Practice Problem Set 8 Solutions

Problem 1

Two objects are moving towards each other with the same speed v. They eventually reach each other and experience a completely **in**elastic collision, losing half of the initial kinetic energy in the process. What is the ratio of their masses? (see Wolfson 9.62)

Solution

The overall goal is to use (1) momentum, and (2) energy, to find relationships between the initial and final velocities. This will then give information about the masses and we will find that their ratio has a numerical value independent of velocity.

Linear momentum is conserved in this system so we have in general

$$\begin{aligned} \boldsymbol{P}_i &= \boldsymbol{P}_f \\ m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 &= (m_1+m_2)\,\boldsymbol{v}_f \qquad \text{but } \boldsymbol{v}_1 = -\boldsymbol{v}_2 = \boldsymbol{v}_i \\ m_1\boldsymbol{v}_i - m_2\boldsymbol{v}_i &= (m_1+m_2)\,\boldsymbol{v}_f \\ (m_1-m_2)\,\boldsymbol{v}_i &= (m_1+m_2)\,\boldsymbol{v}_f \end{aligned}$$

Since this system is in one dimension we can drop the vector notation and just focus on the speeds. So far we have from momentum conservation

$$v_f = \frac{m_1 - m_2}{m_1 + m_2} v_i \tag{1}$$

Energy is generally not conserved during inelastic collisions – and this is no exception – however we are told specifically how much is lost in the process (i.e. one half). So

$$\begin{aligned} E_f &= \frac{1}{2} E_i \\ \frac{1}{2}(m_1+m_2)\,v_f^2 &= \frac{1}{2}\left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right) \qquad \text{but } v_1 = v_2 = v_i \\ (m_1+m_2)\,v_f^2 &= \frac{1}{2}m_1 v_i^2 + \frac{1}{2}m_2 v_i^2 \\ (m_1+m_2)\,v_f^2 &= \frac{1}{2}(m_1+m_2)\,v_i^2 \\ v_f^2 &= \frac{1}{2}v_i^2 \\ v_f &= \pm\frac{1}{\sqrt{2}}v_i \end{aligned} \tag{2}$$

Choosing either root (they correspond to which mass is being divided by which in the ratio) and equating the two relationships between velocities gives

$$\begin{aligned}
v_f &= v_f \\
\frac{m_1 - m_2}{m_1 + m_2} v_i &= \frac{1}{\sqrt{2}} v_i \qquad \text{now cancel } v_i \\
\sqrt{2}\left(m_1 - m_2\right) &= m_1 + m_2 \\
\sqrt{2} m_1 - m_1 &= \sqrt{2} m_2 + m_2 \\
m_1 \left(\sqrt{2} - 1\right) &= m_2 \left(\sqrt{2} + 1\right) \\
\frac{m_1}{m_2} &= \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \approx 5.83 \qquad \text{OR the inverse: } \approx 0.172
\end{aligned}$$

Problem 2

A 400-mg popcorn kernel is skittering across a non-stick frying pan at 8.2 cm/s. It heats up to the point where it bursts, and splits into two pieces of popcorn with equal mass. If one of the two pieces happens to remain at rest immediately after the burst, how much energy was released during the process? (see Wolfson 9.66)

Solution

First thing to note is that since one piece is at rest after the burst, the other particle is forced to travel in the same direction as the initial kernel – and hence the problem is entirely one-dimensional. Additionally, since the pieces are of equal mass the initial kernel must have mass $2m$. Like the previous problem, we similarly use momentum and energy to find relationships. Defining the initial speed as v_i and arbitrarily letting piece 1 be the one that is at rest, we have the conservation of linear momentum:

$$\begin{aligned}
P_i &= P_f \\
(2m) v_i &= \cancelto{0}{m v_1} + m v_2 \\
2 v_i &= v_2
\end{aligned}$$

Now we look at the initial and final energy:

Initial Energy

$$\begin{aligned}
K_i &= \frac{1}{2}(2m) v_i^2 \\
&= m v_i^2
\end{aligned}$$

Final Energy

$$\begin{aligned}
K_f &= K_1 + K_2 \\
&= \cancelto{0}{\frac{1}{2} m v_1^2} + \frac{1}{2} m v_2^2 \\
&= \frac{1}{2} m \left(2 v_i\right)^2 \\
&= 2 m v_i^2
\end{aligned}$$

where in the last line we substituted $v_2 = 2v_i$. So the difference in energy is (with $m = 0.0002$ kg)

$$\begin{aligned}
\Delta K &= K_f - K_i \\
&= 2 m v_i^2 - m v_i^2 \\
&= m v_i^2 \\
&= (0.0002 \text{ kg})(0.082 \text{ m/s})^2 \\
&\approx 1.3 \times 10^{-6} \text{ J} \\
&= 1.3\ \mu\text{J}
\end{aligned}$$

Problem 3

An 80-kg astronaut working on the International Space Station is going on a routine space walk. While working 200 m away from the entry point – and at rest relative to it – her safety line becomes detached and her oxygen tank indicates she has 4 minutes of usable air remaining. To get herself back to the entry point, she throws her 10-kg tool kit exactly away from the entry point which then travels at 8 m/s relative to herself. Will she make it back to the station in time? (see Wolfson 9.89)

Solution

The conservation of linear momentum gives

$$\begin{aligned} P_i &= P_f \\ 0 &= m_t v_t + m_a v_a \end{aligned}$$

However these velocities are with respect to the station (i.e. the rest frame). We only know how fast the toolbox is travelling with respect to the astronaut, v_{ta}. So using the relative velocity formula

$$v_{ta} = v_t - v_a \qquad \Rightarrow \qquad v_t = v_{ta} + v_a$$

and plug back into the momentum relation:

$$\begin{aligned} m_t (v_{ta} + v_a) + m_a v_a &= 0 \\ (m_t + m_a) v_a &= -m_t v_{at} \\ v_a &= -\frac{m_t}{m_t + m_a} v_{at} \end{aligned}$$

Now if we define the positive direction to be pointing from the astronaut to the entry point, then $v_{at} = -8$ m/s and the speed of the astronaut relative to the station is

$$v_a = -\frac{m_t}{m_t + m_a} v_{at} = -\frac{10 \text{ kg}}{90 \text{ kg}} (-8 \text{ m/s}) = \frac{8}{9} \text{ m/s} \approx 0.89 \text{ m/s}$$

the time T it takes for her to reach the station is then

$$T = \frac{d}{v_a} \approx \frac{200 \text{ m}}{0.89 \text{ m/s}} \approx 225 \text{ s} = 3.75 \text{ min}$$

So yes, the astronaut makes it back with $0.25 \cdot 60 = 15$ seconds to spare.