Practice Problem Set 8

March 12, 2017

Problem 1

A 250g mass is mounted on a spring with a spring constant of $k = 3.3$ N/m. The damping constant for this system is $b = 8.4 \text{ x } 10^{-3}$kg/s. Is the system underdamped, critically damped, or overdamped? How many oscillations will the system undergo during the time it takes the amplitude to decay to $1/e$ of its original value?

Solution

Compare b with $2m\omega_0$

$$2m\omega_0 = 2m\sqrt{\frac{k}{m}} = 1.8\text{kg/s} \tag{1}$$

With $b = 8.4 \text{ x } 10^{-3}$kg/s, $b \ll 2m\omega_0$ so the motion is underdamped.
The system with oscillate with:

$$x(t) = A\exp\left(-\frac{b}{2m}t\right)\cos(\omega t + \delta) \tag{2}$$

$$\omega = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}} \tag{3}$$

For oscillations to decay to $1/e$ of their original value we have

$$A\exp\left(-\frac{b}{2m}t\right) = A\exp\left(-1\right) \tag{4}$$

$$t = \frac{2m}{b} = 59.5\text{s} \tag{5}$$

The periods of the oscillations is

$$T = \frac{2\pi}{\omega} = 1.7\text{s} \tag{6}$$

Thus, the number of oscillations is 34

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Problem 2

A block with mass M rests a frictionless surface inclined 30° and is connected to a horizontal spring of force constant k. The other end of the spring is attached to a wall. A second block with mass m rests on top of the first block. The coefficient of static friction between the blocks is μ_s. Find the maximum amplitude of the oscillation such that the top block will not slip on the bottom block.

Solution

The force on the small box is

$$\begin{aligned} F_{net} &= F_f + F_g \\ &= mg\mu_s - mg\sin(\theta) \\ &= ma \end{aligned} \tag{7}$$

The equation for the movement of the spring is

$$x(t) = A\sin(\omega t) \tag{8}$$

$$\omega = \sqrt{\frac{k}{m+M}} \tag{9}$$

The acceleration is the second derivative of the position

$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t) \tag{10}$$

The max acceleration is

$$a_{max}(t) = -A\omega^2 \tag{11}$$

Plug this into the force equation

$$A_{max} = \frac{g(m+M)(\mu_s - \mathrm{Sin}(\theta))}{k} \tag{12}$$

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Problem 3

A steel beam of mass M and length L is suspended at its midpoint by a cable and executes torsional oscillations. If two masses m are now attached to either end of the beam and this reduces the frequency by 10% what is the ratio m/M?

Solution

The frequency is proportional to $I^{-1/2}$ where I is the moment of inertia.

$$\frac{\omega_f}{\omega_i} = \sqrt{\frac{I_i}{I_f}} \tag{13}$$

The moments of inertia for the beam and the system if beam plus balls are

$$I_{beam} = I_i = \frac{ML^2}{12} \tag{14}$$

$$I_{sys} = I_f = \frac{ML^2}{12} + \frac{mL^2}{2} \tag{15}$$

Plugging this into the ratio of frequencies we can solve for the ratio m/M

$$\frac{m}{M} = \frac{1}{6}\left(\frac{1}{0.9^2} - 1\right) = 0.039 \tag{16}$$

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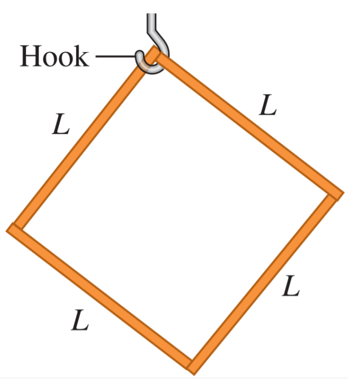
Problem 4

A square object of mass m is constructed of four identical uniform thin sticks, each of length L, attached together. This object is hung on a hook at its upper corner. If it is rotated slightly to the left and then released, at what frequency will it swing back and forth?



Solution

The frequency of a physical pendulum is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}} \tag{17}$$

where $d = L\cos(45°)$ is the distance from the pivot(hook) to the center of mass of the square loop, and I is the moment of inertia.

The moment of inertia is given by

$$\begin{aligned} I &= I_{cm} + md^2 \\ &= I_{cm} + \frac{mL^2}{2} \\ &= 4\left(\frac{1}{12}\left(\frac{m}{4}\right)L^2 + \frac{m}{4}\left(\frac{L}{2}\right)^2\right) + \frac{mL^2}{2} \\ &= \frac{5}{6}mL^2 \end{aligned} \tag{18}$$

Plugging this into the equation for the frquecny we get

$$f = \frac{1}{2\pi}\sqrt{\frac{6g}{5\sqrt{2}L}} \tag{19}$$

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