Practice Problem Set 4

1. Wolfson 21.74

2. Flatland question

Coulomb's law is a consequence of Gauss' law, one of the four fundamental laws of electromagnetism. In this problem, we will use Gauss' law to think about electric fields in a hypothetical two-dimensional (2D) space.

(a) Does Coulomb's law take the same form in 2D space? Use Gauss' law to find the electric field due to a point charge q existing in 2D space.

(b) Use Gauss' law to find the electric field due to an infinite, thin rod with uniform linear charge density λ existing in 2D space.

(c) Repeat part (**b**), this time using your result from part (**a**) and integrating over the charged rod. You may find it useful to know that

$$\frac{\mathrm{d}}{\mathrm{d}u}\arctan u = \frac{1}{1+u^2}. \tag{1}$$

3. Gauss's Law For Gravitation Question

An analogy can be made between electric fields and graviational fields, which both obey r^{-2} laws:

$$\vec{E} = \frac{kq}{r^2}\hat{r}, \qquad \vec{g} = -\frac{GM}{r^2}\hat{r}. \tag{2}$$

In particular, just as we did for electric fields, we could explore gravitational field lines, gravitational flux through surfaces, and so on.

(a) Based solely on comparison between the two laws appearing in Eq. 2, deduce the gravitational analogue of Gauss' law. Remember that Gauss' law for electric fields is

$$\oint_S \vec{E}\cdot \mathrm{d}\vec{A} = \frac{1}{\epsilon_0} q_{\text{enclosed}}, \tag{3}$$

where S is a closed surface, and $k = 1/(4\pi\epsilon_0)$.

(b) Take the Earth to be a ball of uniform mass density ρ. What is the gravitational field as a function of radius within the ball?