi_0(t)} e^{-i\epsilon\varphi_l(t)}$$

  - $\omega_0 = e^* V/\hbar$: Josephson frequency

- q.p. for edge $l$ with unitary Klein factors $F_l$

$$\psi_l^\dagger \propto F_l e^{i\varphi_l},$$

$$F_l F_m = e^{-i\alpha_{lm}} F_m F_l$$

$\alpha_{02} = \alpha_{21} = \alpha_{01} = \theta$, $\alpha_{lm} = -\alpha_{ml}$

# Edge states for primary Jain sequence

- Chiral boson Lagrangian (charge mode $\phi_c$, topological modes $\phi_N$) (Lopez and Fradkin, 99)

$$\mathcal{L}_0 = \frac{1}{4\pi\nu}\partial_x\phi_c(-\partial_t\phi_c - \partial_x\phi_c) + \frac{1}{4\pi}(\partial_x\phi_N\partial_t\phi_N)$$

# Edge states for primary Jain sequence

- Chiral boson Lagrangian (charge mode $\phi_c$, topological modes $\phi_N$) (Lopez and Fradkin, 99)

$$\mathcal{L}_0 = \frac{1}{4\pi\nu}\partial_x\phi_c(-\partial_t\phi_c - \partial_x\phi_c) + \frac{1}{4\pi}(\partial_x\phi_N\partial_t\phi_N)$$

- Quasi particle at $x = 0$:

$$\psi^\dagger(t) \propto e^{i(\frac{1}{p}\phi_c + \sqrt{1+\frac{1}{p}}\phi_N)} \equiv e^{i\varphi(t)}$$

$$\langle\psi(t)\psi^\dagger(0)\rangle = e^{\langle\varphi(t)\varphi(0)\rangle} = C(t)e^{-i\frac{\theta}{2}\mathrm{sgn}(t)}, \quad C(t) \equiv \left|\frac{\frac{\pi\tau_0}{\beta}}{\sinh(\frac{\pi}{\beta}t)}\right|^K$$

$\frac{K}{2} = \frac{1}{2p(2np+1)}$ : scaling dimension, $\beta = 1/k_B T$

# Perturbative calculation

- $S^{\tilde{\epsilon}}(t)$ to lowest nontrivial order

$$\propto \tilde{\epsilon} \int dt_i^2 \cos[\omega_0(t - t_1 - \tilde{\epsilon} t_2)](C(t-t_1)C(t_2))^2 \left\{ \left( \frac{C(t-t_2)C(t_1)}{C(t)C(t_1-t_2)} \right)^{\tilde{\epsilon}} \sum_{\eta_1,\eta_2} \chi(\theta) - 1 \right\}$$

# Perturbative calculation

- $S^{\tilde{\epsilon}}(t)$ to lowest nontrivial order

$$\propto \tilde{\epsilon} \int dt_i^2 \cos[\omega_0(t - t_1 - \tilde{\epsilon} t_2)] (C(t-t_1)C(t_2))^2 \left\{ \left( \frac{C(t-t_2)C(t_1)}{C(t)C(t_1-t_2)} \right)^{\tilde{\epsilon}} \sum_{\eta_1,\eta_2} \chi(\theta) - 1 \right\}$$

$\eta = +/-$: forward/backward Keldysh time contour

$\tilde{\epsilon} = +/-$: relative tunneling orientation

## Perturbative calculation

- $S^{\tilde{\epsilon}}(t)$ to lowest nontrivial order

$$\propto \tilde{\epsilon} \int dt_i^2 \cos[\omega_0(t - t_1 - \tilde{\epsilon} t_2)](C(t-t_1)C(t_2))^2 \left\{ \left( \frac{C(t-t_2)C(t_1)}{C(t)C(t_1-t_2)} \right)^{\tilde{\epsilon}} \sum_{\eta_1,\eta_2} \chi(\theta) - 1 \right\}$$

$\eta = +/-$: forward/backward Keldysh time contour

$\tilde{\epsilon} = +/-$: relative tunneling orientation

- The phase sum $\sum_{\eta_1,\eta_2} \chi(\theta) = \sum_{\eta_1,\eta_2} \eta_1 \eta_2 e^{i\Phi_{\tilde{\epsilon}}^{\eta_1,\eta_2}[R_\zeta]}$

1) comes from contour ordering

2) carries the information of statistics

# Perturbative calculation

- $S^{\tilde{\epsilon}}(t)$ to lowest nontrivial order

$$\propto \tilde{\epsilon} \int dt_i^2 \cos[\omega_0(t-t_1-\tilde{\epsilon}t_2)](C(t-t_1)C(t_2))^2 \left\{ \left( \frac{C(t-t_2)C(t_1)}{C(t)C(t_1-t_2)} \right)^{\tilde{\epsilon}} \sum_{\eta_1,\eta_2} \chi(\theta) - 1 \right\}$$

$\eta = +/-$: forward/backward Keldysh time contour

$\tilde{\epsilon} = +/-$: relative tunneling orientation

- The phase sum $\sum_{\eta_1,\eta_2} \chi(\theta) = \sum_{\eta_1,\eta_2} \eta_1 \eta_2 e^{i\Phi_{\tilde{\epsilon}}^{\eta_1,\eta_2}[R_\zeta]}$

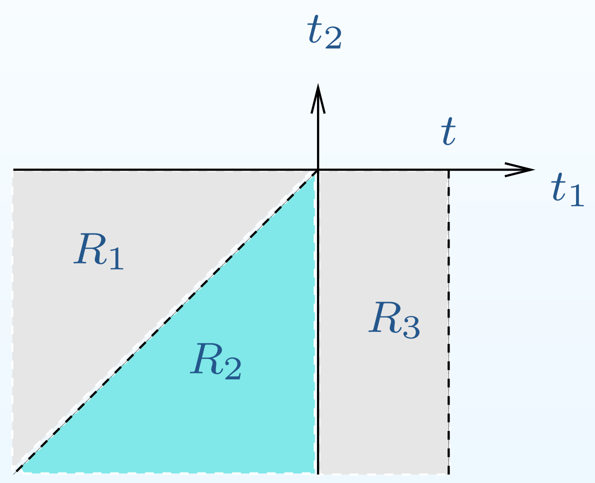
  1) comes from contour ordering

  2) carries the information of statistics

$$\Rightarrow \boxed{S(t) = \mathcal{A}(\omega_0 t; T/T_0, K) + \cos\theta \, \mathcal{B}(\omega_0 t; T/T_0, K)}$$

# Anatomy of the phase factor

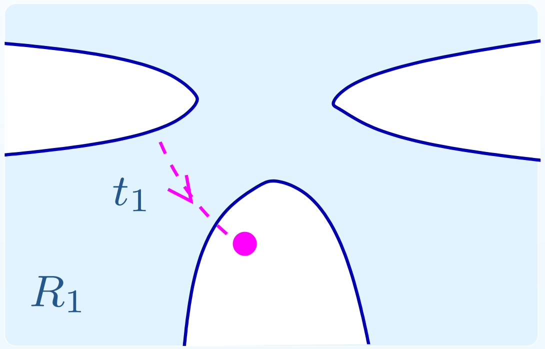
- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

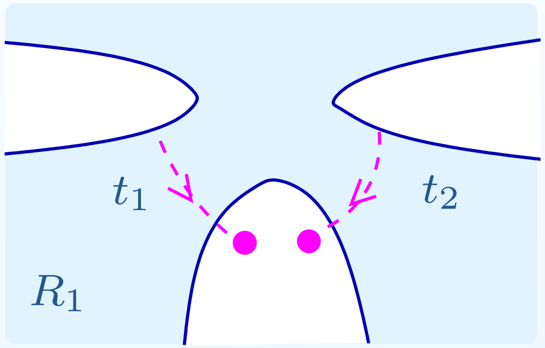
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

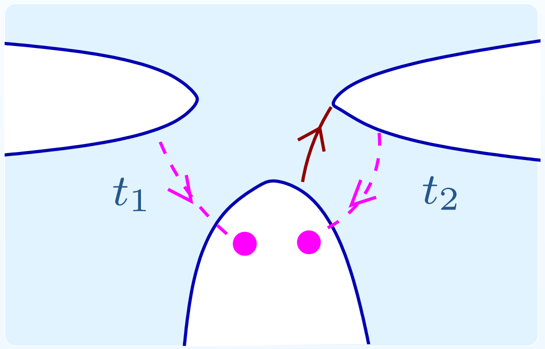
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

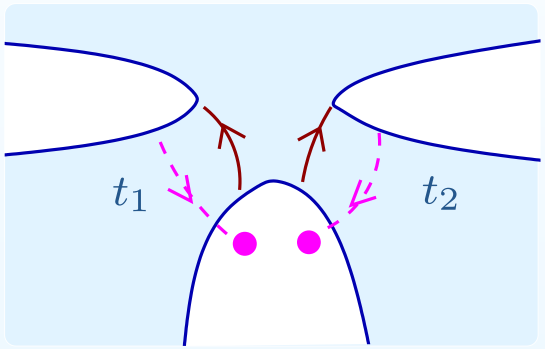
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

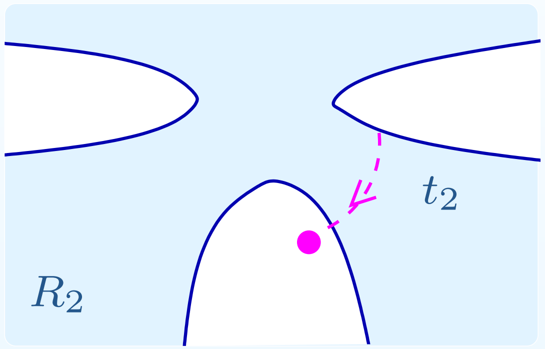
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

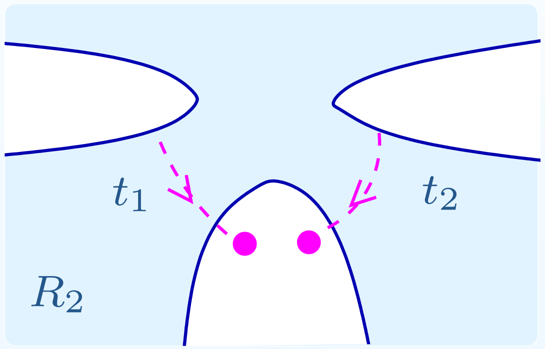
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

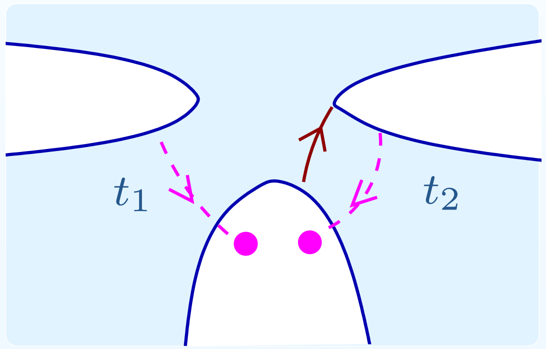
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

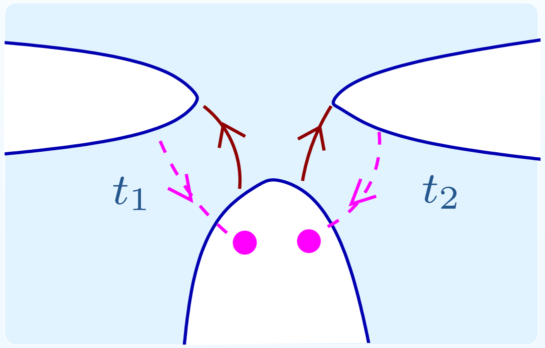
$\tilde{\epsilon} = +$



# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

$\tilde{\epsilon} = +$



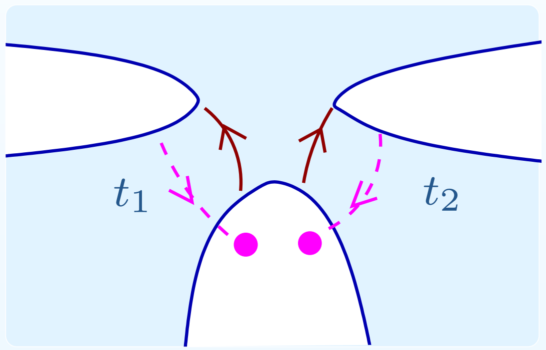

virtual exchange of qp's

$\Rightarrow \chi[R_2;\eta] = e^{i\theta\eta}\chi[R_1;\eta]$

# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

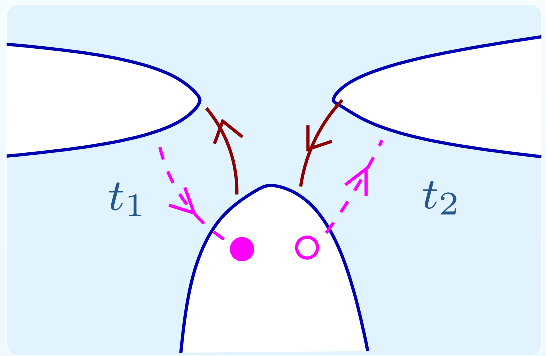
$\tilde{\epsilon} = +$




virtual exchange of qp's

$\Rightarrow \chi[R_2;\eta] = e^{i\theta\eta}\chi[R_1;\eta]$

$\tilde{\epsilon} = -$



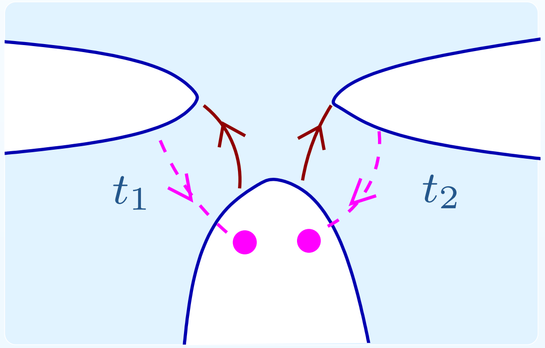

virtual exchange of p-h's

$\Rightarrow \chi[R_2;\eta] = e^{-i\theta\eta}\chi[R_1;\eta]$

# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

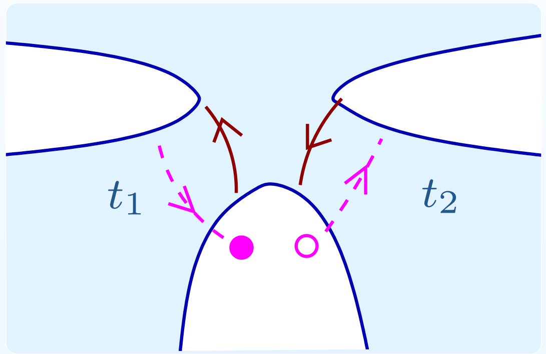
$\tilde{\epsilon} = +$




virtual exchange of qp's

$\Rightarrow \chi[R_2;\eta] = e^{i\theta\eta}\chi[R_1;\eta]$

$\tilde{\epsilon} = -$




virtual exchange of p-h's

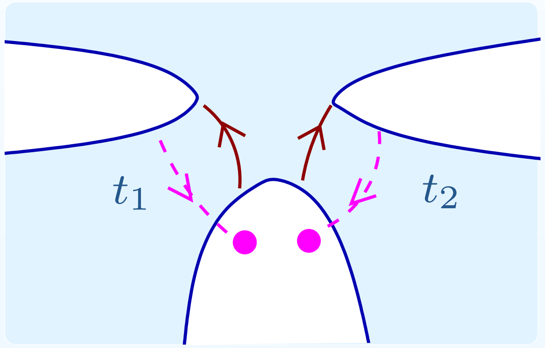
$\Rightarrow \chi[R_2;\eta] = e^{-i\theta\eta}\chi[R_1;\eta]$

- Phase factor sum in $R_1$ and $R_2$

# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

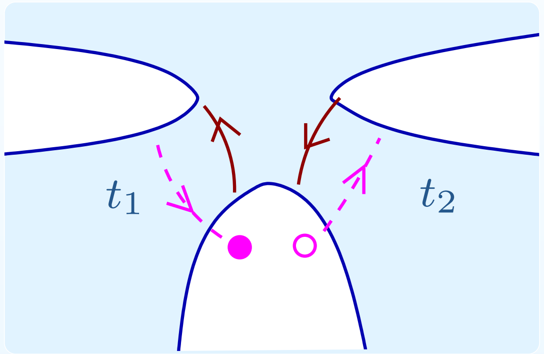
$\tilde{\epsilon} = +$




virtual exchange of qp's

$\Rightarrow \chi[R_2;\eta] = e^{i\theta\eta}\chi[R_1;\eta]$

$\tilde{\epsilon} = -$




virtual exchange of p-h's

$\Rightarrow \chi[R_2;\eta] = e^{-i\theta\eta}\chi[R_1;\eta]$

- Phase factor sum in $R_1$ and $R_2$

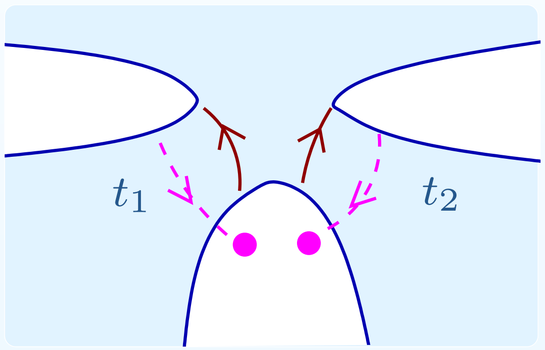
$$\sum_{\eta=\pm} \chi[R_1;\eta](=e^{i\theta\eta}) \propto \sin\theta$$

$$\sum_{\eta=\pm} \chi[R_2;\eta](=e^{i(\theta+\theta)\eta}) \propto \sin\theta\,\cos\theta$$

# Anatomy of the phase factor

- $R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.

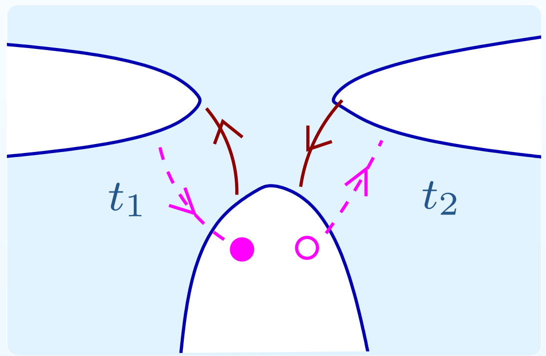
$\tilde{\epsilon} = +$




virtual exchange of qp's

$\Rightarrow \chi[R_2;\eta] = e^{i\theta\eta}\chi[R_1;\eta]$

$\tilde{\epsilon} = -$




virtual exchange of p-h's

$\Rightarrow \chi[R_2;\eta] = e^{-i\theta\eta}\chi[R_1;\eta]$

- Phase factor sum in $R_1$ and $R_2$

$$\sum_{\eta=\pm} \chi[R_1;\eta](=e^{i\theta\eta}) \propto \sin\theta$$

$$\sum_{\eta=\pm} \chi[R_2;\eta](=e^{i(\theta+\theta)\eta}) \propto \sin\theta\,\cos\theta$$

$$\sum_{\eta=\pm} \chi[R_1;\eta](=\eta e^{i\theta\eta}) \propto \sin\theta$$

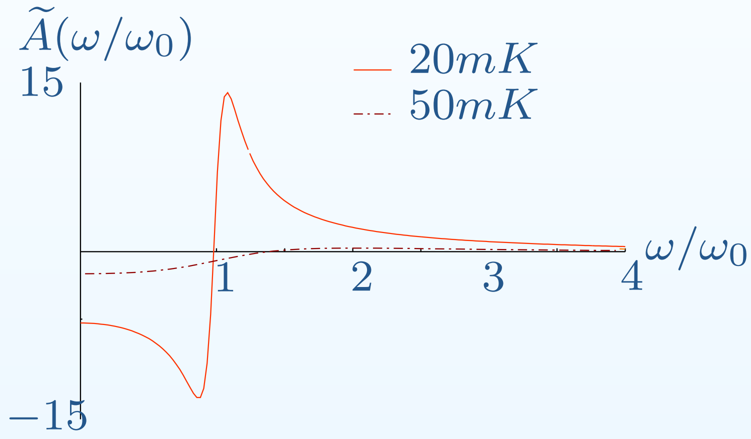
$$\sum_{\eta=\pm} \chi[R_2;\eta](=\eta e^{i(\theta-\theta)\eta}) = 0$$

## Frequency spectrum

- Direct term $\widetilde{A}$ v.s. exchange term $\widetilde{B}$

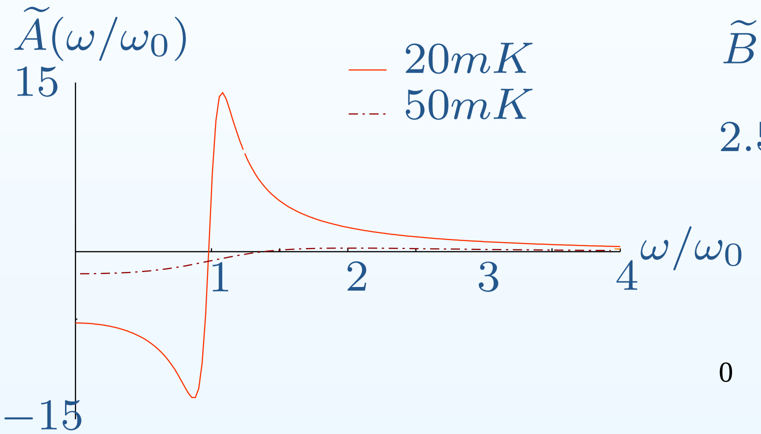
# Frequency spectrum

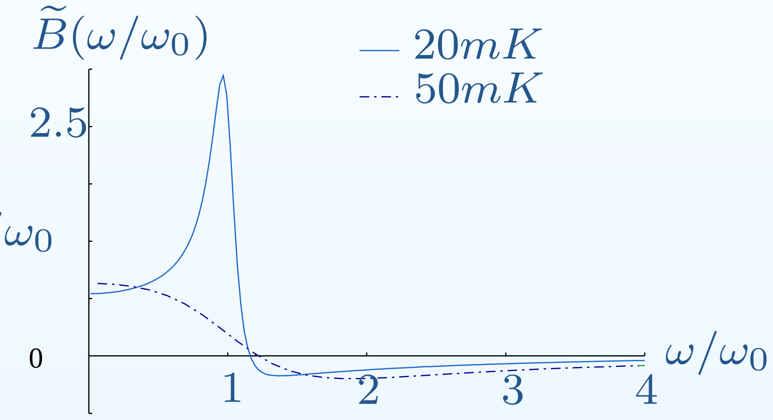
- Direct term $\widetilde{A}$ v.s. exchange term $\widetilde{B}$



# Frequency spectrum

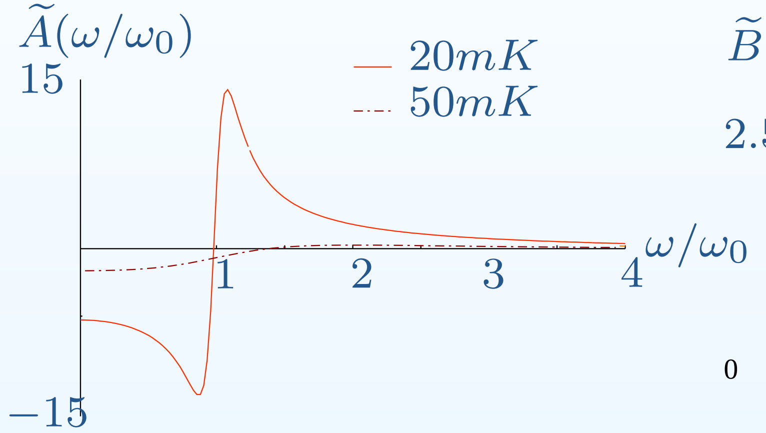
- Direct term $\widetilde{A}$ v.s. exchange term $\widetilde{B}$

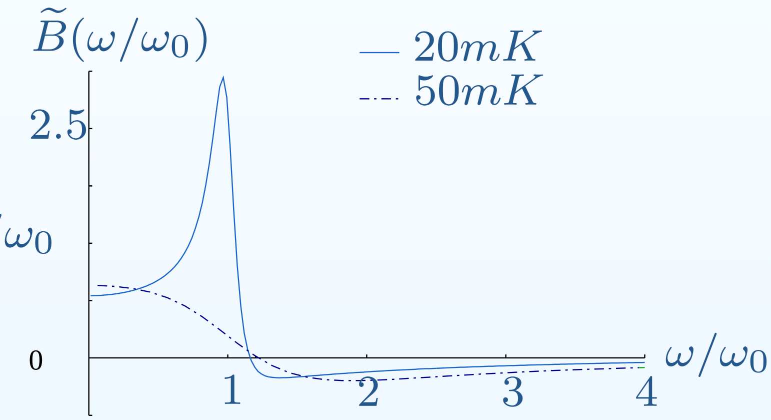


# Frequency spectrum

- Direct term $\widetilde{A}$ v.s. exchange term $\widetilde{B}$

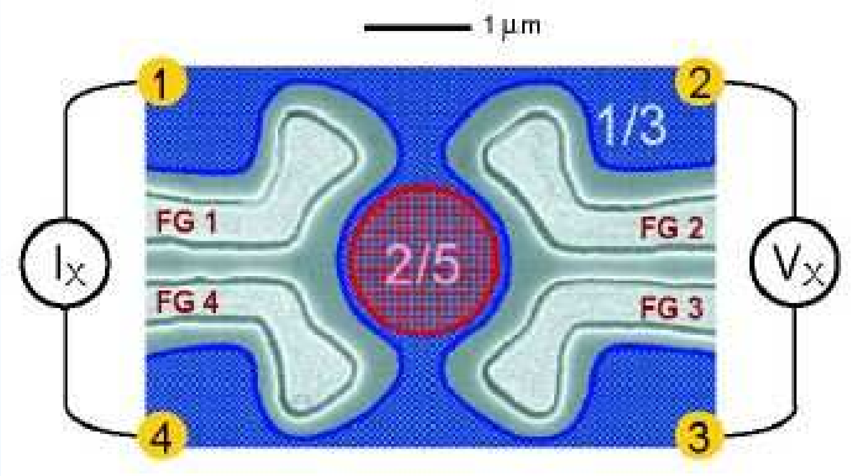



- $\widetilde{S}(\omega/\omega_0;T) = \widetilde{A} + \cos\theta\ \widetilde{B}$ .
- "Bunching" Laughlin qp ($\theta < \pi/2$) v.s."anti-bunching" non-Laughlin qp ($\theta > \pi/2$).

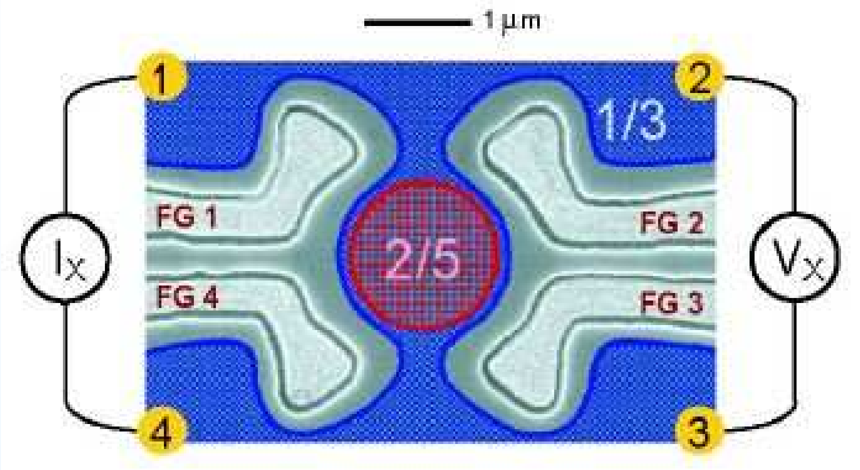
# Contents

# Superperiod Aharonov-Bohm effect (Goldman, 05)

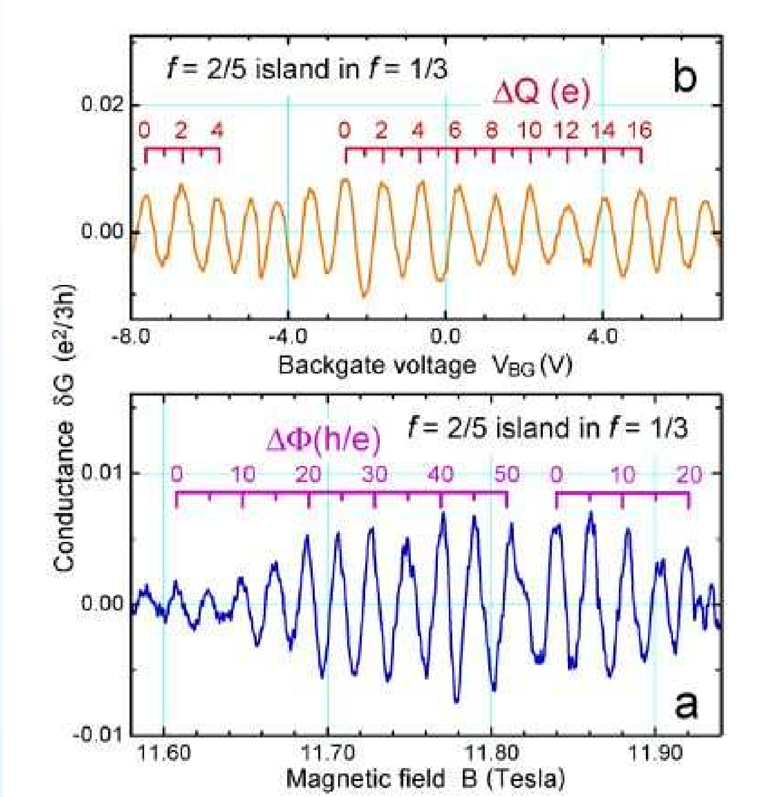



Four terminal measurements.

# Superperiod Aharonov-Bohm effect (Goldman, 05)

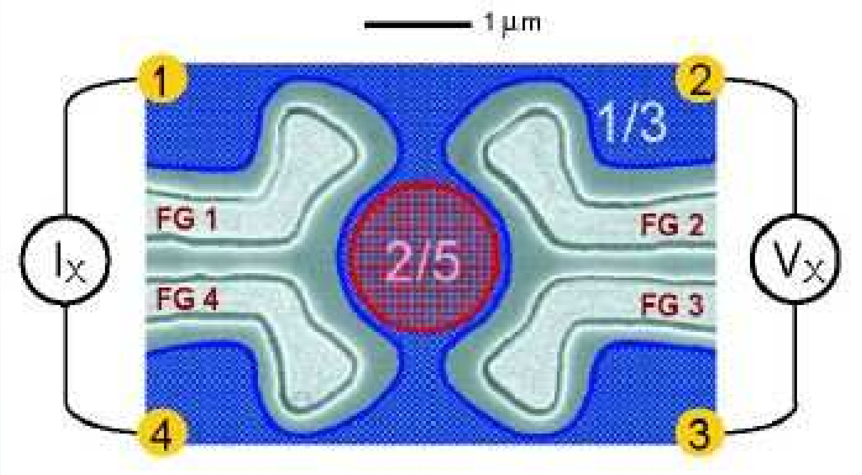



Four terminal measurements.

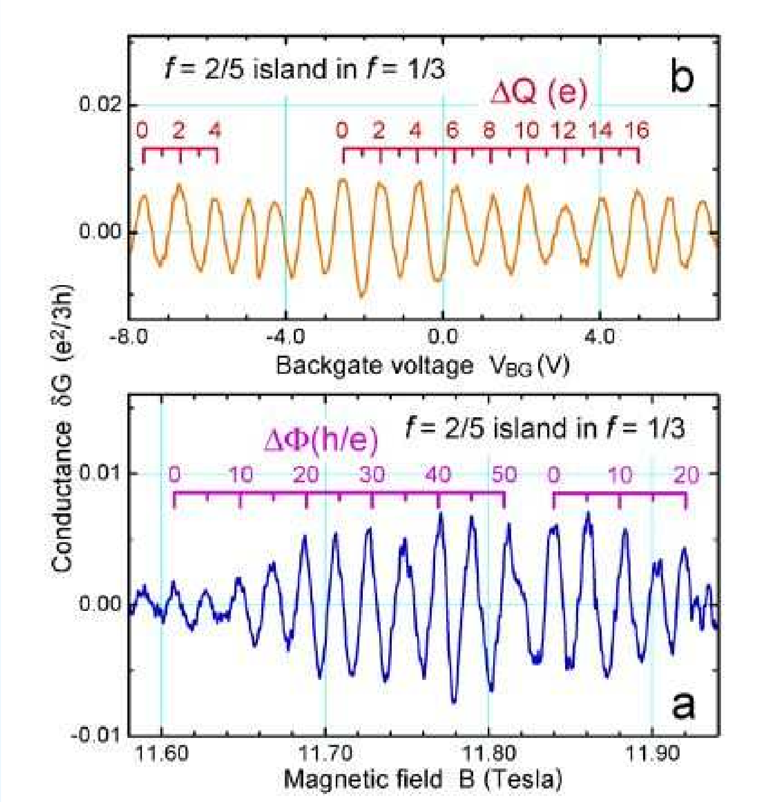



Superperiod oscillation with $\Delta\Phi = 5\phi_0$.

# Superperiod Aharonov-Bohm effect (Goldman, 05)




Four terminal measurements.



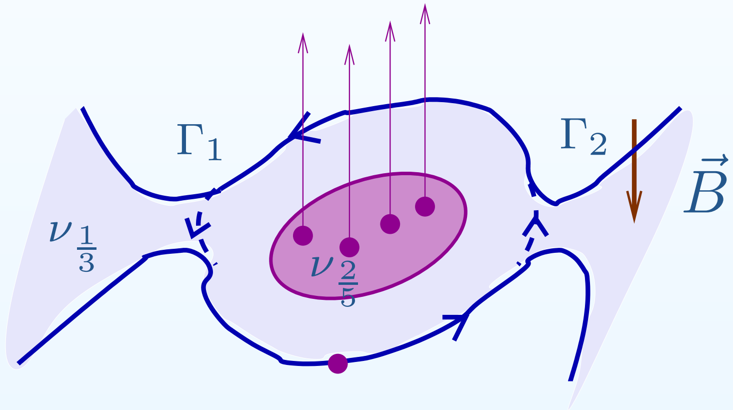

Superperiod oscillation with $\Delta\Phi = 5\phi_0$.

**Is this REALLY a measure of fractional statistics?**

# Our model

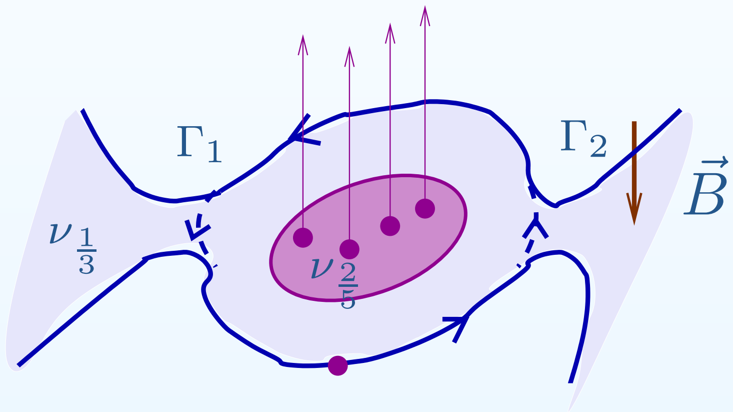
- Double point contact interferometer

  (Related works E.-A. Kim et al, PRL 03; Chamon et al, PRB 97)



# Our model

- Double point contact interferometer
  (Related works E.-A. Kim et al, PRL 03; Chamon et al, PRB 97)




- Assumptions:
  - No direct tunneling between outer edge and the inner puddle.
  - Coherent propagation of $1/3$ qp along outer edge.
  - Incompressibility of $2/5$ puddle.
  - Fractional statistics between $1/3$ qp's with $\theta = \pi/3$.

# Incompressibility of $2/5$ FQH liquid

- Hierarchical picture:

  $1/3$ qp's condense to form a puddle of $2/5$ state

# Incompressibility of $2/5$ FQH liquid

- Hierarchical picture:

  $1/3$ qp's condense to form a puddle of $2/5$ state

- Incompressibility of $2/5$ state $\Rightarrow$ flux superquantization

# Incompressibility of $2/5$ FQH liquid

- Hierarchical picture:

  $1/3$ qp's condense to form a puddle of $2/5$ state

- Incompressibility of $2/5$ state $\Rightarrow$ flux superquantization
  - $\frac{(\text{total charge of puddle } Q)/e}{Bs/\phi_0} = \nu_{2/5}$
  - $B \Uparrow$ require $Q \Uparrow$ :$N$ extra $1/3$ qp condense to the puddle of area $s$

$$\nu_{1/3}\frac{Bs}{\phi_0} + \frac{1}{3}N = \nu_{2/5}\frac{Bs}{\phi_0}$$

(Jain, Kivelson, Thouless,1993)

# Incompressibility of $2/5$ FQH liquid

- Hierarchical picture:

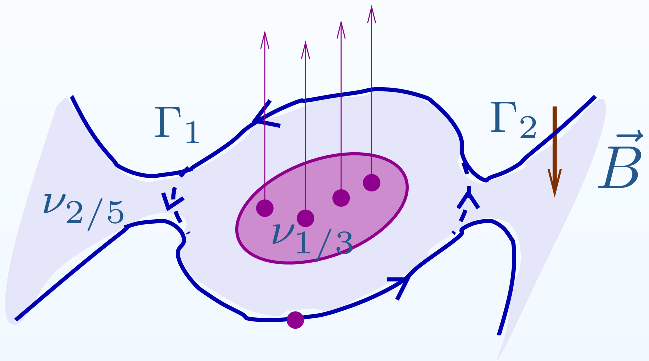
  $1/3$ qp's condense to form a puddle of $2/5$ state

- Incompressibility of $2/5$ state $\Rightarrow$ flux superquantization
  - $\frac{(\text{total charge of puddle } Q)/e}{Bs/\phi_0} = \nu_{2/5}$
  - $B \Uparrow$ require $Q \Uparrow$ :$N$ extra $1/3$ qp condense to the puddle of area $s$

$$N = \left[\frac{|B|s}{5\phi_0}\right]$$

(E.-A. Kim, in preparation)

# Interference conditions



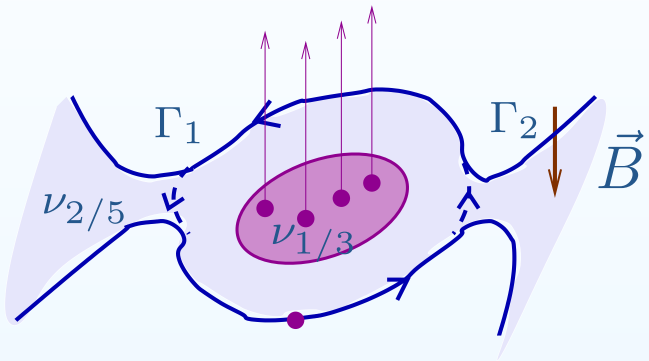

- Two independent periods:
  (1) Ahranov-Bohm phase due to flux through the area $S$

  $$2\pi \frac{e^*}{e} \frac{|B|S}{\phi_0} = 2\pi \frac{1}{3} \frac{|B|S}{\phi_0}$$

  (2) Statistical phase due to $N$ qp's in the puddle of area $s$

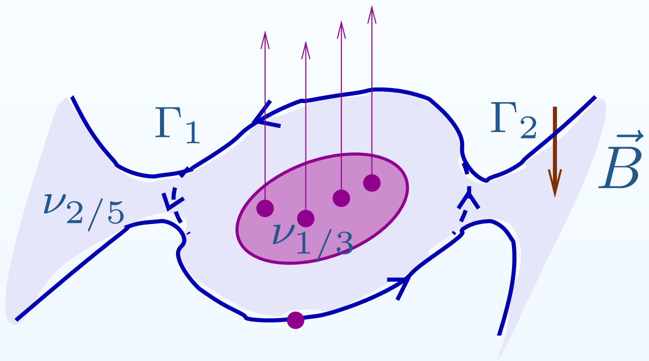
  $$-2\theta N = -\frac{2\pi}{3} N = -\frac{2\pi}{3} \left[ \frac{|B|s}{5\phi_0} \right]$$

# Interference conditions




- Two independent periods:
  (1) Ahranov-Bohm phase due to flux through the area $S$
  $$2\pi \frac{e^*}{e} \frac{|B|S}{\phi_0} = 2\pi \frac{1}{3} \frac{|B|S}{\phi_0}$$
  (2) Statistical phase due to $N$ qp's in the puddle of area $s$
  $$-2\theta N = -\frac{2\pi}{3} N = -\frac{2\pi}{3}\left[\frac{|B|s}{5\phi_0}\right]$$
- Interference condition (E.-A. Kim, in preparation):
  - $\Delta \frac{\gamma}{2\pi} = \left(5\frac{e^*}{e}\frac{S}{s} - \frac{\theta}{2\pi}\right)\Delta\left[\frac{|B|s}{5\phi_0}\right] = \left(\frac{5}{3}\frac{S}{s} - \frac{1}{3}\right)\Delta\left[\frac{|B|s}{5\phi_0}\right] =$ integer

# Interference conditions

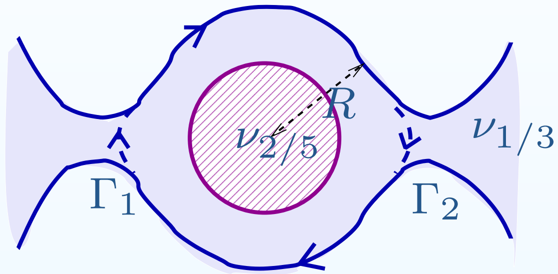



- Two independent periods:
  (1) Ahranov-Bohm phase due to flux through the area $S$
  $$2\pi \frac{e^*}{e}\frac{|B|S}{\phi_0} = 2\pi \frac{1}{3}\frac{|B|S}{\phi_0}$$
  (2) Statistical phase due to $N$ qp's in the puddle of area $s$
  $$-2\theta N = -\frac{2\pi}{3}N = -\frac{2\pi}{3}\left[\frac{|B|s}{5\phi_0}\right]$$

- Interference condition (E.-A. Kim, in preparation):
  - $\Delta\frac{\gamma}{2\pi} = \left(5\frac{e^*}{e}\frac{S}{s} - \frac{\theta}{2\pi}\right)\Delta\left[\frac{|B|s}{5\phi_0}\right] = \left(\frac{5}{3}\frac{S}{s} - \frac{1}{3}\right)\Delta\left[\frac{|B|s}{5\phi_0}\right] =$ integer
  - $S/s = 1.43 \sim 7/5 \Rightarrow \boxed{\Delta|B|s = 5\phi_0}$:
    consistent with the experiment

# Temperature dependence of oscillation amplitude

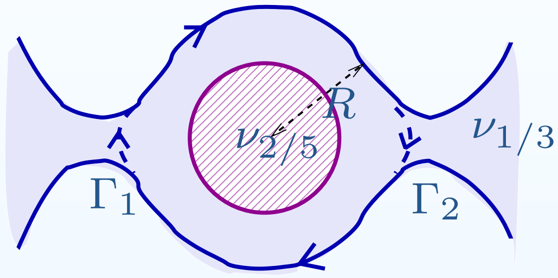
- Perturbative calculation to leading order in $\Gamma$




$$H_t = \frac{\Gamma_1}{2} e^{-i\omega_J t} \psi^\dagger_{R,1} \psi_{L,1} + e^{i\gamma} \frac{\Gamma_2^*}{2} e^{i\omega_J t} \psi^\dagger_{L,2} \psi_{R,2} + h.c.$$

# Temperature dependence of oscillation amplitude

- Perturbative calculation to leading order in $\Gamma$

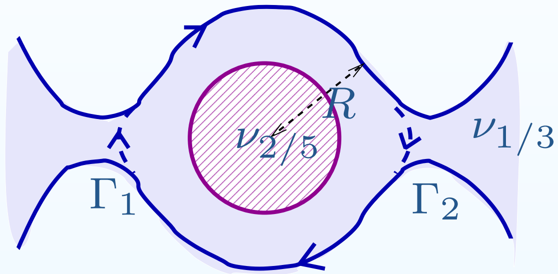



$$H_t = \frac{\Gamma_1}{2} e^{-i\omega_J t} \psi^\dagger_{R,1} \psi_{L,1} + e^{i\gamma} \frac{\Gamma_2^*}{2} e^{i\omega_J t} \psi^\dagger_{L,2} \psi_{R,2} + h.c.$$

- The conductance; data v.s. theory
  $G(\omega_0, v/R, T) = \bar{G}(\omega_0/T) + \cos\gamma\ \delta G(\omega_0, v/R, T)$, $\omega_0 = e^* V/\hbar$

# Temperature dependence of oscillation amplitude

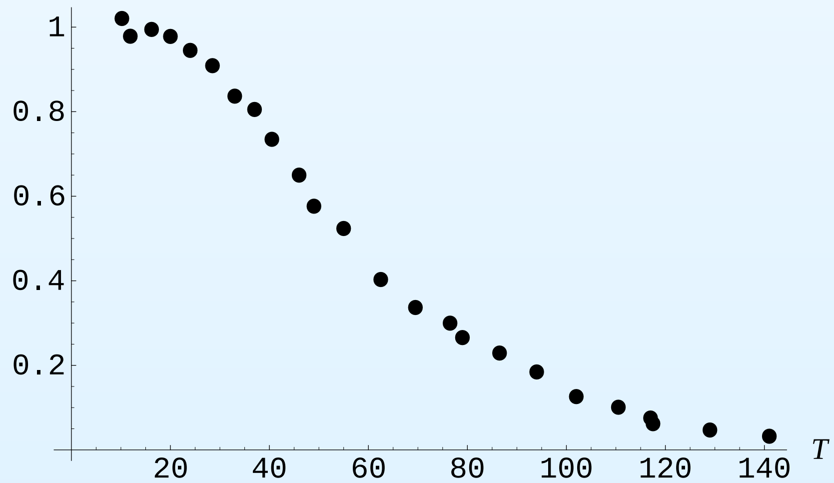
- Perturbative calculation to leading order in $\Gamma$




$$H_t = \frac{\Gamma_1}{2} e^{-i\omega_J t} \psi^\dagger_{R,1} \psi_{L,1} + e^{i\gamma} \frac{\Gamma_2^*}{2} e^{i\omega_J t} \psi^\dagger_{L,2} \psi_{R,2} + h.c.$$

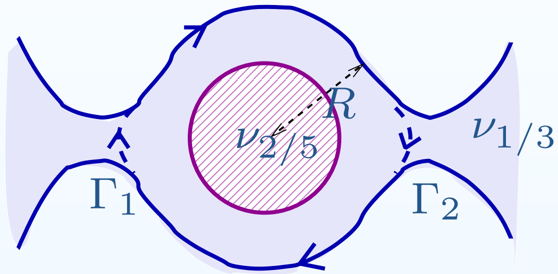
- The conductance; data v.s. theory
  $G(\omega_0, v/R, T) = \bar{G}(\omega_0/T) + \cos\gamma\ \delta G(\omega_0, v/R, T)$, $\omega_0 = e^* V/\hbar$

$\delta G(T)/\delta G(T = 11mK)$



# Temperature dependence of oscillation amplitude

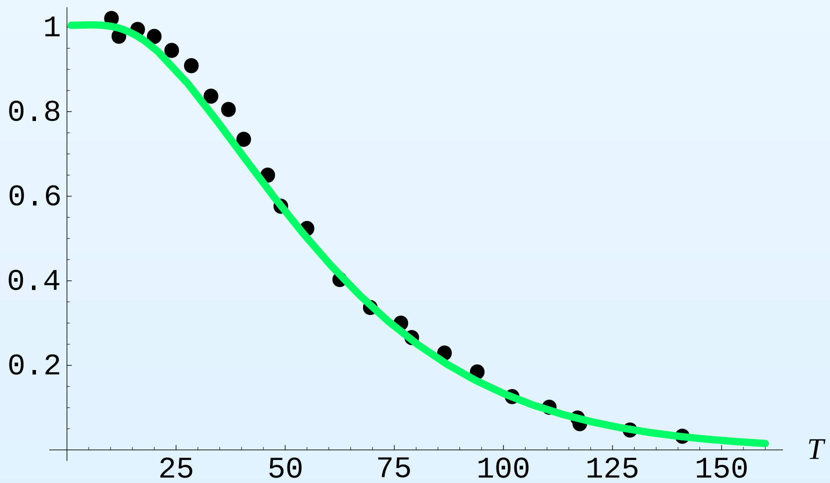
- Perturbative calculation to leading order in $\Gamma$




$$H_t = \frac{\Gamma_1}{2} e^{-i\omega_J t} \psi^\dagger_{R,1} \psi_{L,1} + e^{i\gamma} \frac{\Gamma_2^*}{2} e^{i\omega_J t} \psi^\dagger_{L,2} \psi_{R,2} + h.c.$$

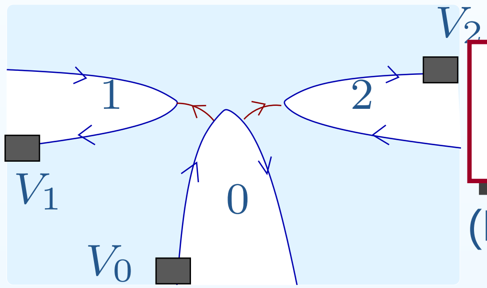
- The conductance; data v.s. theory
$G(\omega_0, v/R, T) = \bar{G}(\omega_0/T) + \cos\gamma\ \delta G(\omega_0, v/R, T)$, $\omega_0 = e^* V/\hbar$

$\delta G(T)/\delta G(T=11mK)$



# Summary: Anyon there?

- T-junction proposal: the cross current noise $S(\omega)$

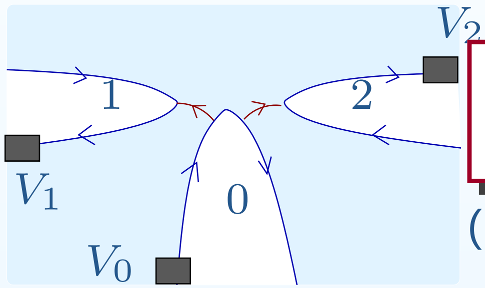



$$\frac{\tilde{S}(\omega/\omega_0)}{\langle I_1\rangle\langle I_2\rangle} = \tilde{A}(\omega/\omega_0, T/T_0) + \cos\theta \tilde{B}(\omega/\omega_0, T/T_0)$$

(E.-A. Kim et al, PRL 05)

# Summary: Anyon there?

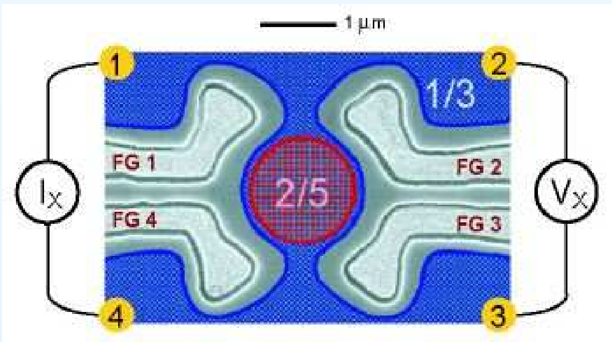
- T-junction proposal: the cross current noise $S(\omega)$



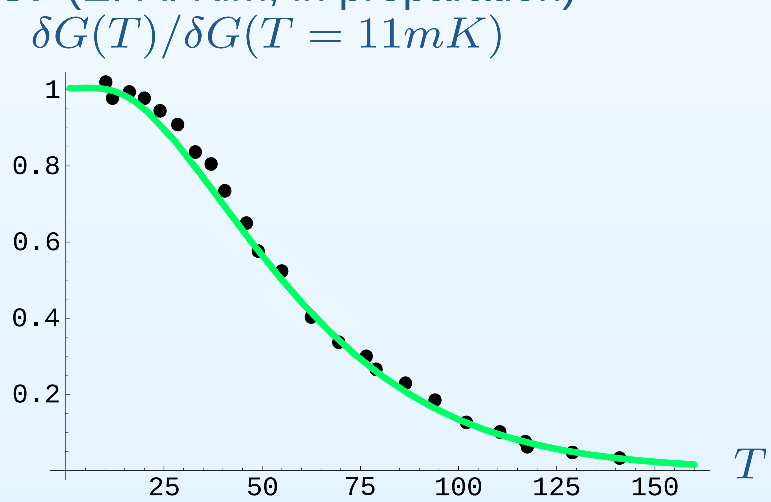

$$\frac{\tilde{S}(\omega/\omega_0)}{\langle I_1\rangle\langle I_2\rangle} = \tilde{A}(\omega/\omega_0, T/T_0) + \cos\theta\,\tilde{B}(\omega/\omega_0, T/T_0)$$

(E.-A. Kim et al, PRL 05)

- Quantum Hall interferometer (E.-A. Kim, in preparation)



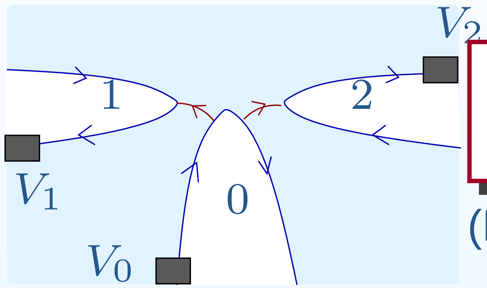

The set up.




The fit

# Summary: Anyon there?

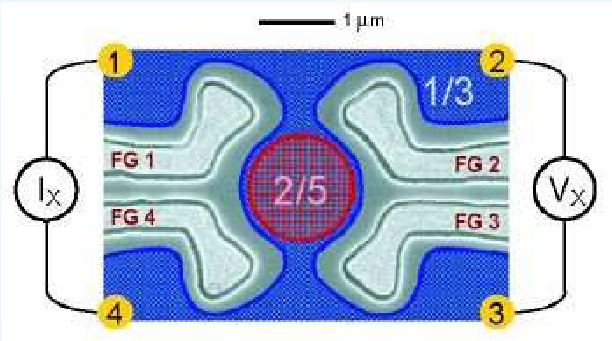
- T-junction proposal: the cross current noise $S(\omega)$



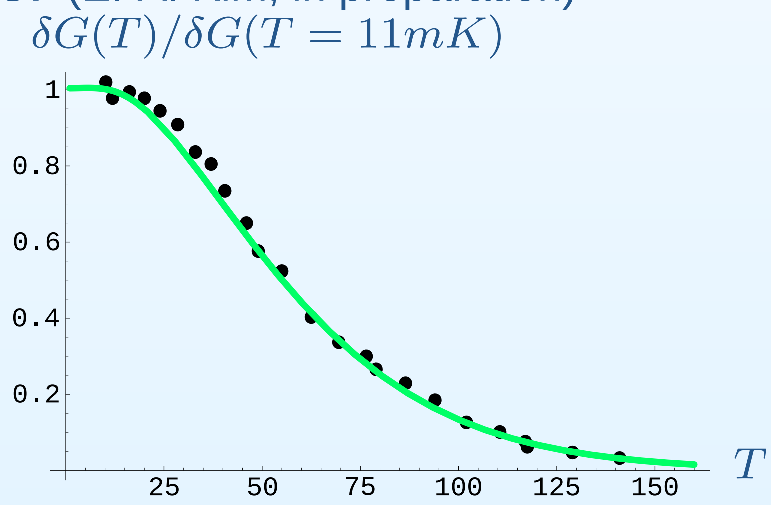

$$\frac{\tilde{S}(\omega/\omega_0)}{\langle I_1\rangle\langle I_2\rangle} = \tilde{A}(\omega/\omega_0, T/T_0) + \cos\theta\,\tilde{B}(\omega/\omega_0, T/T_0)$$

(E.-A. Kim et al, PRL 05)

- Quantum Hall interferometer (E.-A. Kim, in preparation)




The set up.




The fit

- Future: non-Abelian statistics

# Summary: Anyon there?

If they are there, we can now manipulate them!

# References


- E.-A. Kim, M. Lawler, S. Vishveshwara and E. Fradkin, “A Proposal for Measuring Fractional Charge and Statistics in Fractional Quantum Hall States in Noise experiments”, PRL **95**, 176402 (2005)
- Physical Review Focus Story 14, 2 Nov 2005
- E.-A. Kim, M. Lawler, S. Vishveshwara, E. Fradkin, “Cross Current Noise in a Fractional quantum Hall T-junction”, in preparation.
- E.-A. Kim, "Superperiodic interference effects and fractional statistics", in preparation.