Stretched-Pulse Additive Pulse Mode-Locking in Fiber Ring Lasers: Theory and Experiment

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Abstract— **Stretched-pulse additive pulse mode-locking uses segments of fiber of large positive and large negative group velocity dispersion (GVD) in the cavity. The changes in pulse width per pass due to the varying GVD can be an order of magnitude or more. A theory is developed based on the master equation that covers this case of large pulse changes in one transit. The general predictions of the theory are verified by experimental results.**

I. Introduction

PASSIVELY mode-locked erbium-doped fiber lasers will most likely fill a niche in the newly developing technology of 1.54 μm optical components. The erbium gain bandwidth has been demonstrated to support sub-100 fs pulses [1]–[3] with pulse energies of over 0.5 nJ [4]. The systems can be pumped with diode lasers at either 0.98 or 1.48 μm, giving them the potential to become compact, inexpensive, and robust sources of ultrashort pulses. Furthermore, the potential for low noise from diode pumping make fiber lasers especially attractive as sources for pump-probe or fiber squeezing experiments [5].

A variety of methods have been used to mode-lock erbium-doped fiber lasers. Several groups have employed semiconductor saturable absorbers in the cavity and demonstrated pulses as short as 200 fs in a polarization maintaining (PM) fiber system [6]. Another promising approach is based on additive pulse mode-locking (APM) [7]. Early work in APM fiber lasers was performed in neodymium-doped fiber systems at 1.06 μm. The systems demonstrated the use of nonlinear loop reflectors [8]–[14] and nonlinear polarization rotation [15]–[20] (polarization APM [14], [21]) as effective self-stabilized passive mode-locking methods for producing ultrashort pulses. These techniques were successfully extended to erbium-doped fiber lasers in standing-wave [1], [22]–[27], "figure eight" [28]–[30], and unidirectional ring cavity configurations [3], [4], [23], [31]–[38]. To date, the shortest pulses have been generated from APM systems [3].

Much of the work in APM erbium-doped fiber systems has concentrated on using solitons for ultrashort pulse generation [1], [2], [28]–[30], [34]–[37]. A serious difficulty in these lasers has been the large nonlinearity in cavities that are over several meters in length. A soliton becomes unstable as the peak phase shift in one transit approaches 2π. For a soliton with a "soliton period" of Z_0, this is equivalent to the distance $8Z_0$, since this is the distance over which a soliton accumulates a phase shift of 2π. The instability arises from the periodic perturbations that the pulse experiences as it circulates in the laser cavity. It is the same instability that has been well-known in soliton communications, where the data stream experiences periodic perturbations from periodic amplification [39]–[41].

The problem has been recently examined in the context of fiber lasers in order to explain the narrow side-bands which are observed in the spectra of fiber soliton lasers [42]–[47]. A soliton has a propagation constant k that is larger than the linear propagation constant at the soliton carrier frequency due to self-phase modulation, and strong phase matching occurs with dispersive waves at predictable resonant wavelengths. This eventually limits the pulse width. To avoid this limitation in a laser cavity of length L, solitons must be characterized by $8Z_0 \gg L$. Typically, the shortest pulses observed in actual systems satisfy $8Z_0 \geq 4L/8L$. This limit on duration in turn sets a limit on single pulse energy because the soliton energy $E_{\text{sol}} \propto 1/\tau$, where τ is the pulse width. Thus in fiber soliton lasers, low energy multiple pulses of relatively long duration (>300 fs) are most often observed [48] unless very short cavities [1], [24] or cavities with low average anomalous group velocity dispersion (GVD) are used [1], [2].

Previously, we reported a polarization APM erbium-doped soliton ring laser [36], which was shown to be fully self-starting due to the unidirectional operation [23]. That system produced stable 452 fs pulses at the fundamental period of 24 ns. The operation of the laser was explained using the master equation for APM of a soliton traveling in the ring [14], [49]. Recent experiments with an erbium-doped fiber ring laser with sections of large positive and large negative GVD have led to much shorter output pulses of ~75 fs duration [3], [4], [50]. A conceptual diagram is illustrated in Fig. 1. Due to the large GVD of the fibers, the pulse is temporally stretched and compressed in one transit. This serves to lower the average peak power compared to that which would be experienced by a transform-limited pulse of the same bandwidth. The output pulse is, in general, chirped. The GVD in the output coupling fiber can be adjusted to compensate the chirp, as illustrated in Fig. 2.

Fig. 3 shows the output spectra of the soliton ring system and a stretched-pulse ring system, which illustrates the large bandwidth increase obtained from the stretched-pulse technique. The soliton spectral width was 9 nm and the stretched-

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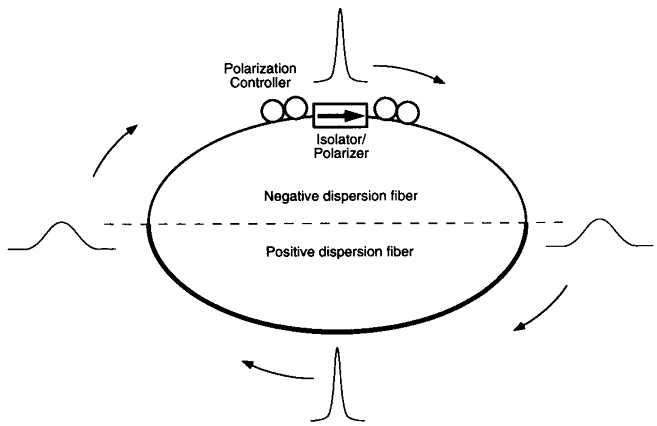

Fig. 1. Stretched-pulse additive pulse mode-locked fiber laser schematic.

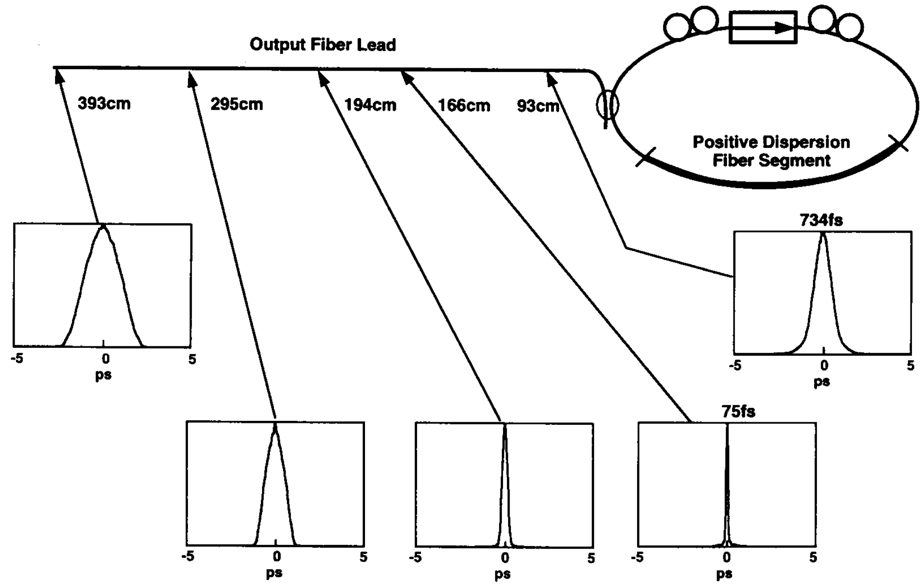

Fig. 2. Chirp compensation in a stretched-pulse additive pulse mode-locked fiber laser. Output coupling fiber lead is used as dispersive delay line.

pulse spectral width was 49 nm. The corresponding output pulse energies were 7 and 29 pJ, respectively. The lengths of the cavities for Fig. 3 are 4.8 and 4.2 m, respectively. The only significant difference between them are the GVD's of the erbium-doped fibers, which are approximately -0.014 ps^2/m in the first case and $+0.075$ ps^2/m in the second case. The total GVD's in the soliton system and the stretched-pulse system were estimated to be -0.096 ps^2 and $+0.004 \pm 0.006$ ps^2, respectively. Fig. 4 shows autocorrelation traces of the pulses obtained from the soliton system and a stretched-pulse system. The stretched pulses in this case were 90 fs in minimum duration assuming a Gaussian profile. The background created in the soliton system due to the periodic perturbations clearly shows as an exponentially decaying pedestal accompanying the main pulse. The beats result from the wavelength separation between the first and second-order sidebands on either side of the pulse spectrum. The background is greatly reduced in the stretched-pulse case because $k(\omega)$ is dynamic and thus phase-matching to dispersive waves is reduced [50]. Thus, stretched-pulse APM is effective in helping produce pulses with cleaner spectra than when operating in the soliton regime. Note that recently intracavity filtering was also demonstrated as an effective means of reducing sidebands in soliton fiber lasers with no expense in pulse duration [51].

From the pulse widths observed at the output of the stretched-pulse system, one may estimate the stretching factor, which we define as the ratio of maximum to minimum pulse width within the laser cavity. For highly stretched systems, we have estimated stretching factors >20, thus there are large changes in pulse width in one round-trip [52]. Krausz *et al.* [52] and Hofer *et al.* [20] have theoretically and experimentally investigated mode-locking when the pulse experiences large changes per pass. The different elements in the system act on the pulse via operators that do not commute. This means

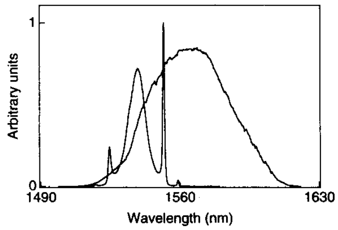

Fig. 3. Comparison of spectra from soliton fiber ring laser and stretched-pulse fiber ring laser. Both cavities were nearly identical except for the dispersion of the erbium-doped fiber. Soliton pulse width was 450 fs and stretched-pulse pulse width was 90 fs assuming sech.

that the ordering of the elements in the resonator affects the performance of the system. It is difficult to arrive at general criteria in this case, and analytic results cannot be obtained.

It is possible, however, to develop an analytic theory in the case of the stretched-pulse all-fiber ring, if one limits the investigation to a reasonable scenario. One acknowledges that the pulse changes greatly due to GVD in one round-trip. However, one still insists that the change of the pulse due to all other effects is small within one pass. This is a reasonable requirement because usually large nonlinear changes of the pulse per pass are to be avoided to obtain good pulse shapes, spectra and pulse stability. The stretching and compressing of the pulse is a linear, reversible process and thus is not harmful. To the contrary, it is useful since it reduces the self-phase modulation of the fiber ring. The net self-phase shift in one round-trip is the integral of the phase changes per unit length. This point of view is analogous to the one used in interpreting long distance pulse propagation through a cascade of amplifiers, with large changes of amplitude between amplifiers, as soliton propagation [39]–[41], [53]. As long as the soliton effects within one amplifier spacing are small, these effects can be averaged and the propagation can be considered as one along a uniform transmission medium.

In this paper, we develop an analytic theory for stretched-pulse APM where we use a chirped Gaussian pulse model. The Gaussian model is justified on theoretical grounds and the experimentally observed pulse-shapes are Gaussian over most of their temporal profile. Section I considers a model of self-phase modulation (SPM) when the pulse amplitude experiences large changes in one transit. This is used to derive parameters for the SPM and self-amplitude modulation (SAM) coefficients for the master equation in Section II. We give qualitative theoretical predictions for pulse energy, pulse width, and stability as a function of the net cavity GVD. Experimental results are given in Section III and compared to theory.

II. Operation of the Fiber Ring

Consider the unidirectional ring of Fig. 1, ignoring at first all nonlinear effects. The GVD's of the two segments, one of positive GVD, the other of negative GVD, are assumed to balance perfectly. If one assumes that a transform-limited Gaussian pulse exists at a given cross-section in the positive GVD segment as shown, then the pulse disperses as it travels away from the position of minimum width. It becomes transform-limited in the fiber segment of negative GVD at the symmetric location shown. The width averaged over one round-trip is a function of this location as well as the minimum pulse width. The average pulse width is minimized if the minimum width locations are in the plane of symmetry of the ring. For a given pulse energy, minimization of the average width maximizes the nonlinear effects of the pulse. If these nonlinear effects lead to APM action, the APM action is maximized. Because APM acts as an artificial saturable absorber, the loss is minimized. Hence, it is to be expected that the minimum-width location of the pulse will be in the symmetry plane, at least when the operation is near mode-locking threshold. This is the starting point of the analysis. We begin by assuming a transform-limited Gaussian pulse in the symmetry plane of the ring and introduce GVD imbalance and nonlinearity as perturbations.

A. Group Velocity Dispersion Imbalance

If the GVD of the two segments is not balanced and there are no other effects, the pulse cannot reproduce itself in one round-trip. It can reproduce itself however if other effects act upon it as well, as they do in an actual fiber laser. The GVD imbalance, if not too large, produces a change of the pulse that can be represented by the operator

$$jD\frac{d^2}{dt^2} = j\left\{\left(\frac{k''_{\mathrm{p}}L_{\mathrm{p}}}{2}\right) - \left|\frac{k''_{\mathrm{n}}L_{\mathrm{n}}}{2}\right|\right\}\frac{d^2}{dt^2} \tag{1}$$

where k''_i is the GVD, L_i is the fiber length, and the subscripts "p" and "n" denote positive and negative GVD, respectively. The imbalance of the ring, leading to a GVD parameter D, causes asymmetric behavior of the pulse in the two parts of the ring. This affects the SAM and SPM parameters. However, in the master equation formalism, which assumes small changes per pass from the reference plane back to the reference plane, the introduction of the different parameters can be viewed as a Taylor expansion to first order in all mode-locking parameters. Interactions between the different effects are of higher order and are ignored.

B. The Accumulated Nonlinear Phase

We shall treat the nonlinear effects as perturbations. The equation for the pulse propagation of elliptic polarization in terms of the circularly polarized basis E_+, E_- is:

$$\frac{\partial}{\partial z}E_\pm = j\frac{k''}{2}\frac{\partial^2}{\partial t^2}E_\pm - j\frac{4\pi}{3\lambda}\frac{n_2}{\mathcal{A}_{\mathrm{eff}}}\{|E_\pm|^2 + 2|E_\mp|^2\}E_\pm \tag{2}$$

where λ is the wavelength, $\mathcal{A}_{\mathrm{eff}}$ is the effective mode cross-sectional area, and n_2 is the Kerr coefficient. We have normalized the E-field so that its absolute square is power. Following the polarization transformer in Fig. 1, the polarization in the linear basis is,

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} re^{-j\phi} \\ j\sqrt{1-r^2} \end{bmatrix} A_o \tag{3}$$

where A_o is the field amplitude. It is more convenient to analyze polarization-APM in the circular basis because the

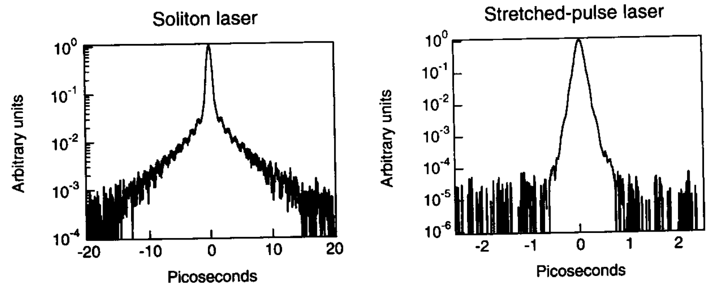

Fig. 4. Autocorrelations from soliton fiber ring laser and stretched- pulse fiber ring laser. The pulse width for the soliton system was 450 fs assuming sech and 90 fs for the stretched-pulse system, assuming Gaussian.

coherence term is not present (2). Equation (3) can be transformed to the circular basis E_+ and E_- using the unitary transformation U,

$$U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \tag{4}$$

and

$$\begin{bmatrix} E_+ \\ E_- \end{bmatrix} = U \begin{bmatrix} E_x \\ E_y \end{bmatrix}. \tag{5}$$

This gives for the initial amplitudes E_+^o and E_-^o,

$$E_\pm^o = \frac{1}{\sqrt{2}}(re^{-j\phi} \mp \sqrt{1-r^2})A_o. \tag{6}$$

To determine the nonlinear phase shift, we compute the accumulated phase shift across the pulse, integrated around the loop, assuming that the pulse shape is determined by the linear dispersive propagation:

$$E_\pm(z,t) \propto \frac{\tau_o}{\sqrt{\tau_o^2 + jk''z}} e^{-\frac{t^2}{2(\tau_o^2 + jk''z)}}. \tag{7}$$

The net phase shift is then:

$$\Phi_\pm(t) = |A_o|^2 \frac{2\pi}{\lambda}\frac{n_2}{A_{\text{eff}}}(1 \pm \eta) \oint dz \frac{1}{\sqrt{1+(z/b)^2}} e^{-\frac{t^2/\tau_o^2}{\left(1+\frac{z^2}{b^2}\right)}} \tag{8}$$

where

$$b = \frac{\tau_o^2}{|k''|}$$

$$\eta = \frac{2}{3} r\sqrt{1-r^2}\cos\phi.$$

We now make an approximation that we shall find well justified: we expand the exponential to order t^2. The expansion is justified by the fact that the nonlinear phase shift is the result of the successive self-phase modulations of a pulse that varies in width. Thus the phase profile extends over a time interval large compared with the minimum pulse width. This in turn leads to an equation that resembles the forced modelocking equation with Gaussian solutions. Hence the starting assumption of a Gaussian pulse profile is justified at this point. If the pulse is greatly stretched in one round-trip, the limits $-L/2, L/2$ on the integral multiplying t^2 over the two segments can be extended to infinity and one finds

$$\begin{aligned} \Phi_\pm(t) &= 4\kappa|A_o|^2\, b\,(1 \pm \eta)\log\left(\frac{L}{2b}\right)\left[1 - \frac{t^2/\tau_o^2}{\log\left(\frac{L}{2b}\right)}\right] \\ &= \Phi_\pm^o\left[1 - \mu\frac{t^2}{\tau_o^2}\right] \end{aligned} \tag{9}$$

with

$$\kappa \equiv \frac{2\pi}{\lambda}\frac{n_2}{A_{\text{eff}}}$$

$$\Phi_\pm^o \equiv \frac{4\kappa|A_o|^2 b}{\mu}(1 \pm \eta)$$

$$\mu \equiv 1/\log(L/2b).$$

A factor of 2 has been included to account for the occurrence of two pulse-width minima in a single round-trip. Here $\Phi_\pm^o$ is the time independent part of the phase shift, and $\mu < 1$ is the curvature of the parabolic time dependence. One may note that the curvature of the parabola, the expansion of the Gaussian to order t^2, has been reduced ($\mu < 1$) by the stretching of the pulse. This is, of course, the case for all higher-order expansion terms; they are smoothed by the averaging over the propagation around the ring. Further one may note that the contribution to the parabolic phase is proportional to the distance parameter b. This parameter is equal to the distance within which the pulse width increases by a factor $\sqrt{2}$. Thus the shaping of the phase profile occurs within a distance much shorter than the overall fiber length. One may treat the phase profile as if it were produced by a lumped nonlinear element at the position of the minimum pulse width. Afterwards, the pulse with SPM propagates in a linear dispersive medium. The fields $E_\pm$ after the nonlinear phase shift are:

$$E_\pm = E_\pm^o e^{-j\Phi_\pm^o} e^{-Q\frac{t^2}{2}} \simeq E_\pm^o\left[1 - j\Phi_\pm^o\left(1 - \mu\frac{t^2}{\tau_o^2}\right)\right] e^{-\frac{t^2}{2\tau_o^2}} \tag{10}$$

$$Q = \frac{1}{\tau_o^2}\left(1 + j\mu\Phi_\pm^o\right). \tag{11}$$

Here we have expanded the exponential in the nonlinear phase shift so as to represent the effect of SPM as a modulating multiplier in the master equation to be discussed in Section II.

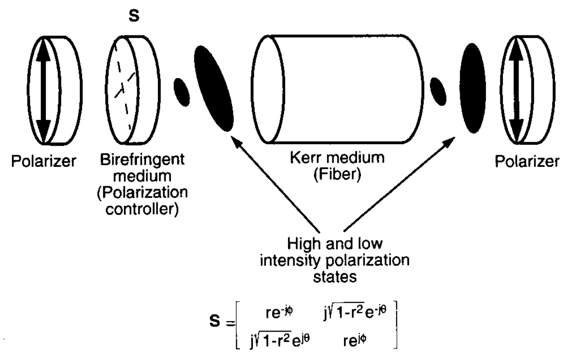

Fig. 5. Polarization APM structure.

C. The APM Action

We are now ready to consider the APM action. A basic polarization-APM structure is illustrated in Fig. 5. Linearly polarized light is transformed into elliptically polarized light by the polarization transformer. The ellipse rotates via the Kerr effect, and the angle of rotation is proportional to intensity. If properly biased, higher intensities pass through the output polarizer with lower loss. A more general polarization-APM structure would contain a second polarization transformer after the Kerr medium, which could relinearize the polarization and make the structure lossless for a certain intensity. For simplicity, we limit the analysis to the simpler structure of Fig. 5 since none of the important physics is lost.

We have previously analyzed the structure of Fig. 5. If we assume the Kerr medium is isotropic, the treatment of the nonlinear phase as a first order perturbation is very simple [14]. In the circular polarization basis, there is no coherence term (2). The Kerr effect can be modeled as nonlinear phase terms multiplying E_+ and E_- with the phases given by Φ_+ and Φ_-, respectively. Although real fibers always have some linear birefringence, if the birefringence is weak and the fiber lengths are much less than a beatlength, the isotropic approximation is justified. When the birefringence is significant, Jacobian elliptic functions must be used to derive exact expressions for the SAM and SPM parameters [16]. Nevertheless, the isotropic model still offers qualitative insight on the system operation.

Let ℓ represent the loss, $\gamma|a|^2$ the SAM coefficient, and $\delta|a|^2$ the SPM coefficient of the structure in Fig. 5. From [14], they are given by,

$$\ell = 1 - r \tag{12}$$

$$\gamma|a|^2 = -\sqrt{1-r^2}\sin\phi\left(\frac{\Phi_+ - \Phi_-}{2}\right) \tag{13}$$

$$\delta|a|^2 = \left(\frac{\Phi_+ + \Phi_-}{2}\right) \tag{14}$$

where a represents the field. These expressions still apply for stretched-pulse APM. As always, common phases of E_+ and E_- do not affect the polarization in a way that results in SAM, only differences do.

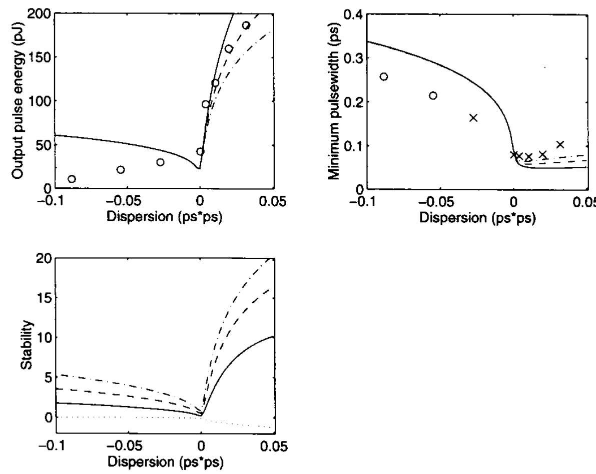

Fig. 6. Energy, transform-limited pulse width, and stability plotted against dispersion for $\gamma_o' = 0.01$ (solid), 0.2 (dashed), 0.3 (dot-dashed), and $\gamma_o' = 0$ (dotted). Data points are taken from [50]. In the pulse-width plot, circles indicate the assumption of sech pulse shapes and crosses indicate the assumption of Gaussian when extracting the pulse width from the measured autocorrelation.

If the polarizer is placed as shown in Fig. 1 at the position at which the pulse is transform-limited (except for the SPM term), from (9), (13), and (14),

$$\gamma|a|^2 = -\frac{4\kappa|A_o|^2 b}{3\mu}\, r(1-r^2)\sin(2\phi)\left(1 - \mu\frac{t^2}{\tau_o^2}\right) \tag{15}$$

$$\delta|a|^2 = \frac{4\kappa|A_o|^2 b}{\mu}\left(1 - \mu\frac{t^2}{\tau_o^2}\right). \tag{16}$$

From the sign of the term with the parabolic time dependence, we see that this is similar to the usual APM action where the peak of the pulses is enhanced with respect to the wings.

III. The Master Equation

We are now ready to write down the master equation. Denoting the self-consistent pulse solution by $a(t)$, we have

$$\left[(g-\ell) + \left(\frac{g}{\Omega_g^2} + jD\right)\frac{d^2}{dt^2} + \gamma_o|A_o|^2\left(1 - \mu\frac{t^2}{\tau_o^2}\right) - j\delta_o|A_o|^2\left(1 - \mu\frac{t^2}{\tau_o^2}\right)\right]a(t) = -j\psi a(t) \tag{17}$$

where

$$\gamma_o \equiv -\frac{4\kappa b}{3\mu}\, r(1-r^2)\sin(2\phi)$$

$$\delta_o \equiv \frac{4\kappa b}{\mu}.$$

Here g is the (amplitude) gain, ℓ is the loss per pass given in (12), Ω_g is the gain bandwidth, D is the net GVD operator, and (15) and (16) have been used for the SAM and SPM coefficients.

The parabolic shaping profile of amplitude and phase introduced by the nonlinearity calls for Gaussian solutions. Using the Ansatz:

$$a(t) = A_o e^{-Q\frac{t^2}{2}} \tag{18}$$

and equating terms of equal power in t, one obtains two complex equations for the pulse parameter $Q \equiv Q_R + jQ_I$:

$$g - \ell - (G + jD)Q + \gamma|A_o|^2 - j\delta|A_o|^2 = -j\psi \tag{19}$$

$$(G + jD)Q^2 - \mu|A_o|^2(\gamma_o - j\delta_o)\frac{Q_R}{|Q|^2} = 0 \tag{20}$$

with

$$G \equiv \frac{g}{\Omega_g^2}.$$

Here we have replaced the inverse square of the pulse width τ_o by $Q_R/|Q|^2$, which is the transform-limited pulse width.

We find for Q_R and Q_I,

$$Q_R = \sqrt{\frac{4\kappa|A_o|^2}{k''(1 + \tan^2(\alpha))}\sqrt{\frac{\gamma_o'^2 + 1}{G^2 + D^2}}} \tag{21}$$

$$Q_I = Q_R \tan(\alpha) \tag{22}$$

where,

$$\alpha = \pi - \frac{1}{2}\left[\tan^{-1}\left(\frac{D}{G}\right) + \tan^{-1}\left(\frac{1}{\gamma_o'}\right)\right]$$

$$\gamma_o' = \frac{2}{3}r(1 - r^2)\sin(2\phi).$$

The stability of the pulses against growth of noise requires that the gain as determined from (19) be less than the loss. This gives the equation

$$-GQ_R + DQ_I + \gamma_o|A_o|^2 > 0. \tag{23}$$

Thus far we supposed that the polarizer responsible for the APM action is placed symmetrically as shown in Fig. 1. This is not generally necessary, since the results are invariant to the placement, at least as far as the approximate analysis presented here applies. The argument is as follows. The reader will have noticed the close analogy of the Gaussian pulse propagation in the two dispersive fibers and that of a beam of Gaussian shape in a system of two lenses. In fact, an exact theory for the self-phase modulation of Gaussian beams in the tight focusing limit was derived in [54]. This is the spatial analog of rapid temporal stretching of Gaussian pulses in a Kerr medium. The Q-parameter is analogous to $1/q$ of the Gaussian beam problem. The reversal of the dispersion is analogous to a reversal of the phase-front curvatures by a lens. The Q-parameter of (18) evolves according to an [ABCD] formalism like the parameter $1/q$ of the Gaussian beam. At the symmetric position in the ring, the Q-parameter repeats in the absence of SPM (ignoring the slight imbalance of the dispersions, i.e., the D-parameter). The effect of SPM on the pulse at and near the pulse-width minimum positions producing changes $\delta(Q_+)$ and $\delta(Q_-)$ in the Gaussian pulses is analogous to Kerr lensing of the Gaussian beams of opposite circular polarizations, producing changes $\delta(1/q_+)$ and $\delta(1/q_-)$ in the circularly polarized beams. These changes evolve as perturbations of the [ABCD] matrix formalism. The polarizer subtracts these two changes according to (15). Even though this difference is a function of the positions along the ring, referred back into the symmetry position, it is independent of the position at which the difference is taken, i.e., independent of the positions of the polarizer.

IV. Solution Of Master Equation

In solving the master equation one must decide on the choice of parameters. It is found in fiber lasers that the shortest and most energetic pulses are obtained just before multiple pulsing and pulse break-up occur. This is attributed to excessive nonlinear phase shifts in the laser which cause sideband generation and pulse-shape deterioration. When the laser operates in the stretched pulse regime, the sideband generation is greatly suppressed. However, the pulse intensity clamping due to the sinusoidal response of the nonlinear transmissivity cannot be eliminated. Indeed, if the phase bias is changed so as to decrease the nonlinear response, then the APM action is rendered too weak for self-starting. Hence, one may consider $|A_o|^2$ to be fixed at some maximum value, while varying the remaining parameters. Fig. 6 shows the pulse energy, transform-limited pulse width, and stability against cw plotted against GVD. These curves were calculated using (21) with $G = 0.001$, $\kappa|A_o|^2 = 1.0$, $\pi|A_o|^2 = 250$ pJ/ps, $k'' = 0.075$ ps^2/m, and $\delta = 2.14$. The values were estimated from experimental data reported in [50].

Although it is difficult to precisely estimate the parameter values from real data, one finds general agreement with the observed trends. We find that the shortest pulses are predicted for small net positive GVD. The pulse energy also sharply increases on the positive GVD side. This is in agreement with what was reported in [50]. The data for experimentally observed minimum pulse width versus GVD is plotted against the theory for comparison. We also plot the maximum output pulse energy versus GVD. The agreement in the energy curve is good for near zero and positive GVD. In the negative GVD regime, we find a decrease of energy with increasing negative GVD in the data, while the theory predicts increasing energy. The theory neglects sideband generation. In the soliton regime, as the group velocity dispersion increases, the sidebands move closer to the spectrum, get stronger, and parasitically limit the energy that can be achieved with a given gain. The decrease in energy leads to longer pulsewidths. From the measured time-bandwidth products, one expects the Gaussian theory to be most accurate for small net negative GVD. In the positive GVD regime, nonlinear chirp on the pulse causes deviations from the ideal Gaussian pulse model and the time-bandwidth products increase to 0.60.

One may be tempted to consider other ways of reducing the nonlinearity. For example, it is well known that highly chirped pulses are solutions of the master equation of APM [55], [49] if the dispersion is made positive and large. For a given bandwidth, such chirped pulses have reduced peak intensity and thus experience reduced nonlinearity in a fiber ring. However, it is also found that bandwidth is sacrificed, [49, Fig. 2], unless the self-amplitude modulation coefficient γ is increased correspondingly. However, γ can be only increased by strengthening the interferometric action of Polarization-APM. This causes overdrive of the APM action at lower peak intensities, thus limiting the achievable pulse energy. Stretched pulse APM operates near the effective zero dispersion point, and thus leads to broad pulse bandwidth with an acceptably low value of the self-amplitude modulator coefficient. Another virtue of the stretched pulse system is that the effective self-amplitude modulation coefficient γ changes from a higher value during pulse build-up, when the pulse shape is roughly constant around the loop, to a much smaller value in steady state operation, when the short pulse occupies only a small portion of the ring. It is as if the length of the ring were changing from build-up to steady state operation. In this way self-starting is guaranteed and the overdive of the nonlinear transmissivity is effectively postponed.

The experiments in [50] extract the pulse in a stretched-pulse APM laser after passage through the positive GVD segment of the loop. The chirp on the pulse is highly linear and efficient external compression is achieved over a large range of net intracavity GVD. The theory yields Gaussian solutions and the experiment confirms the Gaussian nature of the pulse over most of the time interval of the pulse. This Gaussian character of the pulse leads to a linear chirp (as function of time) thus leading to better compressibility.

In real systems, the distributed nature of GVD and SPM affect the pulse dynamics in an important way, which the first order analytic theory here does not address. In particular, we expect the location of the pulse minima within the loop to shift depending on the relative strengths of the soliton compression in the negative GVD segment and the enhanced pulse spreading in the positive GVD segment. Higher-order theories for stretched-pulse APM should consider the role of these effects.

V. Conclusion

We have developed a formalism which describes analytically the mode-locking of pulses in a fiber ring for the case when the pulse undergoes major changes due to dispersion in one round-trip around the ring. It was pointed out that this situation is beneficial in all-fiber erbium ring lasers in which the gain requirement calls for relatively long fiber lengths. These lengths lead to excessive nonlinear effects that are reduced by the pulse lengthening due to dispersion. It is likely that similar considerations will also lead to improved performance of the figure eight laser configurations.

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