Practice Problem Set 1 - **SOLUTIONS**

PHY152H1S Winter 2016

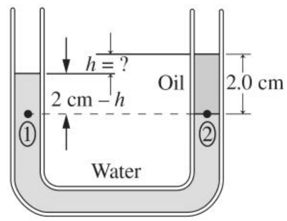
1. Wolfson 15.39

- **INTERPRET** This problem involves calculating the area needed for a given pressure to produce a given force. We are given the mass and the gauge pressure of the tires, and we want to find the total tire area that's in contact with the road.
- **DEVELOP** As shown in Equation 15.1, pressure measures the normal force per unit area exerted by a fluid. For this problem, the fluid is air. The force exerted on the road by the tires is the weight of the car, $F = mg$.
- **EVALUATE** With a gauge pressure of $p = 230$ kPa, the contact area is
- $$A = \frac{F_g}{p} = \frac{mg}{p} = \frac{(1950\text{ kg})(9.8\text{ m/s}^2)}{23\times10^4\text{ Pa}} = 0.0830\text{ m}^2 = 830\text{ cm}^2$$
- **ASSESS** Our result implies that the contact area of each wheel is about 200 cm^2, or the area of a 25 × 8 cm^2 rectangular surface, which seems reasonable.

2. Wolfson 15.29 (Also, what is the volume of the most massive block you can carry underwater?)

- **INTERPRET** This problem involves the buoyancy force, which will help us to carry a concrete block if it is submerged in water. We can use Archimedes's principle and Newton's second law to calculate the most massive concrete block we could lift underwater. We are given the mass of the largest block we can carry on land and the density of the concrete.
- **DEVELOP** Make a free-body diagram of the situation (see Figure 15.8). Applying Newton's second law to the concrete block gives
- $$\begin{aligned} \vec{F}_{\text{net}} &= m_{\text{w}}\vec{a} \\ F_{\text{b}} + F_{\text{app}} - m_{\text{w}}g &= 0 \\ F_{\text{app}} &= m_{\text{w}}g - F_{\text{b}} \end{aligned}$$
- where the subscript w indicates the mass we can carry under water. The maximum force we can apply is $F_{\text{app}} = m_{\text{L}}g$, where $m_{\text{L}} = 25$ kg is the maximum mass we can carry on land. The buoyancy force on a block is $F_{\text{b}} = \rho_{\text{w}}gV_{\text{c}}$, where $\rho_{\text{w}} = 1.0\times10^3$ kg/m^3 is the density of water and V_{c} is the volume of the concrete block, which is given by $V_{\text{c}} = m_{\text{w}}/\rho_{\text{c}}$ where $\rho_{\text{c}} = 2200$ kg/m^3 is the density of the concrete. We can solve this expression for the maximum mass m_{w} that we can carry in water.
- **EVALUATE** Inserting the given quantities in the expression for F_{app} and solving for m_{w} gives
- $$\begin{aligned} m_{\text{L}}g &= m_{\text{w}}g - \rho_{\text{w}}gV_{\text{c}} = m_{\text{w}}g - \rho_{\text{w}}g\left(\frac{m_{\text{w}}}{\rho_{\text{c}}}\right) = m_{\text{w}}g\left(1-\frac{\rho_{\text{w}}}{\rho_{\text{c}}}\right) \\ m_{\text{w}} &= m_{\text{L}}\left(\frac{\rho_{\text{c}}}{\rho_{\text{c}}-\rho_{\text{w}}}\right) = 46\text{ kg} \end{aligned}$$
- The volume is 46 / 2200 = 0.021 m^3. This is the volume of a cube of concrete that is **28 cm** on each side.
- **ASSESS** We can check this solution by looking at what happens if $\rho_{\text{c}} = \rho_{\text{w}}$. In this case, the "block" would have neutral buoyancy and we would be able to lift any size.

3. Wolfson 15.43

- **INTERPRET** The U tube contains two liquids, oil and water, in hydrostatic equilibrium. We want to find their height difference.
- **DEVELOP** The pressure at point 2 in the figure below, which is the oil-water interface, is
  - $$p_2 = p_{\text{atm}} + \rho_{\text{oil}} g l$$
- where $l = 2.0$ cm. The pressure at point 1, which is at the same height as point 2, is
  - $$p_1 = p_{\text{atm}} + \rho_{\text{w}} g (l - h)$$
- From Equation 15.3, $p = p_0 + \rho g h,$ we see that the pressure at points at the same height are the same, so $p_1 = p_2$. Using the information that $\rho_{\text{oil}} = 0.82\rho_{\text{w}}$ allows us to solve for h.
  - 

- **EVALUATE** Equating the two pressures leads to $\rho_{\text{oil}} g l = \rho_{\text{w}} g (l - h)$ or
  - $$h = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{w}}}\right) l = \left(1 - \frac{0.82\rho_{\text{w}}}{\rho_{\text{w}}}\right) l = (1 - 0.82)(2.0\ \text{cm}) = 3.6\ \text{mm}$$
- **ASSESS** Note that the final answer does not depend on the atmospheric pressure, p_{atm}, because this pressure pushes down equally on both the oil and the water. The U tube can be used to measure the density of a fluid, if we know the height difference h and the density of the other fluid.