PHY152 – Practice Problem Set 7 Solutions

1) Wolfson 13.55

This problem is about simple harmonic motion of a pendulum. We are asked to solve for the period, first by treating it as a simple pendulum and then as a physical pendulum.

Period for a simple pendulum is:

$$T = 2\pi \sqrt{\frac{mL^2}{mgL}} = 2\pi \sqrt{\frac{L}{g}}$$

$$T = 2\pi \sqrt{\frac{.8}{9.8}} = 1.795\ s$$

For the physical pendulum we will need to invoke Parallel-Axis Theorem to account for the rotational inertia of the ball/sphere (Table 10.2) about the pivot point:

$$I = I_{cm} + md^2 = \frac{2}{5}mR^2 + mL^2$$

Hence,

$$T = 2\pi \sqrt{\frac{\frac{2}{5}mR^2 + mL^2}{mgL}} = 2\pi \sqrt{\frac{\frac{2}{5}R^2 + L^2}{gL}}$$

$$T = 2\pi \sqrt{\frac{\frac{2}{5} * .075^2 + .8^2}{9.8 * .8}} = 1.798\ s$$

$$\%\ Difference = \left|\frac{1.798 - 1.795}{1.798}\right| 100\% = \mathbf{.17}\%$$

2) Wolfson 13.65

This problem involves determining # of oscillations for the amplitude to decay to 1/e of its initial value in a spring-damper system. The mass (0.25kg) is mounted on a spring (3.3 N/m) and the damping constant is b = 8.4 x 10^{-3} kg/s.

Examining the equation which models the damped oscillations:

$$x(t) = Ae^{-\frac{bt}{2m}}\cos(\omega t + \phi)$$

the term responsible for decay is

$$e^{-\frac{bt}{2m}}$$

Hence,

$$e^{-\frac{bt}{2m}} = e^{-1}$$

$$\frac{bt}{2m} = 1$$

$$t = \frac{2m}{b} = \frac{2(.25)}{8.4\ x\ 10^{-3}} = 59.5\ s$$

Angular frequency:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.3}{.25}} = 3.6\frac{rad}{s}$$

Period:

$$T = \frac{2\pi}{\omega} = 1.75\ s$$

Hence,

$$\#oscillations = \frac{t}{T} = \frac{59.5}{1.75} = \mathbf{34\ oscillations}$$

3) Wolfson 13.82

This problem involves torsional oscillation of a disk with two masses attached at the end. We are given that the disk has diameter 50cm (R=0.25m), mass of 340g (M=0.34kg) and is suspended by a wire with torsional constant (κ=5N m/rad). Once the birds land on opposite sides, frequency of oscillation is (f=2.6Hz).

For torsional oscillation:

$$\omega = \sqrt{\frac{\kappa}{I}}$$

Total rotational inertia will consist of:

$$I = I_{disk} + I_{birds}$$

$$I = \frac{1}{2}MR^2 + 2mR^2$$

So,

$$2\pi f = \sqrt{\frac{\kappa}{\frac{1}{2}MR^2 + 2mR^2}}$$

$$m = \frac{1}{2R^2}\left\{\kappa\left(\frac{1}{2\pi f}\right)^2 - \frac{1}{2}MR^2\right\}$$

$$m = \frac{1}{2\,(.25^2)}\left\{5\left(\frac{1}{2\pi(2.6)}\right)^2 - \frac{1}{2}.34(.25^2)\right\}$$

$$m = \mathbf{0.065\ kg}$$