Quantum Critical Behaviour Near the Nematic Instability of a Fermi Fluid

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(cf. Phys. Rev. B **73**, 085101 (2006) and `cond-mat/0605203`)

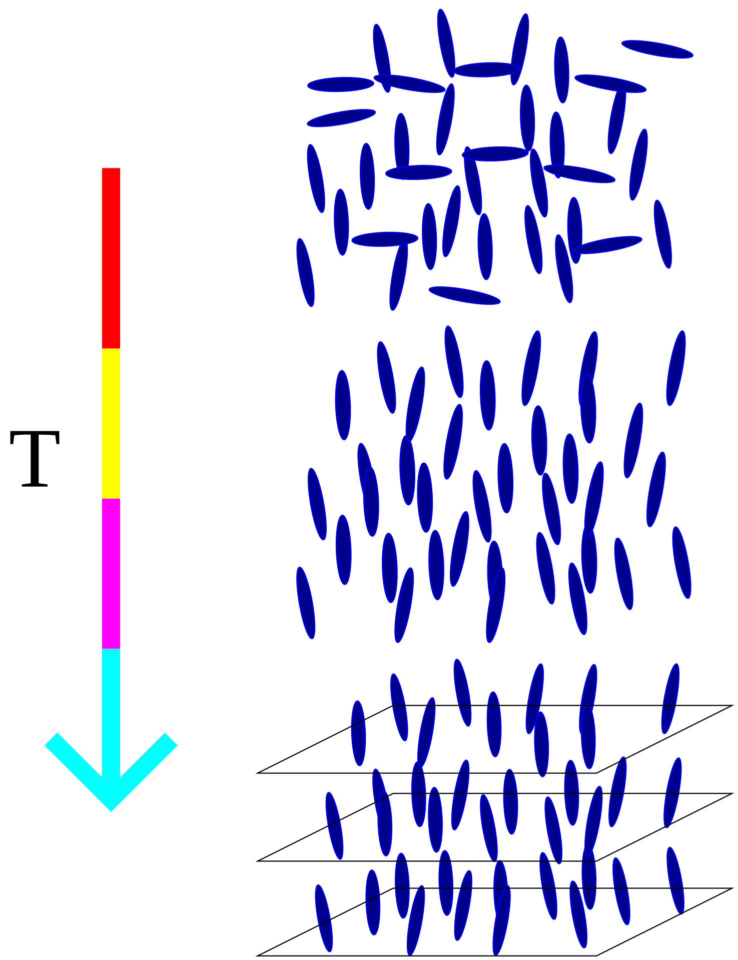
Table of Contents

- Introduction to quantum anisotropic phases
- Theory of the nematic Fermi fluid
- The method of high dimensional bosonization
- Results at criticality and into the nematic phase
  - Quasi-particle properties
  - Heat capacity
- Conclusions and future directions

Why anisotropic fermion phases?

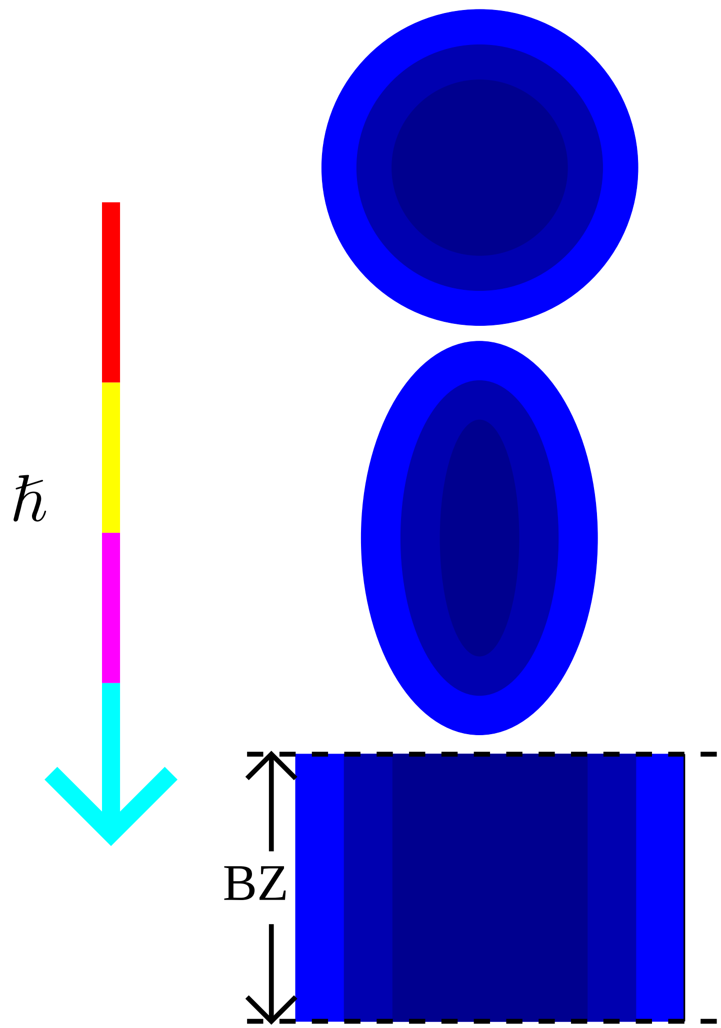
- Experimental evidence
  - Stripe phases / nematic phases in strongly correlated lattice systems
  - Quantum Hall anisotropic phases in the higher Landau levels
- Theoretical questions
  - Are these phases non-Fermi liquids?
  - How can one reliably describe them?
  - How can they be understood from microscopics?

The Freezing of Nematogens



- Gas / liquid phase (Isotropic)
- Nematic phase (Rotational symmetry breaking)
- Smectic phase (Translational symmetry breaking)

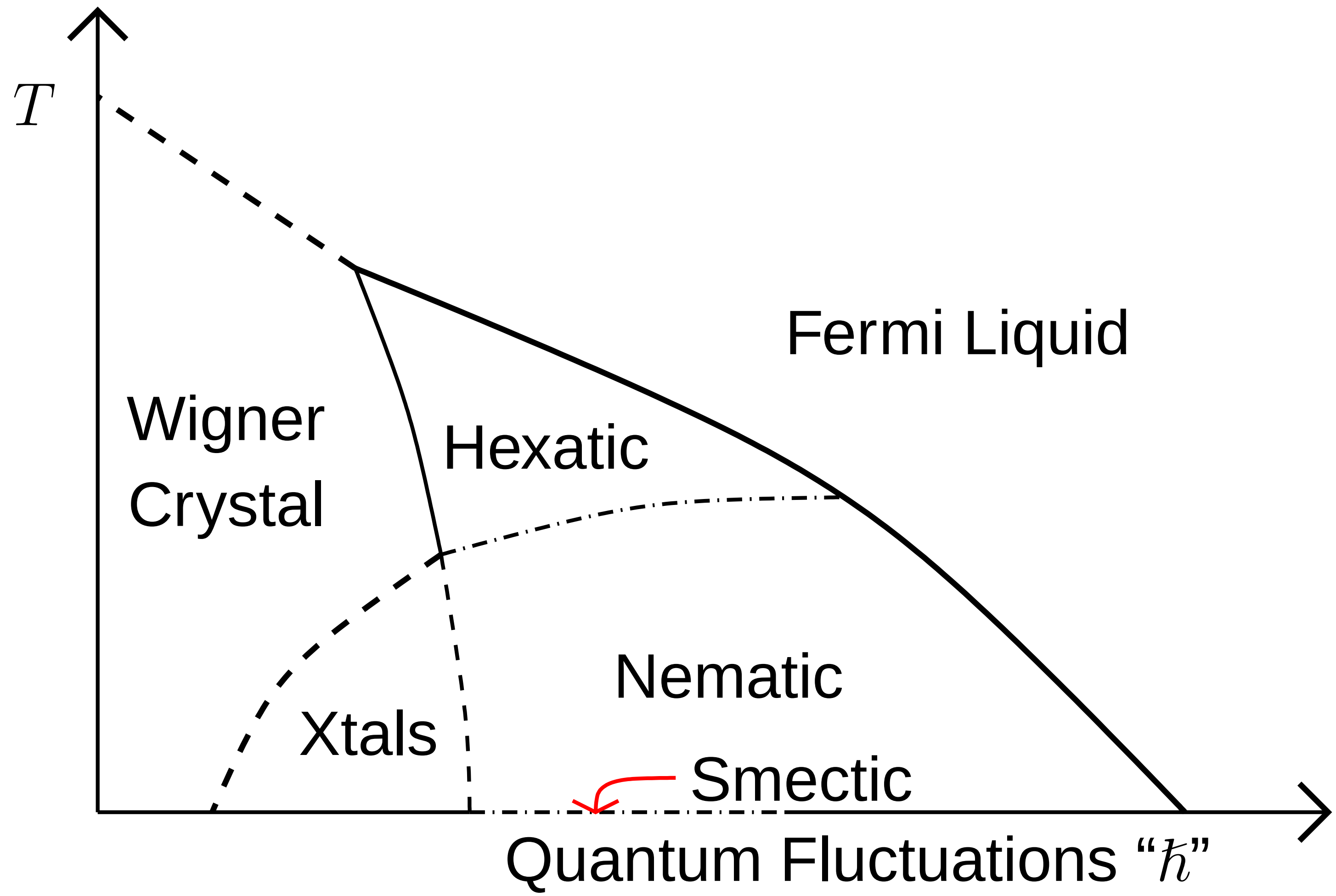
“Freezing” the Fermi Liquid



- Fermi liquid phase
- Nematic Fermi fluid
- Smectic Fermi fluid

Phase diagram for 2D fermions

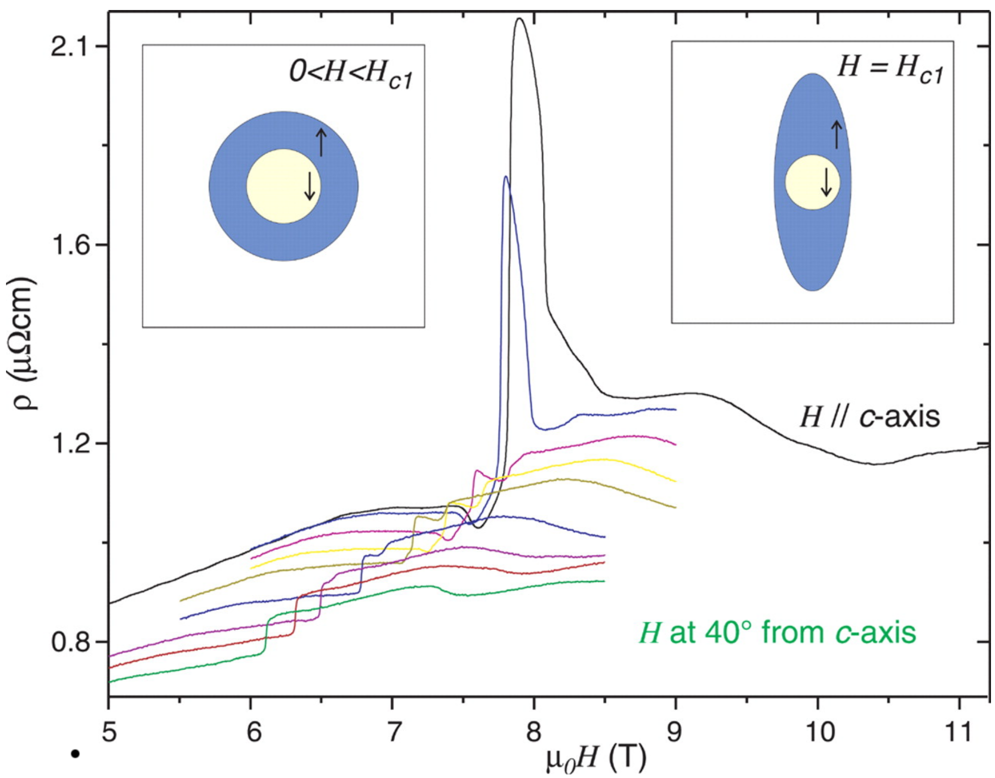
Kivelson, Fradkin and Emery, nature, **393**, 550 (1998)



Introduction

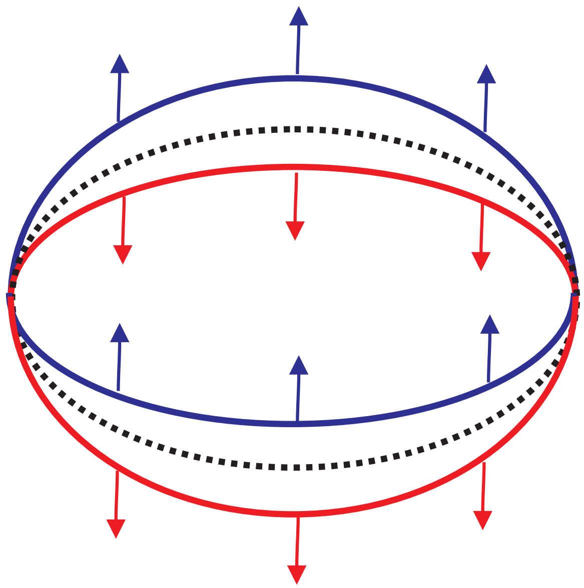
Lattice nematic candidates?

$Sr_3Ru_2O_7$



2D Nematic
(MacKenzie and Co, 2004)

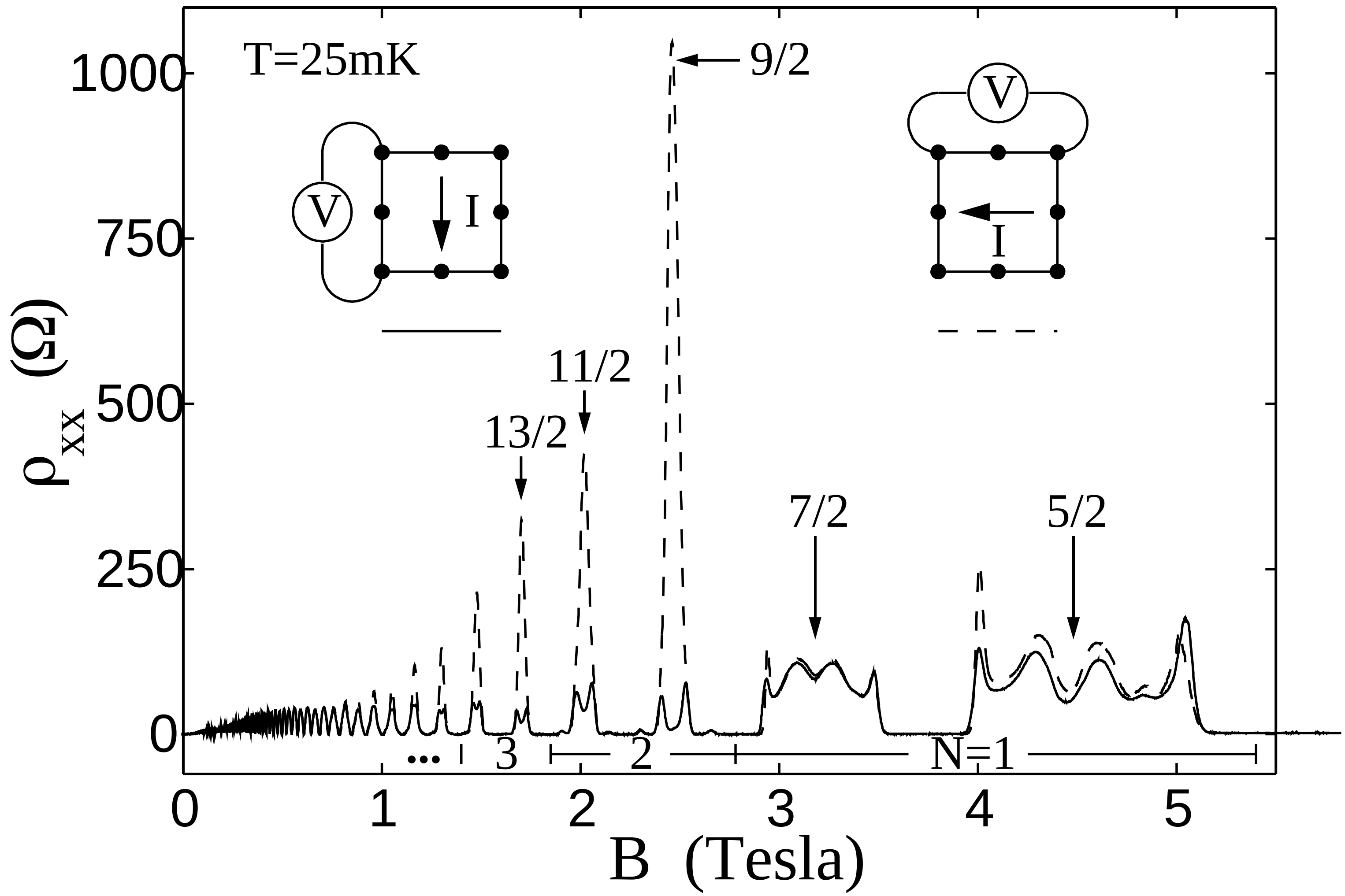
URu_2Si_2



Nematic Spin Nematic
(Wu and Zhang 04,
Varma and Zhu 05)

Introduction

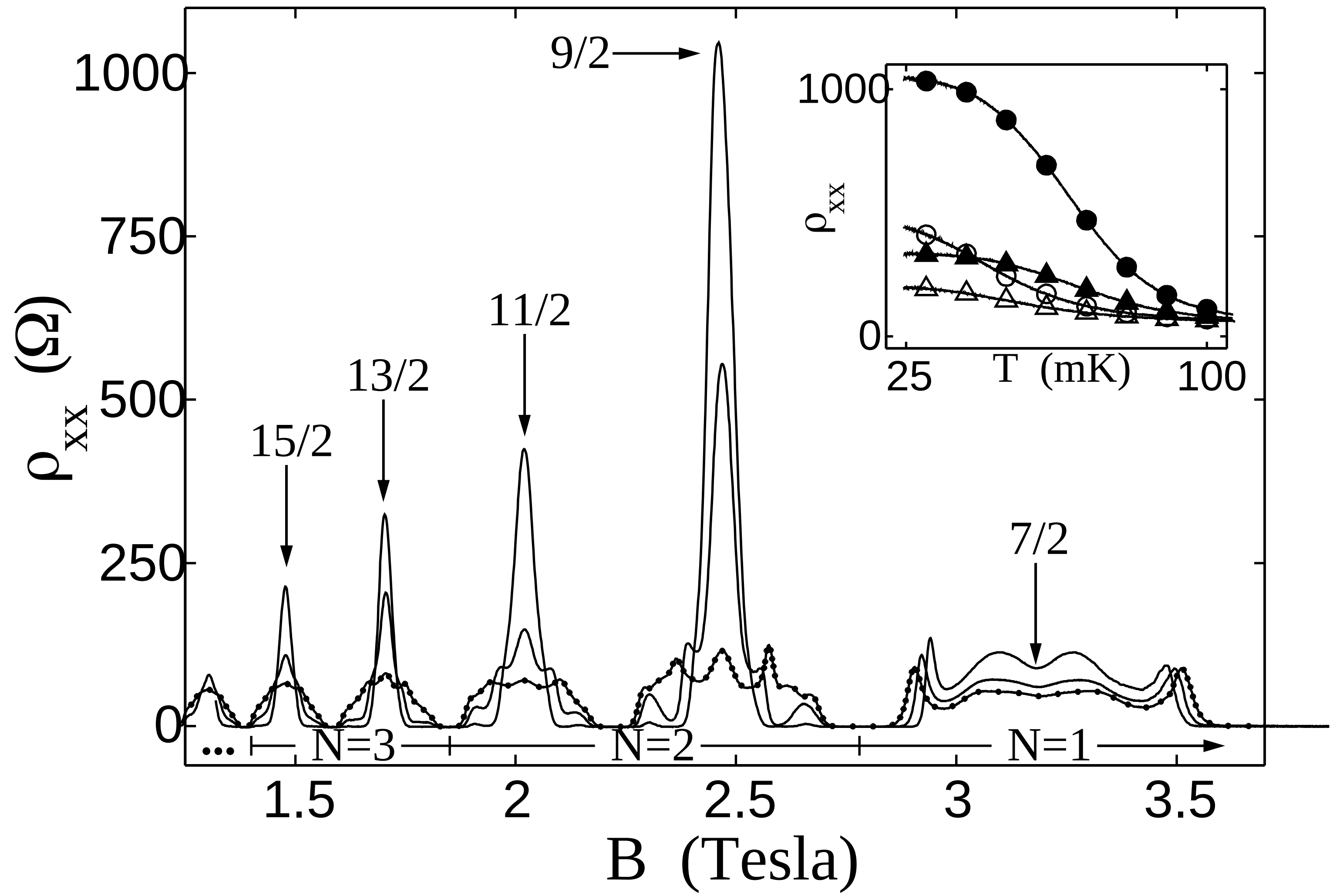
Quantum Hall Liquid Crystals



(Lilly et. al., Phys. Rev. Lett. 82, 394 (1999))

Introduction

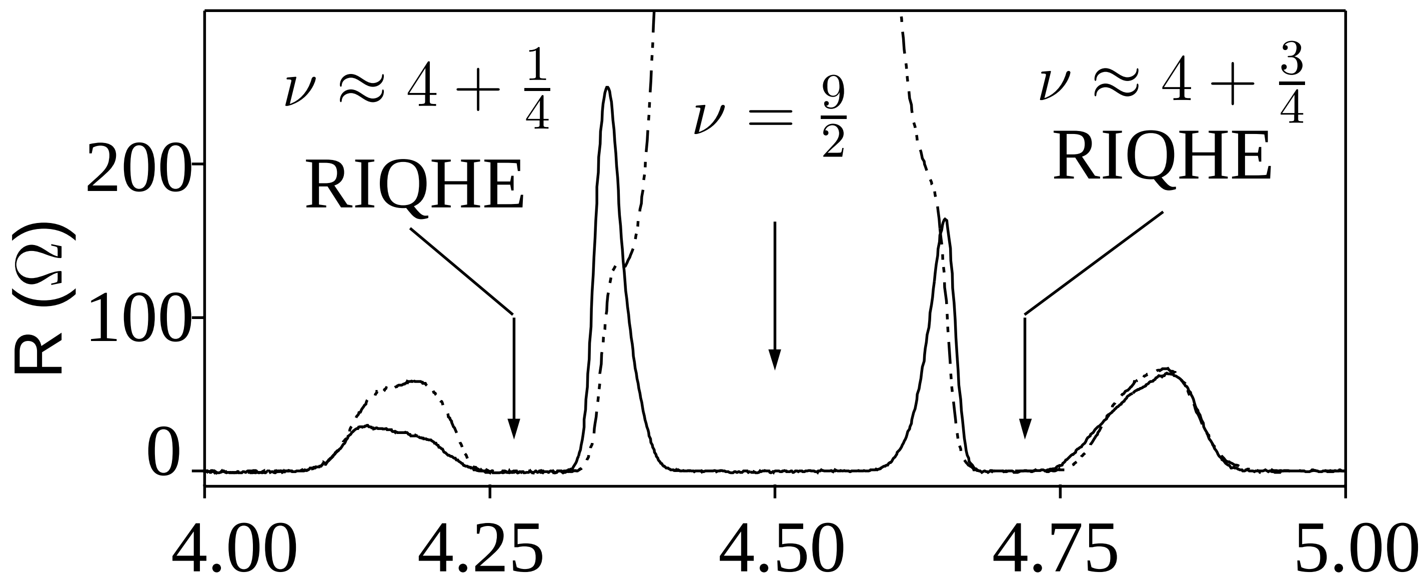
Quantum Hall Liquid Crystals



(Lilly et. al., Phys. Rev. Lett. 82, 394 (1999))

Introduction

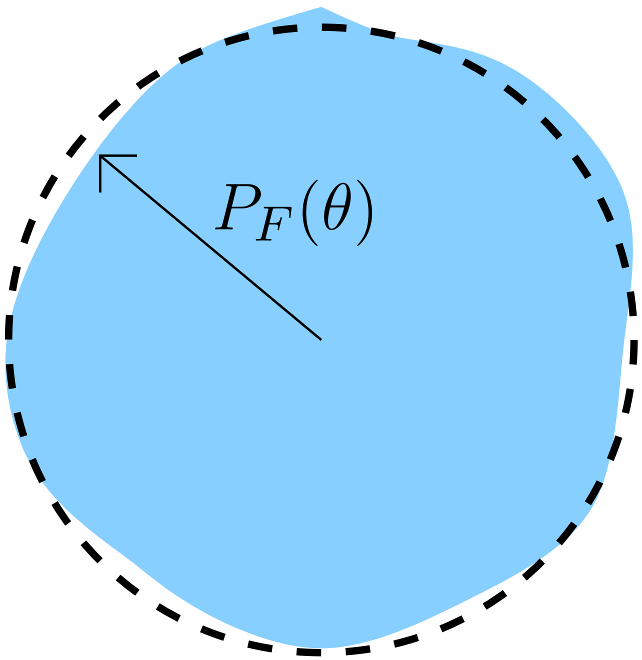
Quantum Hall Liquid Crystals



Longitudinal resistances in ultra-clean samples. Solid line in the $[110]$ direction, dashed line in the $[1\bar{1}0]$ direction. Taken at $T = 50$mK.

(Cooper et. al., Phys. Rev. Lett. 90, 226803 (2003))

Pomeranchuk Stability Criterion



Landau Energy Functional

$$E[\delta n_p] = \sum_p \varepsilon_p \delta n_p + \frac{1}{2} \sum_{p,p'} f_{p,p'} \delta n_p \delta n_{p'}$$

- Let $n_p = 1$ for $|p| < |P_F(\theta)|$ and zero otherwise
- $E[P_F^\ell] \propto \sum_{\ell,m} |P_F^\ell|^2 (1 + N(0) f_\ell)$
- Stable for $F_\ell \equiv N(0) f_\ell > -1$

This is the Landau theory of phase transitions applied to the Landau Fermi liquid!

The Nematic Instability (Oganesyan et. al. (2001))

- A natural order parameter is the quadrupole density:

$$\hat{\mathbf{Q}}(\mathbf{r}) = -\frac{1}{k_F^2}\hat{\psi}^\dagger(\mathbf{r})\begin{pmatrix}\partial_x^2 - \partial_y^2 & 2\partial_x\partial_y \\ 2\partial_x\partial_y & \partial_y^2 - \partial_x^2\end{pmatrix}\hat{\psi}(\mathbf{r})$$

which measures quadrupolar distortions of the FS.

- A simple model is fermions interacting only via

$$\hat{\mathbf{V}} = \frac{1}{4}\int d^2r d^2r' F_2(|\mathbf{r}-\mathbf{r}'|)\mathbf{t}r\hat{\mathbf{Q}}(\mathbf{r})\hat{\mathbf{Q}}(\mathbf{r}')$$

where we keep a finite length ($\sim \sqrt{\kappa}$) interaction:

$$F_2(q) = \frac{F_2}{1+\kappa|F_2|q^2}$$

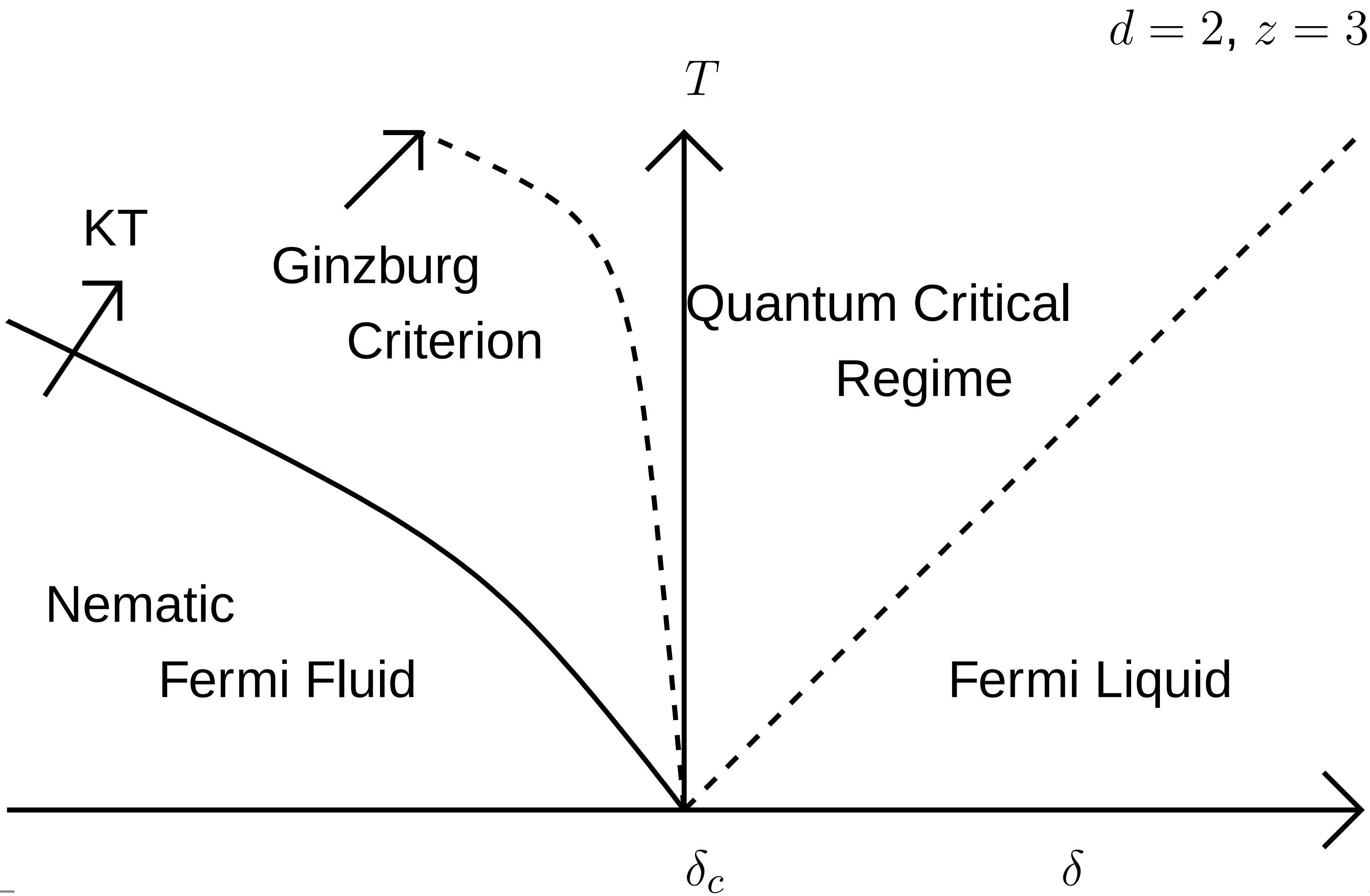
Hertz's RPA Approach

- Find an order parameter theory, for example
  - Split up interactions ($\hat{\mathrm{V}}$) using Hubbard-Stratonovich
  - Integrate out fermions and expand result.
- Consider $F_2(0) \rightarrow -1^+$, $\delta_2(q) \equiv 1 + F_2(q) \rightarrow \kappa q^2$
- Discover three d-wave modes:

$$
\begin{aligned}
\omega_q^{(1)} &= (v_F/\sqrt{2})q & \propto \quad q \\
\omega_q^{(2)} &= (v_F\sqrt{\delta_2(q)}/2)q & \propto \quad q^2 \\
\omega_q^{(3)} &= i(v_F\delta_2(q)/2)q & \propto \quad q^3
\end{aligned}
$$

- $z = 3$ and effective dimension $d_{\text{e}ff} = 2 + 3 = 5$.

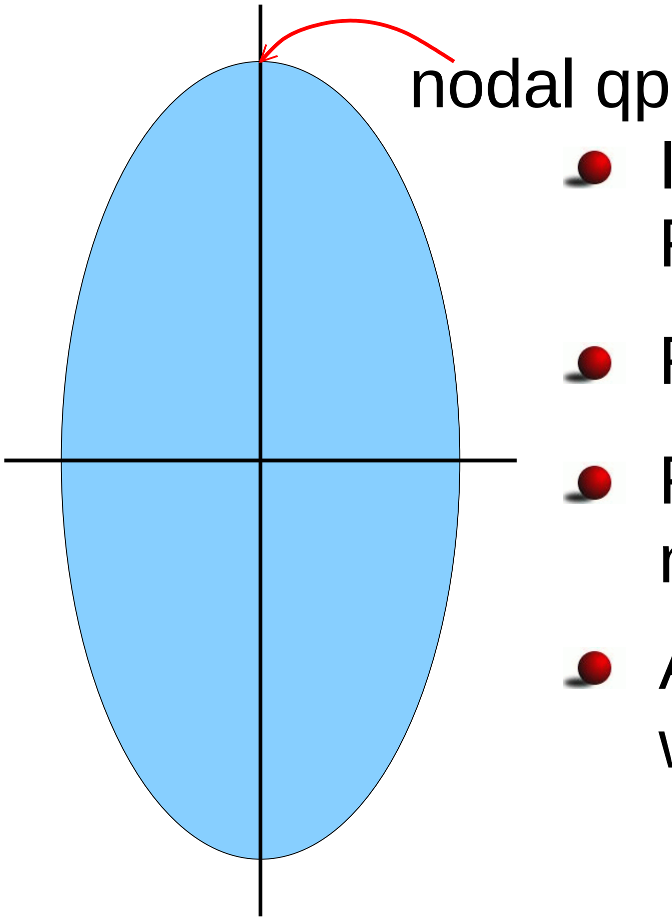
Phase Diagram near QCP



Theory of the nematic Fermi fluid

RPA+Hartree-Fock

Compute the inverse lifetime $\Gamma(\omega) = -2\mathcal{I}m\Sigma(\omega)$



- Indicates a break down of the Fermi liquid.
- Find $\Gamma(\omega) \sim \omega^{2/3}$ at $F_2(0) = -1$
- Find $\Gamma(\omega) \sim |\sin 2\theta|^{4/3}\omega^{2/3}$ in nematic phase.
- At $\theta = \{0, \pm\pi/2, \pi\}$, $\Gamma(\omega) < \omega$ and we have nodal quasi-particles.

Our Method

High Dimensional Bosonization

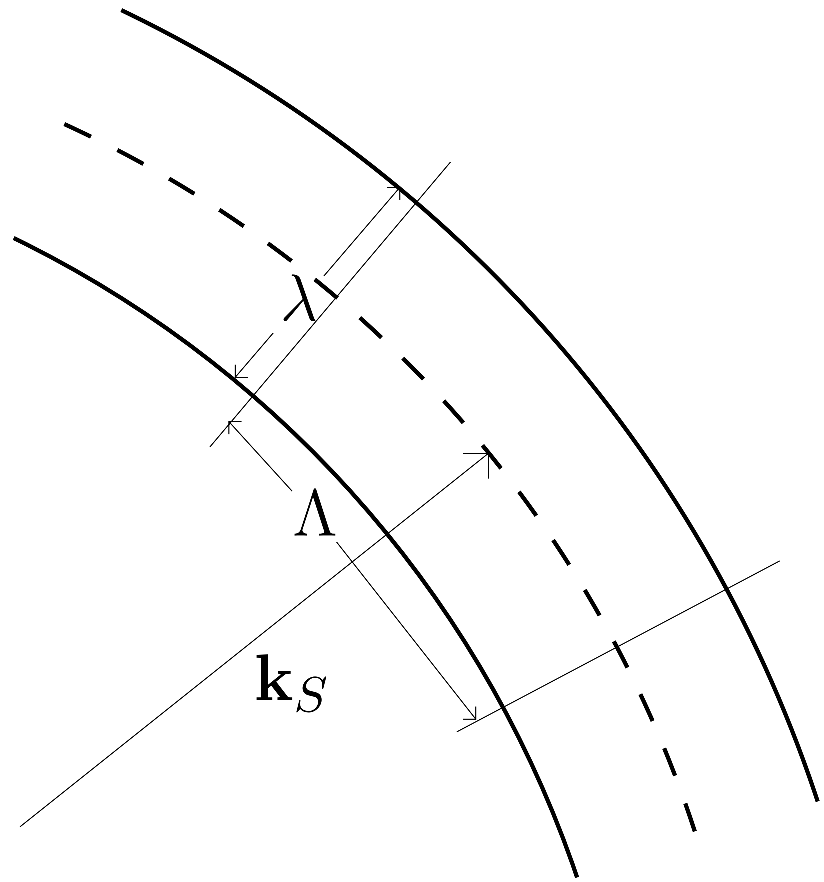
- Fermion Operator

$$\psi(\mathbf{r}) = \sum_S \frac{1}{\sqrt{N}} \psi_S(\mathbf{r}) e^{i\mathbf{k}_S\cdot\mathbf{r}}$$

$$\psi_S(\mathbf{r}) = \sqrt{\frac{N}{L^D}} \sum_{q\in\mathcal{P}_S} c_{\mathbf{k}_S+\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}}$$

- Expand the energy dispersion about $\mathbf{k}_S$

$$\varepsilon_{\mathbf{k}_S+\mathbf{q}} - \mu = \mathbf{v}_S \cdot \mathbf{q} + q^2/2m$$



Our Method

High Dimensional Bosonization

- Define the patch density: $\delta\hat{n}_S(\mathbf{r}) \equiv \hat{\psi}^\dagger_S(\mathbf{r})\hat{\psi}_S(\mathbf{r})$
- These obey the uncertainty relation

$$\Delta k_S \Delta r \approx (\hbar\Lambda)\lambda^{-1} \geq \hbar$$

- They are naturally represented by coherent states
- In the low energy limit, they obey the linearized collisionless Boltzmann equation

$$\partial_t \delta\hat{n}_S(\mathbf{r},t) + \mathbf{v}_S \cdot \nabla \delta\hat{n}_S(\mathbf{r},t) + \\ \mathbf{v}_S \cdot \nabla \sum_T \int d^2r' F_{S-T}(r-r')\delta\hat{n}_S(\mathbf{r}',t) = 0$$

Our Method

Reproduce Known Results

Within high dimensional bosonization

- Specific heat is linear in T.
- Bosonization reproduces RPA in the spinless case with linearized energy dispersion. Why?
  - Both are exact in low energy, long wavelength limit.
  - this is therefore an important check.
- Pomeranchuk's argument is recovered at mean field
- *In the Fermi liquid phase*, quasi-particles are long lived.
- The fermion residue is $0 < Z < 1$ and Green functions have no anomalous dimensions.

Hence, bosonization nicely describes Fermi liquid theory!

Order parameter theory

- Introduce the order parameter in terms of the densities δn_S:

$$Q(\mathbf{r}) = \sum_S \frac{1}{N} e^{i2\theta_S} \delta n_S(\mathbf{r})$$

- This is the $\ell = 2$, d-wave case.
- Integrate out all other angular momentum channels
- Recover the order parameter theory of Oganesyan et. al.
- Bosonization agree's with the Hertz-Millis approach when applied to the nematic quantum critical point.

Fermion Correlations

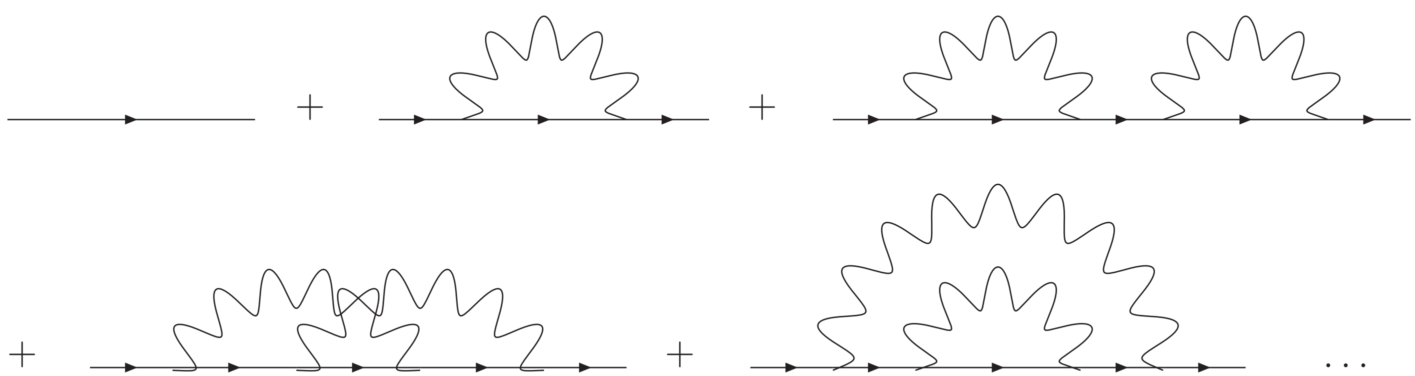
- Within bosonization, the fermion operator is

$$\hat{\psi}_S(\mathbf{r}) = \eta_S(\mathbf{r}_t)\sqrt{N(0)v_F\lambda}\, : e^{-i\hat{\varphi}_S(\mathbf{r})/\hbar} :$$

- So the fermion Green function is of the form

$$G_{F(S)}(\mathbf{r},t) \equiv -i\langle \psi_S(\mathbf{x},t)\psi_S^\dagger(\mathbf{0},0)\rangle = G^0_{F(S)}(\mathbf{r},t)e^{iG^I_{B(S)}(\mathbf{r},t)}$$

- Expanding in powers of G^I_B gives:



where the interaction is the RPA interaction.

Our Results

The Fermion Green Function

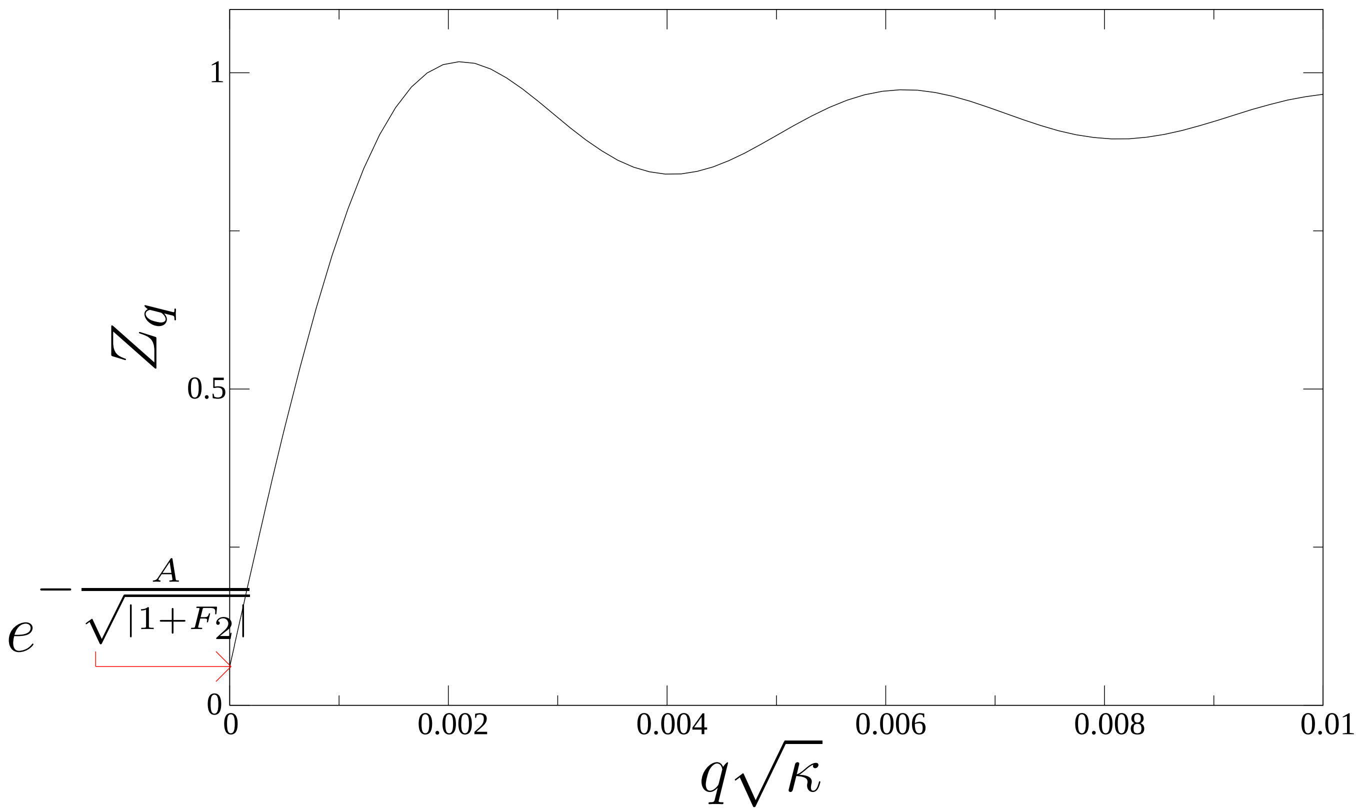
At equal Times

- Fermi Liquid $$G_F(\mathbf{x},0) = \frac{Z}{|\mathbf{x}|^{3/2}} + reg(\mathbf{x})$$
- Nematic QCP $$G_F(\mathbf{x},0) = \frac{C}{|\mathbf{x}|^{3/2}} e^{-A|\mathbf{x}|^{1/3}}$$
- Nematic Phase $$G_F(\mathbf{x},0) = \frac{C}{|\mathbf{x}|^{3/2}} e^{-A|\sin 2\theta_x|^{4/3}|\mathbf{x}|^{1/3}}$$

Our Results

Fermion Residue

- *Near the nematic quantum critical point*



Migdal (1957): $Z_q \equiv n_{\mathbf{k}_F - \mathbf{q}} - n_{\mathbf{k}_F + \mathbf{q}}$

- *In the nematic phase*, residue vanishes except along the principal axes.

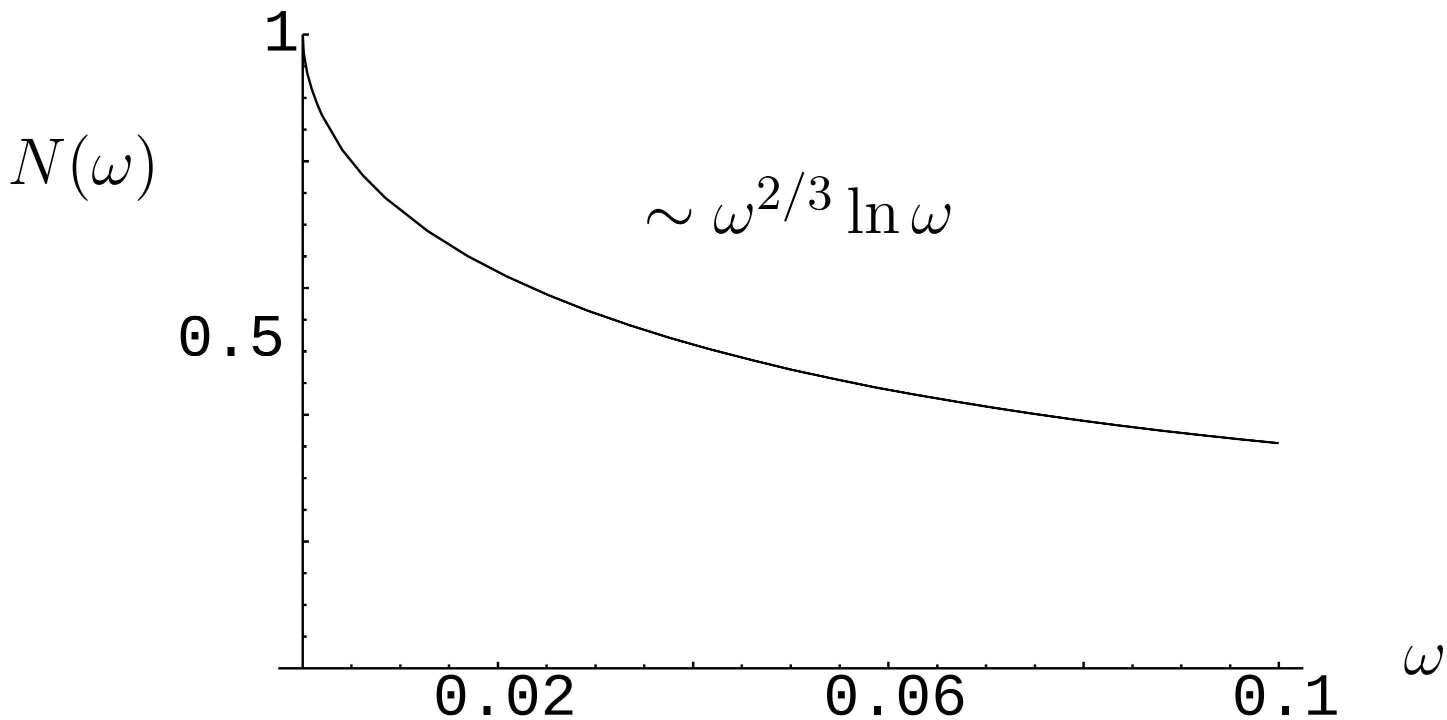
The Fermion Green Function

At equal positions

- Fermi Liquid $$G_F(0,t) = \frac{C}{t} + reg(t)$$
- Nematic QCP $$G_F(0,t) = \frac{C}{t} e^{-A\frac{\ln t}{|t|^{2/3}}}$$
- Nematic Phase $$G_F(0,t) = \frac{C}{t}\left\langle e^{-A|\sin 2\theta|^{4/3}\frac{\ln t}{|t|^{2/3}}}\right\rangle_\theta$$

Fermion Density of States

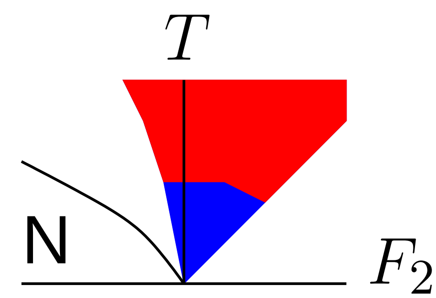
$$N(\omega) \equiv -\frac{1}{\pi}\mathcal{I}mG_F(x,x,\omega)$$



$$\left.\frac{\partial N(\omega)}{\partial\omega}\right|_{\omega\to 0} \to \infty$$

Local Quantum Critical Behavior

"Above" the quantum critical point:
(At $F_2 = -1, T_F \gg T \gg T_\kappa$)



- Order parameter fluctuates on scale of $\xi \gg a$
- Fermion correlations ultra local in space:

$$G_F(x,0) = \frac{1}{|\mathbf{x}|^{3/2}} \exp\left(-ATx^2 \ln\left(\frac{\xi}{x}\right)\right)$$

- But well behaved in time:

$$G_F(0,t) = finite \text{ as } \xi \to \infty$$

- In nematic phase, $\xi = \infty$ protected by symmetry!
- Similar to the quantum Lifshitz model!

(see Ghaemi, Vishwanath and Sentil PRB **72** 024420 (2005))

Our Results

Heat Capacity

At the nematic quantum critical point ($F_2 = -1$)

- $C_V = C_V^0 + C_V^I$
- $C_V^0 = 2\zeta(2)\big(N(0)L^2 k_B T\big)k_B$ as expected ($\zeta(2) = \pi^2/6$)
- and C_V^I is

$$C_V^I = \tfrac{1}{3}(2-\tfrac{1}{3})\Gamma(2-\tfrac{1}{3})\zeta(2-\tfrac{1}{3})\frac{N(0)L^2}{\sqrt{\kappa}k_F}k_B T\left(\frac{\hbar v_F/\sqrt{\kappa}}{k_B T}\right)^{1/3} k_B$$

- Heat Capacity goes like $\sim T^{2/3}$

Conclusions

In Summary

- We have investigated the Fermi liquid-to-nematic QCP and the nematic phase non-perturbatively.
- The theory of bosons
  - naturally describes Pomeranchuk instabilities.
  - recovers Hertz's order parameter theory directly.
  - gives a $T^{2/3}$ heat capacity in the nematic phase and the nematic QCP.
- Results:
  - demonstrates that the nematic QCP and the nematic phase is a theoretically accessible non-Fermi liquid.
  - Local quantum criticality: Fermion correlations are ultra short ranged at the quantum critical point and into the nematic phase.