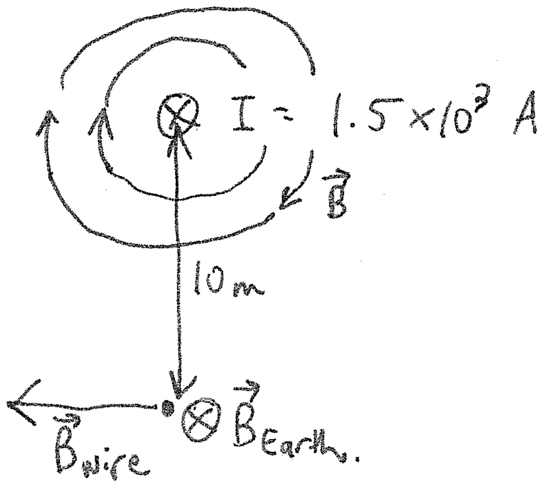
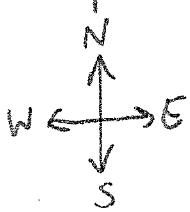
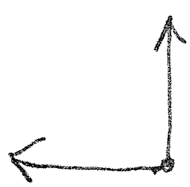
Practice Problem Set 6
Q.1 Wolfson 26.60

← West ⊗ into page = "magnetic North" ie toward the North Magnetic Pole.
→ East



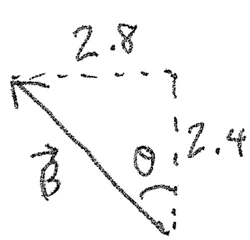
Top View:





$B_{Earth} = 0.24\ G = 2.4\times10^{-5}\ T$

$$B_{wire} = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times10^{-7}\,(1.5\times10^3)}{2\pi\cdot(10)}$$
$$= 2.8\times10^{-5}\ T$$



$$\theta = \tan^{-1}\left(\frac{2.8}{2.4}\right) = 49.399^\circ$$
$$B = \sqrt{2.4^2 + 2.8^2}\times10^{-5}$$
$$= 3.6878\times10^{-5}\ T$$

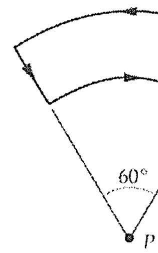
$\vec{B} = 3.7\times10^{-5}$ T, 49° West of North.

The compass will NOT help me find my way.

(2)

Practice Problem Set 6
Q. 2

2. The closed loop shown in the figure carries a current I in the counterclockwise direction. The radius of the inner arc is R_1 and that of the outer arc is R_2. Find the magnetic field at point P.



We need to use Biot-Savart law to find the magnetic field.

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

where $\hat{r}$ is the unit vector and r is the distance from an element $d\vec{l}$ on the wire to point P. For the arcs, the distance from the wire to the point is constant. $r = R_1$ for the inner arc and $r = R_2$ for the outer arc.

For the straight lines, the magnetic field at P is zero because $d\vec{l}$ and $\hat{r}$ are parallel. For the inner arc, $d\vec{l} \times \hat{r} = \hat{k} dl$, where $\hat{k}$ is into the page (assume the loop is in xy-plane). For the outer arc, $d\vec{l} \times \hat{r} = -\hat{k} dl$.

Now we can calculate the magnetic field of the inner arc by substituting $r = R_1$ and $d\vec{l} \times \hat{r} = \hat{k} dl$ into the equation of the Biot-Savart law.

$$\vec{B}_{inner} = \frac{\mu_0}{4\pi}\int \frac{I dl}{R_1^2}\hat{k} = \frac{\mu_0 I}{4\pi R_1^2}\hat{k}\int dl$$

$$\int dl = \int_0^{\frac{\pi}{3}} R_1 d\theta = \frac{\pi R_1}{3}$$

$$\vec{B}_{inner} = \frac{\mu_0 I}{4\pi R_1^2}\frac{\pi R_1}{3}\hat{k} = \frac{\mu_0 I}{12 R_1}\hat{k}$$

Similarly,

$$\vec{B}_{outer} = \frac{\mu_0}{4\pi}\int \frac{I dl}{R_2^2}(-\hat{k}) = -\frac{\mu_0 I}{4\pi R_2^2}\frac{\pi R_2}{3}\hat{k} = -\frac{\mu_0 I}{12 R_2}\hat{k}$$

$$\vec{B} = \vec{B}_{inner} + \vec{B}_{outer} = \frac{\mu_0 I}{12}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\hat{k}$$

(7)

Practice Problem Set 6

Q.3.

3. Consider a simple classical model for a hydrogen atom. Suppose an electron (of charge $-e$ and mass m_e) orbits the nucleus (of charge $+e$) and is confined purely by electrostatic forces to a circular orbit of radius R. What is the magnetic dipole moment of the electron?

The centripetal force is provided by the Coulomb force between the electron and the nucleus.

$$\frac{m_e v^2}{R} = \frac{ke^2}{R^2}$$

$$v = \sqrt{\frac{ke^2}{m_e R}}$$

A current of one ampere is one coulomb of charge going past a given point per second. The period of the electron is $\frac{2\pi R}{v}$, so the current is $\frac{e}{T} = \frac{ev}{2\pi R}$. The magnetic dipole moment is

$$\mu = IA = \frac{ev}{2\pi R}\pi R^2 = \frac{e}{2\pi R}\sqrt{\frac{ke^2}{m_e R}}\pi R^2 = \frac{e^2}{2}\sqrt{\frac{kR}{m_e}}$$

Because the eletron carries a negative charge, the direction of the electron is opposite to the direction in which the electron moves. Using the right hand rule, the direction of the magnetic dipole is opposite to the angular velocity of the electron.