Practice Problems - Week 7

1 Dipole Beside A Wire

An infinitely long wire with current $I\hat{y}$ runs along the y-axis. A magnetic dipole of moment μ lies a distance x on the x-axis in the xy-plane. Which direction (in xyz) should the dipole be oriented to maximize the force it experiences from the wire? What is the magnitude and direction of this force?

1.1 answer

The dipole should be oriented in the positive or negative z direction. The force will be away from the wire ($\hat{x}$) in the former case and toward it ($-\hat{x}$) in the latter. In either case, the magnitude of the force will be $F = \frac{\mu\mu_0 I}{2\pi x^2}$.

2 Cyclotron Frequency

An electron is moving vertically and westward, 20^o west of vertical, at a speed of 10^4 m/s (ignore the effects of gravity and relativity). Upon entering a region with a constant eastward magnetic field of magnitude 600 mT, the electron begins to follow a corkscrew trajectory. The electron performs three full rotations before exiting the region of magnetic field. How wide is this region?

2.1 answer

The cyclotron period is $T = \frac{2\pi m}{qB}$. The component of the velocity moving in the westward direction is $v\sin(20^o)$. Therefore the distance travelled in three rotations is $d = 3Tv_{west} = 3\left(\frac{2\pi m}{qB}\right) v\sin(20^o)$. Putting the numbers in gives 611 nm.

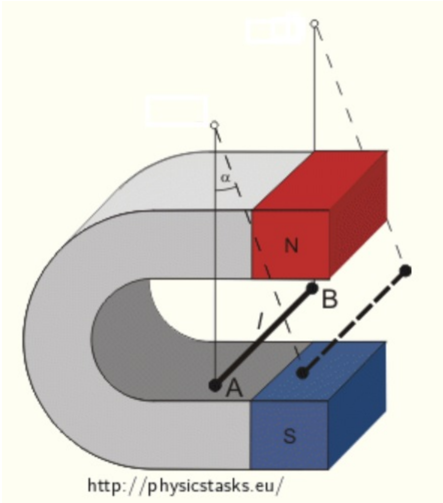
3 Biot-Savart Law

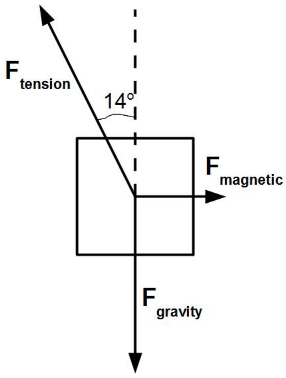
A wire having a length of 10 cm and a weight of 20 g is part of a circuit, hanging on two thin conductors of negligible mass. If the wire carries a current of 0.4 A it deflects from the vertical in homogeneous magnetic field by an angle of 35^o. Derive the expression for the magnetic field.

3.1 answer

The magnetic field runs from north to south and the current goes either into or out of the page; in any case they are 90^o to each other. If the current is into the page the resulting force is to the left by the right hand rule, and to the right if the current comes out of the page. Based on the deflection of the wire we conclude the current flows from B to A, out of the page.
In the y direction we have $F_T\cos\alpha = F_g = mg$. In the x direction we have $F_T\sin\alpha = F_m = I\,l\,B$.
Combining, we get $\tan\alpha = \frac{I\,l\,B}{m\,g}$, which can be rearranged to get

$$B = \frac{m\,g}{I\,l}\tan\alpha. \tag{3.1}$$





Substituting in the values from the question gives

$$B = \frac{(0.02kg)(9.8m/s^2)}{(0.4A)(0.1m)} \tan(35^o) = 3.43T. \tag{3.2}$$

The magnetic field is 3.43 T [downward].

4 Hall Effect

A rectangular aluminium strip measures 1.0 mm in the direction of a uniform 4.2 T magnetic field. When the strip carries a 8.5 A current perpendicular to the field, a 1.2 μV Hall potential develops across the strip. Find the number density of free electrons in the aluminium.

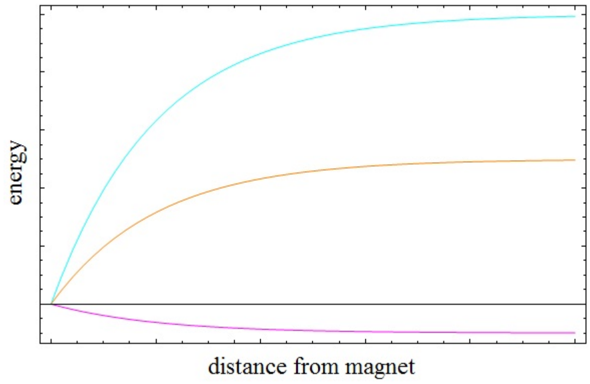
4.1 answer

Upon solving $F_{magnetic} = F_{electric}$ we arrive at the equation for the Hall potential, $V_H = \frac{IB}{nqt}$. Rearranging gives

$$n = \frac{IB}{V_H qt} = \frac{(8.5A)(4.2T}{(1.2 \times 10^{-6}T)(1.6 \times 10^{-19}C)(0.001m)} = 1.86 \times 10^{29}/m^3. \tag{4.1}$$

The number density of electrons in this aluminium is approximately 18.6 x $10^{28}/m^3$.

5 Types of Magnetism



You bring three objects near a strong magnet. The figure above shows a potential energy plot of the three objects as a function of distance from the magnet. Which of the three plots corresponds to an object that is ferromagnetic? Paramagnetic? Diamagnetic?

5.1 answer

These potentials are entirely made up. The only thing to note is that generally ferromagnetism is greater than paramagnetism and both are attractive. Diamagnetism is repulsive and weak. Therefore ferromagnetism is cyan, paramagnetism is orange, and diamagnetism is magenta.