PHY151 PRACTISE PROBLEMs FOR WEEK 5 PRACTICALS

Q1: Consider the situation that a box of $10kg$ mass needs to be dragged across a distance of $1000m$ to reach its receiver, whom demands to receive the box within $100s$. Now, assume the ideal case that the surface between the ground and the box is frictionless. What is the minimum constant force needed to be applied to the box, so the time condition would be satisfied? The box start at rest.

Q1 solution: Without friction the only force on the box would be the applied force F_a. Then the acceleration would be given by Newton's 2nd law as $F_{net} = F_a = ma$. Then since there is no initial velocity, the displacement is simply $d(t) = \frac{1}{2}at^2$. The condition would require $d(t = 100s) = 1000m$, so plug in the numbers we would have $1000m = \frac{1}{2}a \times 100s^2 \Rightarrow a = 1/5ms^{-2} \Rightarrow ma = F_{net} = F_a = 2N$. So a $2N$ constant force is enough. (This is quite a small force.)

Q2: Now get closer to the real world by considering the coefficient of friction on the surface between the box and the ground is $\mu = 0.5$ instead of 0. What is the minimum constant force needed to satisfy the time condition in this situation? Here assume the gravitational acceleration $g = 10$ from here till the end of this practise problem set for the sake of simplicity.

Q2 solution: In this situation the only change is the additional friction force that acts against the applied force. Then the net force would be $F_{net} = F_a + f$, where the applied force F_a and the friction force f acts in opposite direction (and therefore are in opposite signs). The constant acceleration required to satisfied the time condition would be the same, so we would still have $F_{net} = 2N$; and with $\mu = 0.5$ we would have $f = \mu F_n$ with $F_n = mg$ the normal force. With $\mu = 0.5$, $m = 10$, and $g = 10$ we would have $f = 50N$ and therefore $F_a = F_{net} + f = 52N$ is the applied force required. (We should see that here the majority of force is spend against friction.)

Q3: Now the receiver demands an additional small box of cargo, which has mass $1kg$, to be placed on top of the large box and delivered together. Knowing the coefficient of friction between the large box and the small box is also 0.5, What is the maximum constant acceleration the combination of boxes could achieve before the small box begin to slip? How long it takes to deliver the boxes under this acceleration? Is the time requirement satisfied?

Q3 solution: Here we denote physical quantities associated with the large box with subscript B and those associated with the small box with subscript b. The acceleration of the small box is caused by the friction force f_b between the two boxes. The no-slip condition insists that this friction force f_b is no more than the maximum static friction, which equals $f_{b(max)} = \mu_b m_b g = 5N$. With this force the acceleration of the small box would be $a_b = F_b/m_b = 5ms^{-2}$, which is equal to the acceleration of the large box by the no-slip condition. Then the time required to travel the entire distance would be $1000 = \frac{1}{2} \times 5 \times t^2 \Rightarrow t = 20s$, which satisfies the time requirement. (The net force on the two-box system would be 110N, which is not required to be calculated in this question, but would be useful later.)

Q4: A physicist, who favours the ideal world over the real world, has come to modify the situation to be more ideal by making the surface between the two boxes frictionless. What is the maximum constant acceleration possible for the two boxes now, before the small box begin to slip?

Q4 solution: This is a conceptual question. When $\mu_b = 0$ between the two boxes there will be no friction force and the small box will slip whenever the large box moves. Therefore $a_{max} = 0$ and we cannot carry the small box on top of the large one now. (Here we should be reminded that we cannot move ourselves if there is no friction between us and the ground.)

Q5: To complete the delivery, now the small box is attached at the back of the large box by a spring with negligible mass and spring constant $k = 100N/m$. The coefficient of friction between the small box and the ground is $\mu_b = 0.5$. Again, the delivery needs to be completed within $100s$. What is the force needed to drag the large box? How long will the spring between the boxes stretch during the movement?

Q5 solution: Here the time condition would give the same net acceleration as the one in Q1, which is $a = 1/5$. However, with two boxes on the ground the equation for the net force becomes $F_{net} = m_{net}a + f_{net} = (m_B + m_b)a + f_B + f_b = (m_B + m_b)a + \mu(m_B + m_b)g$. Plug in numbers would give $f_{net} = 55$ and $m_{net}a = 2.2$ which means $F_{net} = 57.2N$. (Note here that this force is different from the one in Q3, due to different constraining conditions.) The force on the spring would be the net force on the smaller box, which is $F_{netb} = m_b a + f_b = 5.2N$, and with $F = kx$ the spring would stretch $0.052m$, or $5.2cm$, during movement.

Q6: The physicist is back with another upgrade: a handle have been installed on the large box with an angle of 45° above the ground. Now the applied pulling force would be along the handle's direction instead of horizontal. What is the force needed now to deliver the boxes on time? Is the force needed smaller now?

Q6 solution: Here we denote the applied pulling force by directions, as F_{ax} and F_{ay}. The handle's angle at 45° means the magnitude of F_{ax} and F_{ay} are equal. Here $F_{netx} = F_{ax} + f_B + f_b$ would provide the net acceleration, while F_{ay} would act against the gravitational force on the large box F_{gB} to reduce friction of the large box f_B. The new friction force on the large box would be $f_B = \mu(m_B g - F_{ay})$. f_b would stay the same as $5N$, since no changes is made on the small box. The time condition is also not changed, so the acceleration would remain as $a = 1/5ms^{-2}$, which gives $F_{netx} = 2.2N$. Substituting f_B into the equation of F_{netx} would give $F_{netx} = F_{ax} + \mu(m_B g - F_{ay}) + f_b$, and with $|F_{ax}| = |F_{ay}|$ we would have $F_{netx} = F_{ax} + \mu(m_B g - F_{ax}) + f_b$ in numbers, where again the applied force and the friction forces would act in opposite directions. Substituting in numbers would give $F_{ax} \approx 38.133N$ and $F_a = F_{ax}/\sin(45°) \approx 53.929N$. So the force do get smaller.