Practice Problem Set 10

Problem 1

A figure skater is rotating at an angular speed of 12.56 rad/s with their arms outstretched. The figure skater then pulls their arms in to their body, changing their moment of inertia from 2.7 kg·m to 1.2 kg·m.

a) Assuming angular momentum is conserved, what is their new angular speed?

Solution: Conservation of angular momentum:

$$L_i = L_f \tag{1}$$
$$I_i\omega_i = I_f\omega_f \tag{2}$$

Therefore:

$$\omega_f = \frac{I_i\omega_i}{I_f} \tag{3}$$
$$= \frac{(12.56\text{rad/s})(2.7\text{kg}\cdot\text{m})}{1.2\text{kg}\cdot\text{m}} \tag{4}$$
$$= 28.3\text{rad/s} \tag{5}$$
$$\doteq 9\pi\text{rad/s} \tag{6}$$

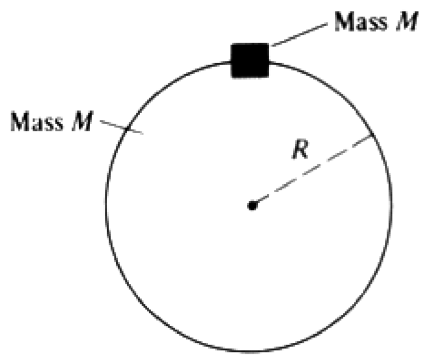
b) If they spin at this new speed for 2s, how many rotations have they completed?

Solution:

$$\theta_f = \omega_f t \tag{7}$$
$$= (9\pi\text{rad/s})(2\text{s}) \tag{8}$$
$$= 18\pi\text{rad} \tag{9}$$

which is equal to full rotations.

Problem 2



A uniform disc of radius R and mass M is free to rotate about its centre-axis. The axis is fixed and horizontal, and the disc rotates in the vertical plane. A small body of mass M is attached to the rim at the highest point above the pivot, and the system is released from rest. Since the system is in an unstable equilibrium, it will soon start to rotate due to some small disturbance. What is the angular velocity of the system when the body reaches the lowest point, directly below the pivot? (HINT: the moment of inertia of a uniform disc about its axis is $I = \frac{1}{2}MR^2$)

Solution: Use conservation of energy:

$$\begin{aligned}
U_i &= U_{i_{mass}} + U_{i_{mass}} && (10)\\
&= 2RMg + RMg && (11)\\
&= 3RMg && (12)\\
U_f &= U_{f_{mass}} + U_{f_{mass}} && (13)\\
&= 0 + RMg && (14)\\
&= RMg && (15)\\
K_i &= 0 && (16)\\
K_f &= \frac{1}{2}I\omega^2 && (17)
\end{aligned}$$

Using the parallel axis theorem:

$$I = I_{disc} + I_{mass} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2 \tag{18}$$

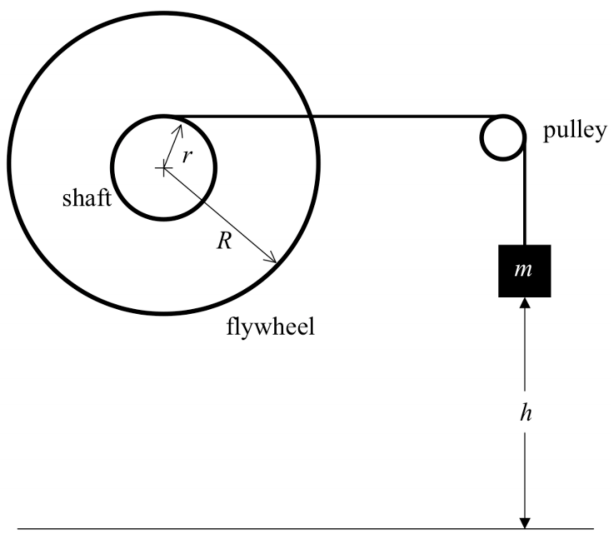
Then,

$$
\begin{aligned}
E_i &= E_f && (19)\\
U_i &= U_f + K_f && (20)\\
3RMg &= \frac{1}{2}\frac{3}{2}MR^2\omega^2 + RMg && (21)
\end{aligned}
$$

Solve for ω:

$$\omega = \sqrt{\frac{8}{3}\frac{g}{R}} \tag{22}$$

Problem 3



A solid flywheel of radius R and mass M is mounted on a light shaft of radius r so that the axis of rotation is horizontal. A light, inelastic rope is wound around the drive shaft and is connected, via a light, frictionless pulley to a mass m, which is suspended a height h above the floor.

a) What is the relationship between torque τ, angular acceleration α and moment of inertia I?

Solution: $\tau = I\alpha$

b) If the mass is released from rest and allowed to fall to the floor, find an expression for the final angular velocity of the flywheel.

Solution: Conservation of energy:

$$\begin{aligned}
E_i &= E_f && (23)\\
U_{mass} &= K_{mass} + K_{wheel} && (24)\\
mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 && (25)\\
&= \frac{1}{2}m(r\omega)^2 + \frac{1}{2}I\omega^2 && (26)
\end{aligned}$$

where $I = \frac{1}{2}MR^2$ for a circular disk. Now solve for ω:

$$\begin{aligned}
\omega &= \sqrt{\frac{2mgh}{mr^2 + I}} && (27)\\
&= \sqrt{\frac{2mgh}{mr^2 + \frac{1}{2}MR^2}} && (28)
\end{aligned}$$

c) How long does it take for the mass to reach the floor?

Solution: Need to find acceleration of the mass. The net force on the mass is given by:

$$ma = -mg + T \qquad (29)$$

For the flywheel:

$$\begin{aligned}
I\alpha &= \tau && (30)\\
&= -rT && (31)\\
I\frac{a}{R} &= -rT && (32)
\end{aligned}$$

Solve for T then substitute into (29):

$$ma = -mg - \frac{I}{r^2}a \qquad (33)$$

Solve for a:

$$a = \frac{-mg}{m + \frac{I}{r^2}} \tag{34}$$

Since acceleration is constant:

$$h = \frac{1}{2}at^2 \tag{35}$$

Solve (35) for a and substitute into (34) to solve for t:

$$t = \sqrt{\frac{2h\left(m + \frac{I}{r^2}\right)}{mg}} \tag{36}$$