## PHY131H1F - Class 15

Today, we are finishing Chapter 9 on Momentum:

Impulse and Momentum
Energy in Collisions


Totally Inelastic Collisions

## Elastic Collisions

Where is the centre of mass of a solid semicircle?


Last day I asked at the end of class:

- Consider the two integrals below. What's the difference?

Impulse (Ch.9)
$\vec{J}=\int \vec{F} d t$
This is the force integrated over time, which gives the change in momentum. [Units: kg m / s]

Work (Ch. 6)
$W=\int \vec{F} \cdot d \vec{r}$
This is the force integrated over distance, which gives the change in energy. [Units: Joules]

$$
\vec{p}=m \vec{v} \text { means } p_{x}=m v_{x} \text { and } p_{y}=m v_{y} .
$$

A basketball with mass 0.1 kg is traveling down and to the right with $v_{x i}=+5 \mathrm{~m} / \mathrm{s}$, and $v_{y i}=-5 \mathrm{~m} / \mathrm{s}$.

Before:


LC Question 1 What is the change in the


It hits the horizontal ground, and then is traveling up and to the right with $v_{x f}=+5$ $\mathrm{m} / \mathrm{s}$, and $v_{\mathrm{yf}}=+4 \mathrm{~m} / \mathrm{s}$. $x$-component of the ball's momentum?

LC Question 2 What is the change in the $y$-component of the ball's momentum?

## Impulse

The impulse upon a particle is:

$$
\begin{aligned}
& J_{x}=\int_{t_{1}}^{t_{2}} F_{x} d t \\
& J_{y}=\int_{t_{1}}^{t_{2}} F_{y} d t
\end{aligned}
$$



- Impulse has units of N s , but you should be able to show that Ns are equivalent to $\mathrm{kg} \mathrm{m} / \mathrm{s}$.
- The impulse-momentum theorem states that the change in a particle's momentum is equal to the impulse on it:

$$
\begin{equation*}
\Delta p_{x}=J_{x} \quad \Delta p_{y}=J_{y} \tag{6}
\end{equation*}
$$



A 0.50 kg cart rolls to the right at $+1.2 \mathrm{~m} / \mathrm{s}$. It collides with a force sensor.
A plot of force versus time (with positive force defined as towards the right) gives an area of -1.0 N s .
What is the velocity of the cart immediately after the collision?

- Consider a car accident in which a car, initially traveling at $50 \mathrm{~km} / \mathrm{hr}$, collides with a large, massive bridge support.
- The car comes to an abrupt stop.

- Why is it better to hit the airbag as opposed to the hard plastic steering wheel or dashboard?
- ANSWER:
- The people must reduce their momentum from $m v$ to zero. This requires a force applied over some amount of time. If the time is very short, the force must be very large (ie hitting steering wheel).
- If the person hits the airbag, this squishes during impact, lengthening the time of the stop. If the stopping process takes longer, then the maximum force is less.

Learning Catalytics Question 3
A 100 g rubber ball and a 100 g damp cloth are dropped on the floor from the same height. They both are traveling at the same speed just before they hit the floor.

The rubber ball bounces, the damp cloth does not.
Which object receives a larger upward impulse from the floor?
A. They receive equal impulses.
B. The damp cloth receives a larger impulse.
C. The rubber ball receives a larger impulse.

## Elastic Collisions

- All quick collisions conserve momentum, because external forces are usually negligible during the collision compared to the collision forces themselves.
- Often, in collisions, kinetic energy after is less than kinetic energy before. That's because energy is used up in deforming or heating the colliding objects.
- If kinetic energy is conserved, the collision is said to be elastic.


## Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Consider a head-on, perfectly elastic collision of a ball of mass $m_{1}$ having initial velocity $v_{1 \mathrm{i}}$, with a ball of mass $m_{2}$ that is initially at rest.


During:
During the collision energy is stored as elastic potential energy.

After:


The balls' velocities after the collision are $v_{1 \mathrm{f}}$ and $v_{2 \mathrm{f}}$.

## Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Momentum conservation: $\quad m_{1} v_{1 \mathrm{f}}+m_{2} v_{2 \mathrm{f}}=m_{1} v_{1 \mathrm{i}}$
Kinetic energy conservation: $\frac{1}{2} m_{1} v_{1 \mathrm{f}}{ }^{2}+\frac{1}{2} m_{2} v_{2 \mathrm{f}}{ }^{2}=\frac{1}{2} m_{1} v_{1 \mathrm{i}}{ }^{2}$
There are two equations, and two unknowns: $v_{1 \mathrm{f}}$ and $v_{2 \mathrm{f}}$.
Solving for the unknowns gives:
Eq. 9.15:

$$
\begin{array}{cl}
v_{1 \mathrm{f}}=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} v_{1 \mathrm{i}} & \begin{array}{l}
\text { (Elastic collision } \\
\text { with ball } 2 \text { initially at } \\
v_{2 \mathrm{f}}=\frac{2 m_{1}}{m_{1}+m_{2}} v_{1 \mathrm{i}}
\end{array} \\
\text { rest.) }
\end{array}
$$

## Elastic Collision in 1 Dimension when ball 2 is initially at rest.

Eq. 9.15

$$
\begin{array}{cl}
v_{1 \mathrm{f}}=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} v_{1 \mathrm{i}} & \text { (Elastic collision } \\
v_{2 \mathrm{f}}=\frac{2 m_{1}}{m_{1}+m_{2}} v_{1 \mathrm{i}} & \text { with ball 2 initially at } \\
\text { rest.) }
\end{array}
$$

These equations come in especially handy, because you can always switch into an inertial reference frame in which ball 2 is initially at rest!

## Demonstration and Example

- A 0.50 kg basketball and a 0.05 kg tennis ball are stacked on top of each other, and then dropped from a height of 0.82 m above the floor.
- How high does the tennis ball bounce?

- Assume all perfectly elastic collisions.



## Demonstration and Example

- Divide motion into segments.
- Segment 1: free-fall of both balls from a height of $\mathrm{h}=0.82 \mathrm{~m}$. Use conservation of energy: $U_{\mathrm{f}}+K_{\mathrm{f}}=U_{\mathrm{i}}+K_{\mathrm{i}}$


$$
\begin{gathered}
0+1 / 2 m v_{\mathrm{f}}^{2}=m g h+0 \\
v_{\mathrm{f}}= \pm[2 g h]^{1 / 2}=-4.0 \mathrm{~m} / \mathrm{s}, \text { for both balls. }
\end{gathered}
$$

- Segment 2: basketball bounces elastically with the floor, so its new velocity is $+4.0 \mathrm{~m} / \mathrm{s}$.


## Demonstration and Example

- Segment 3: A 0.50 kg basketball moving upward at $4.0 \mathrm{~m} / \mathrm{s}$ strikes a 0.05 kg tennis ball, initially moving downward at $4.0 \mathrm{~m} / \mathrm{s}$.

- Their collision is perfectly elastic.
- What is the speed of the tennis ball immediately after the collision?
- A 0.50 kg basketball moving upward at $4.0 \mathrm{~m} / \mathrm{s}$ strikes a 0.05 kg tennis ball, initially moving downward at $4.0 \mathrm{~m} / \mathrm{s}$.
- Their collision is perfectly elastic. What is the speed of the tennis ball immediately after the collision?


## Demonstration and Example

- Segment 4: freefall of tennis ball on the way up. $v_{\mathrm{i} 2}=+10.5 \mathrm{~m} / \mathrm{s}$.
- Use conservation of energy: $U_{\mathrm{f}}+K_{\mathrm{f}}$ $=U_{\mathrm{i}}+K_{\mathrm{i}}$

$$
\begin{gathered}
m g h+0=0+1 / 2 m v_{\mathrm{i}}^{2} \\
h=v_{\mathrm{i}}^{2} /(2 g)=5.6 \mathrm{~m} .
\end{gathered}
$$

- So the balls were dropped from 0.82 m , but the tennis ball rebounds up to 5.6 m ! (Assuming no energy losses.)



## Before Class 16 next Monday Nov. 13

- Complete Problem Set 7 on Chapters 8 and 9 by Monday Nov. 13 at 11:59pm.
- Read the first 3 sections of Chapter 10 and/or watch the Preclass 16 video.
- Have a great Reading Week!


Something to think about: Why is a door easier to open when the handle is far from the hinge, and more difficult to open when the handle is in the middle?


