PHY131H1F - Hour 27

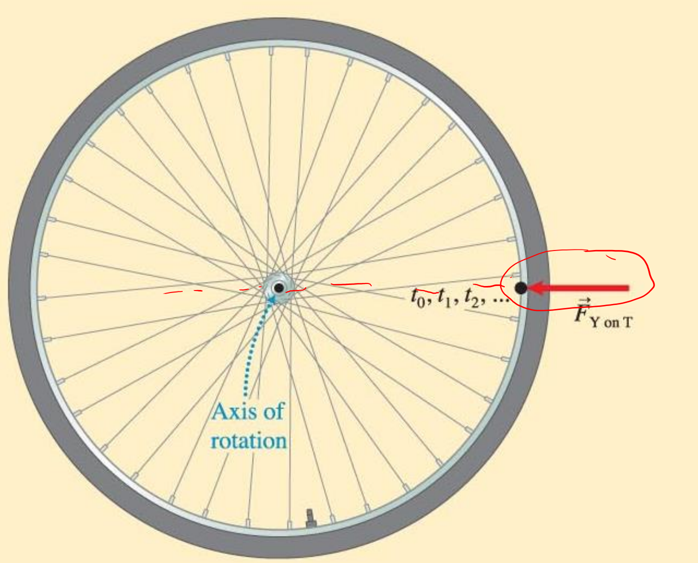
Today:

9.3 Newton's Second Law for Rotational Motion

9.4 Rotational Momentum

Observational Experiment #1

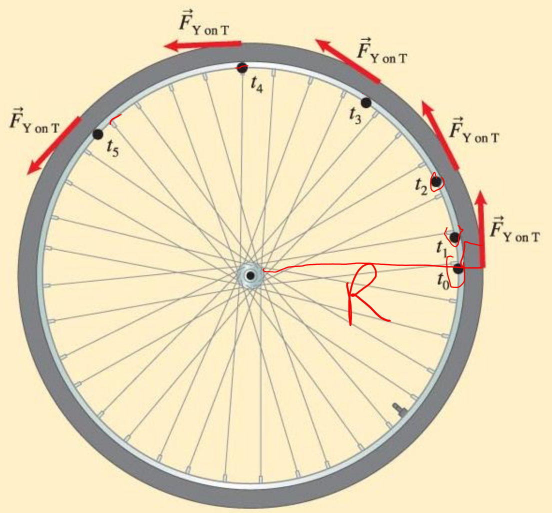
Experiment 1: Your bike sits upside down. You push on the front tire toward the axle. The tire does not turn.



zero torque → no motion.

Observational Experiment #2

Experiment 2: You push lightly and continuously on the outside of the tire in a counterclockwise (ccw) direction tangent to the tire. As you continue to push, the tire rotates ccw faster and faster.



$\tau \overset{+}{=} FR\sin 90^\circ = FR$

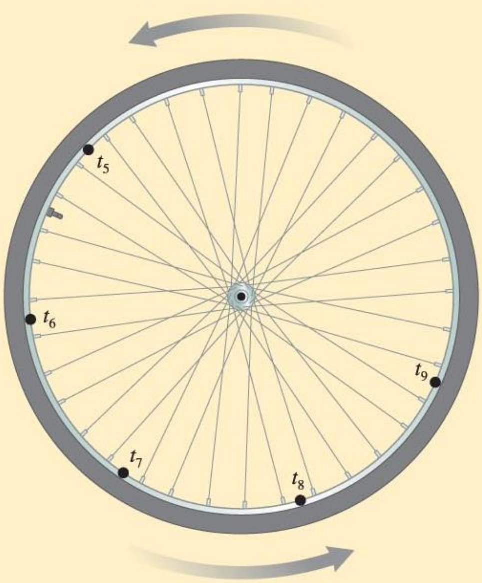
positive external torque

ω increases

$\Rightarrow$ positive angular acceleration

Observational Experiment #3

Experiment 3: You release the spinning tire and watch it. The tire continues rotating ccw at a constant rate.



$\rightarrow$ no torque

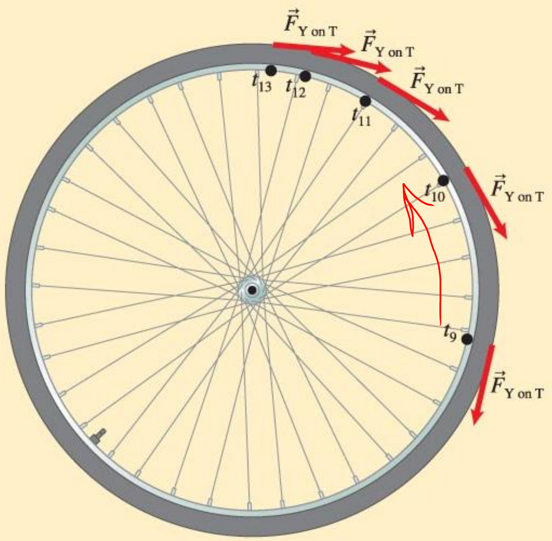
$\sim$ constant ω

$\rightarrow$ zero α

(no angular acceleration)

Observational Experiment #4

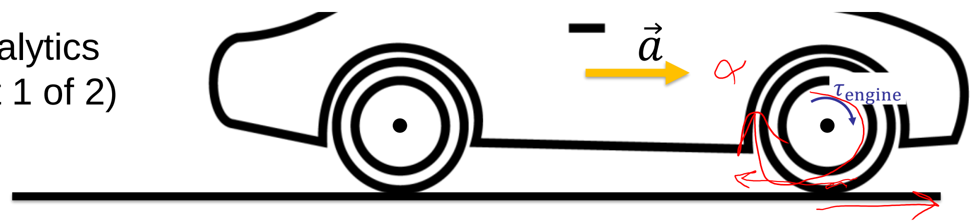
Experiment 4: With the tire still rotating ccw fast, you gently and continuously push clockwise (cw) against the tire. The rotational speed decreases.



Negative torque
ω is positive and decreasing.
→ negative angular acceleration.

→ Angular acceleration is proportional to external torque.

Learning Catalytics Question (part 1 of 2)



You are sitting in your car, and you step on the gas pedal. The car accelerates forward.

Your car has Front Wheel Drive (FWD). That means the front two wheels are connected to the engine, but the back two wheels just freely rotate on their axles.

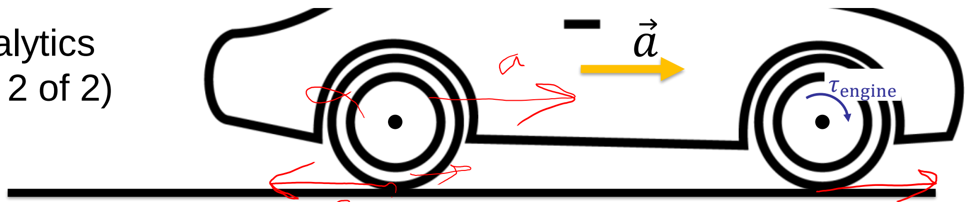
As you accelerate, what is the direction of the force of static friction of the road upon the **front** wheels?

A. Forward

B. Backward

C. The static friction force on the front wheels is zero

Learning Catalytics Question (part 2 of 2)



f_{sb} f_{sf}

$|f_{sf}| >> |f_{sb}|$

You are sitting in your car, and you step on the gas pedal. The car accelerates forward.

Your car has Front Wheel Drive (FWD). That means the front two wheels are connected to the engine, but the back two wheels just freely rotate on their axles.

As you accelerate, what is the direction of the force of static friction of the road upon the **rear** wheels?

A. Forward

B. Backward

C. The static friction force on the rear wheels is zero

Rotational form of Newton's Second Law

- One or more objects exert forces on a rigid body with rotational inertia I that can rotate about some axis.
- The sum of the torques $\Sigma\tau$ (net torque) due to these forces about that axis causes the object to have a rotational acceleration α:

$$\alpha = \frac{1}{I}\Sigma\tau$$

Analogy chart

Linear	**Rotational Analogy**
x	θ
v_x	ω
a_x	α
Force: F_x	Torque: τ
Mass: m	Rotational Inertia: I

Newton's Second Law:

$$a_x = \frac{(F_{net})_x}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$

[Doc Cam Example]

The engine in a small airplane is specified to have a torque of 60.0 N m. This engine drives a propeller whose rotational inertia is 13.3 kg m². On start-up, how long does it take the propeller to reach 200 rpm?

Sketch and translate

$\tau_{engine} = +60\ \text{N}\cdot\text{m}$

$I = 13.3\ \text{kg}\cdot\text{m}^2$

$\omega_f = 200\ \frac{\text{revolutions}}{\text{minute}}$ find $\boxed{t}$

Simplify and diagram

Assume $\omega_0 = 0$.

τ_{engine} causes α.

Use Newton's 2nd Law for rotation:

$\boxed{\alpha = \frac{\tau}{I}}$ Kinematics.

Represent mathematically

$\omega_f = \omega_0 + \alpha t$

$\alpha t = \omega_f - \cancel{\omega_0}^{=0}$

$t = \frac{\omega_f}{\alpha} = \frac{\omega_f I}{\tau}$

Convert ω_f to SI:

$\omega_f = 200\ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)$

Solve and Evaluate

$\omega_f = 20.94\ \text{rad/s}$

$t = \frac{20.94(13.3)}{60} = 4.64\ \text{s}.$

Evaluate

Let's check units

$t = \frac{\omega_f I}{\tau}$ Units: $\frac{\left(\text{rad}/\cancel{\text{s}}\right)\text{kg}\cdot\text{m}^{\cancel{2}}}{\text{N}\cdot\cancel{\text{m}}}$

$t_{units} = \frac{\text{rad}\cdot\text{kg}\cdot\text{m}}{\text{s}\cdot\text{N}}$

$F = ma$

$N = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$

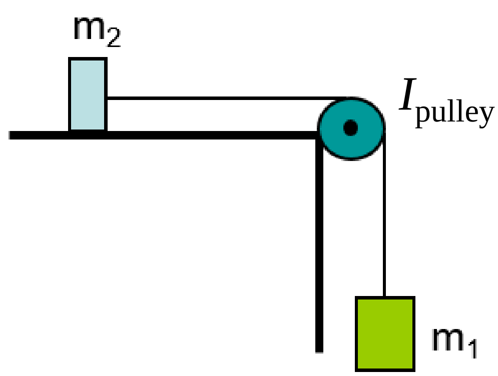
$t_{units} = \frac{\text{rad}\cdot\cancel{\text{kg}}\cdot\cancel{\text{m}}\,\text{s}^{\cancel{2}}}{\cancel{\text{s}}\cdot\cancel{\text{kg}}\cdot\cancel{\text{m}}}$

$t_{units} = \cancel{\text{rad}}\cdot\text{s}$

okay.

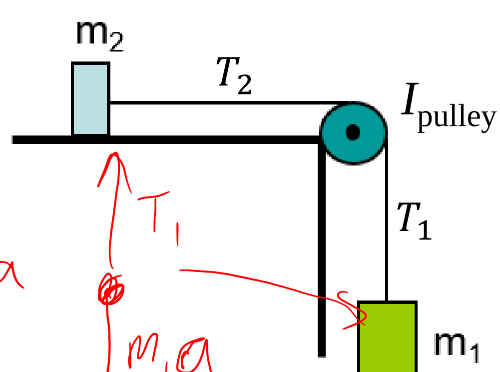
Massive Pulleys

- A real pulley has some rotational inertia, I_{pulley}.
- This means that it will require a net torque in order to cause it to have rotational acceleration.
- For an accelerating system, the tension must **change** as it goes over the pulley.



Learning Catalytics Question

- Mass m_2 is on a frictionless table. It is attached to a cord that wraps around a massive pulley, and is attached to a hanging mass m_1.
- After the system is released, how does the magnitude of the tension in the rope attached to m_1 compare to the magnitude of the force of gravity, $m_1 g$, on m_1?



A. $T_1 > m_1 g$

B. $T_1 < m_1 g$

C. $T_1 = m_1 g$

F_{net} ↓ a ↓

$m_1 g > T_1$

T_1 ↑

$m_1 g$ ↓

Learning Catalytics Question

$\tau_{net} = -RT_1 + RT_2$ ← negative torque,

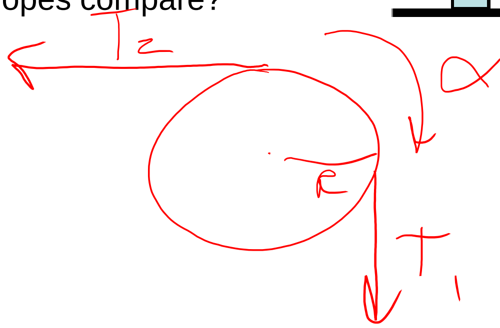
$T_1 > T_2$

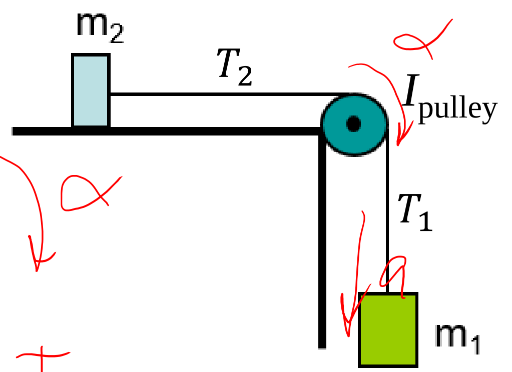
- Mass m_2 is on a frictionless table. It is attached to a cord that wraps around a massive pulley, and is attached to a hanging mass m_1.
- After the system is released, how do the tensions in the two ropes compare?

A. $T_1 > T_2$

B. $T_1 < T_2$

C. $T_1 = T_2$



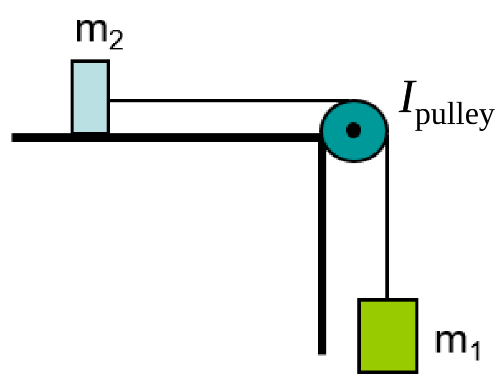


The Massive Pulley Constraints

When an object is attached to a pulley, the distance the object travels is equal to the change in angular position of the pulley times the radius of the pulley.

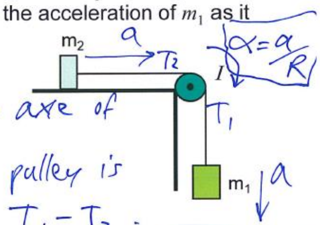
- Distance m_2 and m_1 move is: $s = \theta R$, where θ is the angle the pulley turns through.
- Speeds of m_2 and m_1 are related to the angular speed of the pulley by: $v = \omega R$
- Accelerations of m_2 and m_1 are related to the angular acceleration of the pulley by:

$$\boxed{a = \alpha R}$$ ← useful equation



[Doc Cam Example]

- Mass $m_2 = 2.5$ kg is on a frictionless table. It is attached to a cord that wraps around a massive pulley of radius $R = 0.20$ m, and rotational inertia $I = 0.050$ kg m^2. The same cord is attached to a hanging mass $m_1 = 0.75$ kg. After the system is released, what is the acceleration of m_1 as it falls?



Sketch and translate

- Assume no friction in axle of pulley
- Net torque on pulley is from $T_1 - T_2$.

Simplify and diagram

3 force diagrams

m_2. positive a — N, T_2, $m_2 g$

$\Sigma F = \boxed{T_2 = m_2 a}$

m_1 — T_1, pos. a, $m_1 g$

$\boxed{\Sigma F = m_1 g - T_1 = m_1 a}$

Pulley — T_2, R, R, T_1. Def. pos = clockwise

$$\Sigma \tau = T_1 R \sin 90^\circ - T_2 R \sin 90^\circ = I\alpha$$

3

Represent mathematically

$m_2 a = T_2$ (1)

$m_1 a = m_1 g - T_1$ (2)

$I\alpha = T_1 R - T_2 R$ (3)

$a = \alpha R$ (4) (Pulley constraint)

4 eqs, 4 unknowns: T_1, T_2, a, α

(1) ⇒ $T_2 = m_2 a$ — Solve for a.

(2) ⇒ $T_1 = m_1 g - m_1 a$ — plug in.

(3) & (4) ⇒ $\frac{Ia}{R} = R(T_1 - T_2)$

Solve and Evaluate

$$\frac{Ia}{R^2} = m_1 g - m_1 a - m_2 a$$

$$a\left[\frac{I}{R^2} + m_1 + m_2\right] = m_1 g$$

$$a = \frac{m_1 g}{I/R^2 + m_1 + m_2} = \frac{0.75(9.8)}{(0.05/0.2^2) + 0.75 + 2.5}$$

$\boxed{a = 1.6 \text{ m/s}^2}$ ← less than g.

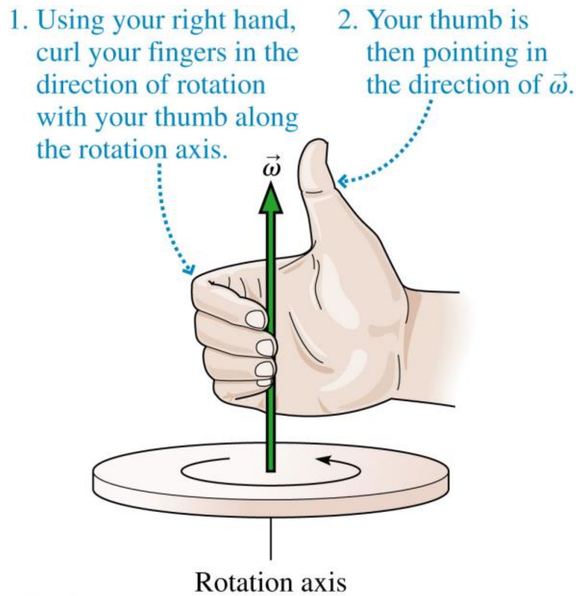
4

The Vector Description of Rotational Motion

- One-dimensional motion uses a scalar velocity v and force F.
- A more general understanding of motion requires **vectors** $\vec{v}$ and $\vec{F}$.
- Similarly, a more general description of rotational motion requires us to replace the scalars ω and τ with the vector quantities $\vec{\omega}$ and $\vec{\tau}$.
- Doing so will lead us to the concept of *angular momentum*.

“The Right Hand Rule”

- The magnitude of the angular velocity $\vec{\omega}$ vector is ω.
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated.



Rotational Momentum

- Rotational momentum

  = rotational inertia × rotational velocity

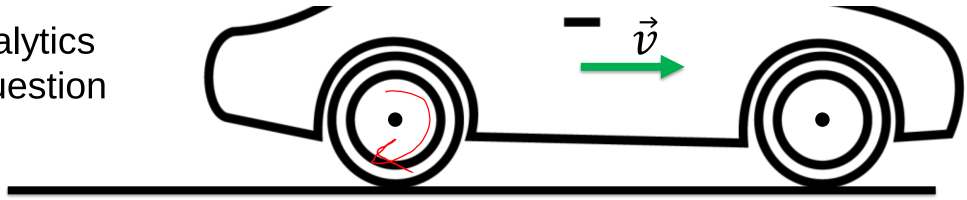
$$\vec{L} = I\vec{\omega}$$

  – This is analogous to

  Linear momentum = mass × velocity

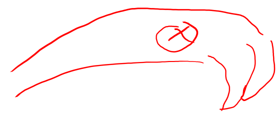
$$\vec{p} = m\vec{v}$$

Learning Catalytics
Discussion Question



You are driving your car towards the right.

What is the direction of the angular momentum, $\vec{L}$, of the wheels?



A. Left

B. Right

C. Up

D. Down

E. Into the page

F. Out of the page

Conservation of Angular Momentum

An isolated system that experiences no net torque has

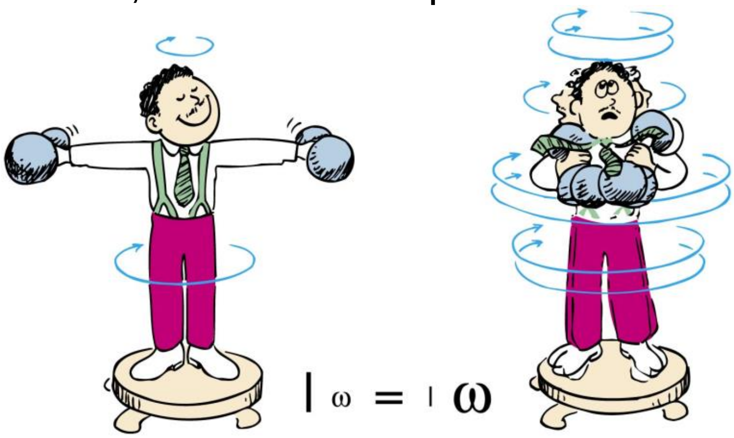
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}} = \vec{0}$$

and thus the angular momentum vector $\vec{L}$ is a constant.

Conservation of Angular Momentum

Example:

- When the professor pulls the weights inward, his rotational speed increases!



Nuclear Magnetic Resonance

- A proton in the nucleus of an atom is rotating like a little spinning top.
- When placed in a strong static magnetic field, the magnetic force produces a torque on the proton, which causes it to precess at a lower rate than the spin; similar to the hanging bicycle wheel demonstration.
- The precession frequency is in the radio-frequency range, which allows the proton to absorb and re-emit radio-waves.
- This allows doctors to image inside the human body using harmless radio waves.