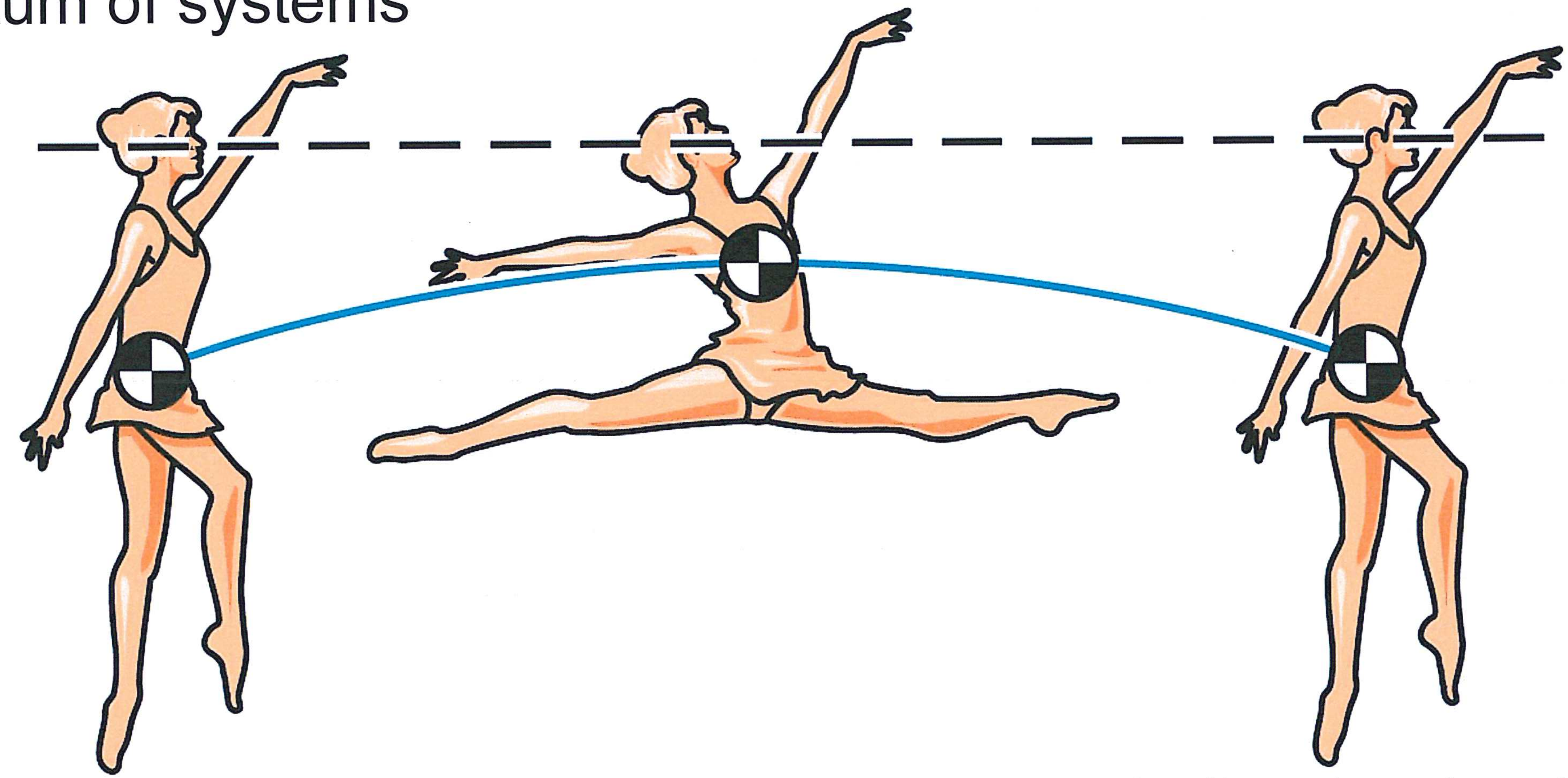
PHY131H1F - Class 14

Today, we are starting Chapter 9 on Momentum:

Hi class!

- Centre of Mass
- Momentum of systems



http://www.chegg.com/homework-help/questions-and-answers/grand-jete-classic-ballet-manoeuvre-dancer-executes-horizontal-leap-moving-arms-legs--cent-q5975347

- *"Will we have to use integration on the exam?"*

- Yes. This is a calculus-based course, and I do expect you to be able to do simple integrals of polynomials, similar to the ones in the examples, Homeworks, and suggested end-of-chapter problems. and sin & cos

- *"If momentum is not conserved when there is a presence of gravity, what explains how the momentum returns to its original value after returning to the launch height?"*

- Remember Momentum includes direction. The y-component of the momentum is **opposite** of its original value when it returns to launch height, so $\Delta \vec{p}_y = -2\vec{p}_{yi}$

- *"Example 9.3 I don't understand how you got the mass density of the wing equation from"*

- It has mass *M*, distributed uniformly over the wing. So the "mass density", or mass per area is $\sigma = M/A$. The units of this is kg / m^2. $\boxed{dA \cdot \sigma = dM}$

1.5

From Fall ~~2017~~ 2015 PHY131H1F Midterm Test 2

Question7

A satellite in an elliptical orbit has a height above the surface of the Earth which ranges from 630 km at its lowest point, up to 7600 km at its highest. When it is at its lowest point (closest to the surface of the Earth), it's moving at 8.7 km/s. How fast is it moving at its highest point? [Assume the Earth is a sphere of radius 6370 km and mass 5.97×10^{24} kg.]

R_E $\quad$ M_E

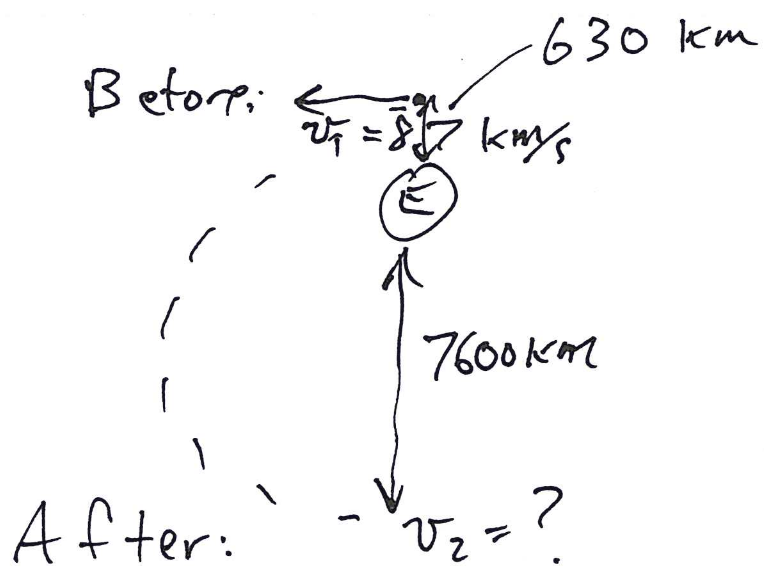
(A) 2.2 km/s (B) 4.4 km/s (C) 6.2 km/s (D) 8.7 km/s (E) 17 km/s

Not circular orbit so $v \neq \sqrt{gr}$

Let's try conservation of energy.

→ Gravity is only force

→ No non-conservative forces.



Use: $E_1 = E_2$

$$\frac{1}{2}\cancel{m}v_1^2 + \left(-\frac{GM_E\cancel{m}}{r_1}\right) = \frac{1}{2}\cancel{m}v_2^2 + \left(-\frac{GM_E\cancel{m}}{r_2}\right)$$

Solve for v_2:

$$\frac{v_2^2}{2} = \frac{v_1^2}{2} + GM_E\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$v_2 = \sqrt{v_1^2 + 2GM_E\left(1/r_2 - 1/r_1\right)}$$

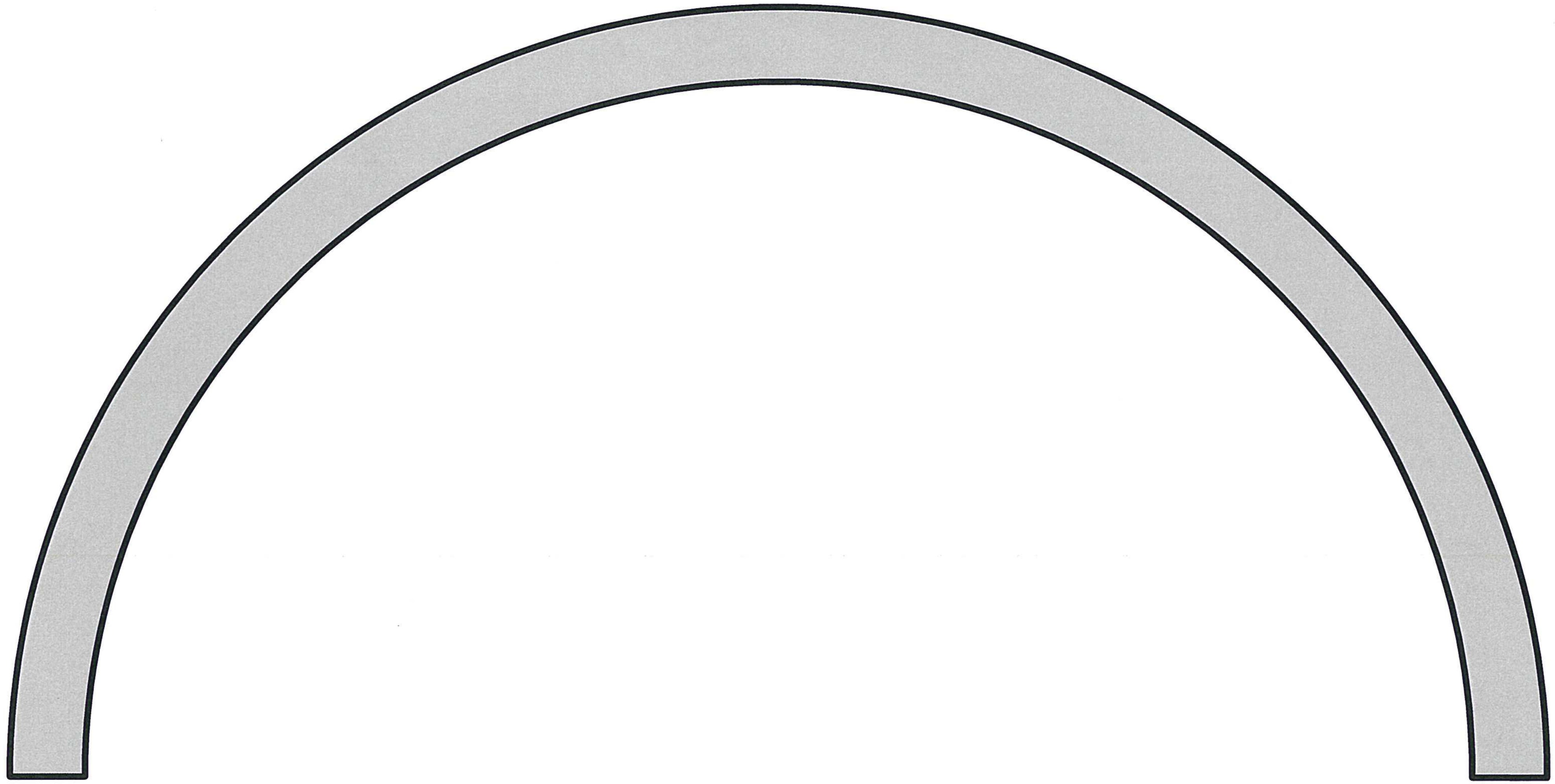
$r_1 = 630\text{ km} + 6370\text{ km} = 7.0 \times 10^6\text{ m}$

$r_2 = 7600\text{ km} + 6370\text{ km} = 1.397 \times 10^7\text{ m}$

$\therefore\ v_2 = 4350.5$ m/s

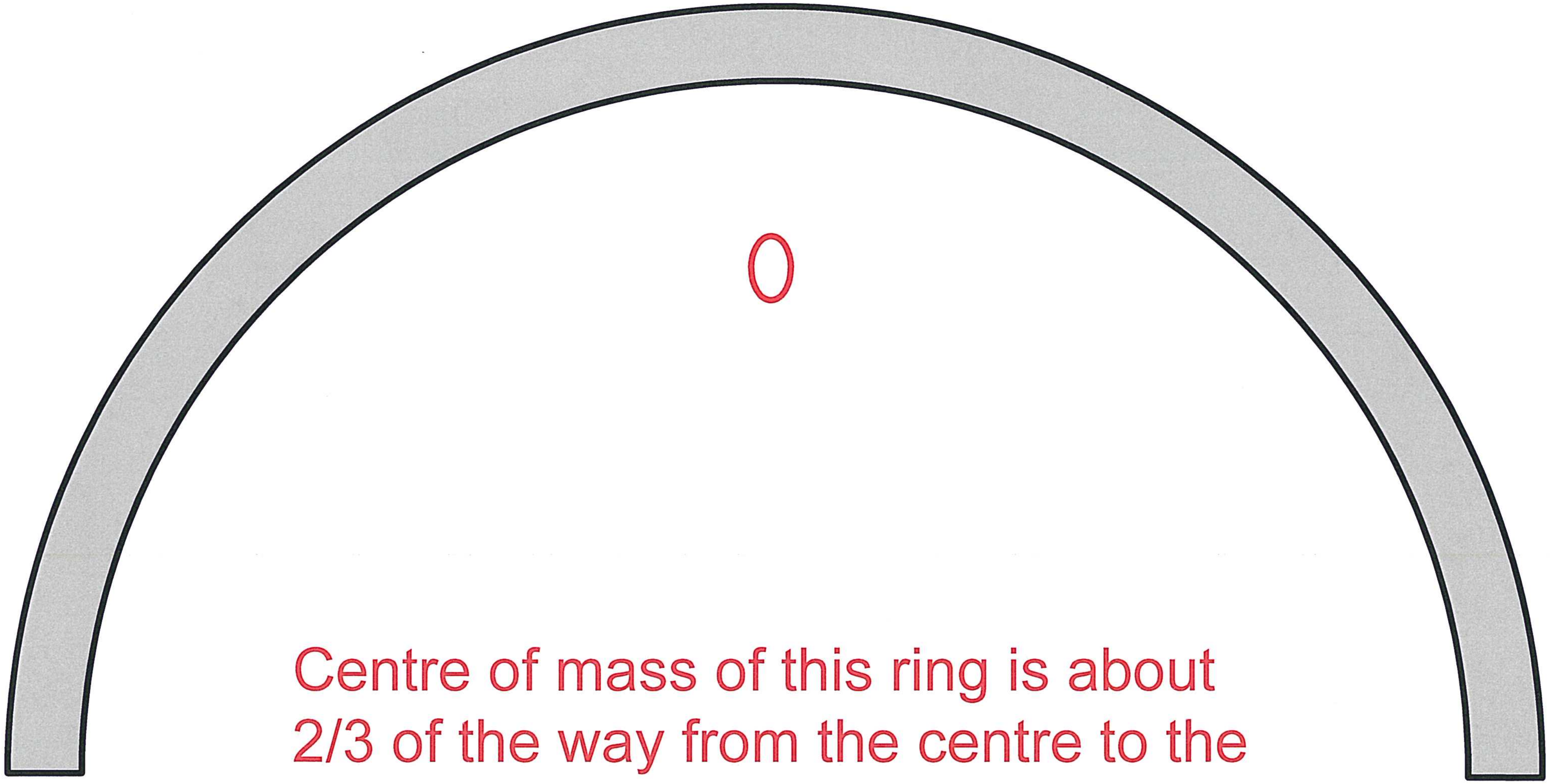
Learning Catalytics Question 1

Where do you think is the centre of mass of this object?



Learning Catalytics Question 1

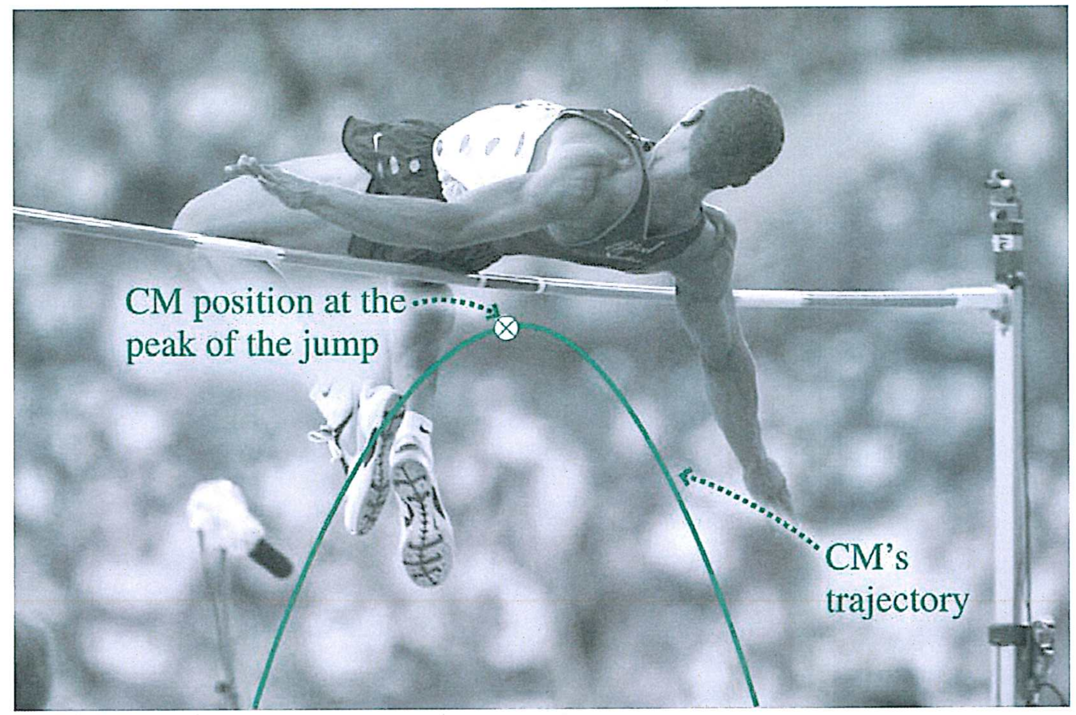
Where do you think is the centre of mass of this object?



Centre of mass of this ring is about 2/3 of the way from the centre to the edge.
($2/\pi$, actually, as we will show…)

Last day I asked at the end of class:

- How is it possible to clear the bar in a high jump if your center of mass does not reach to the height of the bar?
- ANSWER:
- For a projectile, the motion of the centre of mass is governed by the equations of constant acceleration we already know.



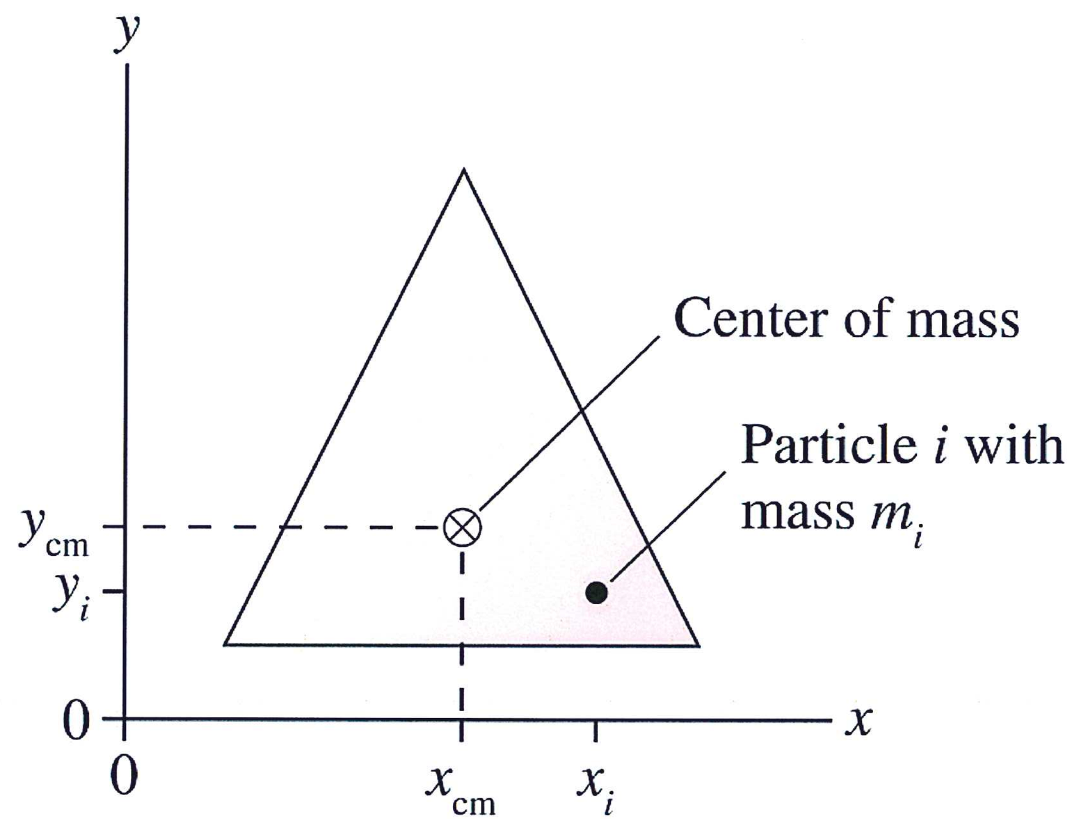
- The center of mass of a complicated shape (like a person doing a back arch) does not need to be within the object.

Center of Mass

Consider an object made of particles. Particle i has mass m_i. The center-of-mass is at

$$x_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$y_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$



Calculating center of mass is much like calculating your grade-point average. Marks in full-courses count twice as much as marks in half-courses. Particles of *higher mass* count *more* than particles of lower mass.

Center of Mass of a Solid Object

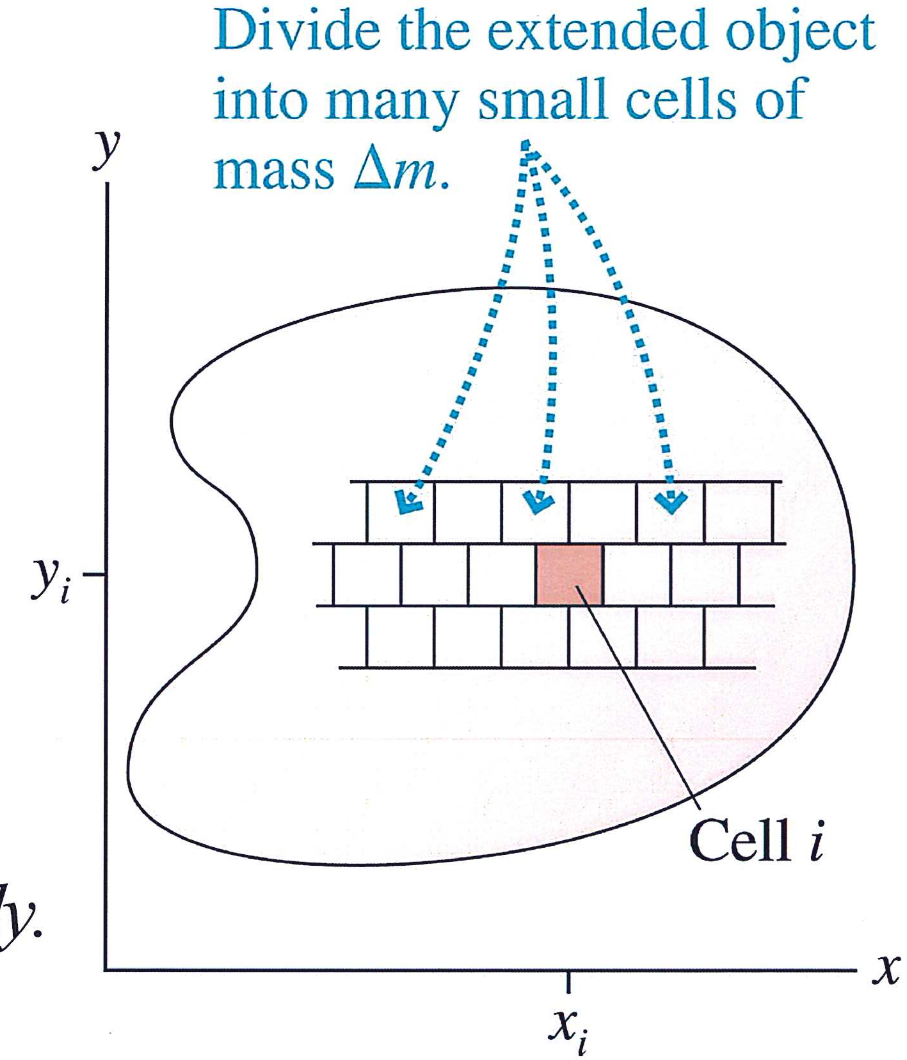
Divide a solid object into many small cells of mass Δm. As $\Delta m \to 0$, and is replaced by dm, the sums become

Summarized by Eq. 9.4 :

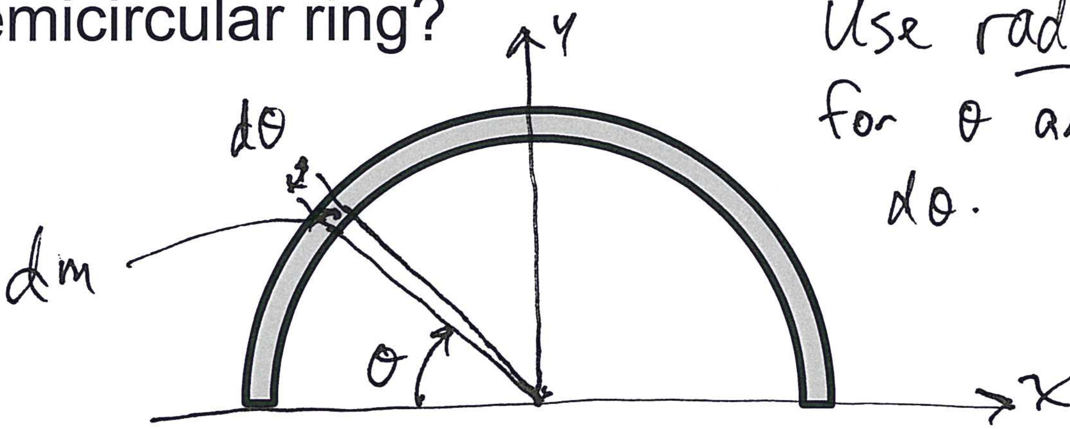
$$x_{cm} = \frac{1}{M}\int x\, dm \qquad \text{and} \qquad y_{cm} = \frac{1}{M}\int y\, dm$$

Before these can be integrated:

- dm must be replaced by expressions using dx and dy.
- Integration limits must be established.



Where is the centre of mass of a semicircular ring?



Use radians for θ and dθ.

$x_{cm} = 0$, by symmetry.

The question is: what is y_{cm}.

Define: dm = small portion of ring. at angle θ. angular width dθ

Mass $dm = \sigma \cdot d\theta$

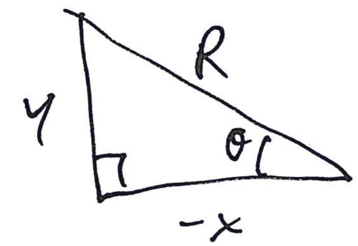
σ = mass density = $\dfrac{M}{\text{angular length of ring}} = \dfrac{M}{\pi}$

since half ring is π radians.

Eq. 9.4

Eq. 9.4 $\quad y_{cm} = \dfrac{1}{M}\int y \cdot dm = \dfrac{1}{M}\int y\left(\dfrac{M\,d\theta}{\pi}\right)$

Note:



$\sin\theta = \dfrac{y}{R}$

$\Rightarrow y = R\sin\theta$

pull constants out of integral

$$y_{cm} = \frac{\cancel{M}}{\cancel{M}\pi}\int_0^{\pi} R\sin\theta\, d\theta$$

$$= \frac{R}{\pi}\int_0^{\pi}\sin\theta\, d\theta = \frac{R}{\pi}\Big[-\cos\theta\Big]_0^{\pi}$$

$\cos\pi = -1$

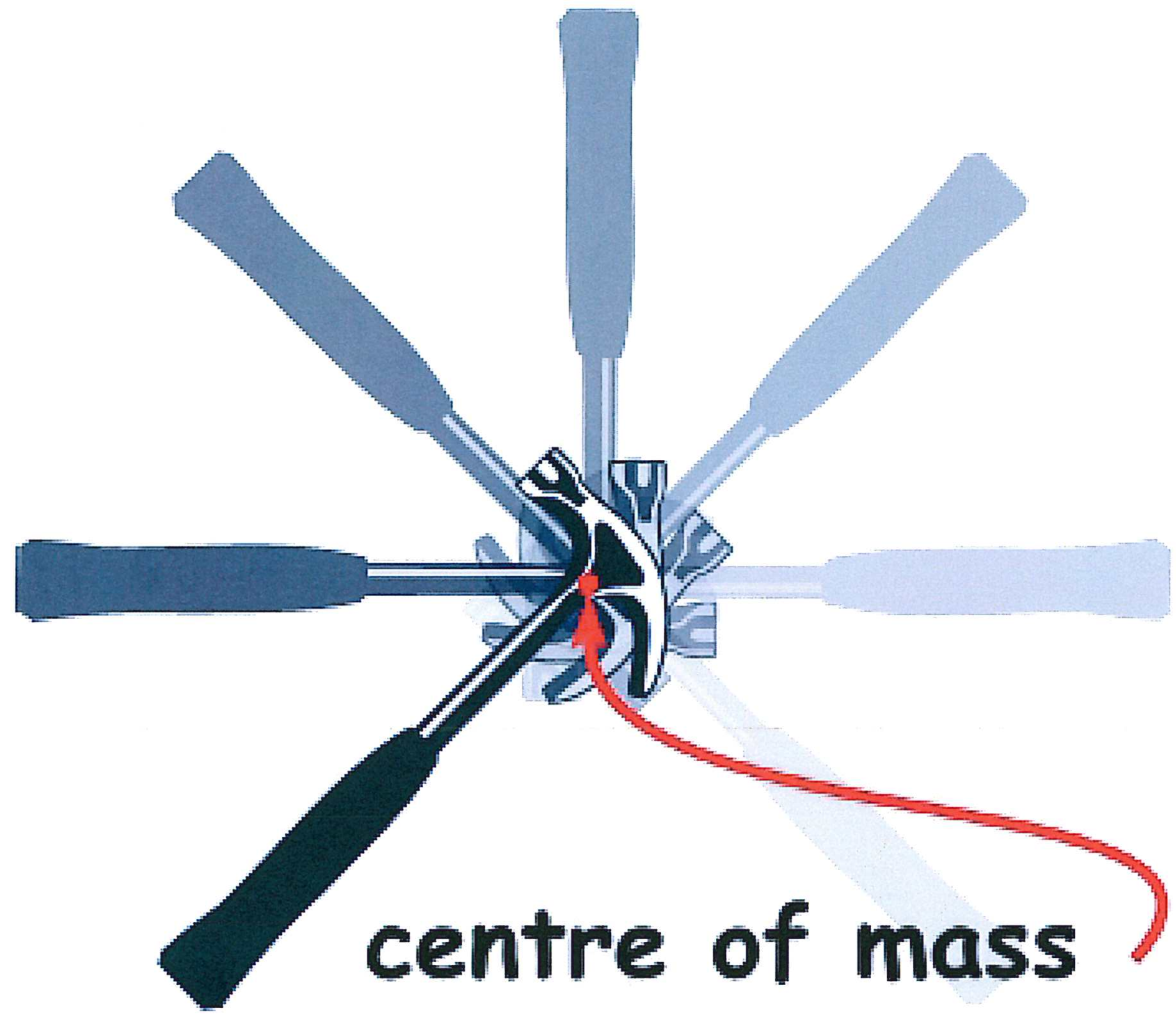
$\cos 0 = +1$

$$y_{cm} = -\frac{R}{\pi}\left[-1-1\right] = \frac{2}{\pi}R$$

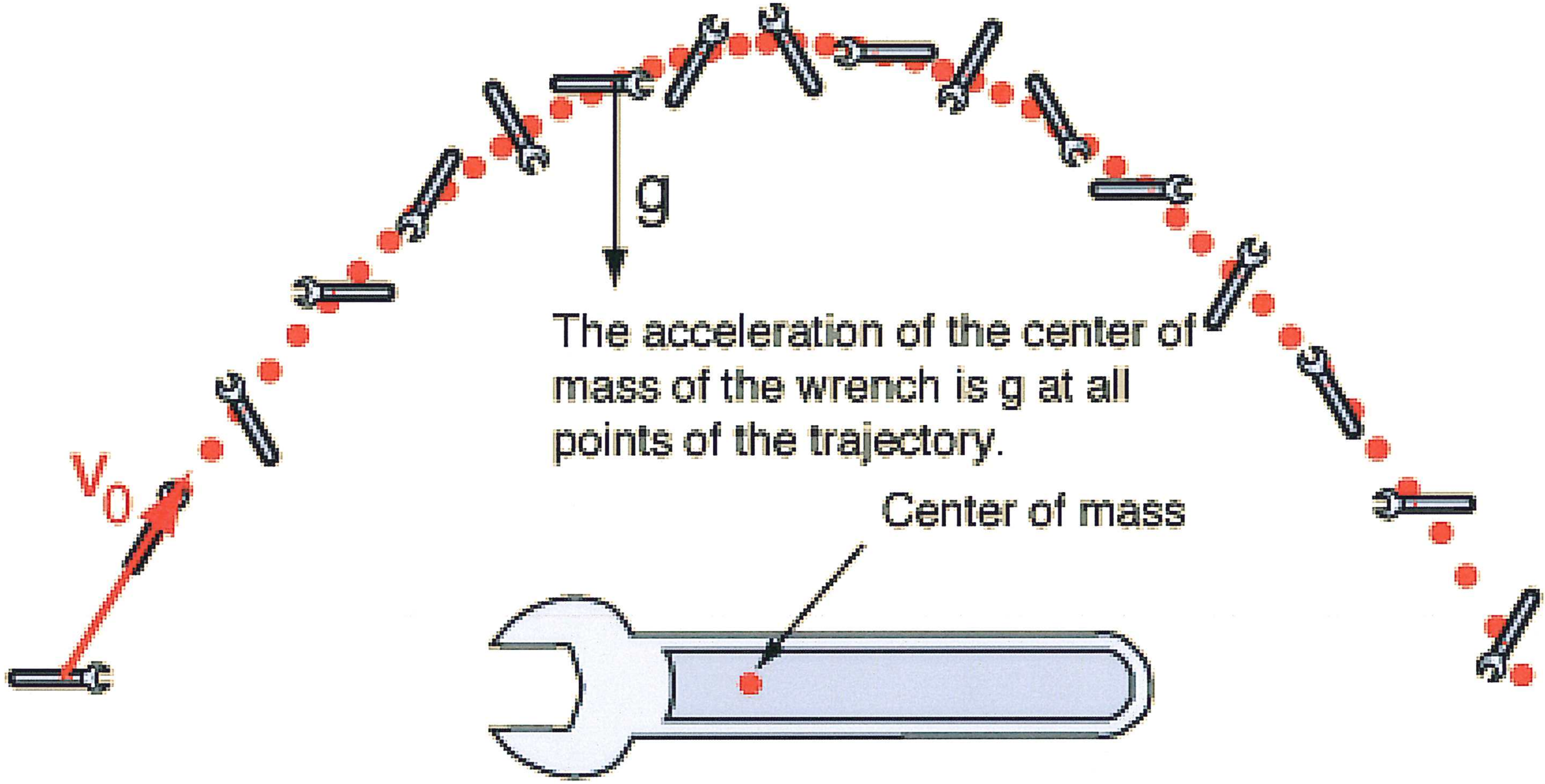
Rotation About the Center of Mass

An unconstrained object (i.e., one not on an axle or a pivot) on which there is no net force rotates about a point called the center of mass.

The center of mass remains motionless while every other point in the object undergoes circular motion around it.

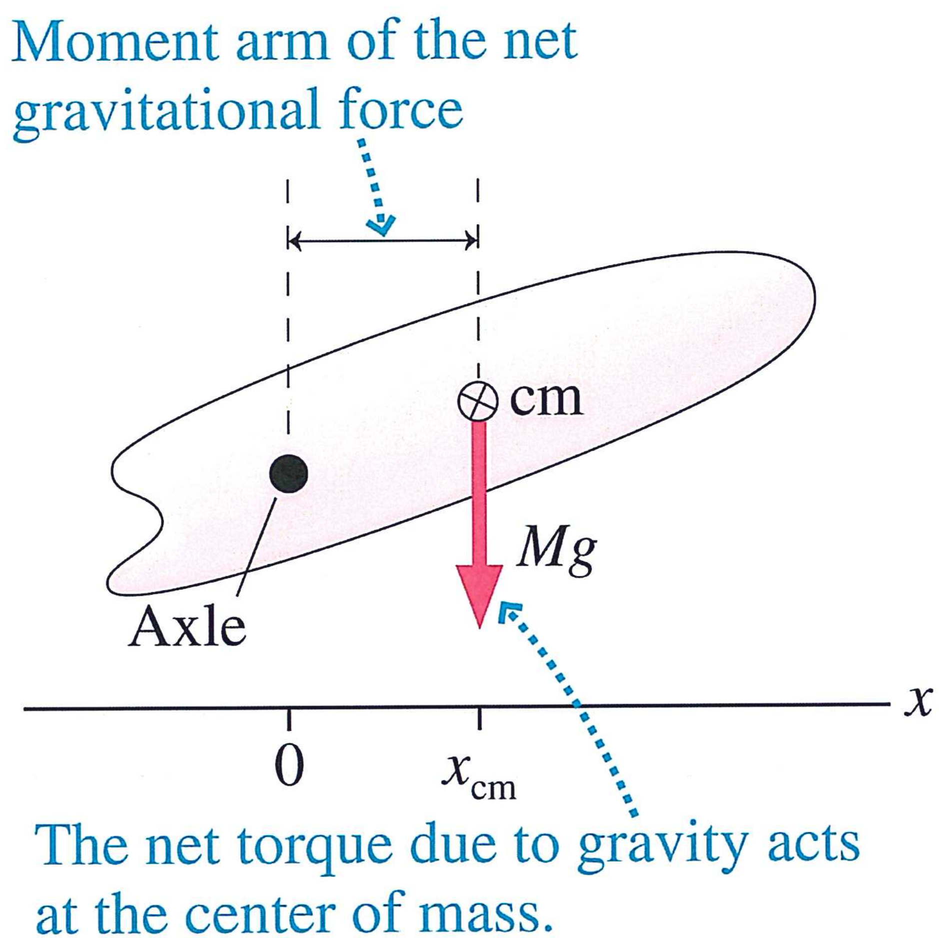


[Image from http://resources.yesican-science.ca/discover_2006_2/images/hammer2.jpg]



[image from http://hyperphysics.phy-astr.gsu.edu/hbase/mechanics/n2ext.html]

Gravitational Torque



- When calculating the torque due to gravity, you may treat the object as if all its mass were concentrated at the centre of mass.
- More about this in Chapter 10!

Chapter 9 BIG IDEA

Law of Conservation of Momentum:

The total momentum $\vec{P} = \sum \vec{p}_i$ of a collection of objects in an isolated system (no external net force on the system) is conserved.

- This is useful in *collisions* and *explosions*, in which the outside forces on the objects are negligible during the collision. So the total momentum *before* (1) equals total momentum *after* (2):

$$\vec{P}_1 = \vec{P}_2$$

- In two-dimensions *x* and *y* this vector equation is actually two equations: $\left(\vec{P}_1\right)_x = \left(\vec{P}_2\right)_x \qquad \left(\vec{P}_1\right)_y = \left(\vec{P}_2\right)_y$

Learning Catalytics Question 2

- Two particles collide, one of which was initially moving, and the other initially at rest. Is it possible for *both* particles to be at rest after the collision? [Assume no outside forces act on the particles.]

A. Yes

B. No

$\vec{p}_1 \neq 0$

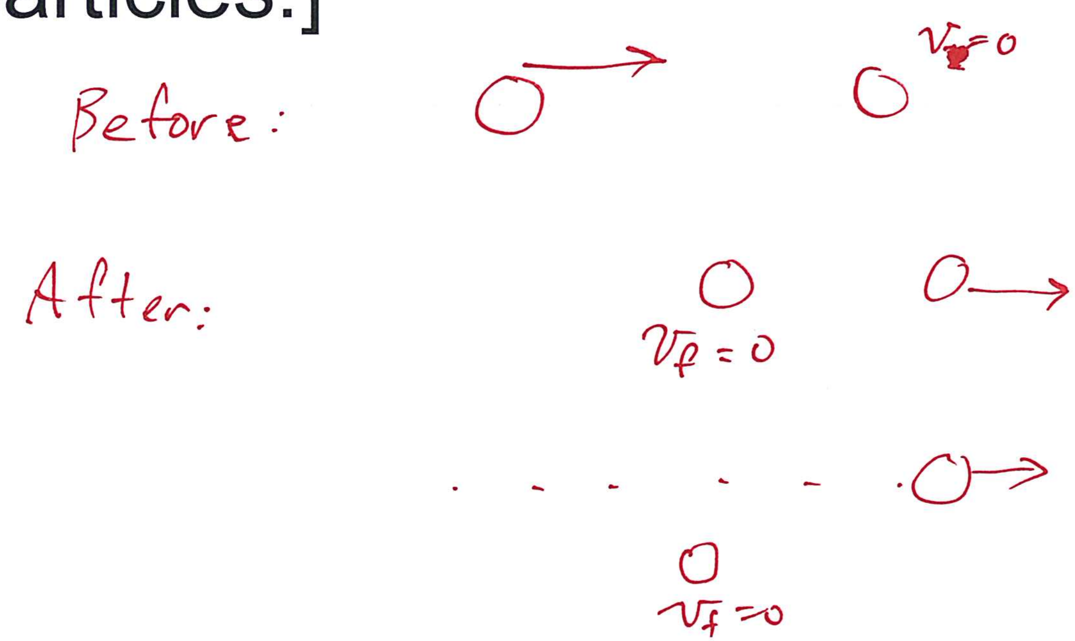
$\vec{p}_2 \neq 0$

Learning Catalytics Question 3

- Two particles collide, one of which was initially moving, and the other initially at rest. Is it possible for *one* particle to be at rest after the collision? [Assume no outside forces act on the particles.]

A. Yes

B. No



A yellow Hummer (m_H = 3900 kg) was driving South and collided with a blue Toyota (m_T = 1200 kg) which was driving East. The speed limit on both roads is 50 km/hr.

After the collision, the two cars stuck together and the combined mass skidded along the ground.

The police measure that the skid marks are a line 10 m long, angled 32° East of South.

The coefficient of kinetic friction between rubber and road is 0.8.

How fast were the cars going before the collision? Who is at fault?

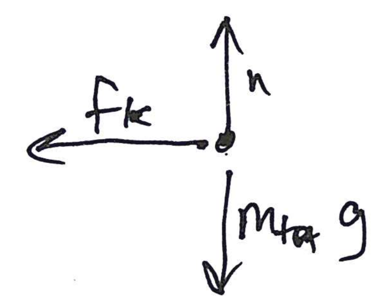
During short collision: use conservation of momentum.

After collision: use kinematics of sliding friction.

Initial velocity of sliding = "after" of collision.

Let's start with kinematics part.

Draw f.bd. of $m_{tot} = m_H + m_T$ stuck together.



$\longrightarrow v$

$n = m_{tot}\, g$

$f_k = \mu_k n = \mu_k m_{tot}\, g$

$(F_{net})_x = -f_k = m_{tot}\, a_x$

$a_x = -\mu_k g$ $\quad \Delta x = +10\text{ m}$

$v_{fx} = 0$ $\quad v_i = ?$

Don't care about t. Let's use

$v_{fx}^2 = v_{ix}^2 + 2 a_x \Delta x$

$$v_{ix} = \sqrt{v_{fx}^2 - 2a_x \Delta x}$$

$$= \sqrt{0 - 2(-\mu_k g)\Delta x}$$

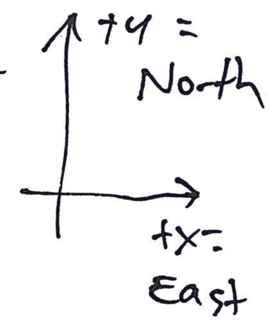
$$= \sqrt{2(0.8)9.8(10)}$$

$v_i = 12.52198$ m/s

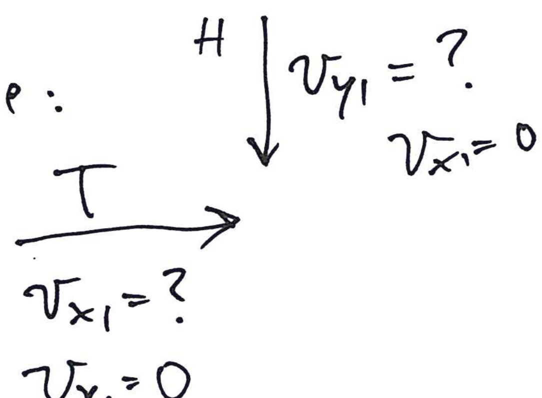
initial speed of sliding = "after" speed of collision

Now let's conserve momentum during collision.

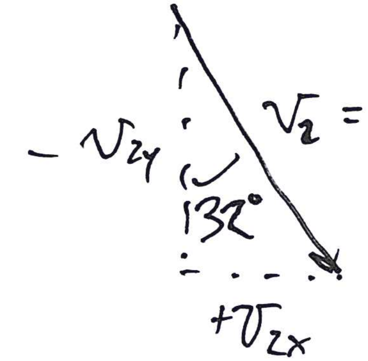
Define



Before:



After:



$v_{2y} = -v_2 \cos\theta$

$v_{2y} = -10.6174$ m/s

$v_{2x} = v_2 \sin\theta = 6.6356$ m/s

x	y
$p_{1x} = p_{2x}$	$p_{1y} = p_{2y}$
$m_T v_{x1} = m_{tot} v_{2x}$	$m_H v_{y1} = m_{tot} v_{2y}$
$v_{x1} = \frac{m_{tot}}{m_T}(6.6356)$	$= \frac{1200+3900}{3900}(-10.6174)$
$\left(\frac{1200+3900}{1200}\right)6.6356$	$= -13.9$ m/s
$v_{x1} = 28.2$ m/s	$\boxed{v_{2y} = -50 \text{ km/hr}}$
Toyota → $\boxed{v_{x1} = 102 \text{ km/hr}}$	

Before Class 15 on Wednesday

- Remember MasteringPhysics.com **Problem Set 6** on Ch.7 is due **tonight** by 11:59pm!!
- Please finish reading Chapter 9 and/or watch Preclass 15 Video:
- Something to think about:
- Consider the two integrals below. What's the difference? [Hint: one is the change in **energy** of an object, and one is the change in **momentum** of an object.]

$$\vec{J} = \int \vec{F} dt$$

$$W = \int \vec{F} \cdot d\vec{r}$$