PHY151 H1F : Midterm 2016.

Solutions by J. Harlow.

1. $a = 3t + (t+1)^{-3}$ $x(0) = 0$, $x(1) = 0.25$ m

Find $v(0)$.

Indefinite integral: $v = \int a\, dt + C_0$

where C_0 = an integration constant with units [m/s]

$$v = \int \left[3t + (t+1)^{-3} \right] dt + C_0$$

$$v = 3 \int t\, dt + \int (t+1)^{-3}\, dt + C_0$$

$$v = \frac{3t^2}{2} - \frac{(t+1)^{-2}}{2} + C_0 \qquad (*)$$

Indefinite integral: $x = \int v\, dt + C_1$

where C_1 = an integration constant with units [m]

$$x = \int \left[\frac{3t^2}{2} - \frac{(t+1)^{-2}}{2} + C_0 \right] dt + C_1$$

$$x = \frac{3}{2} \int t^2\, dt - \frac{1}{2} \int (t+1)^{-2}\, dt + \int C_0\, dt + C_1$$

$$x = \frac{1}{2} t^3 + \frac{(t+1)^{-1}}{2} + C_0 t + C_1$$

Next, let's use the conditions $x(0) = 0$ and $x(1) = 0.25$ m to solve for C_0 and C_1.

cont...

1.... continued

$$x(t=0) = 0 + \frac{1}{2} + 0 + C_1$$

$$\Rightarrow \underline{C_1 = -0.5 \ m}$$

$$x(t=1) = 0.25m = \frac{1}{2}(1)^3 + \frac{(2)^{-1}}{2} + C_0 + C_1$$

$$0.25 = 0.5 + 0.25 + C_0 - 0.5$$

$$\Rightarrow \underline{C_0 = 0 \ m/s.}$$

So, $(*)$ Becomes:

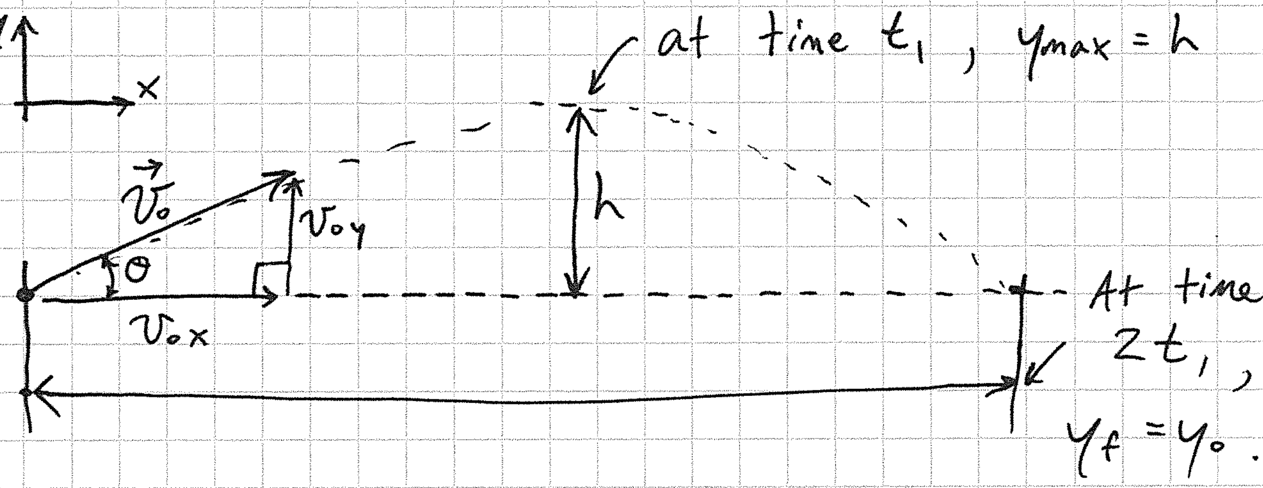
$$V = \frac{3t^2}{2} - \frac{(t+1)^{-2}}{2}$$

at $t=0$:

$$V_0 = 0 - \frac{(1)^{-2}}{2}$$

$$\boxed{V_0 = -0.5 \ m/s}$$

2. (a).



First half of trajectory $t=0$ to t_1:

Initial y-component of velocity $= v_{oy} = v_0 \sin\theta$

Final y-component of velocity $= v_{01} = 0$

Average y-component of velocity, assuming constant acceleration:

$$\bar{v}_y = \frac{v_{oy} + v_{01}}{2} = \frac{v_0 \sin\theta}{2}$$

$$\bar{v}_y = \frac{\Delta y}{\Delta t} = \frac{h}{t_1}$$

$$\frac{v_0 \sin\theta}{2} = \frac{h}{t_1}$$

$$t_1 = \frac{2h}{v_0 \sin\theta} \qquad \text{(Eq. 2.1)}$$

Whole trajectory $t=0$ to $2t_1$:

Constant x-component of velocity $= v_x = v_0 \cos\theta$

$$v_x = \frac{\Delta x}{\Delta t} = \frac{L}{2t_1} = v_0 \cos\theta$$

$$t_1 = \frac{L}{2 v_0 \cos\theta} \qquad \text{(Eq. 2.2).}$$

cont...

2(a).... continued.

Combine Eqs 2.1 & 2.2 :

$$t_1 = t_1$$

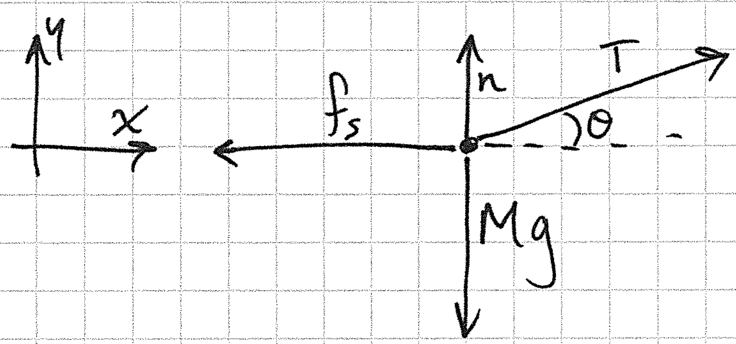
$$\frac{2h}{v_0 \sin\theta} = \frac{L}{2 v_0 \cos\theta}$$

$$\frac{h}{L} = \frac{\sin\theta}{4\cos\theta}$$

$$\boxed{\frac{h}{L} = \frac{1}{4}\tan\theta}$$

2 (b) Same. Note, $h/L = \frac{1}{4}\tan\theta$ depends on initial launch angle only. It does not depend on g. So if you change g (by going to the moon), h/L remains the same!

3 (a). Free-body diagram of box.



n = normal force

If $T = 0$, then the box does not move.

$\Rightarrow T_{min} = 0$.

T can be increased until $f_s = f_{s,max} = \mu_s n$

$(F_{Net})_y = 0$ in equilibrium (no movement),

$\Rightarrow n + T \sin\theta - Mg = 0$, solve for n.

$$n = Mg - T\sin\theta$$

$(F_{Net})_x = 0$ in equilibrium (no movement).

$$T\cos\theta - f_s = 0$$

$$\Rightarrow f_s = T\cos\theta$$

When $T = T_{max}$, $f_s = f_{s,max}$: $\mu_s\left[Mg - T_{max}\sin\theta\right] = T_{max}\cos\theta$

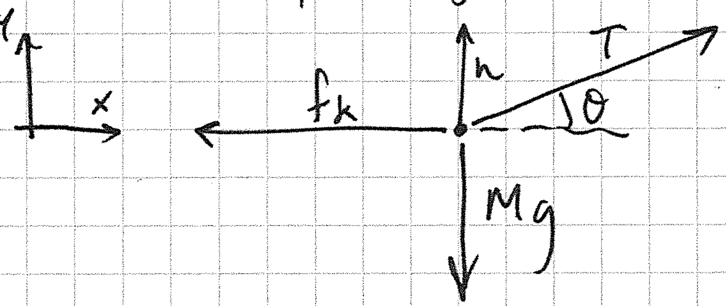
solve for T_{max}:

$$T_{max}(\mu_s \sin\theta + \cos\theta) = \mu_s Mg$$

$$T_{max} = \frac{\mu_s Mg}{\mu_s \sin\theta + \cos\theta}$$

Range: $\boxed{0 < T < \dfrac{\mu_s Mg}{\mu_s \sin\theta + \cos\theta}}$

3 (b). Free-body diagram of box:



$\vec{v}$

Constant speed $\Rightarrow$ Equilibrium. (zero acceleration)

Math is similar to Part (a).

$(F_{net})_y = 0 \Rightarrow \underline{n = Mg - T\sin\theta}$

$(F_{net})_x = 0 \Rightarrow f_k = T\cos\theta$. , $f_k = \mu_k n$

$$\mu_k\left(Mg - T\sin\theta\right) = T\cos\theta, \quad \text{solve for } T$$

$$\boxed{T = \frac{\mu_k Mg}{\mu_k \sin\theta + \cos\theta}}$$

$\leftarrow$ apply this tension to keep a steady speed.

4 (a). Define inertial frame S. Two events in this frame have coordinates (x_1, t_1), (x_2, t_2).

Define $\Delta x = x_2 - x_1$
$\Delta t = t_2 - t_1$

$$\Delta s^2 = \Delta x^2 - c^2 \Delta t^2 \qquad \text{(Eq. 4.1)}$$

Define inertial frame S', moving in the $+x$ direction at speed v, relative to S.

Lorentz Transform the two events.

$$x_1' = \gamma(x_1 - v t_1) \qquad x_2' = \gamma(x_2 - v t_2)$$

$$t_1' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right) \qquad t_2' = \gamma\left(t_2 - \frac{v x_2}{c^2}\right)$$

In Frame S':

$$(\Delta s^2)' = (x_2' - x_1')^2 - c^2(t_2' - t_1')^2$$

$$(\Delta s^2)' = \gamma^2\left[(x_2 - v t_2) - (x_1 - v t_1)\right]^2$$

$$- c^2\gamma^2\left[\left(t_2 - \frac{v x_2}{c^2}\right) - \left(t_1 - \frac{v x_1}{c^2}\right)\right]^2$$

$$(\Delta s^2)' = \gamma^2\left[x_2 - x_1 - v(t_2 - t_1)\right]^2 - c^2\gamma^2\left[t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1)\right]^2$$

$$(\Delta s^2)' = \gamma^2(\Delta x - v\Delta t)^2 - c^2\gamma^2\left(\Delta t - \frac{v}{c^2}\Delta x\right)^2$$

$$(\Delta s^2)' = \gamma^2(\Delta x^2 - 2v\Delta x\Delta t + v^2\Delta t^2) - c^2\gamma^2\left(\Delta t^2 - \frac{2v}{c^2}\Delta x\Delta t + \frac{v^2}{c^4}\Delta x^2\right)$$

.... cont

4 (a) ... continued:

$$(\Delta s^2)' = \gamma^2 \left[\Delta x^2 - \cancel{2v\Delta x \Delta t} + v^2 \Delta t^2 - c^2 \Delta t^2 + \cancel{2v\Delta x \Delta t} - \frac{v^2}{c^2} \Delta x^2 \right]$$

$$(\Delta s^2)' = \left(\frac{1}{\sqrt{1 - v^2/c^2}} \right)^2 \left[\left(1 - \frac{v^2}{c^2}\right) \Delta x^2 + (v^2 - c^2) \Delta t^2 \right]$$

$$(\Delta s^2)' = \frac{1}{(1 - v^2/c^2)} \left[\left(1 - v^2/c^2\right) \Delta x^2 - \left(1 - v^2/c^2\right) c^2 \Delta t^2 \right]$$

$$(\Delta s^2)' = \Delta x^2 - c^2 \Delta t^2$$

Compare with Eq. 4.1:

$$(\Delta s^2)' = \Delta s^2$$ $\Leftarrow$ same in both frames.

4 (b). Define present day Mount Paektu to be the origin $(x, t) = (0, 0)$.

Define "Event 1" to be the Crab Nebula Supernova Explosion.

Let's use units of ly (light years) for distance, and units of yr (years) for time.

In these units, $c = 1$.

$$(x_1, t_1) = (5000, -6000).$$

Define "Event 2" to be the volcanic eruption at Mt. Paektu.

$$(x_2, t_2) = (0, -1070)$$

We wish to find the speed v that a moving frame S' travels such that

$$t_1' = t_2'$$

cont...

4 (b) continued. Lorentz Transform the time coordinates:

$$t_1' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right) = \frac{t_1 - v x_1}{\sqrt{1-v^2}} \quad \text{(since } c=1\text{)}$$

Similarly: $$t_2' = \frac{t_2 - v x_2}{\sqrt{1-v^2}}$$

For the events to be simultaneous in this frame:

$$t_1' = t_2'$$

$$\frac{t_1 - v x_1}{\sqrt{1-v^2}} = \frac{t_2 - v x_2}{\sqrt{1-v^2}}$$

$$t_1 - v x_1 = t_2 - v x_2$$

$$v(x_2 - x_1) = t_2 - t_1$$

$$v = \frac{t_2 - t_1}{x_2 - x_1}$$ ← [Note: this equation looks really weird! But it's okay because $c=1$ ...]

$$v = \frac{-1070 - (-6000)}{0 - 5000} = \frac{-4930}{5000} = -0.986$$

So the speed is 0.986 light years per year:

$$\boxed{v = 0.986\, c}, \text{ or } v = 2.96 \times 10^8 \, m/s$$

PHY 151 H1F Midterm Oct. 26, 2016

Uncertainty & Python Solution.

5. (a) $\bar{x}A$: ~20 : $\boxed{A > C > D > B}$

$\bar{x}B$: ~6

$\bar{x}C$: . 15

$\bar{x}D$: 10

(b) By inspection $\boxed{C > D > B > A}$

(c) Recall from UM Module 2:

"The number of significant figures in an experimental uncertainty is 1 or 2, but never greater than this.".

So : $\boxed{31.4 \pm 0.2}$ either answer is correct.

or : 31.38 ± 0.19

(d) 1. Set acceleration due to gravity to be $-9.8\ m/s^2$

2. Set time-step of numerical integration to be 0.1 seconds.

3. Set the initial time to be 0 s.

4. Set the initial height of the ball to be 3 m

5. Set the initial y-component of the velocity of the ball to be 0 m/s.

6. Start a loop in which $y \geq 0$.

7. Increment t by dt.

... cont

5. (d) continued (11)

8. Add $(v_y \cdot dt)$ to the position.

9. Add $(a_y \cdot dt)$ to the y-comp. of the velocity.

10. Print t, y to the screen.

(e) (0.1, 3.0)
(0.2, 2.902)
(0.3, 2.706)