

Notes for the first lecture on “The growth and morphology of icicles”

This pdf contains a few slides that were not shown in the actual lecture, for lack of time.

This pdf contains live links which are underlined like this.

For data and movies, see the *Icicle Atlas*
https://www.physics.utoronto.ca/Icicle_Atlas

All papers, theses, reports etc on this topic are available at:
https://www.physics.utoronto.ca/nonlinear/papers_icicles.html

YouTube playlist.

Flickr images and movies of icicles.

Flickr images of the icicle machine.

Do not hesitate to contact me with any questions, ideas or whatever
Stephen Morris patternphysics@gmail.com or sw.morris@utoronto.ca
<https://www.physics.utoronto.ca/nonlinear/>

The growth and morphology of icicles

*Interfreeze 2026,
Cargèse, Corsica*

Stephen Morris

*University of
Toronto, Canada*



Icicle (EN)

Stalactite
de glace (FR)

Glaçon (PQ)

Ghiacciolu (Corsica)

つらら (JP)

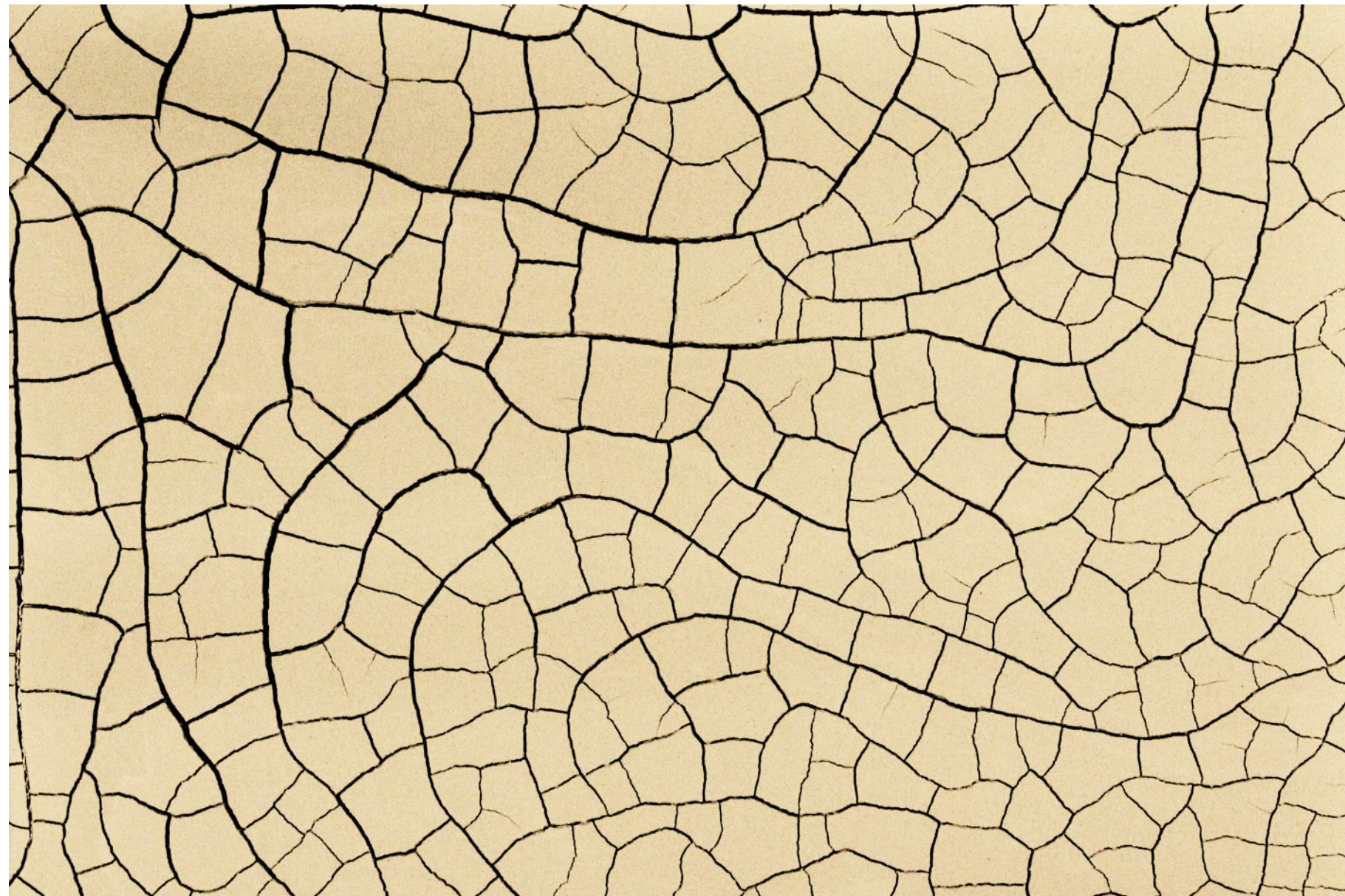


Why study icicles?

For me, this is *blue sky research* ...

Part of a general program applying ideas from nonlinear pattern formation to naturalistic geomorphological systems: **Laboratory geomorphology**.

- Mud cracks

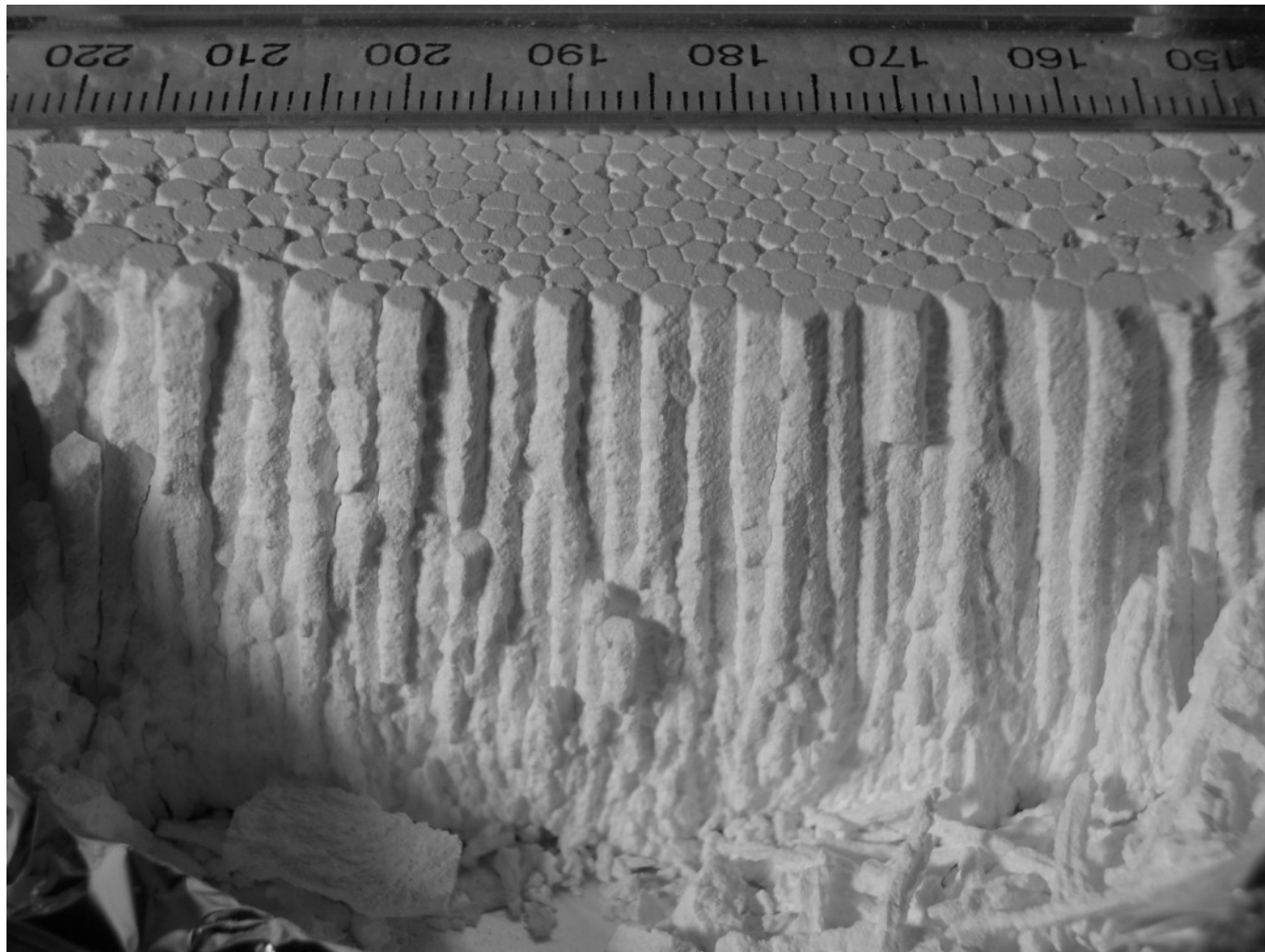


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- Lava (or starch) columns



Why study icicles?

For me, this is *blue sky research* ...

Part of a general program applying ideas from nonlinear pattern formation to naturalistic geomorphological systems: **Laboratory geomorphology.**

- Washboard road



Why study icicles?

For me, this is *blue sky research* ...

part of a general program applying ideas from nonlinear pattern formation to naturalistic geomorphological systems: **Laboratory geomorphology**.

Icicles are unusually amenable to laboratory study, have been almost completely ignored.

Everybody has seen them, but almost nobody has studied them carefully.

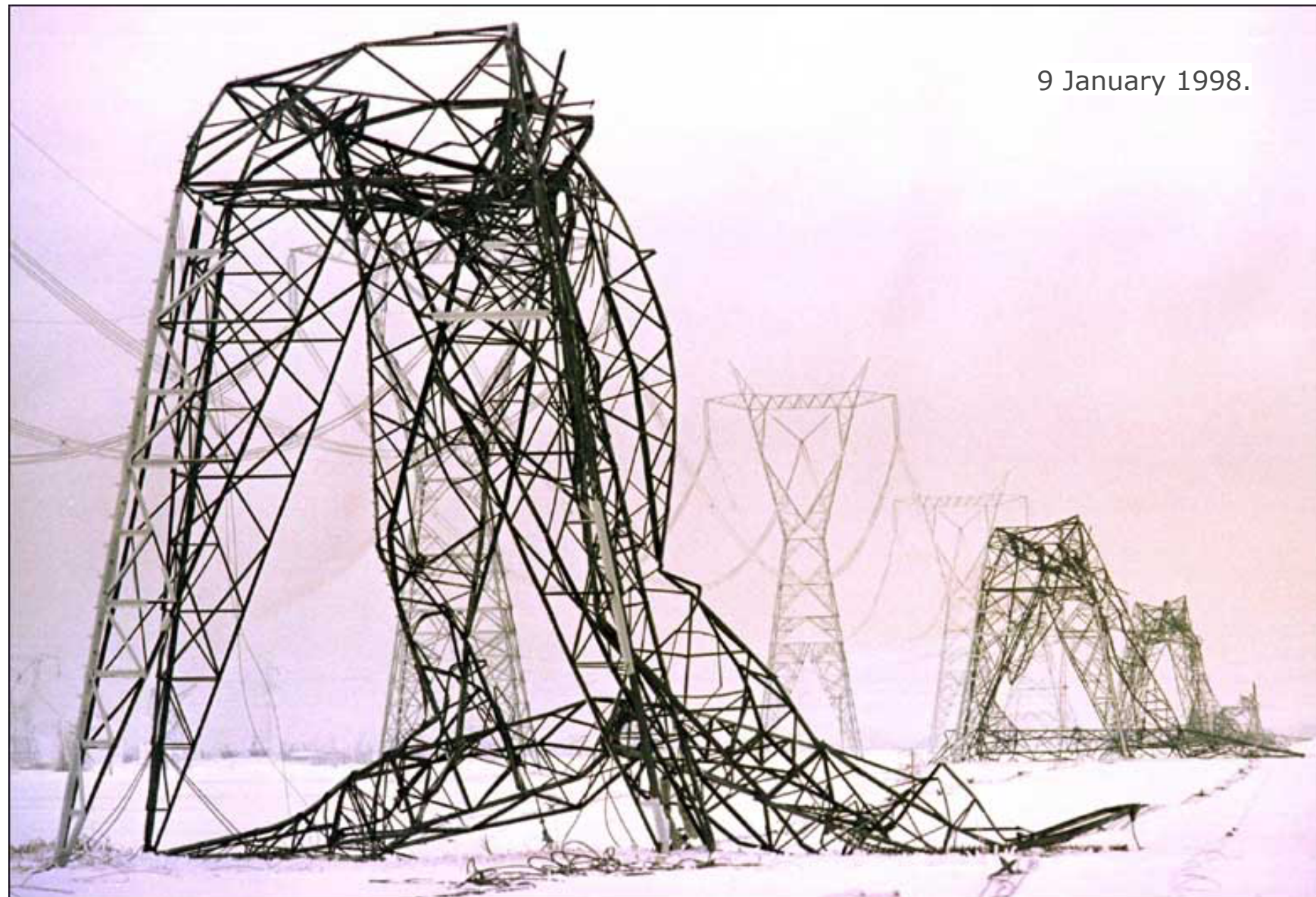


Why study icicles?

For me, this is *blue sky research* ...

but it also has lots of overlap with ice hazard engineering problems.

- Ice storms weigh down power lines



Why study icicles?

For me, this is *blue sky research* ...

but it also has lots of overlap with ice hazard engineering problems.

- Icicles form lightning rods on high voltage equipment



Provocative papers

The theoretical papers that set me on this path:

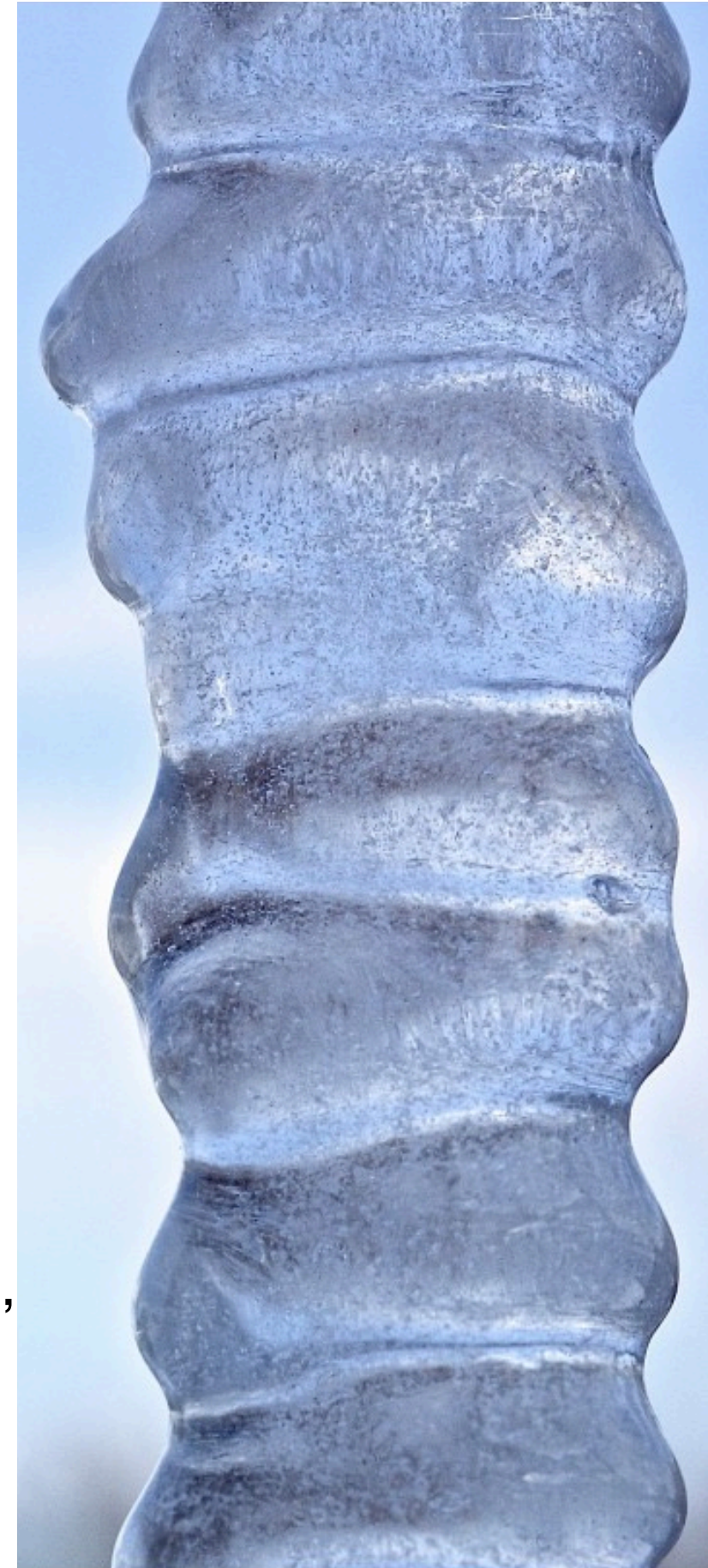
Two theories of icicle ripples

- **N. Ogawa and Y. Furukawa**,
“*Surface instability of icicles*”, Phys. Rev. E 66, 041202 (2002).

- **K. Ueno**,
“*Pattern formation in crystal growth under parabolic shear flow*”,
Phys. Rev. E 68, 021603 (2003), and 6 other papers up to 2011.

A theory of universal “platonic” icicle shapes

- **M. B. Short, J. C. Baygents and R. E. Goldstein**,
“*A free-boundary theory for the shape of the ideal dripping icicle*”,
Phys. Fluids 18, 083101 (2006).



The people who did the work



Antony Szu-Han Chen

Ph.D. 2013

Southern Alberta Institute of Technology

“Experiments on the growth and form of icicles”

Built the Icicle Machine, studied ideal icicle shapes and discovered ripples are due to impurities. Grew over 230 laboratory icicles.



John Ladan

Ph.D. 2023

Vretta Inc

“Experiments on the formation of rippled icicles”

Improved the Icicle Machine, used dye visualization and generalized ripple theory to include impurities. Grew over 180 laboratory icicles.

Also: Roger Mong, Jake R. C. Wells, Ken Liao, Alina Barnett, Josh Calafato ... and probably some others.



Outline

Part 1: **Shape**

- Icicles vs stalactites
- The icicle machine
- Platonic and not so platonic icicles

Part 2: **Ripples**

- Impure icicles are rippled
- Ueno theory of pure rippled icicles
- Ladan theory of impure rippled icicles
- Worster theory of pure rippled icicles

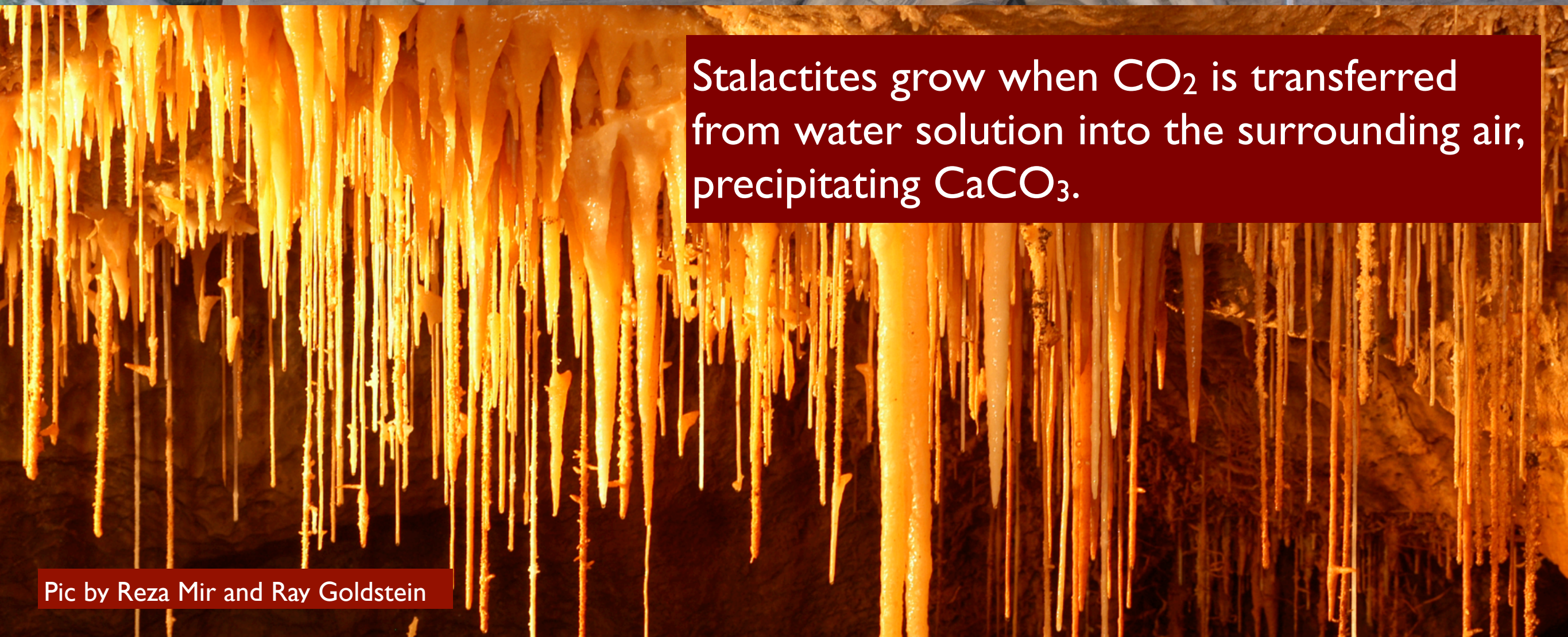
Part 3: **Interior and Flow**

- Internal structure of icicles
- Visualizing surface flow with dye
- Ideas for future work



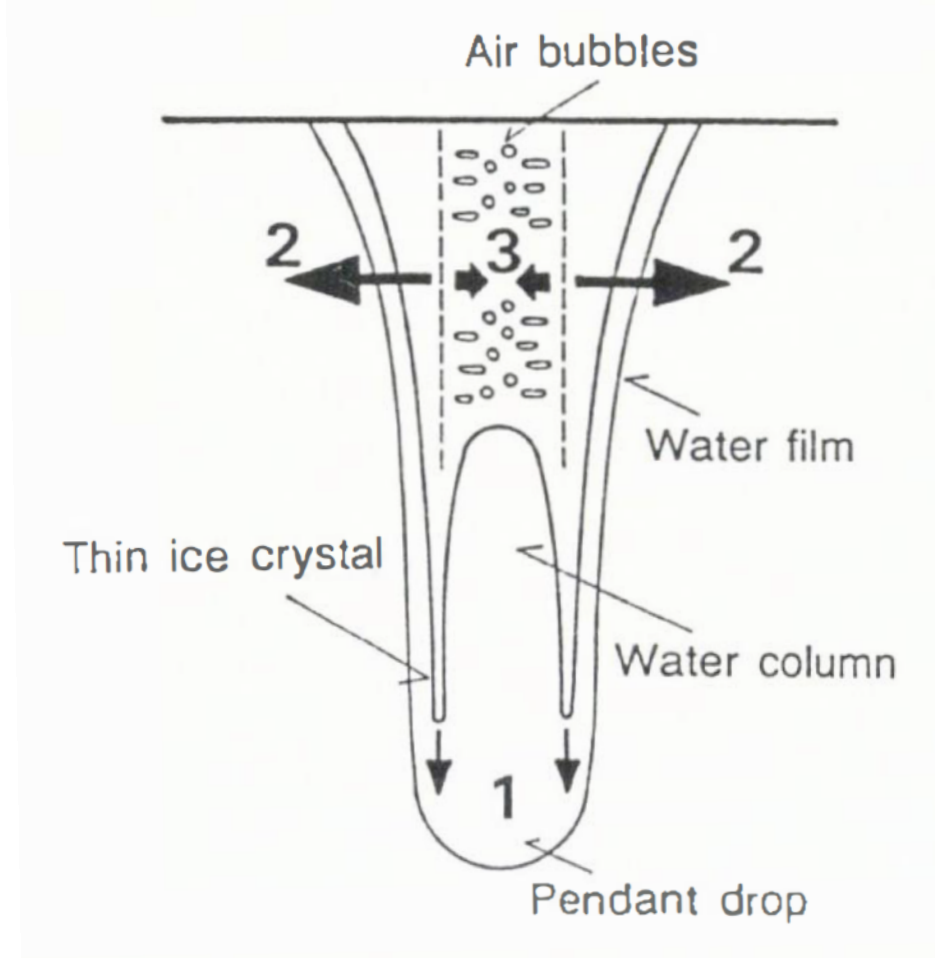


Icicles grow when latent heat is transferred from water flowing over their surface into the surrounding air.



Stalactites grow when CO_2 is transferred from water solution into the surrounding air, precipitating CaCO_3 .

Icicle growth modes

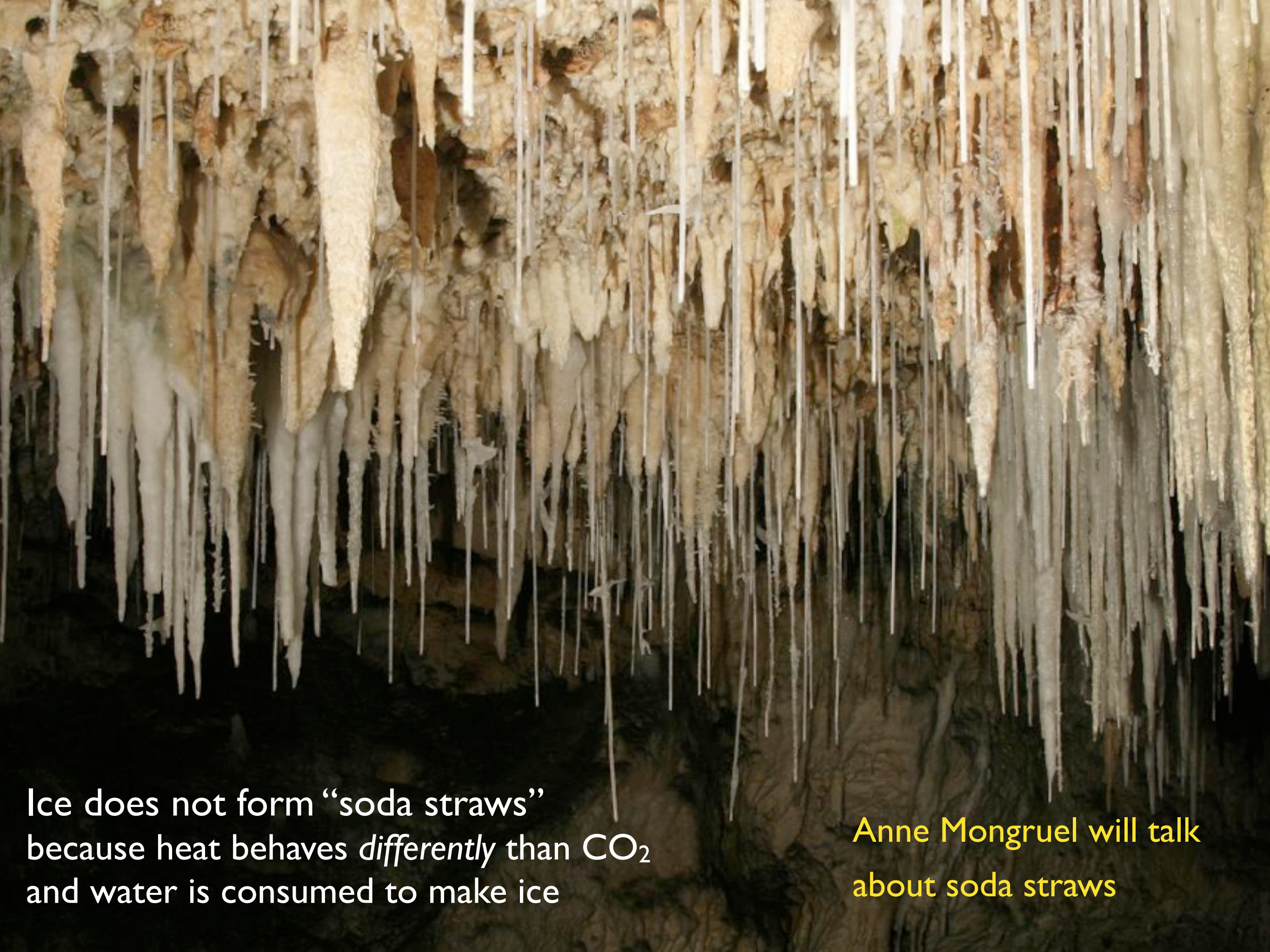


1. Tip growth from pendant water drop
2. Radial growth from surface flow
3. Inward “pipe filling” growth

It is mode 2 that may be unstable
to the formation of ripples

N. Maeno, L. Makkonen, K. Nishimura, K. Kosugi
and T. Takahashi, *J. Glaciol.*, **40**, 319 (1994).





Ice does not form “soda straws”
because heat behaves *differently* than CO_2
and water is consumed to make ice

Anne Mongruel will talk
about soda straws

Cave drapery

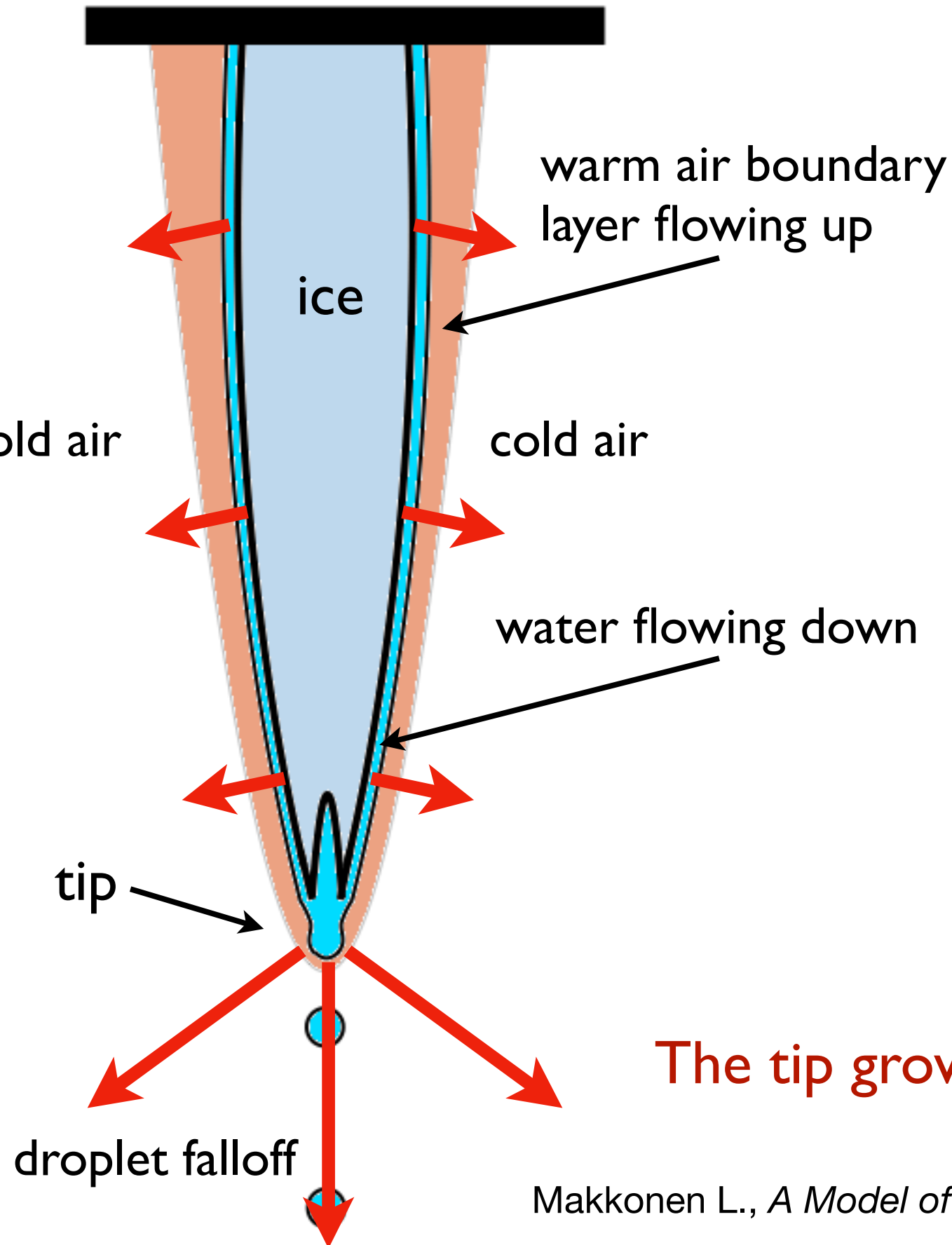


Icicle drapery



Photo by
Dan Rothman, MIT.

Icicle growth schematic

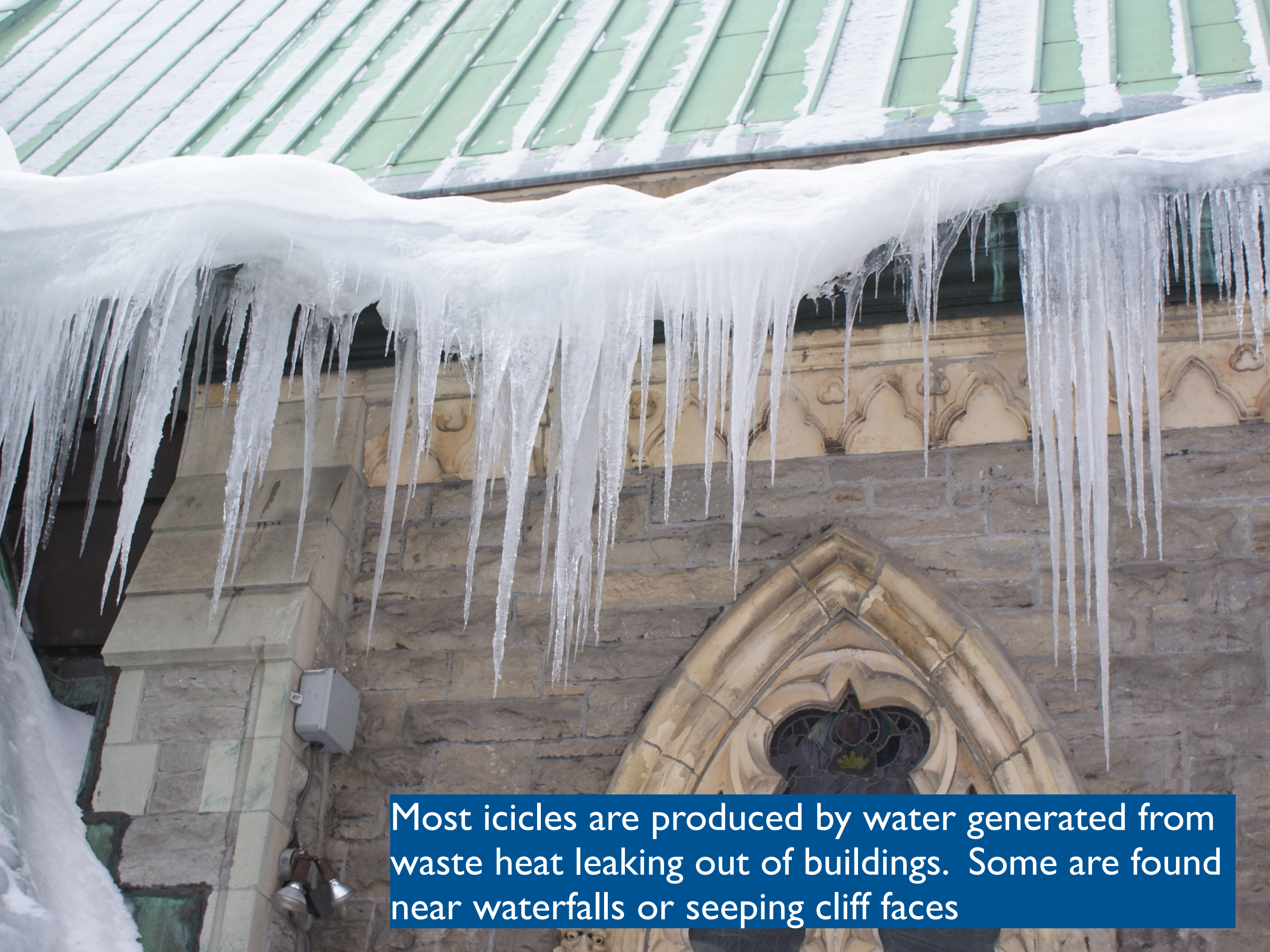


Water flows down the sides of the icicle, releasing latent heat as it freezes. The heat is advected and diffused first by the water and then by the air, which forms a warm rising thermal boundary layer.

The water also cools by radiation and by evaporation, if the humidity is low.

At the tip, heat transfer is more efficient than the sides because the boundary layer is thinner and there are more directions for radiation. In addition, droplets falling off carry away heat.

The tip grows 10 - 100 x faster than the sides



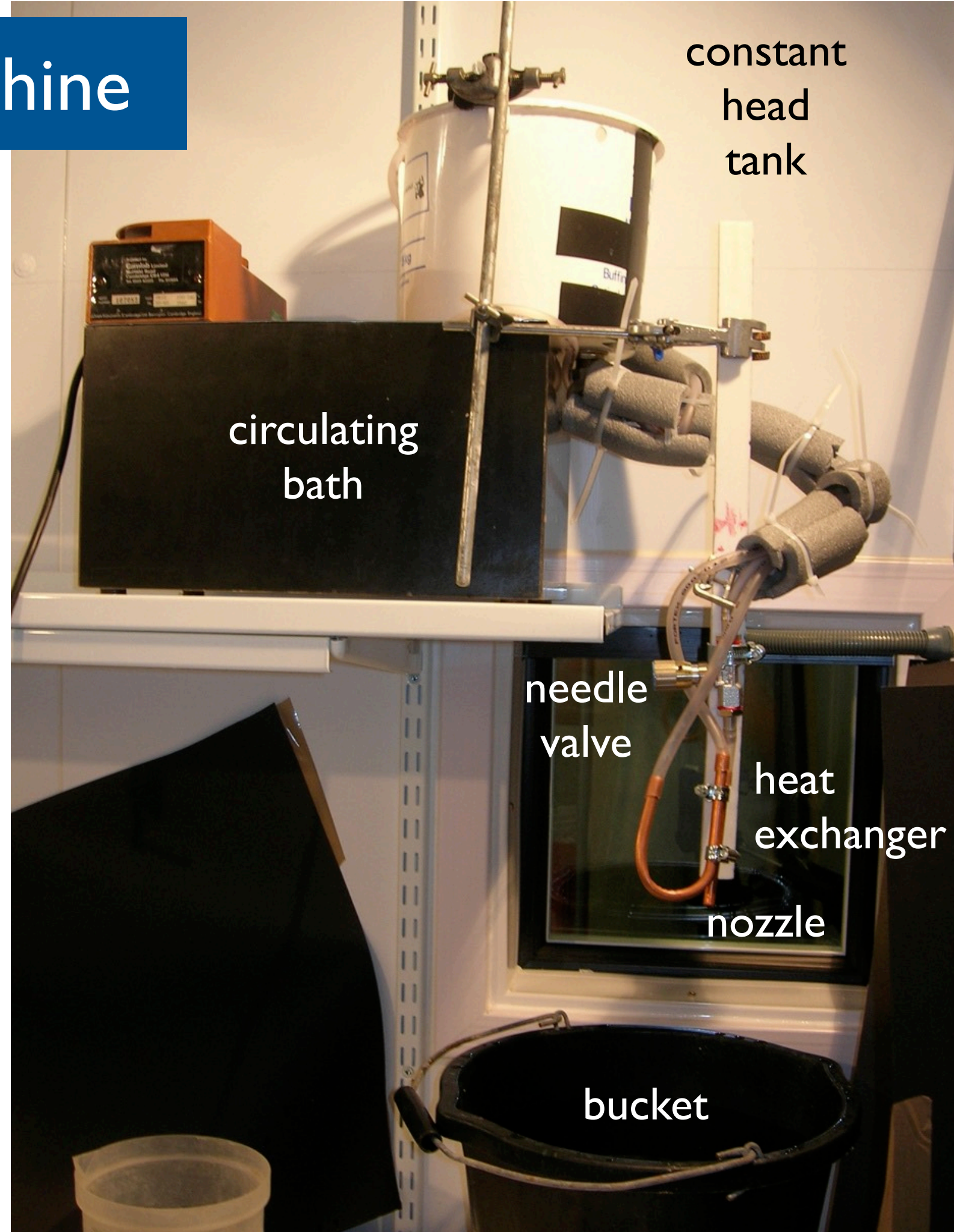
Most icicles are produced by water generated from waste heat leaking out of buildings. Some are found near waterfalls or seeping cliff faces

The first icicle machine

Done in 2006
in the “giant cold
room at Cambridge
University”

- *New Scientist*

Need to deliver a very
slow drip from a pipe that
does not freeze. Used
cooling provided by the
very low humidity, breezy
cold room ...

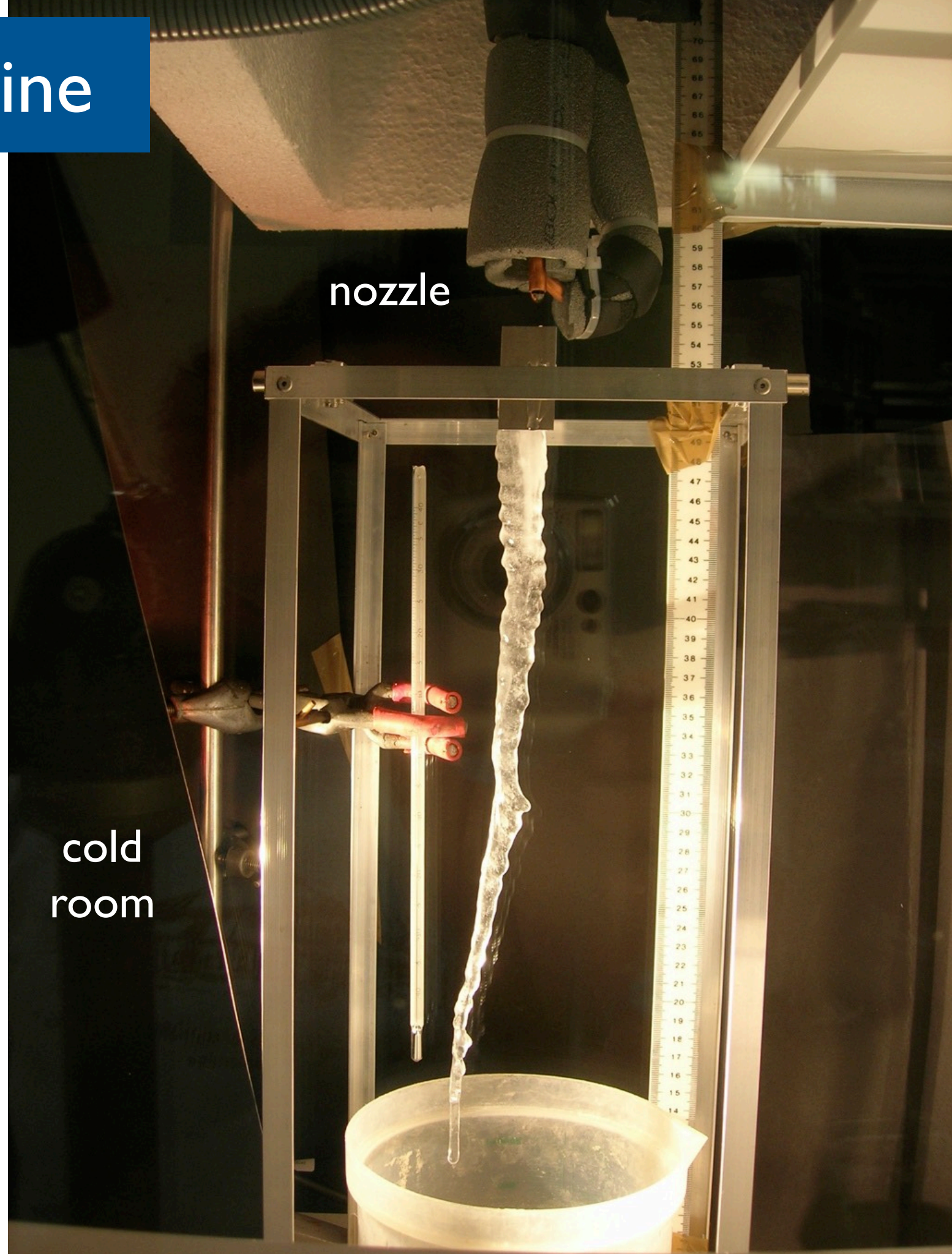


The first icicle machine

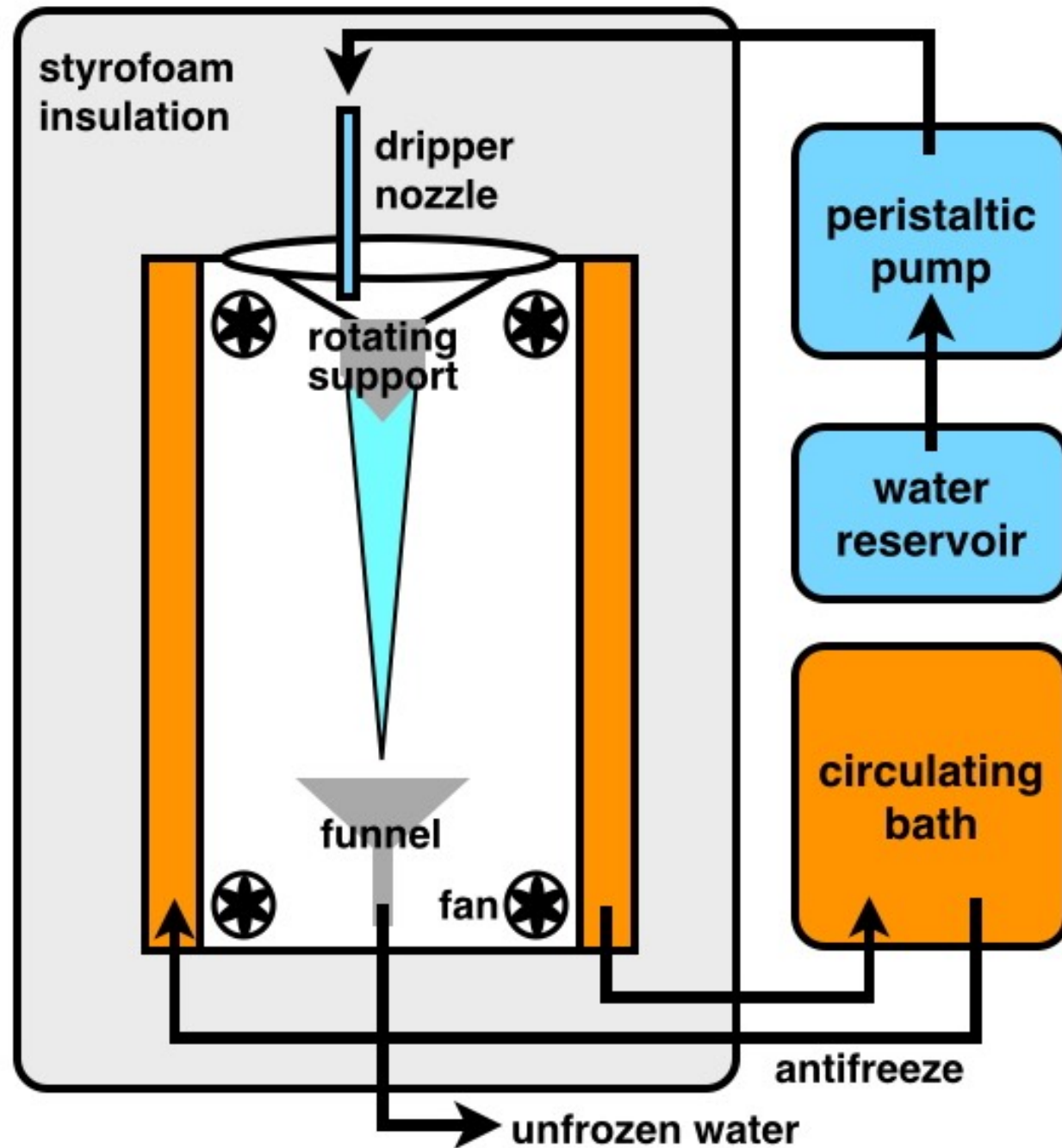
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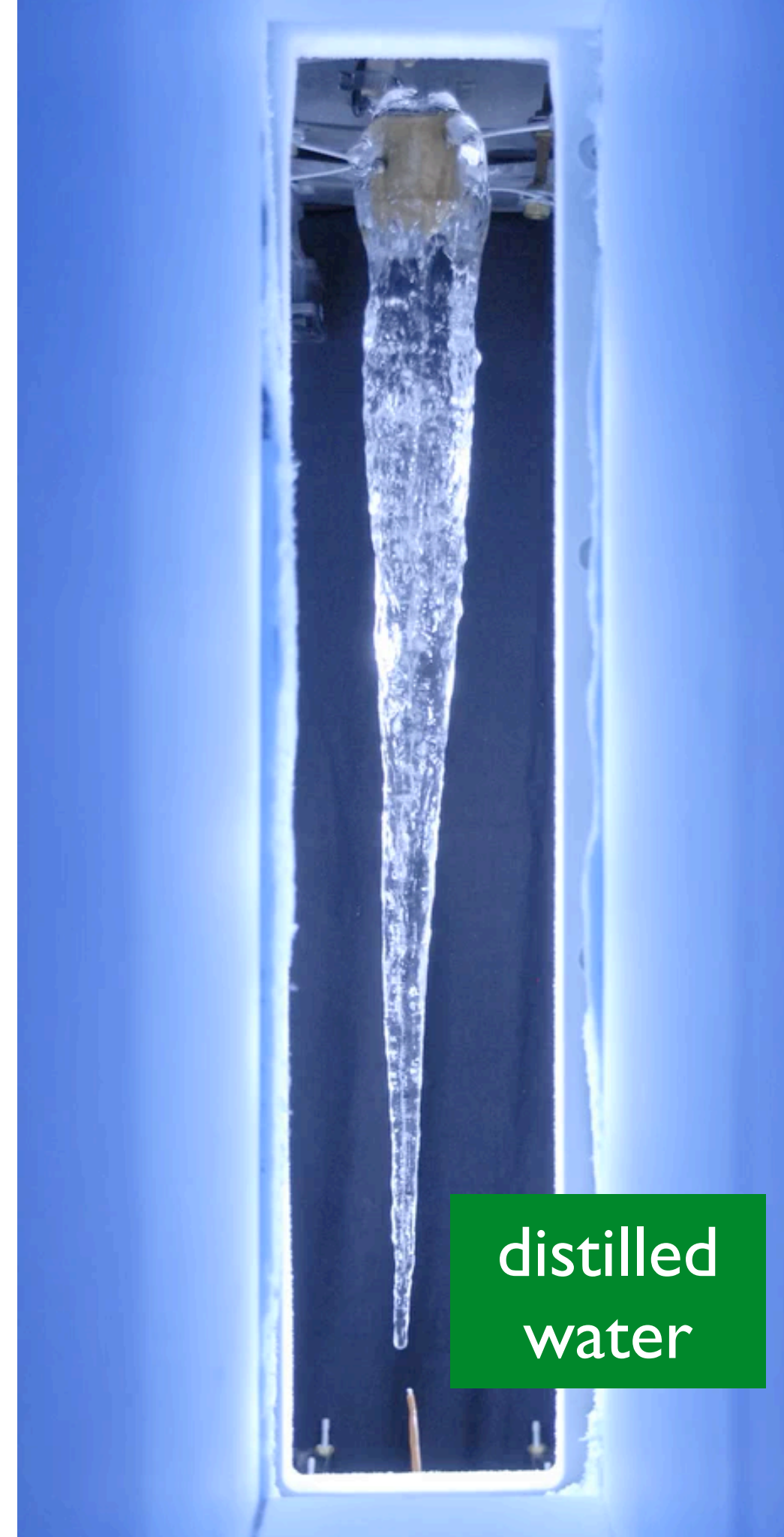


The icicle machine



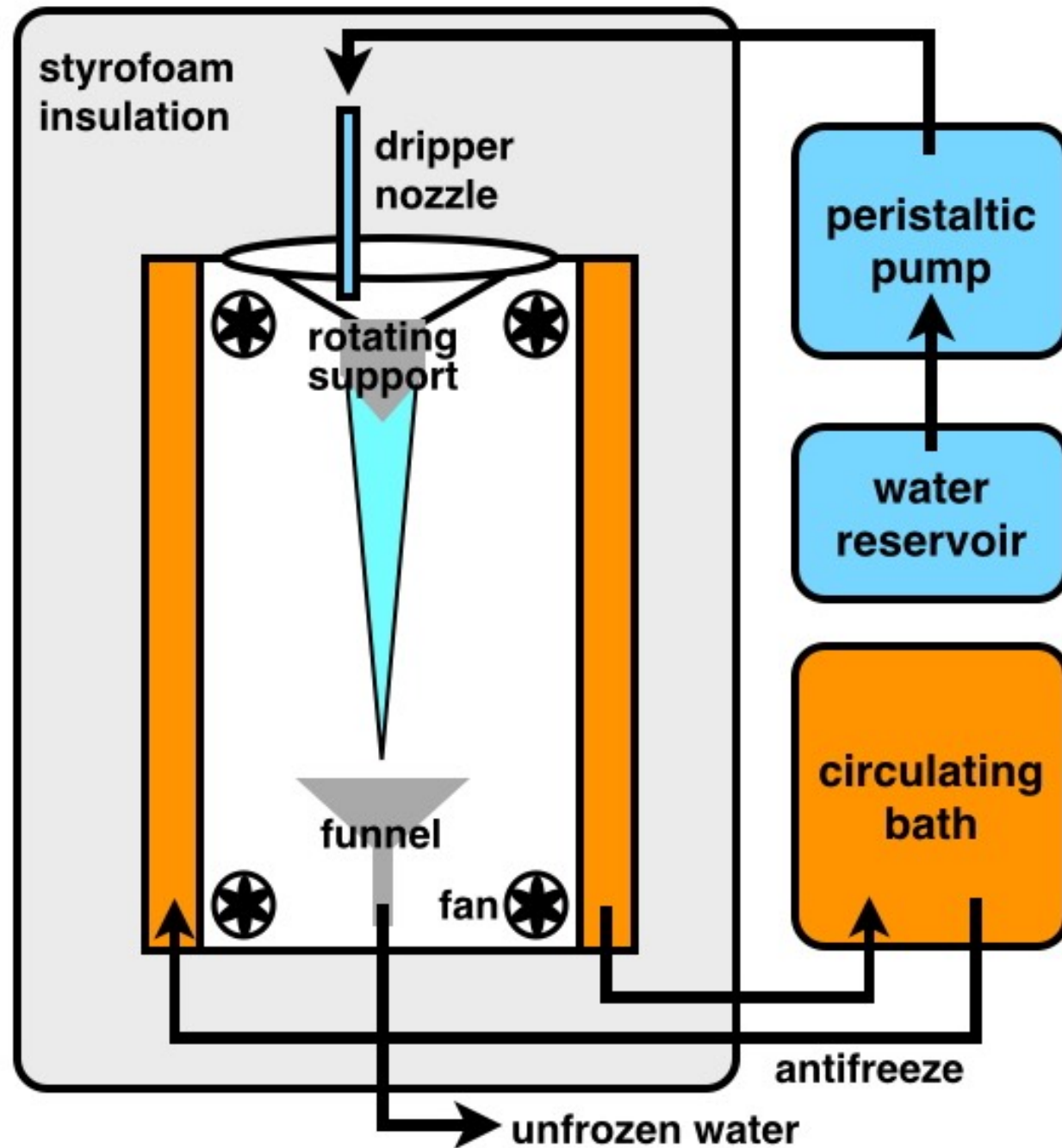
Using time-lapse photography, we can make movies of the icicle's evolving morphology.

1 sec of movie = 20 min real time (1 rotation / 8 min)



distilled
water

The icicle machine



Using time-lapse photography, we can make movies of the icicle's evolving morphology.

Up to 16 rotational views of the same icicle.

tap water

The icicle machine

cold box

antifreeze

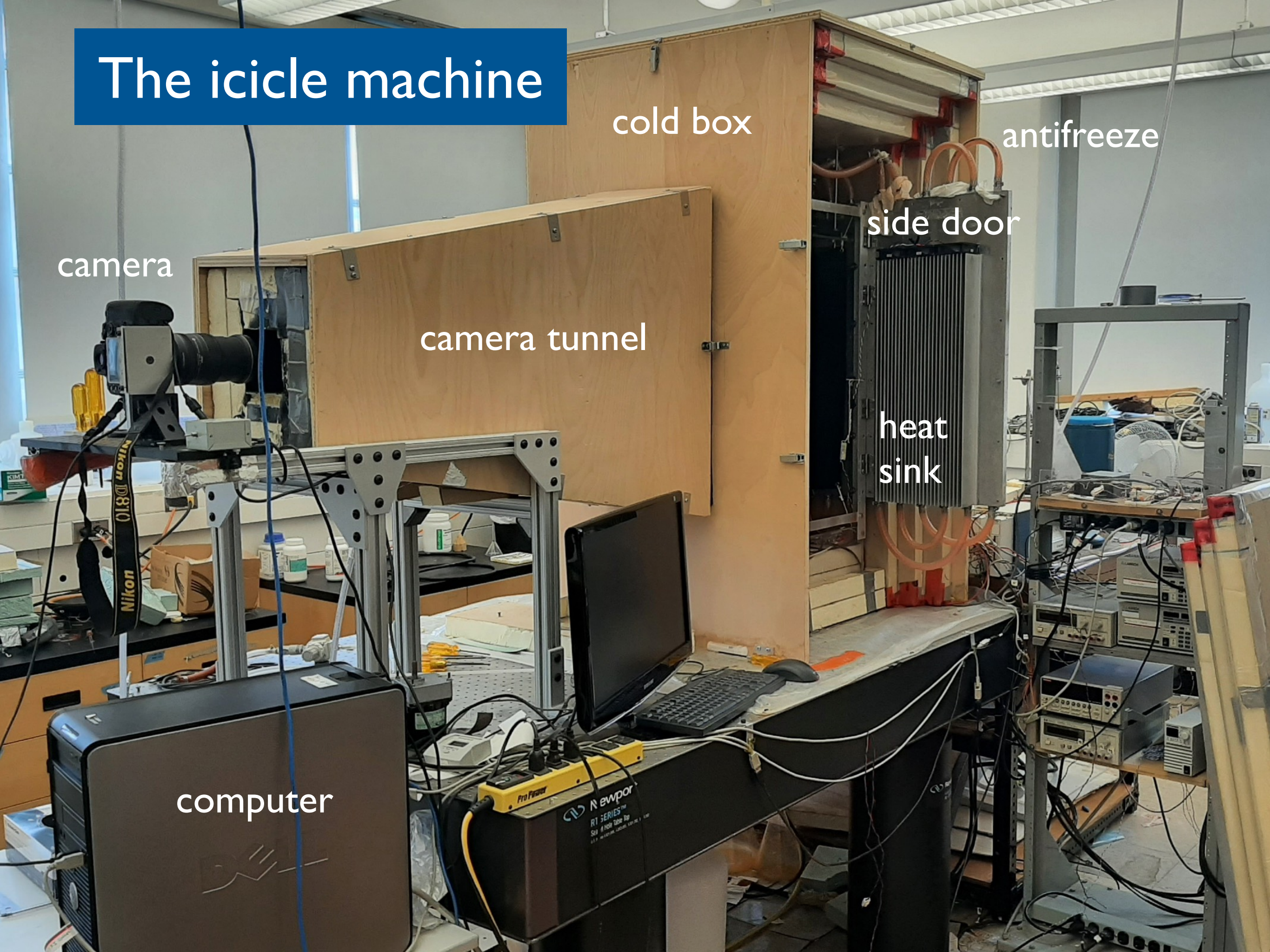
camera

camera tunnel

side door

heat
sink

computer



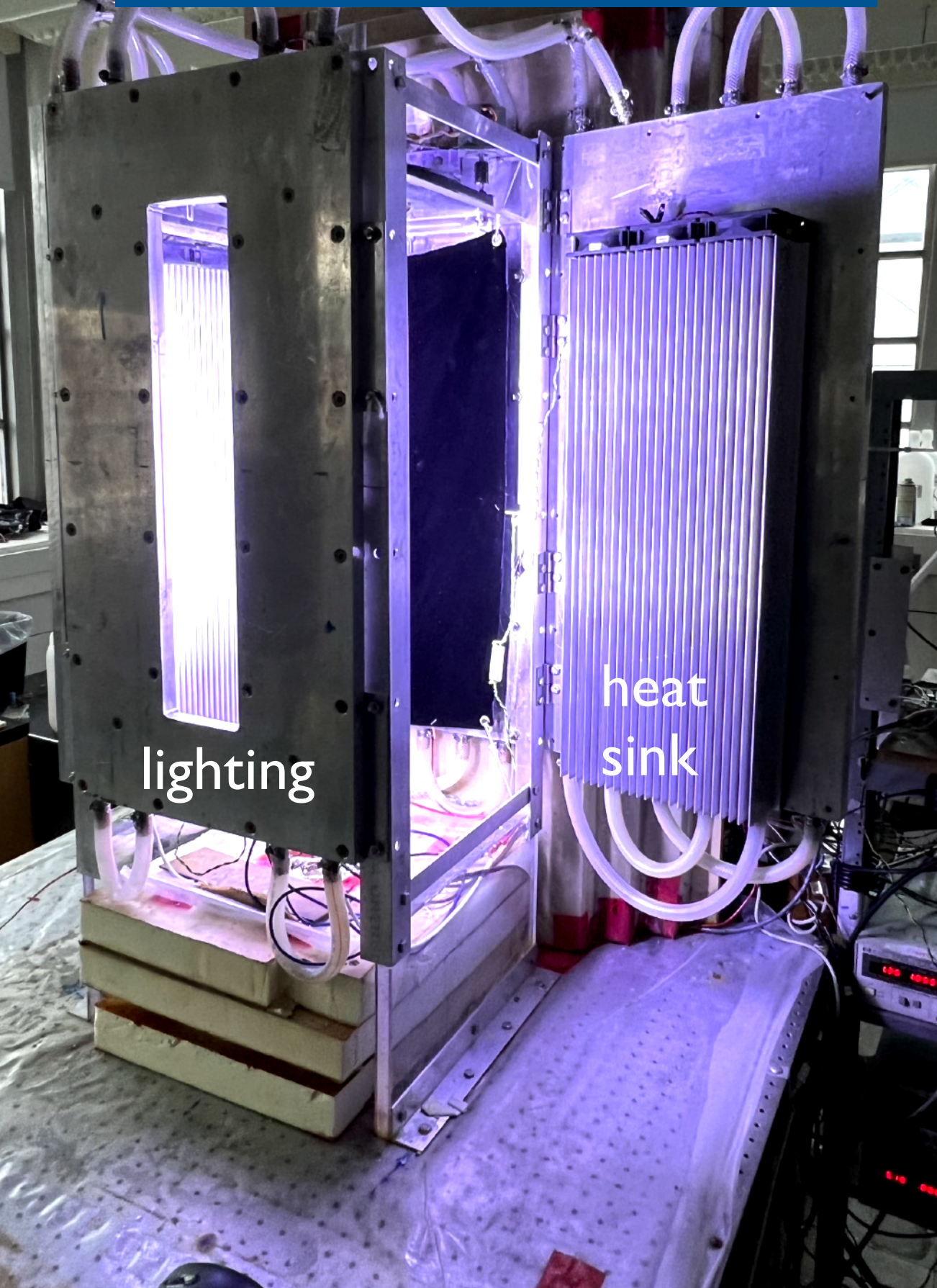
The icicle machine

Cooling of the box is handled by a large commercial circulating bath full of car antifreeze fluid

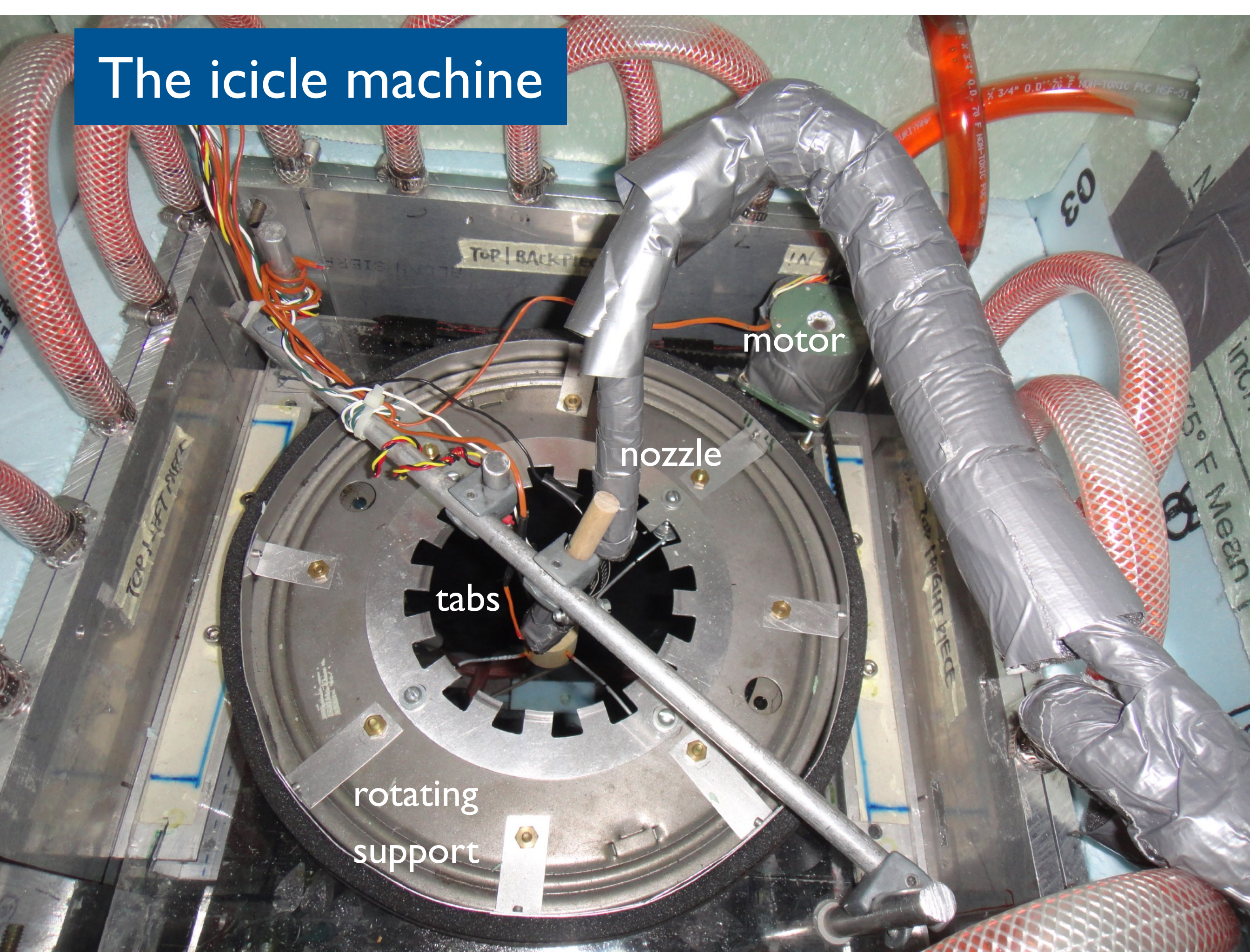
Lowest temperature is about -20 C



The icicle machine



The icicle machine

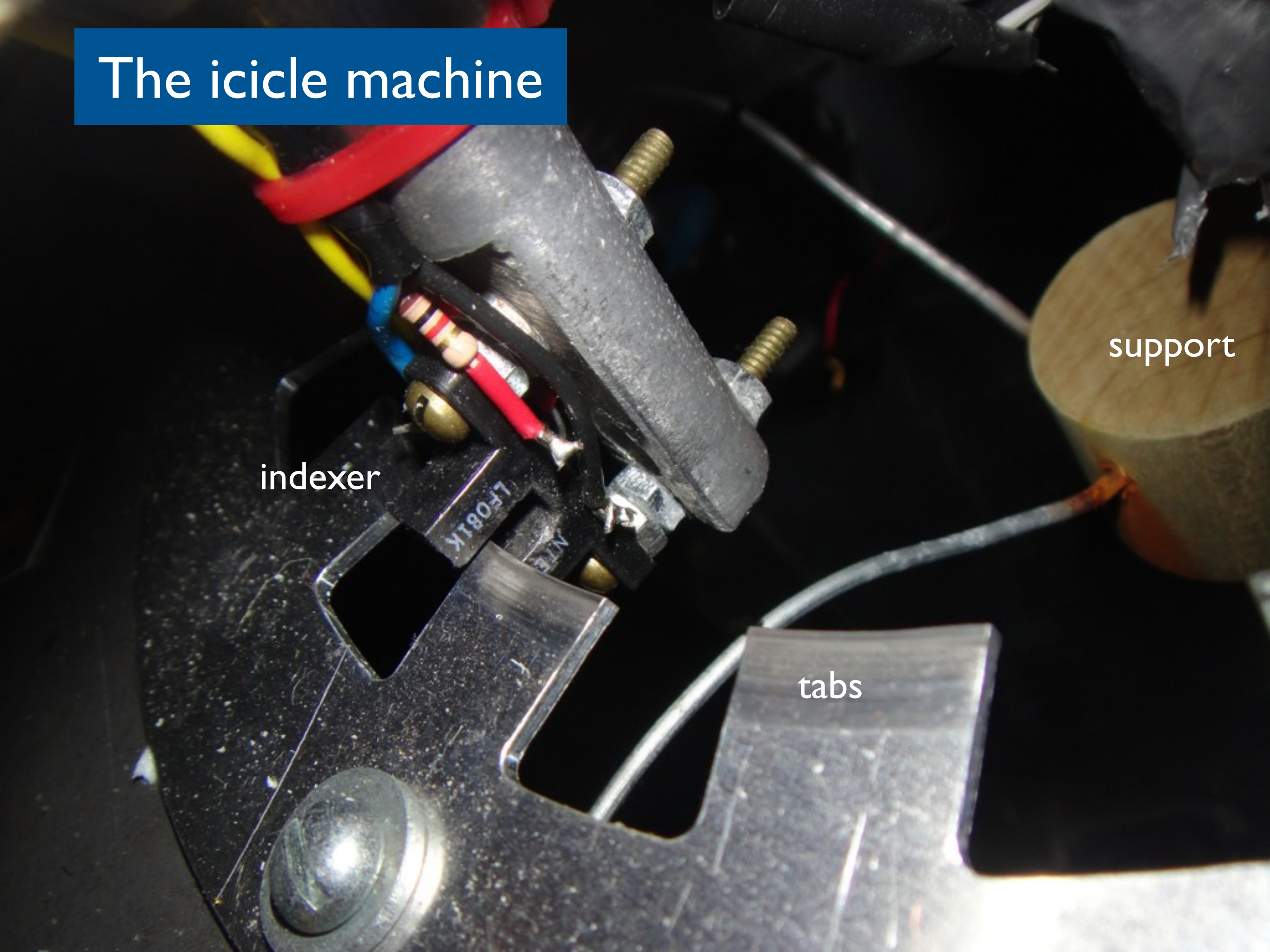


The icicle machine

indexer

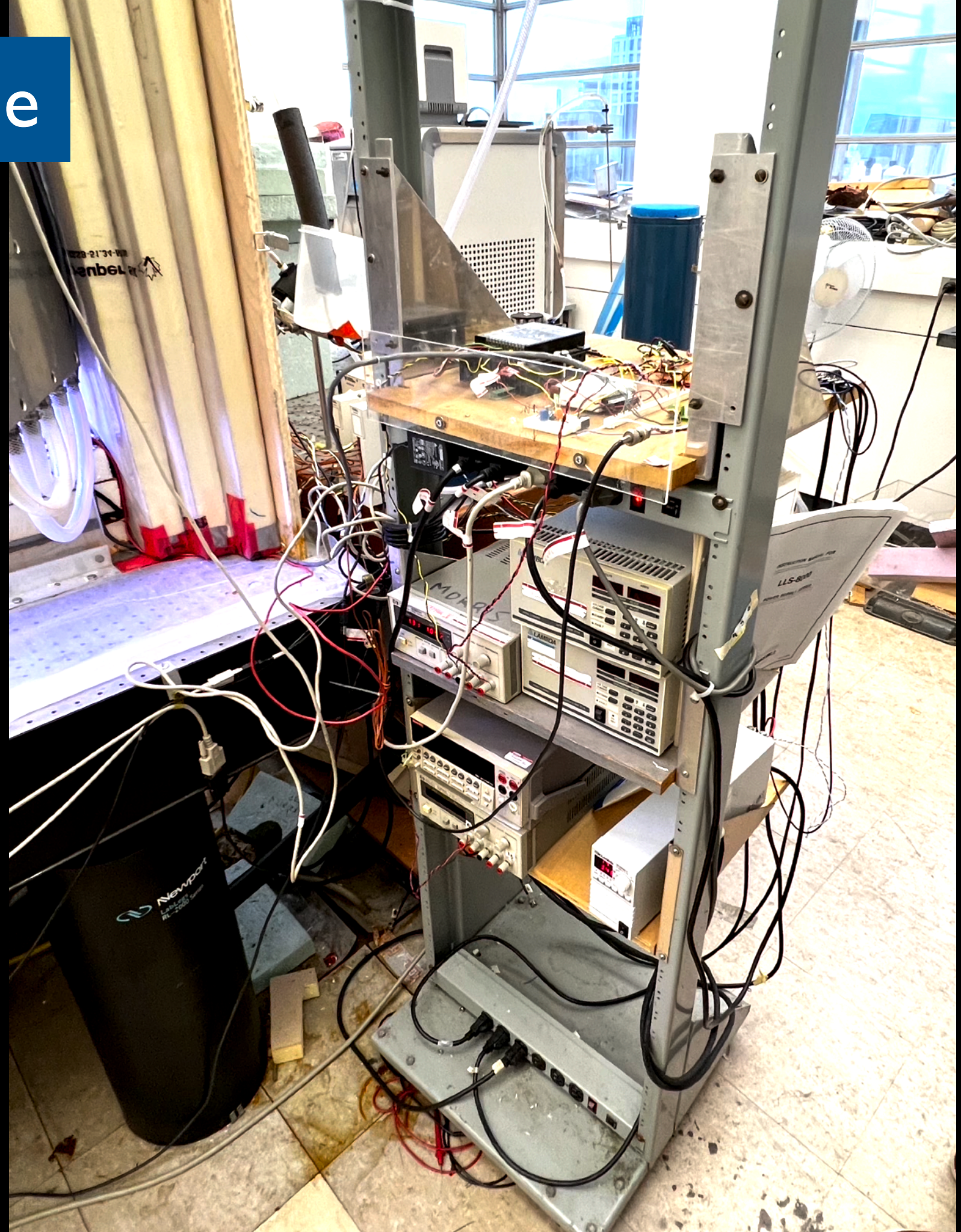
support

tabs



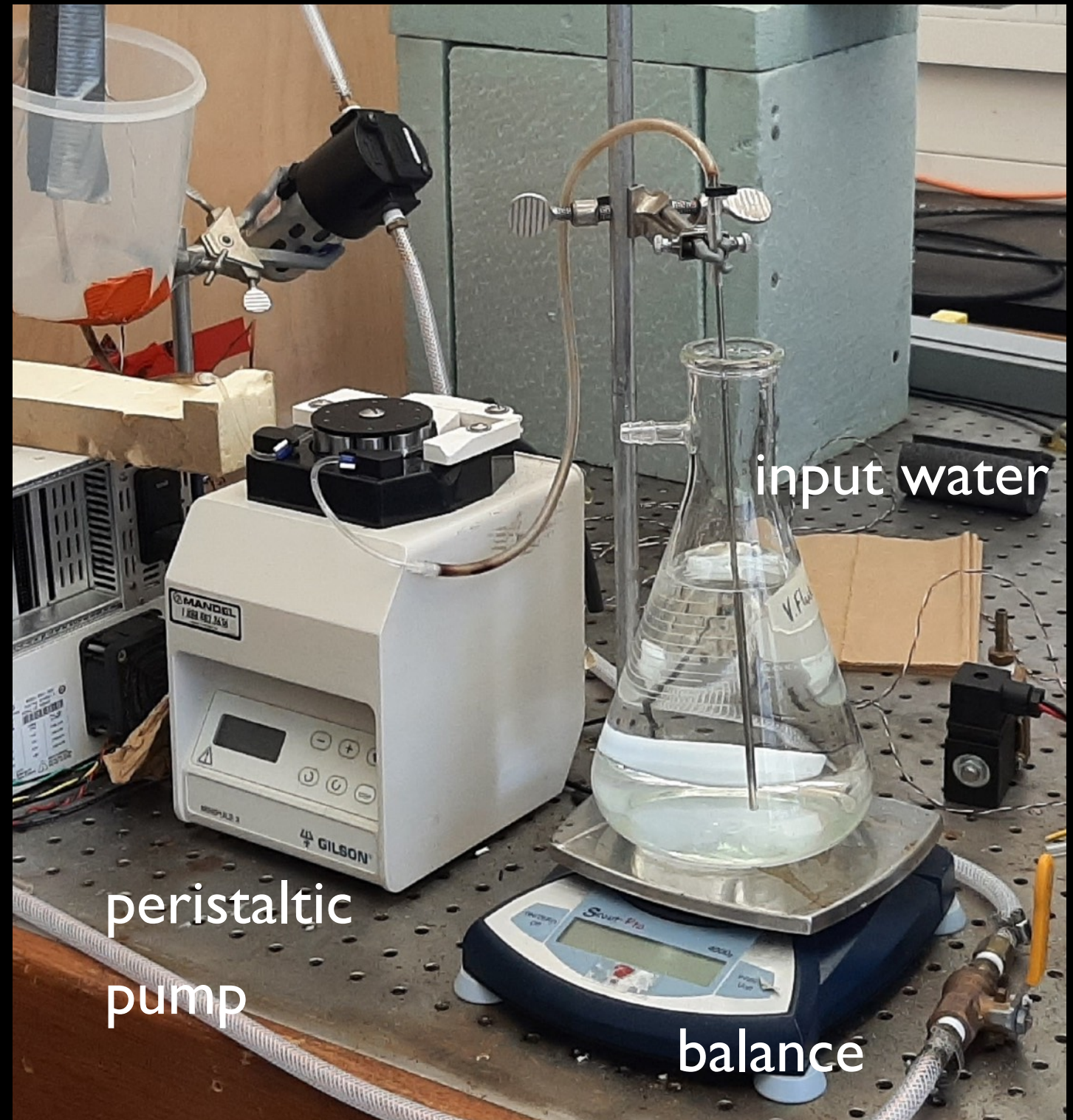
The icicle machine

Dripper nozzle and output funnel are warmed and temperature controlled by the computer



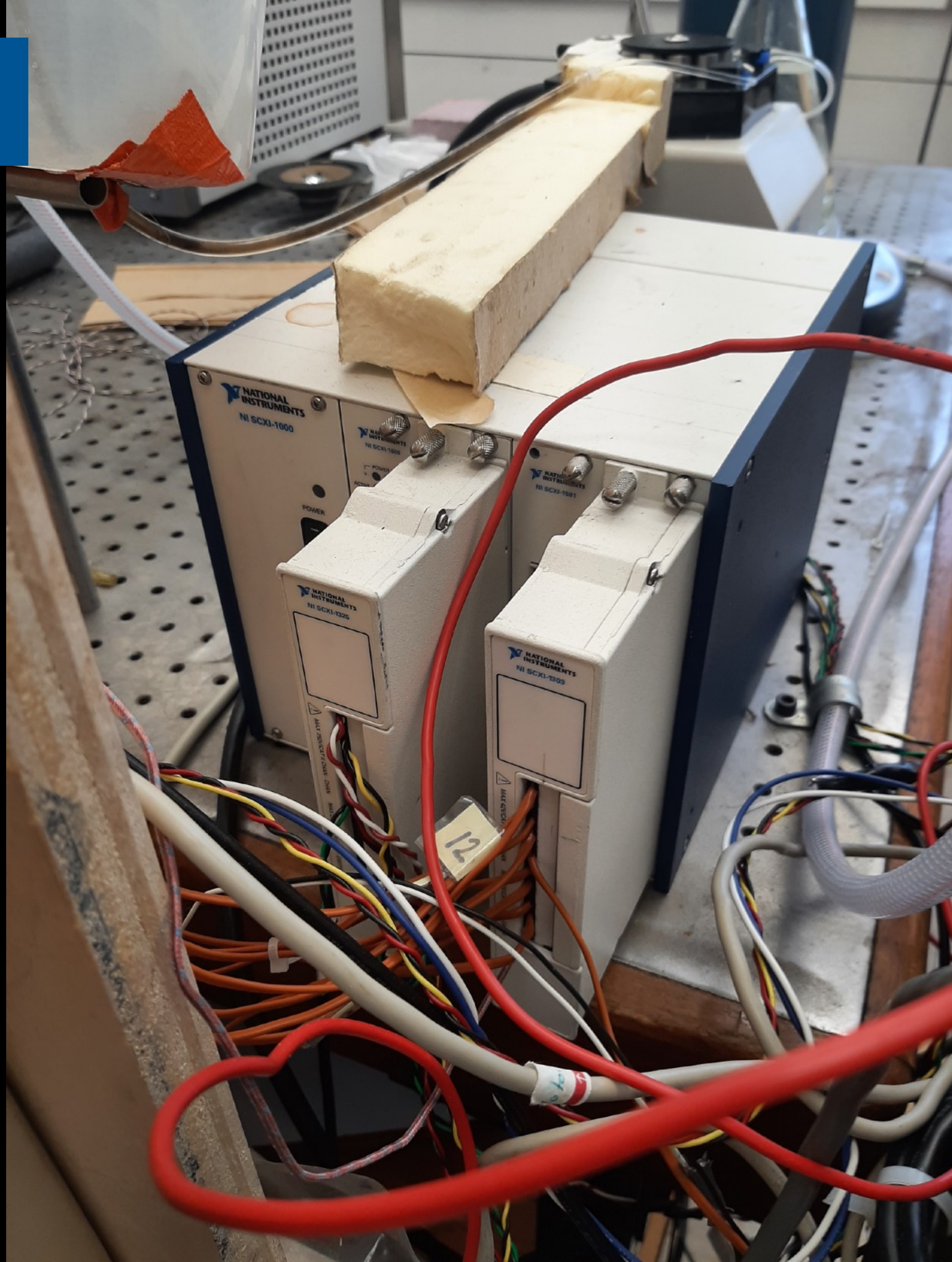
The icicle machine

Input water is continuously weighed by a balance connected to the computer, which also controls the peristaltic pump



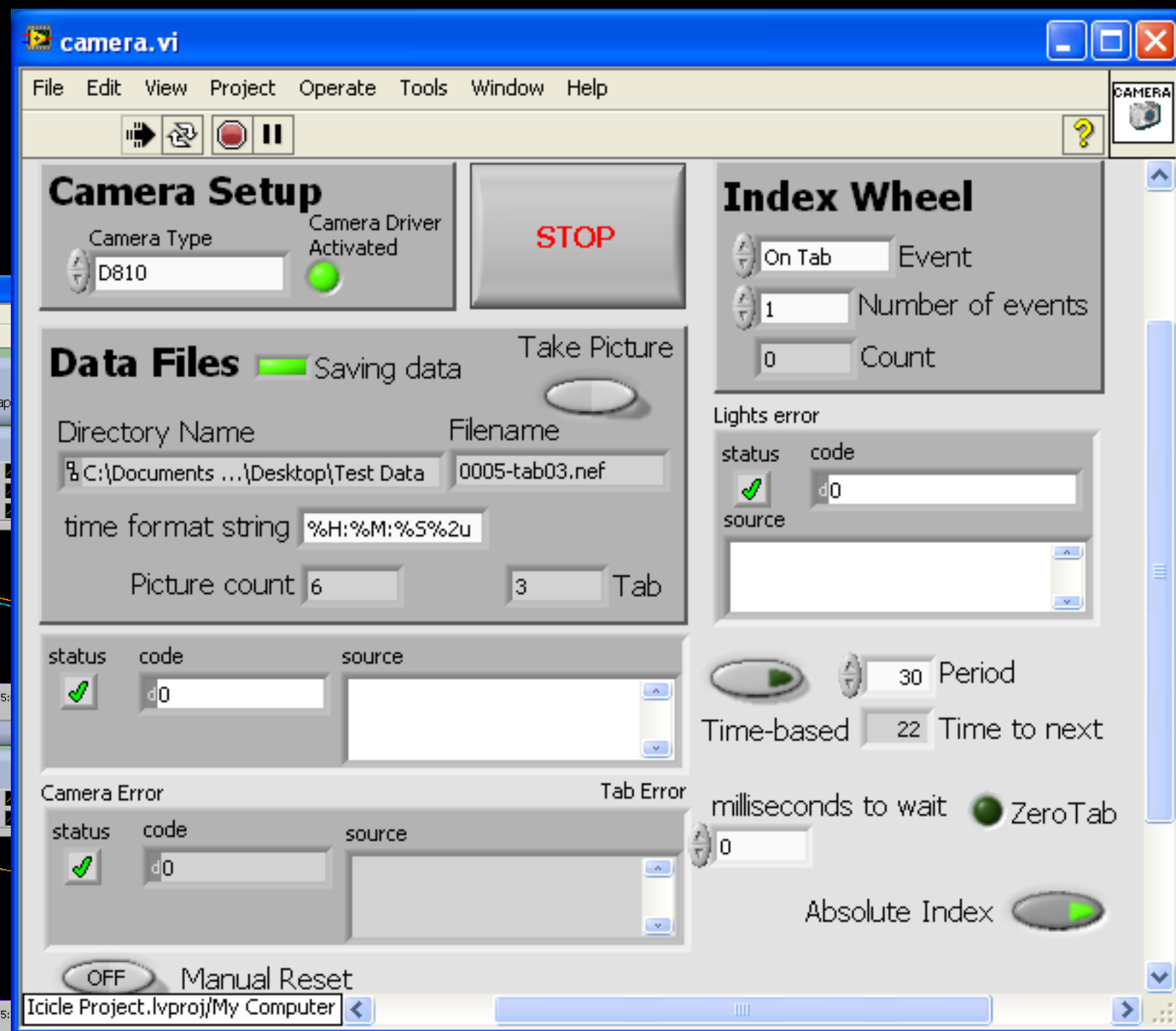
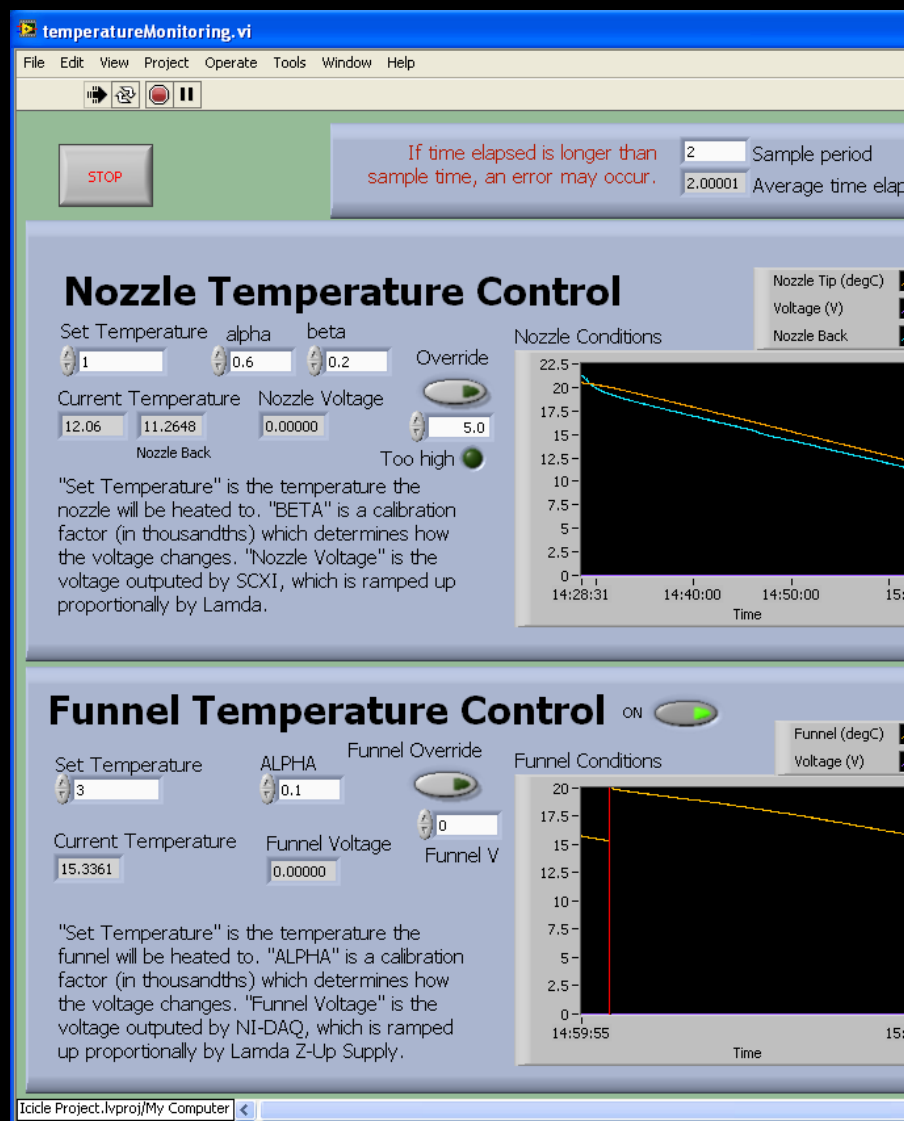
The icicle machine

Temperatures are measured with thermocouples and controlled by the computer via a *National Instruments* data acquisition system using a *LabView* interface



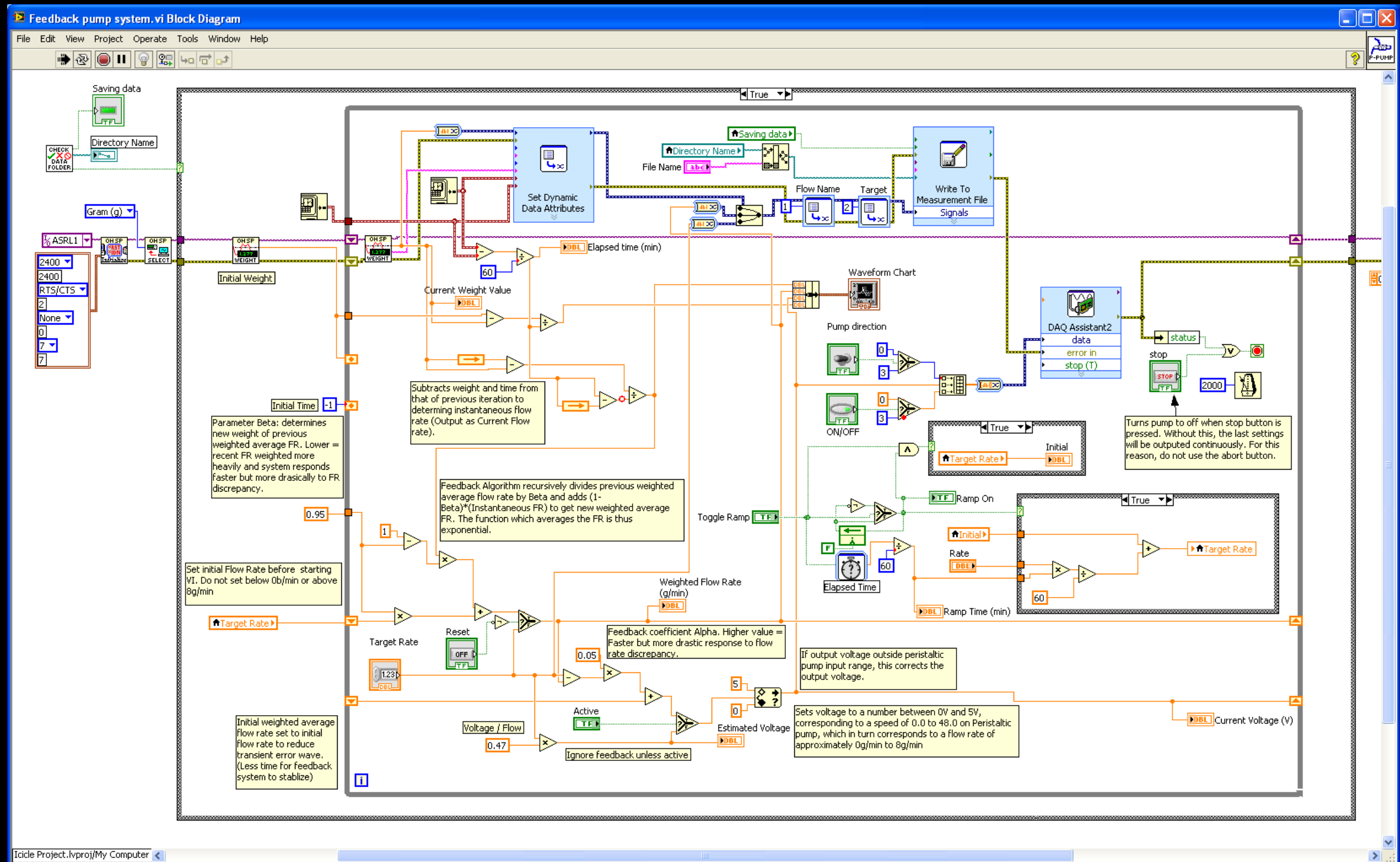
The icicle machine

LabView interfaces



The icicle machine

LabView programming

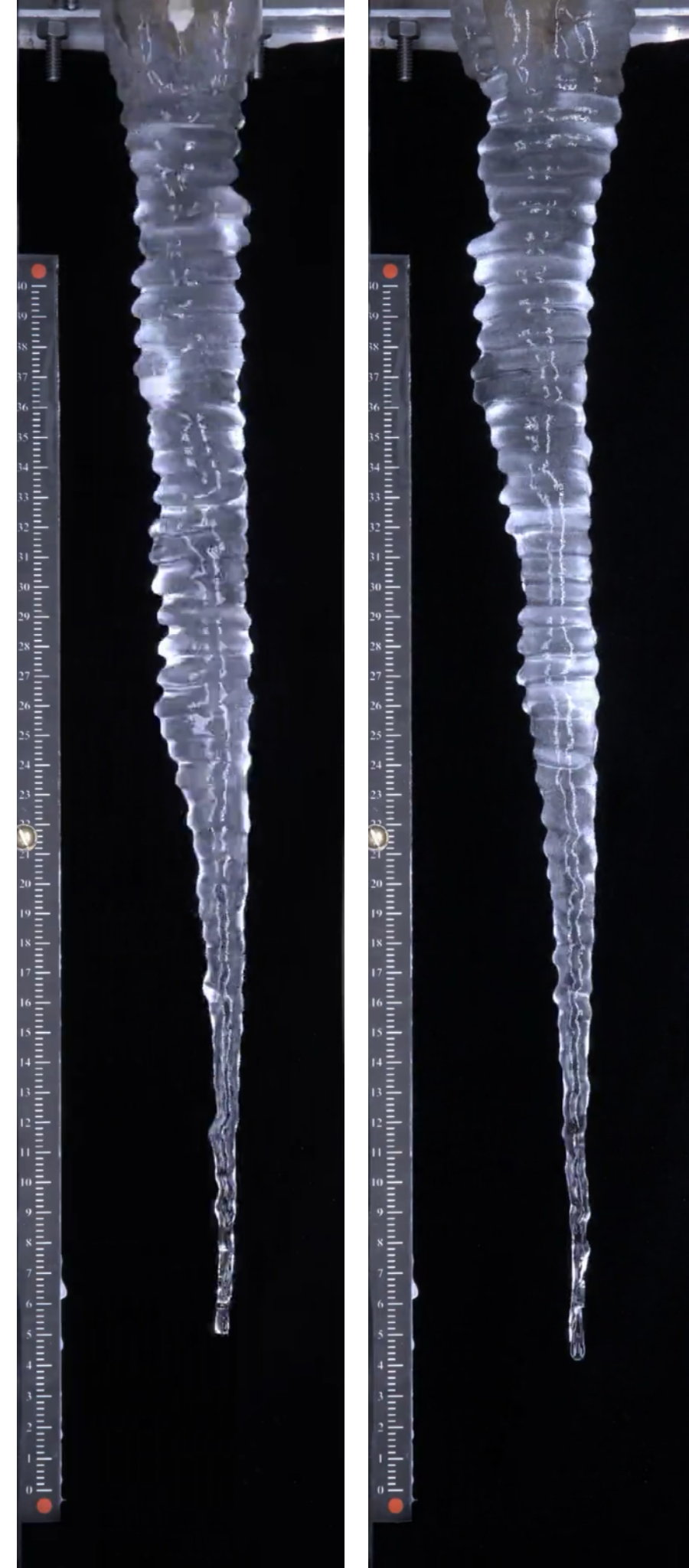


Why rotation?

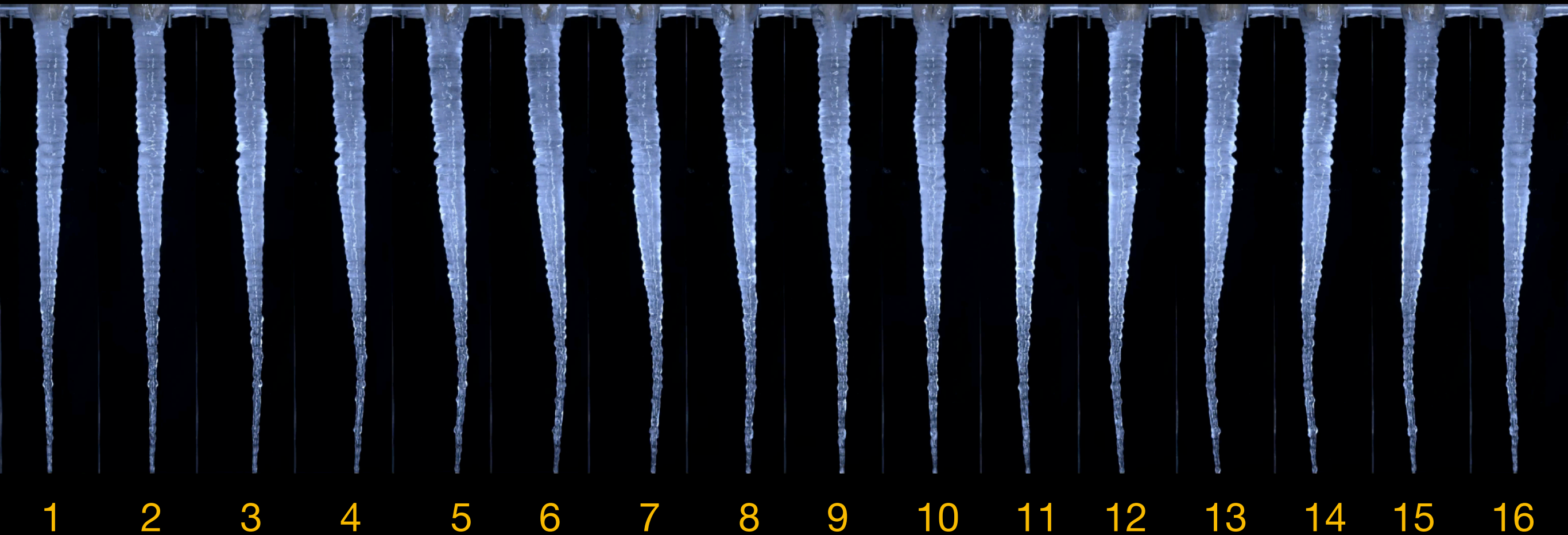
8 minute rotation is fast compared to the growth time (at least 4 hours), but slow compared to fluid mechanical time scale of air motion or water flow (~ 10 seconds).

Centrifugal effects are negligible: $\omega^2 R/g \sim 10^{-5}$

- Encourages axis-symmetry by averaging out radial temperature gradients
- Nozzle is located off axis to distribute feed water symmetrically on average, avoiding drapery *etc.*
- Full 3D shape of icicle can be inferred from rotation and edge detection of images: 8 (Chen) or 16 (Ladan) per rotation



Edge detection



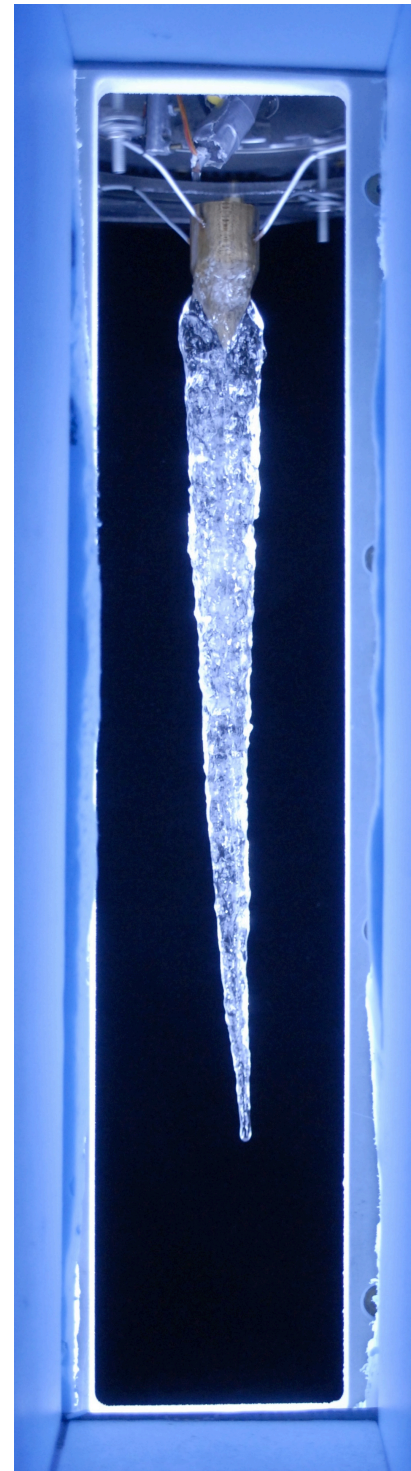
We take images at 8 (Chen) or 16 (Ladan) rotational positions so that the whole 3D shape of the icicle can be reconstructed from measurements of the edges.

Edge detection

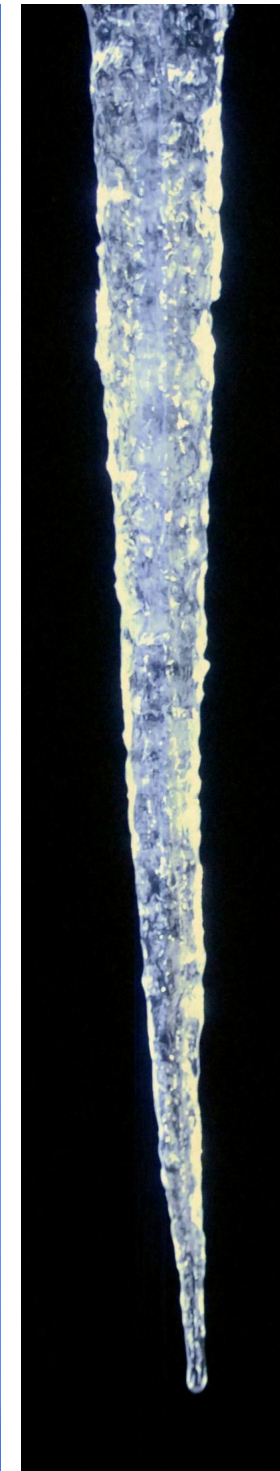
Edges can be detected from every rotational position, to single pixel resolution.

Length and left and right edges can be extracted separately.

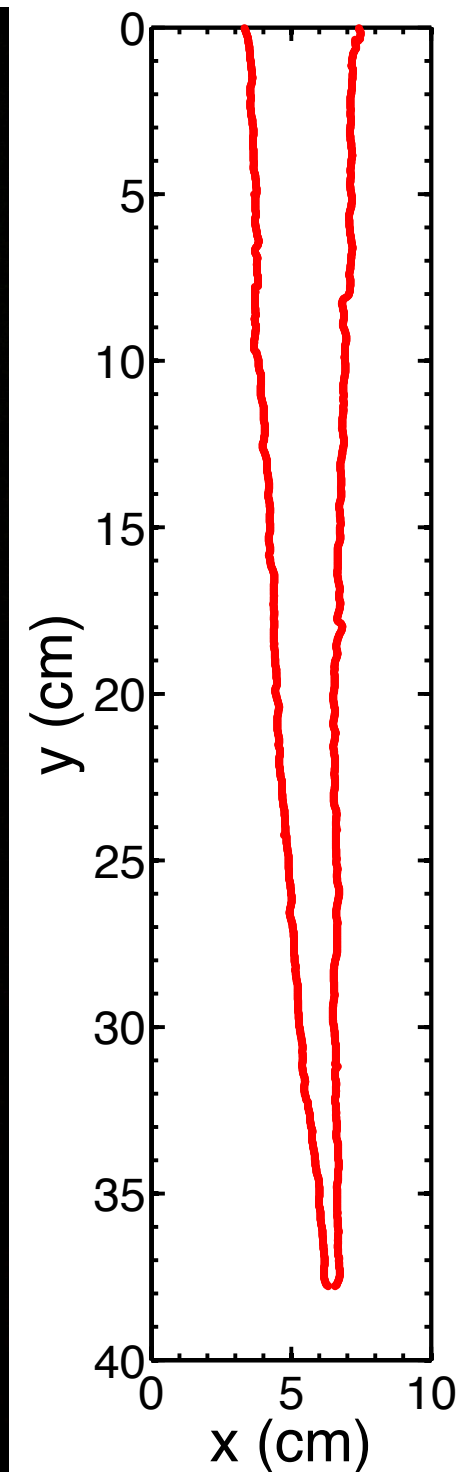
Some icicles bend: centerline is inferred from the average of the left and right edges.



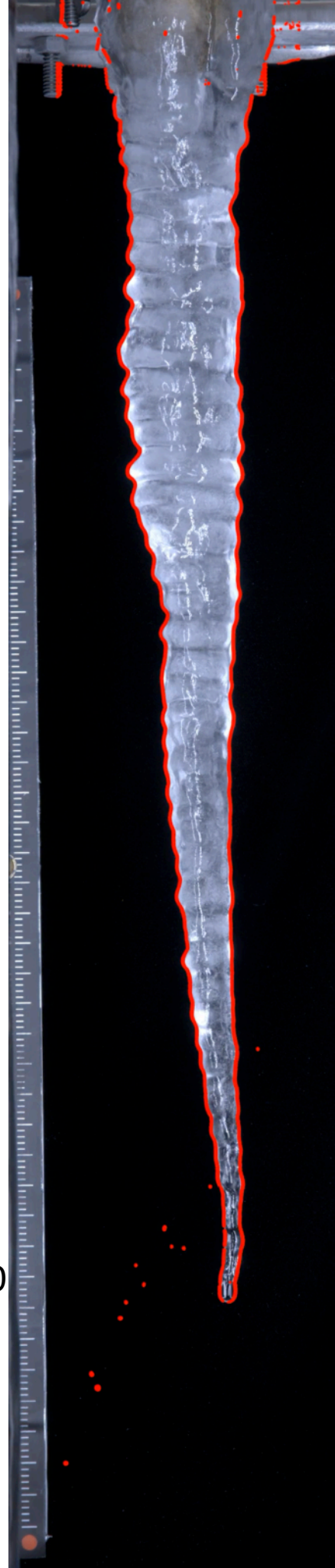
image



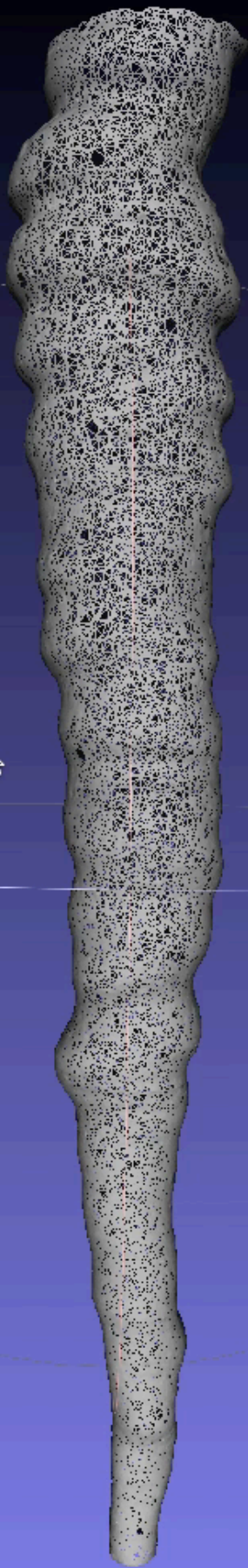
cropped
and bkg
subtracted



edges
detected



3D printable icicles



3D printable icicles

scaled down

*The Semaphore Lab
University of Toronto*



We have a **great deal** of data about the global mass, length and width growth of pure and impure icicles as a function of the basic experimental parameters

- Air temperature $-18^{\circ}\text{C} < T < -8^{\circ}\text{C}$
- Input mass flux $-0.5 \text{ g/min} < Q < 3.5 \text{ g/min}$
- Impurity concentration $(30 - 50) \mu\text{M/Kg (DW)} < c < 44\,000 \mu\text{M/Kg}$
- Impurity species: 7 species with a wide range of charges and molecular weights, including sodium fluorescein dye and a few mixtures

Less interesting parameters: Air motion, nozzle temperature, humidity, rotation ...

The parameter space is **quite large**.

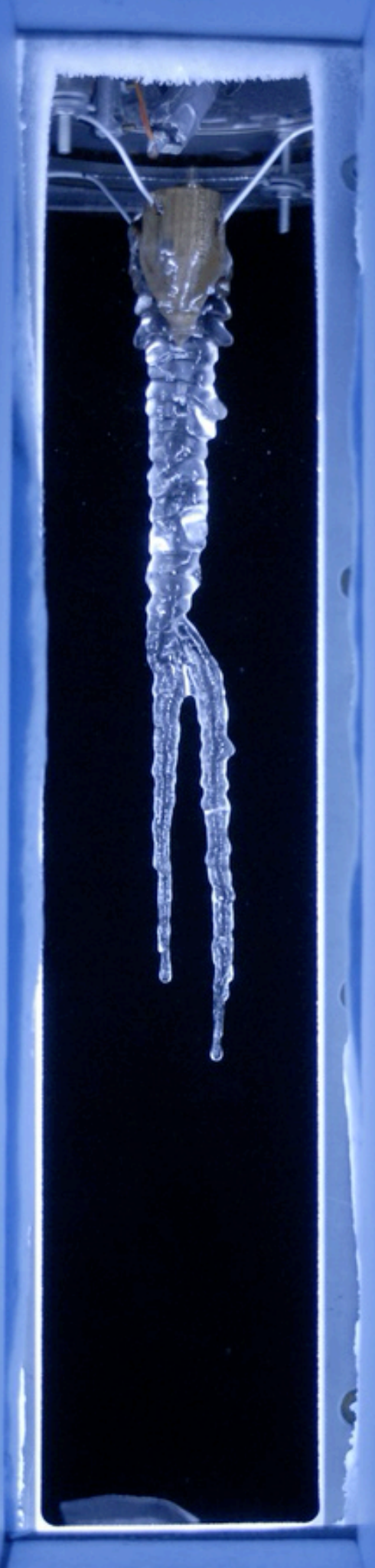
The experiment is **slow**.

Air motion

Icicles tend to form **multiple growing tips** if air is **not** stirred. “Legs” tend to emerge at 60° angles, which suggests crystal structure is involved.

Legs seem to be facilitated by a thick laminar boundary layer and suppressed by a thinner turbulent one.

Henceforth use gently turbulent air: **Fans on.**



Air motion

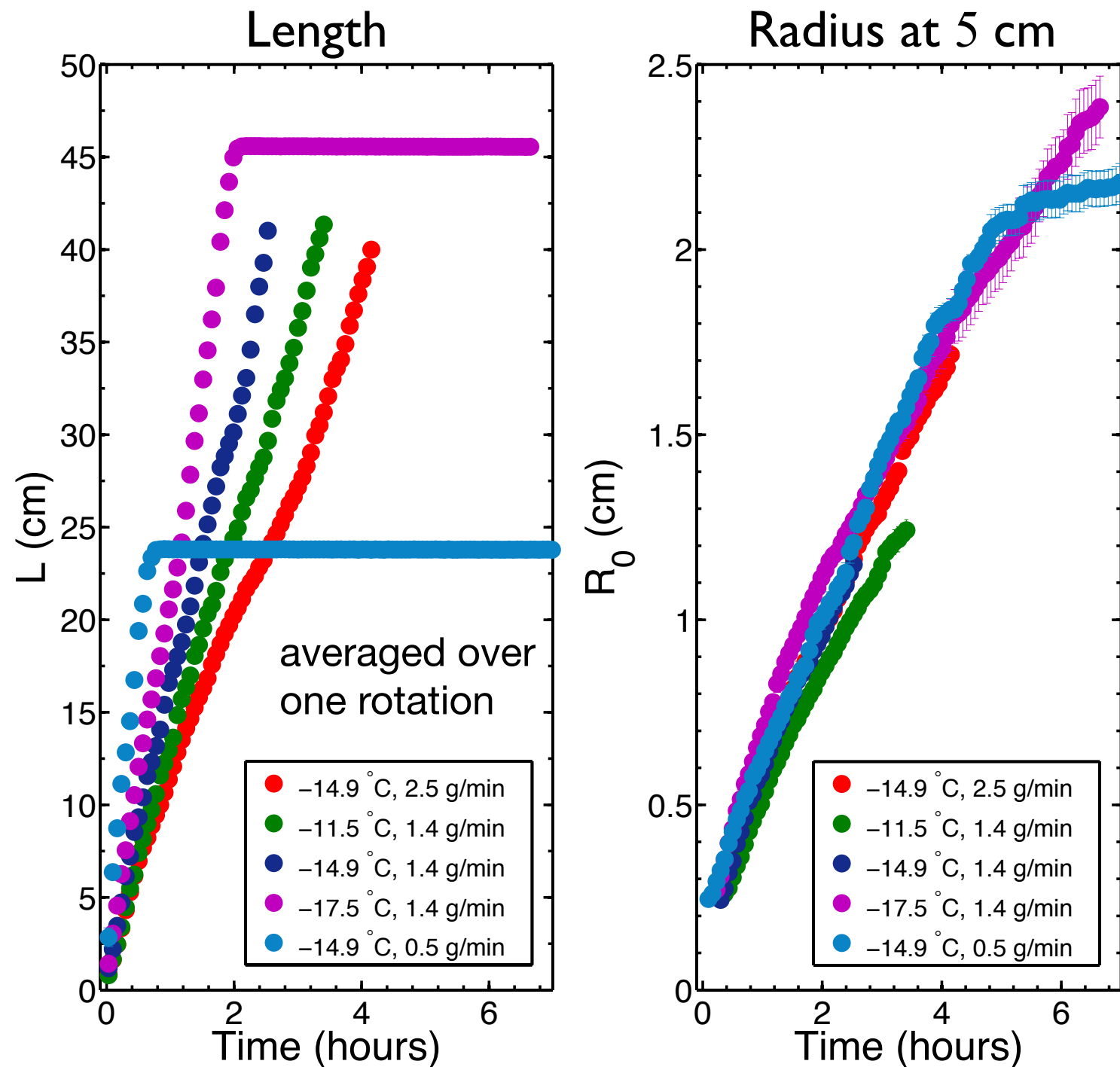
Multiple tips are
sometimes seen on
natural icicles



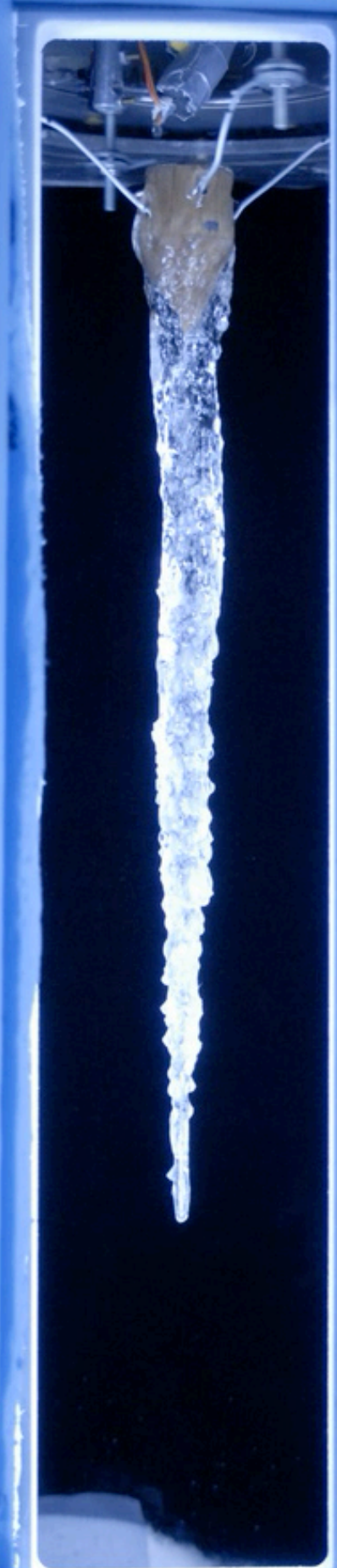
Tim Ingram,
Aberdeen UK

Growth

Distilled water icicles

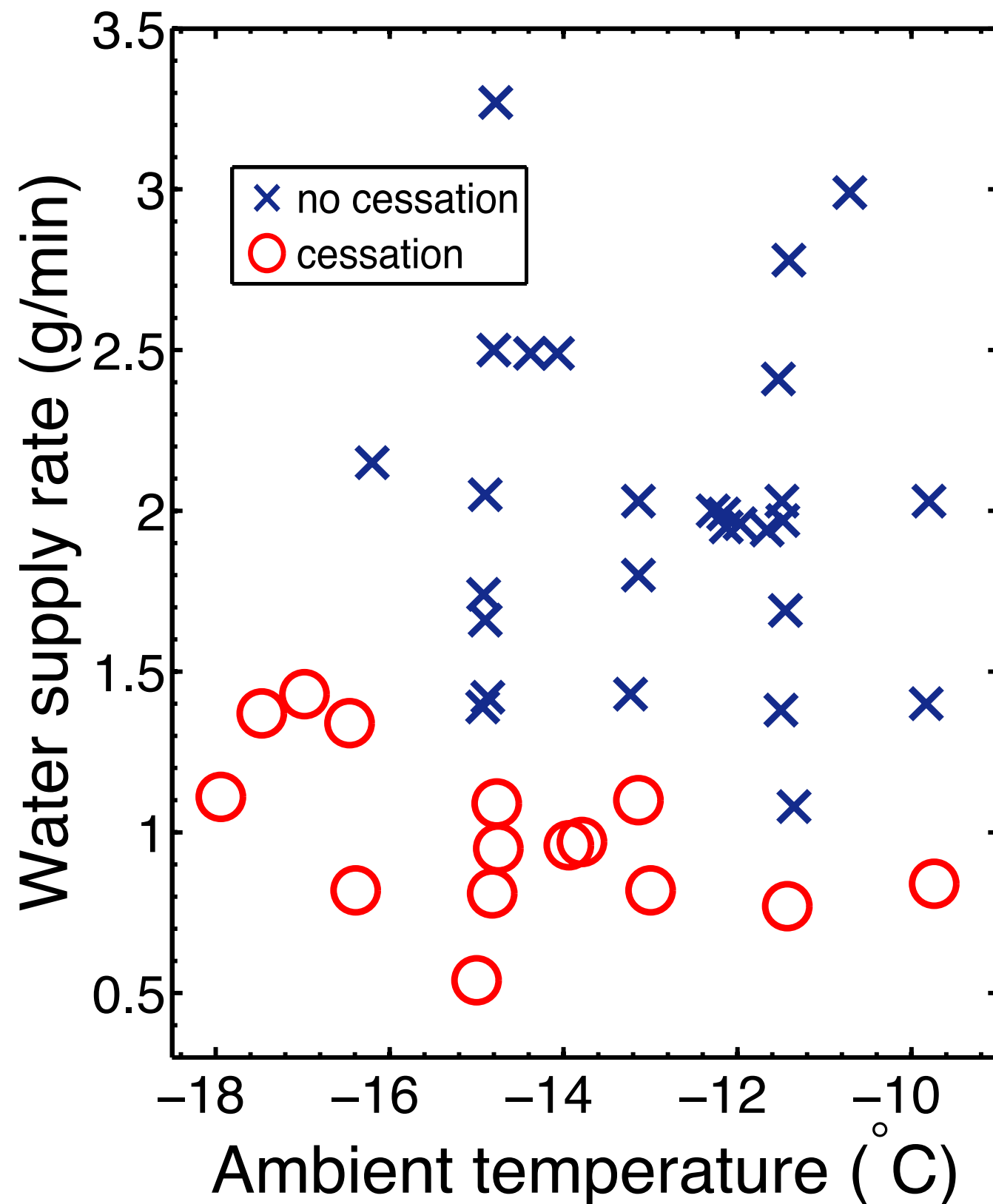


Length growth sometimes *stops*. We call this **cessation**. Radius growth rate depends only weakly on temperature and input flow rate



Cessation

Distilled water icicles



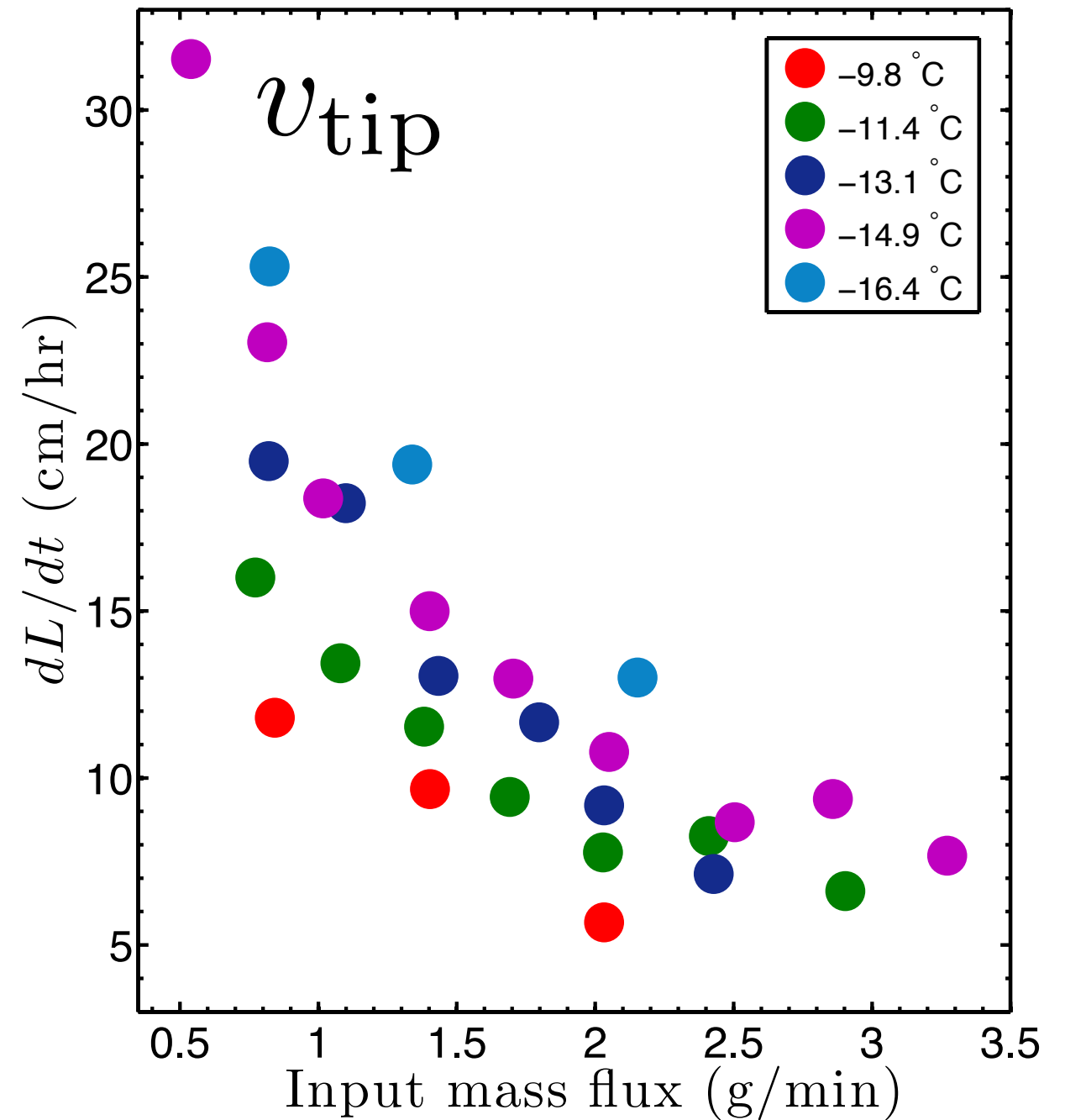
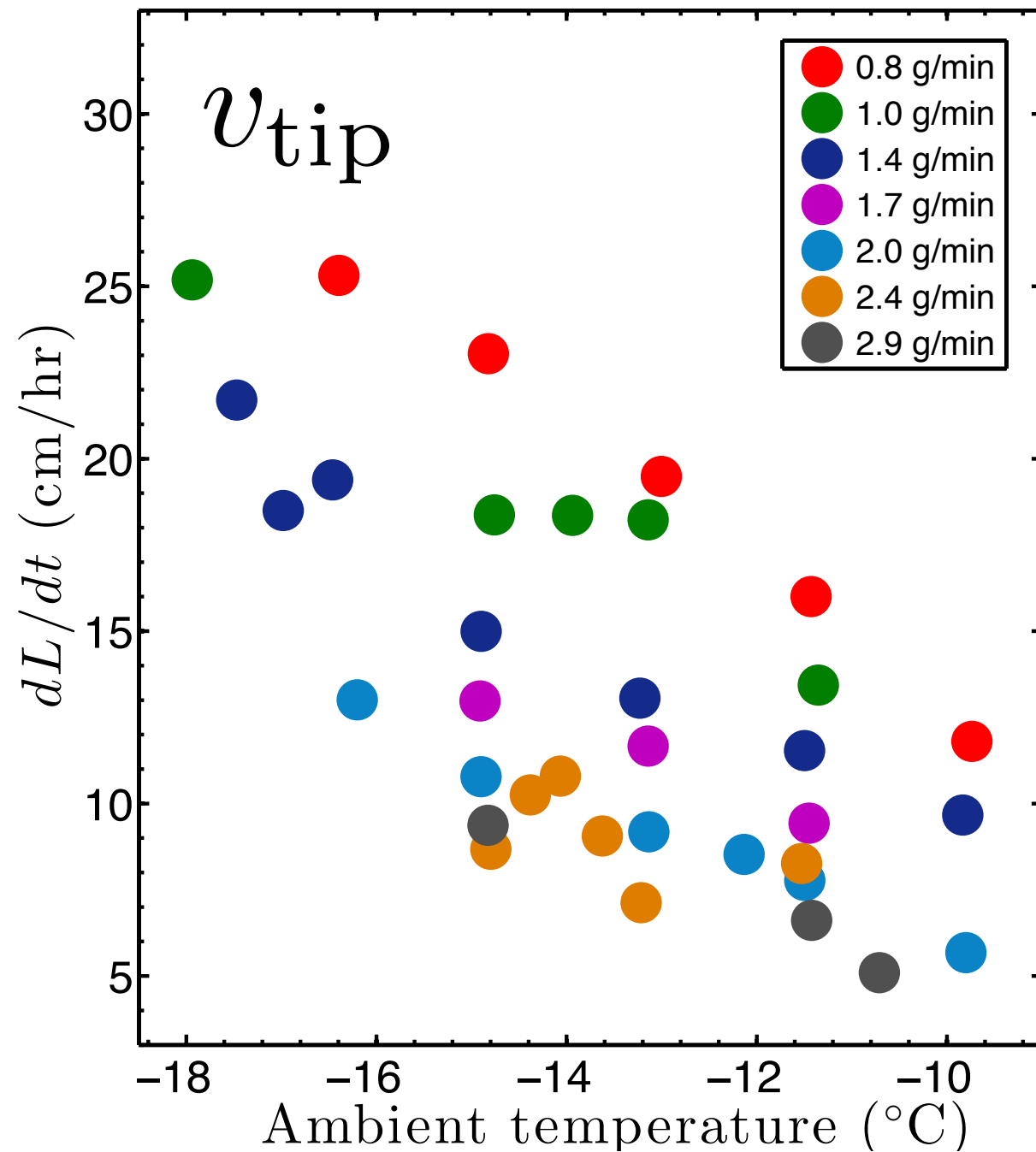
Cessation happens for low flow rate and temperature.

All icicles must undergo cessation eventually, for constant T and Q .

These are just the ones that stop within the time of the experiment.

Growth

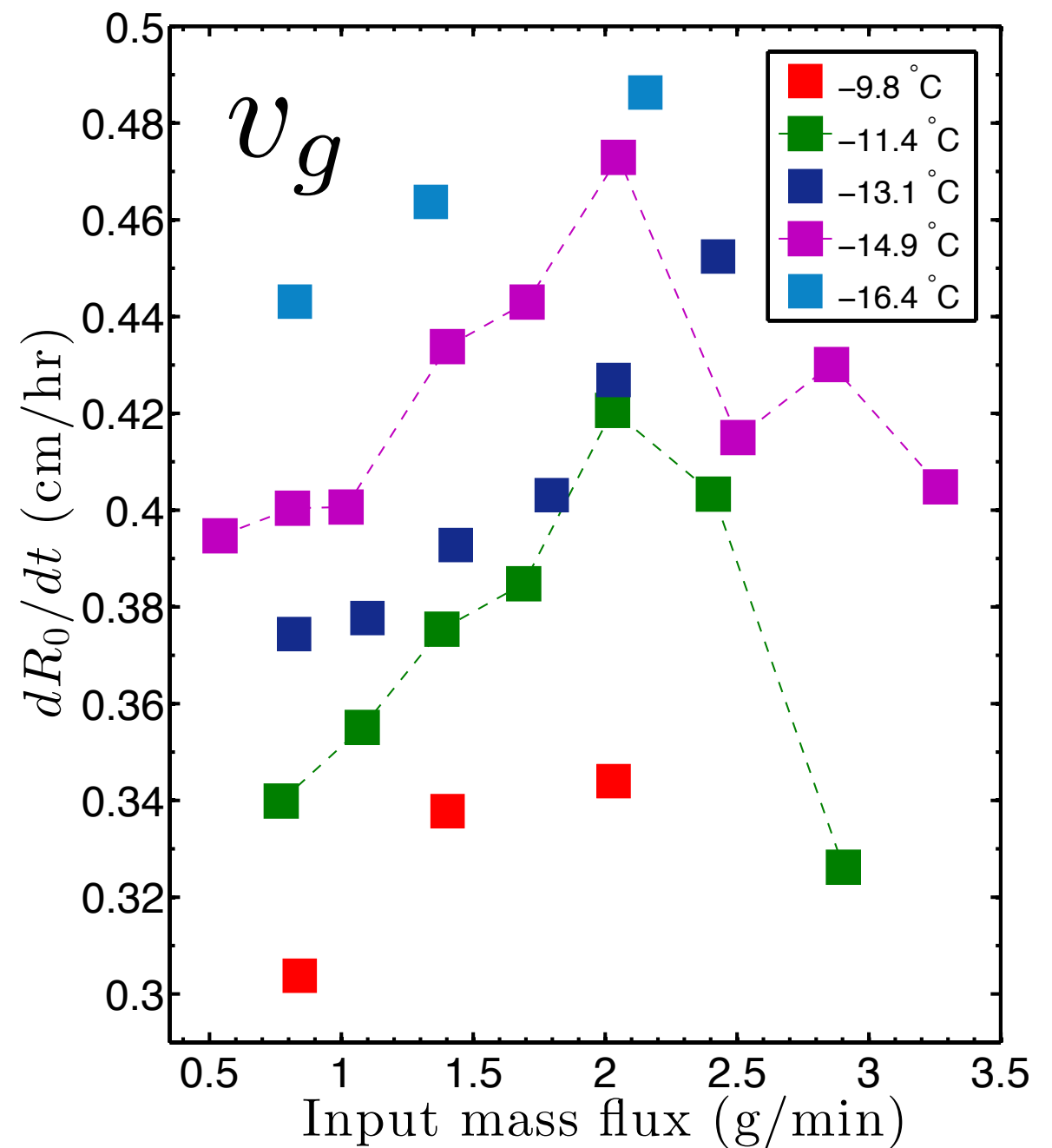
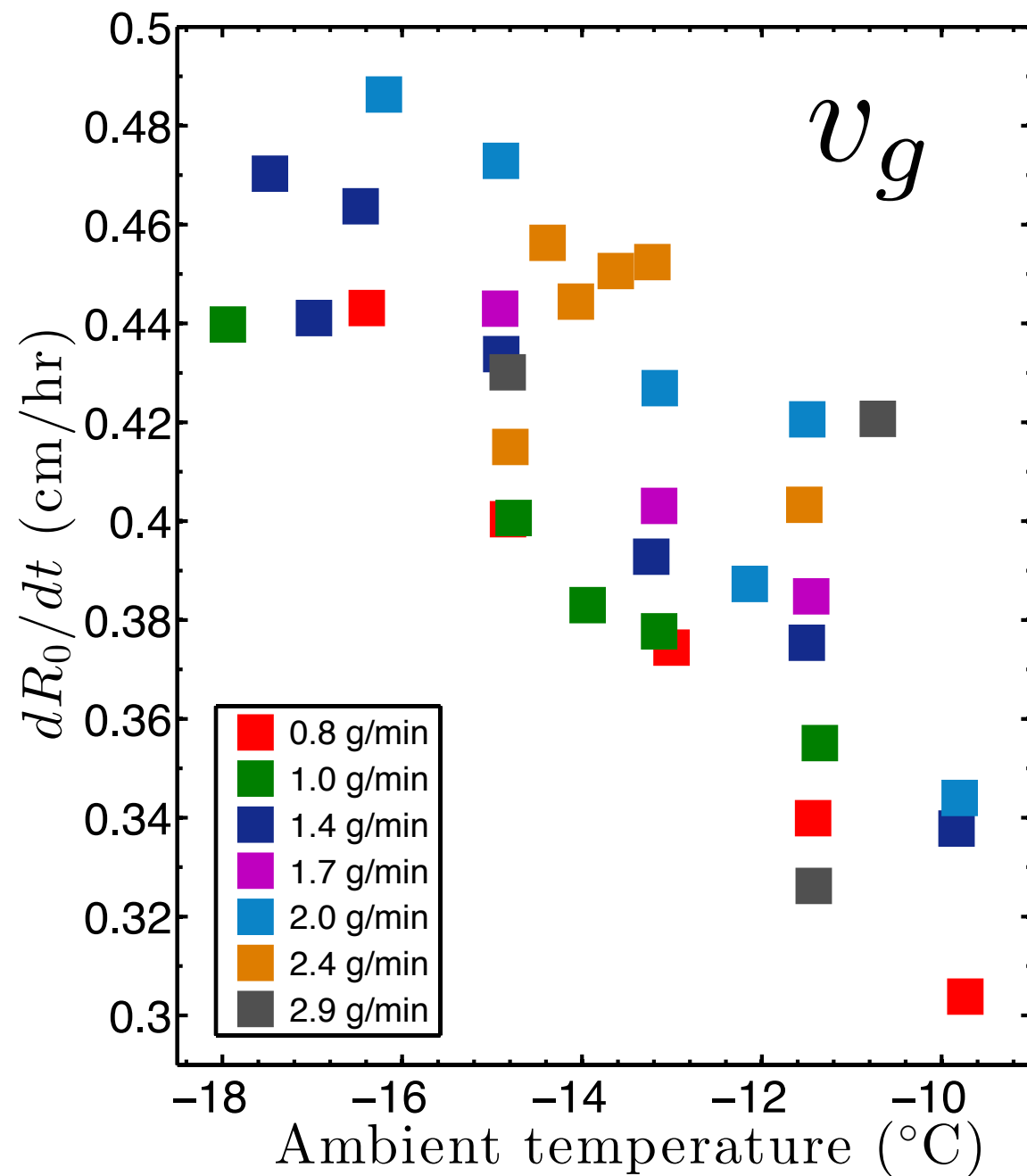
Distilled water icicles



Tip growth speed before cessation vs temperature and flow rate

Growth

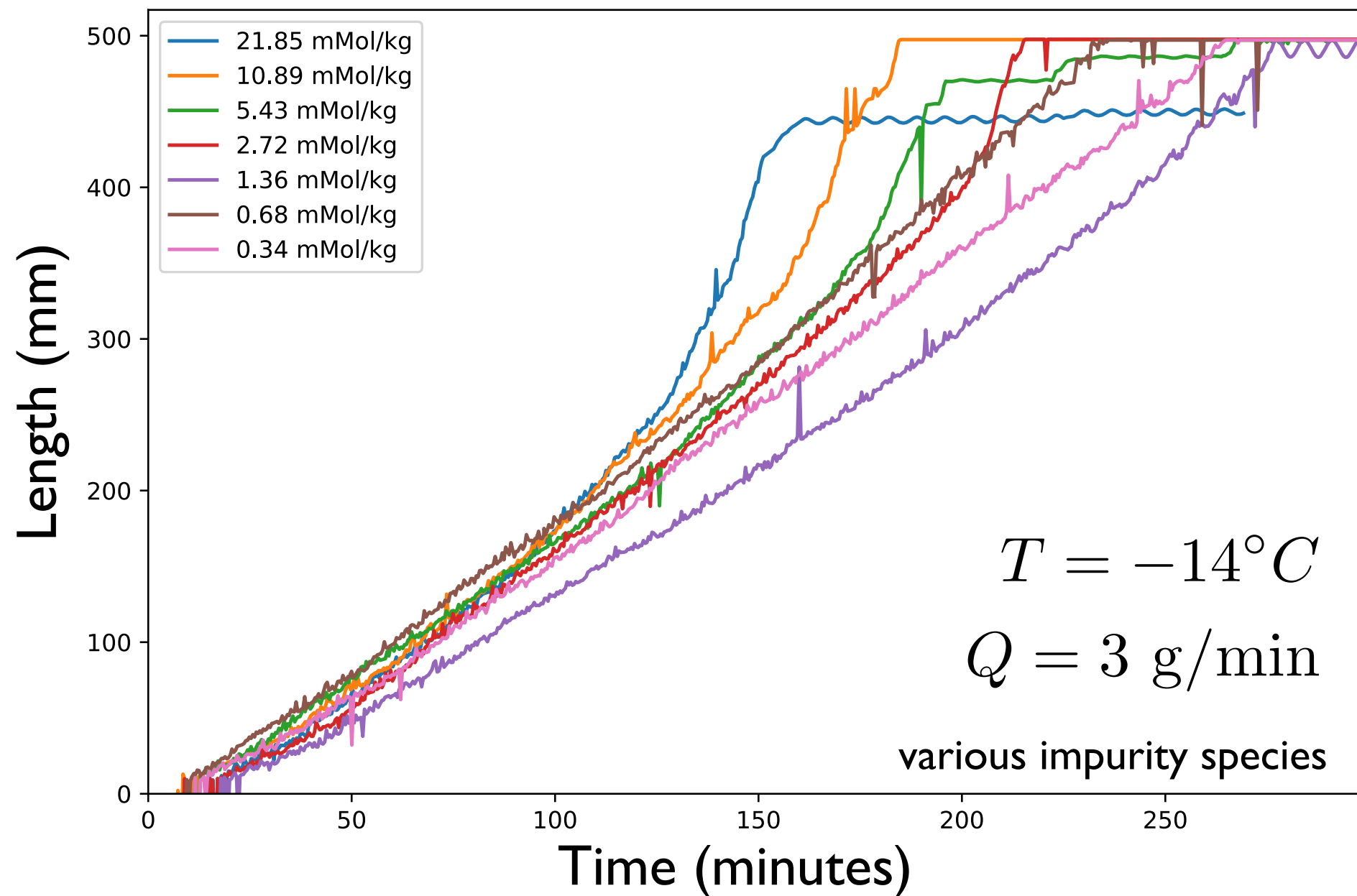
Distilled water icicles



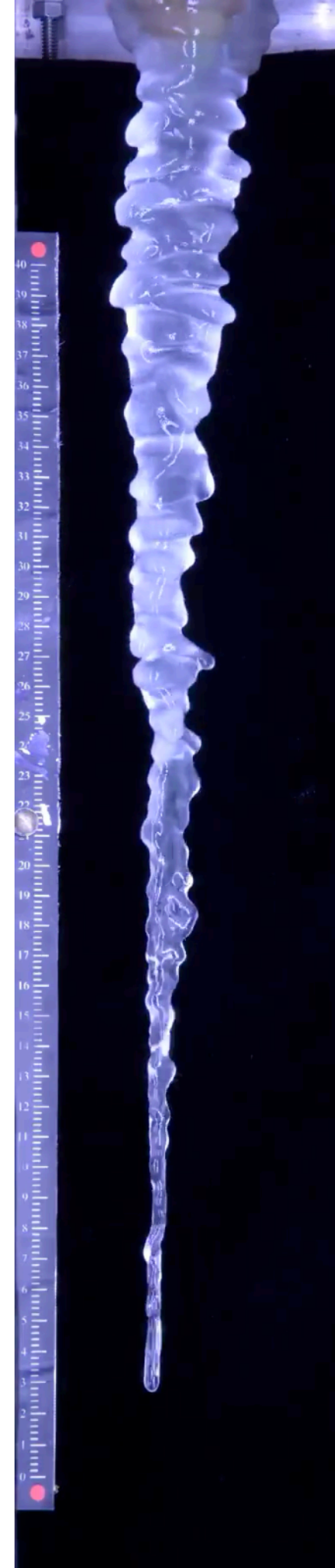
Radial growth speed at $z = 5$ cm before cessation vs temperature and flow rate

Cessation

Impure water icicles

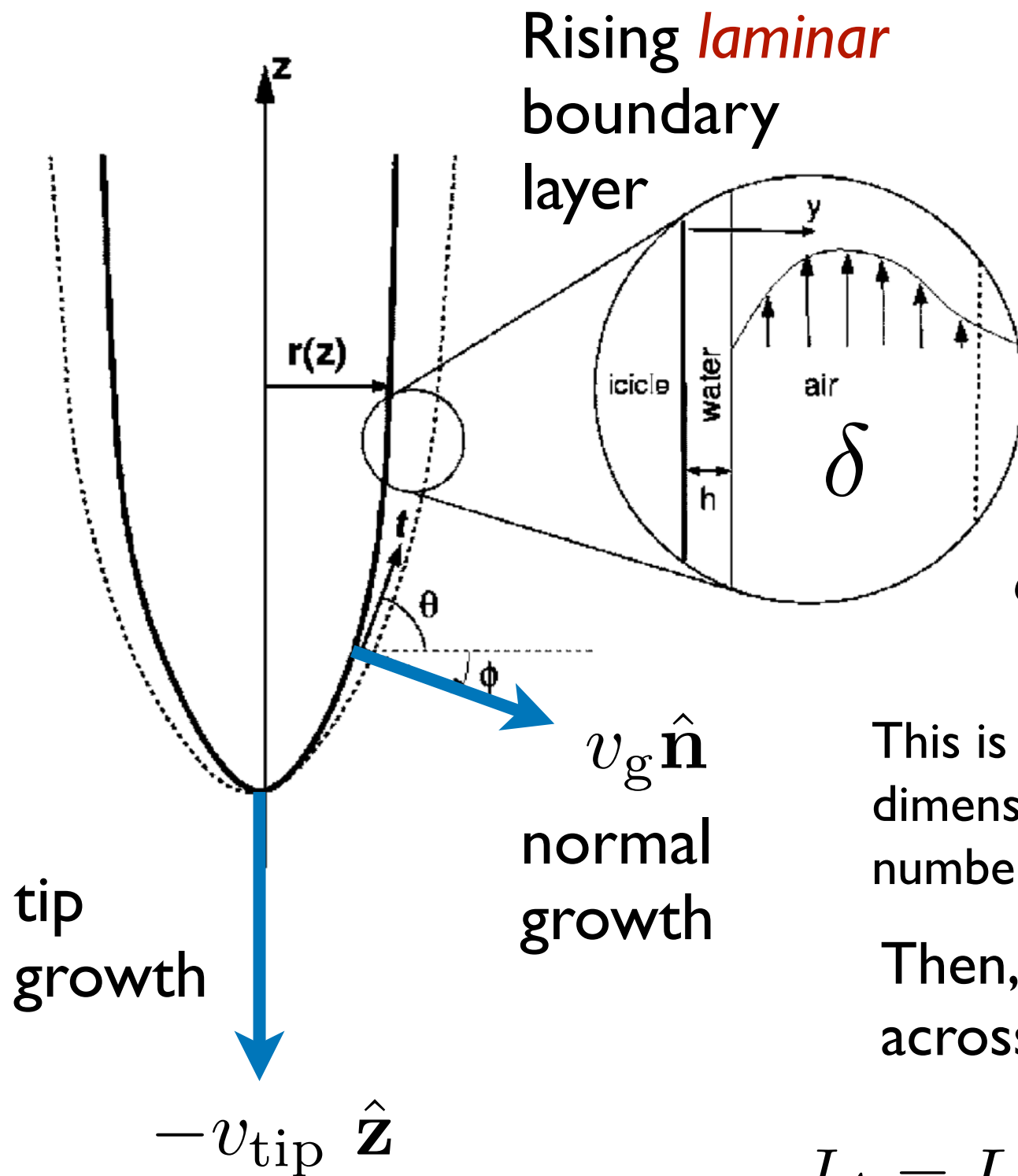


Remarkably, the tip advance of impure icicles *accelerates* before cessation, with higher concentrations beginning acceleration sooner.



Platonic icicle shape theory

M. B. Short, J. C. Baygents, R. E. Goldstein,
Phys. Fluids, **18**, 083101 (2006).



The *axisymmetric* icicle is covered by a thin film of supercooled water, but the temperature drop across the water is negligible. Ice growth is limited by heat transfer across a *laminar* boundary layer in the air with thickness

$$\delta = C\ell \left(\frac{z}{\ell} \right)^{1/4}, \quad \ell = \left(\frac{\nu_a^2}{g\beta_a \Delta T_a} \right)^{1/3}$$

This is the result for a vertical heated plate. C is a dimensionless quantity of order 1 related to the Prandtl number Pr . This will be important later.

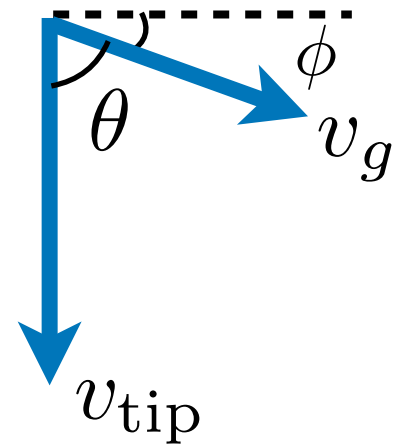
Then, using the Stefan condition, the heat flux across the layer is

$$J_A = L\rho_i v_g = \frac{\Lambda_a \Delta T_a}{\delta} = \frac{\Lambda_a \Delta T_a}{C\ell} \left(\frac{\ell}{z} \right)^{1/4}$$

Platonic icicle shape theory

*M. B. Short, J. C. Baygents, R. E. Goldstein,
Phys. Fluids, **18**, 083101 (2006).*

$$v_g = \frac{J_a}{L\rho_i} = v_c \left(\frac{\ell}{z} \right)^{1/4} \quad \text{with} \quad v_c = \frac{\Lambda_a \Delta T_a}{L\rho_i C \ell}$$



For a uniformly translating invariant shape: $v_g = v_{\text{tip}} \sin \phi$

Write $\sin \phi$ in terms of (dR/dz) and change to dimensionless variables

$\rho = R/a$ and $\zeta = z/a$ gives a **remarkable** dimensionless ODE:

Universal invariant form, with no parameters

$$\frac{d\rho}{d\zeta} = \frac{1}{\sqrt{\zeta^{\frac{1}{2}} - 1}}$$

which has solution

Platonic icicle shape

$$\rho(\zeta) = \frac{4}{3} (\zeta^{\frac{1}{2}} + 2) \sqrt{\zeta^{\frac{1}{2}} - 1}$$

$$\sim \zeta^{\frac{3}{4}} \quad \text{for small } a$$

All lengths scale with this rather strange length:

$$a = \ell \left(\frac{v_c}{v_{\text{tip}}} \right)^4 = \frac{g\beta_a (\Delta T_a)^5}{\nu_a^2} \left(\frac{\Lambda_a}{L\rho C v_{\text{tip}}} \right)^4$$

Fitting the shape

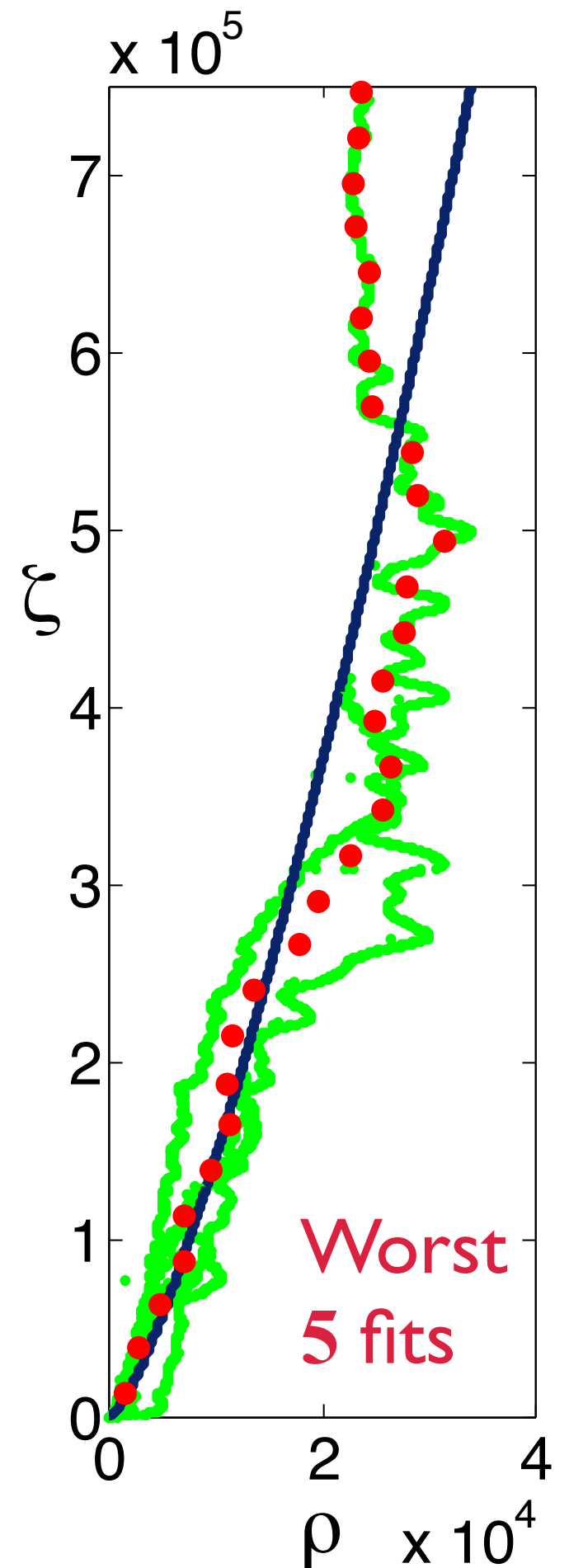
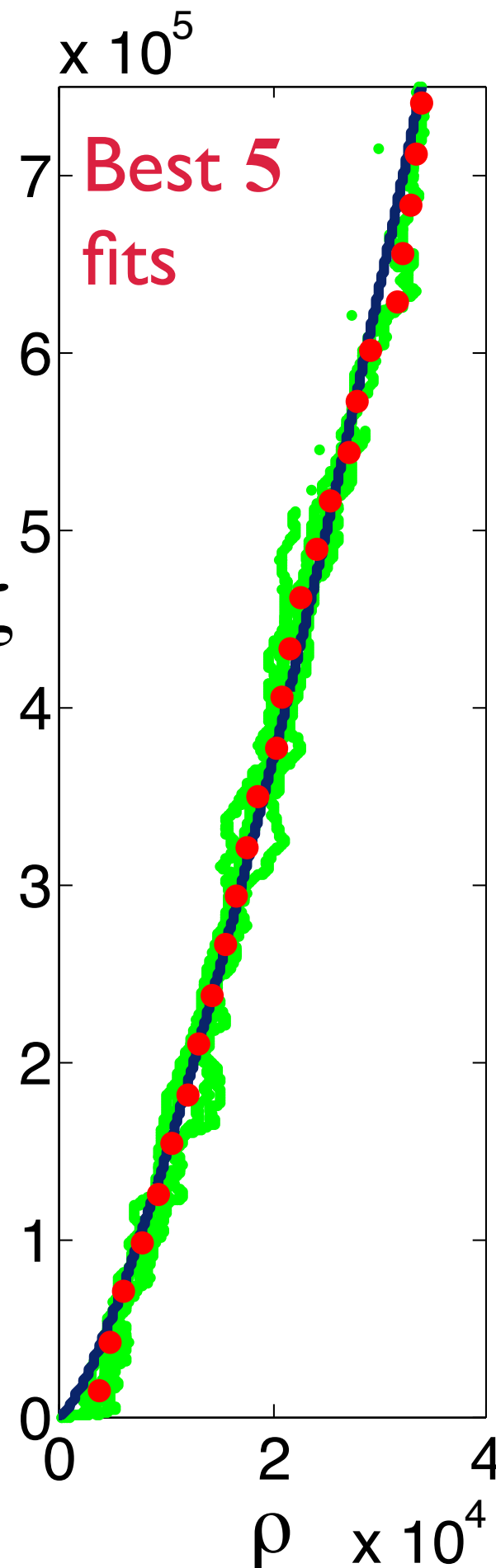
For each icicle, the profile $R(z)$ was fit to the theoretical shape:

$$R = a \left(\frac{4}{3} \left[\left(\frac{z}{a} \right)^{\frac{1}{2}} + 2 \right] \sqrt{ \left(\frac{z}{a} \right)^{\frac{1}{2}} - 1 } \right) \zeta$$

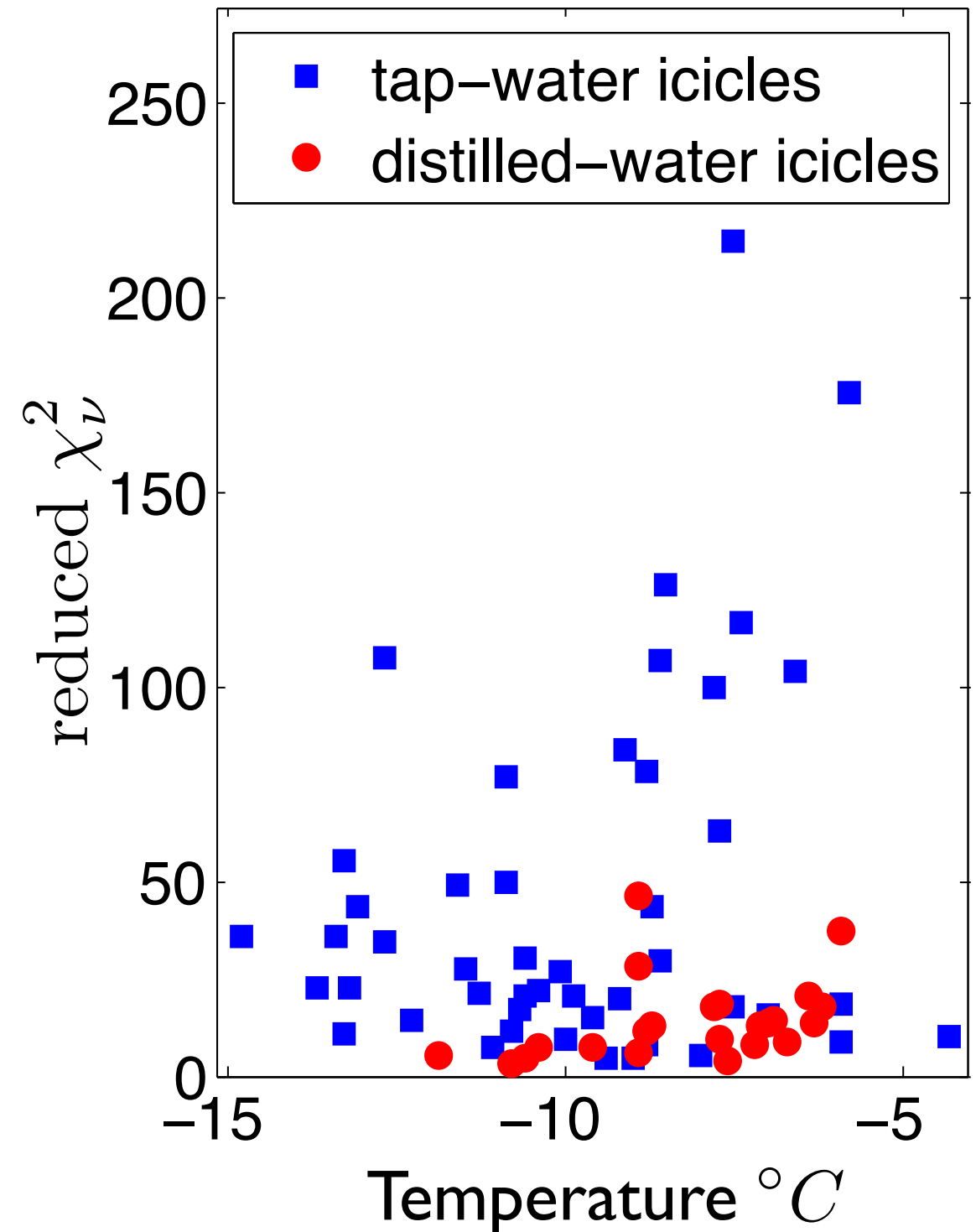
via a least-squares analysis to get the best-fit scaling factor a .

Fitting over 100 icicles, we find that *some* icicles fit the theory to a very remarkable degree, while others *do not*.

A. S. Chen and S.W. Morris,
Phys. Rev. E 83, 026307 (2011).



Not-so-platonic icicles

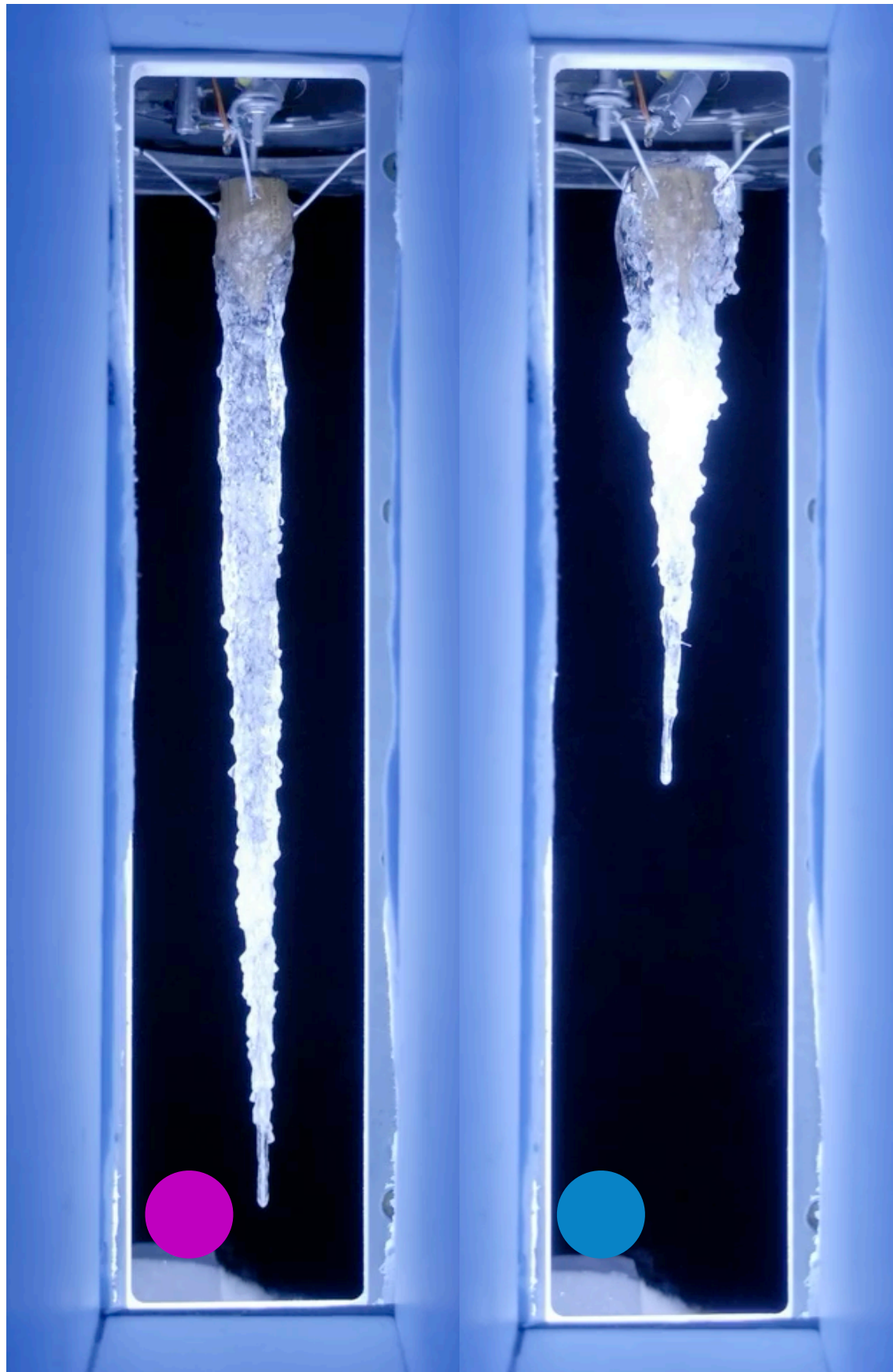


distilled water

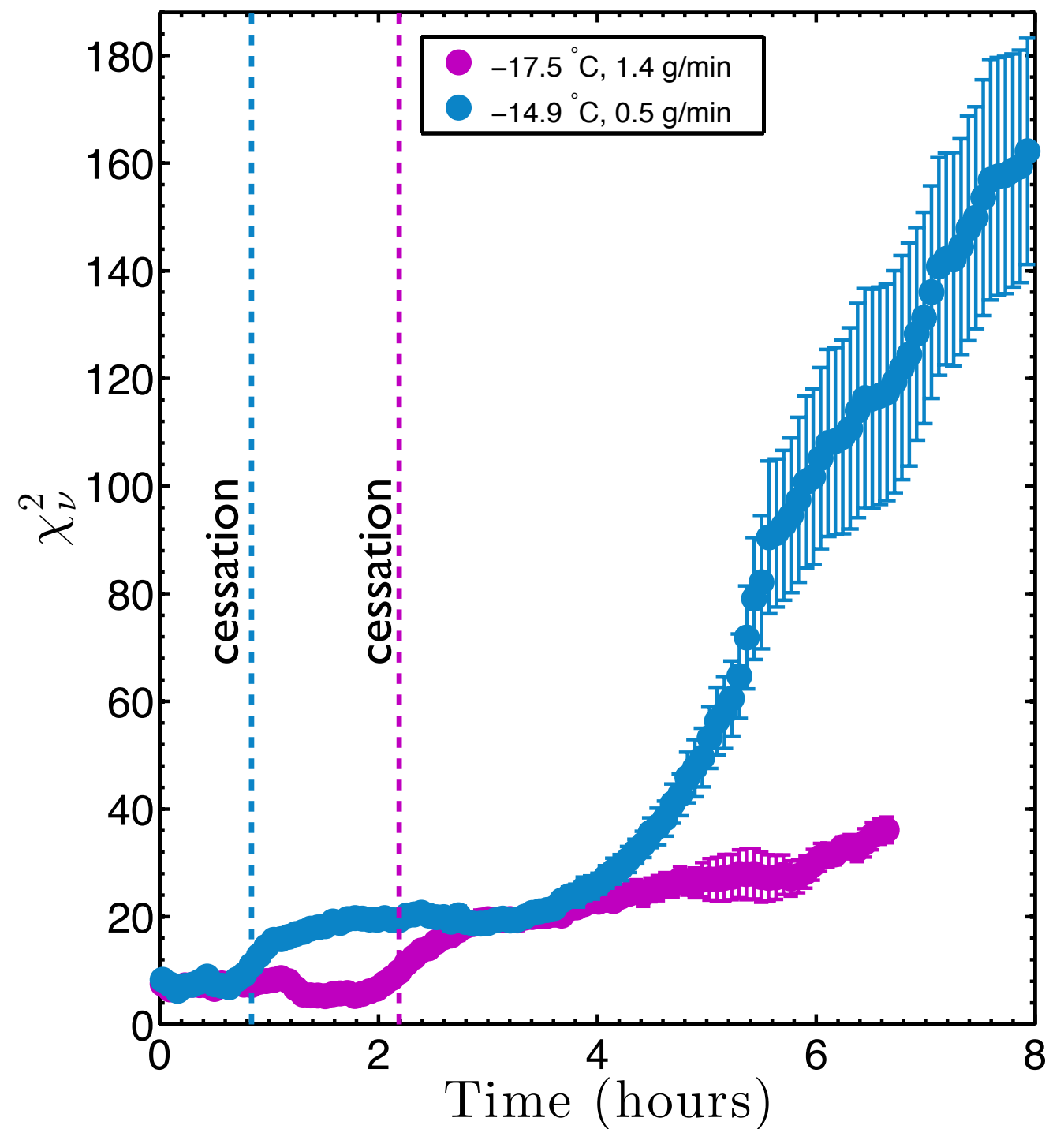
tap water

Water purity is the main factor determining *platonic-ness*

Non-platonic cessation



Distilled water icicles



Icicles quickly become non-platonic when they undergo cessation

The scale factor a

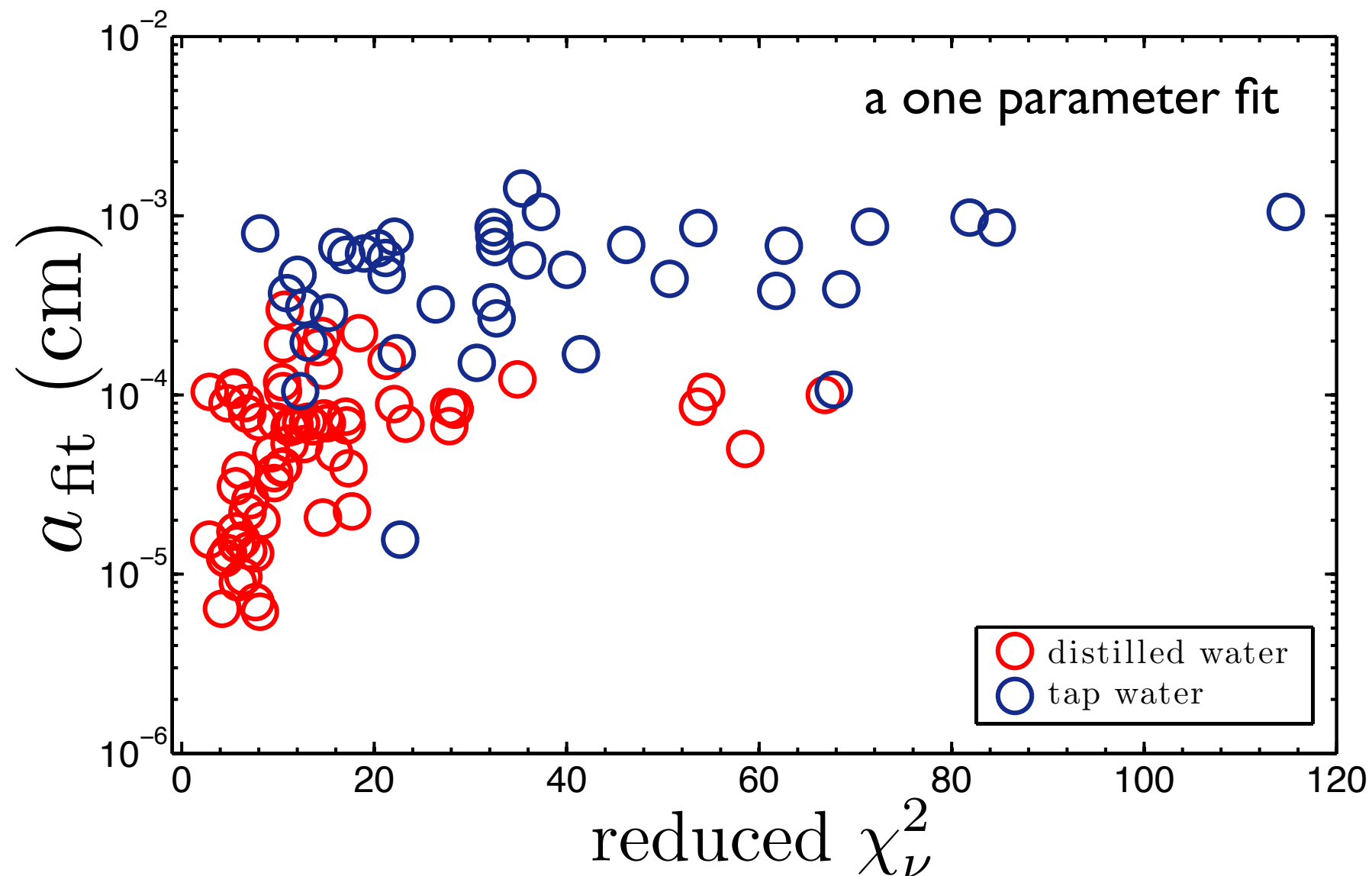
$$a_{\text{theory}} = \frac{g\beta_a(\Delta T_a)^5}{\nu_a^2} \left(\frac{\Lambda_a}{L\rho C v_{\text{tip}}} \right)^4$$

Putting in typical numbers, we find $a_{\text{theory}} \sim 2 \times 10^{-5} \text{ (cm)}$
so a is indeed **very very small**.

The values of a that emerge from fitting the shape are also **very very small** — roughly in the same ballpark.

Since a is so small, we expect that we should be in the small a power law limit:

$$\rho(\zeta) = \frac{4}{3} \zeta^{\frac{3}{4}}$$

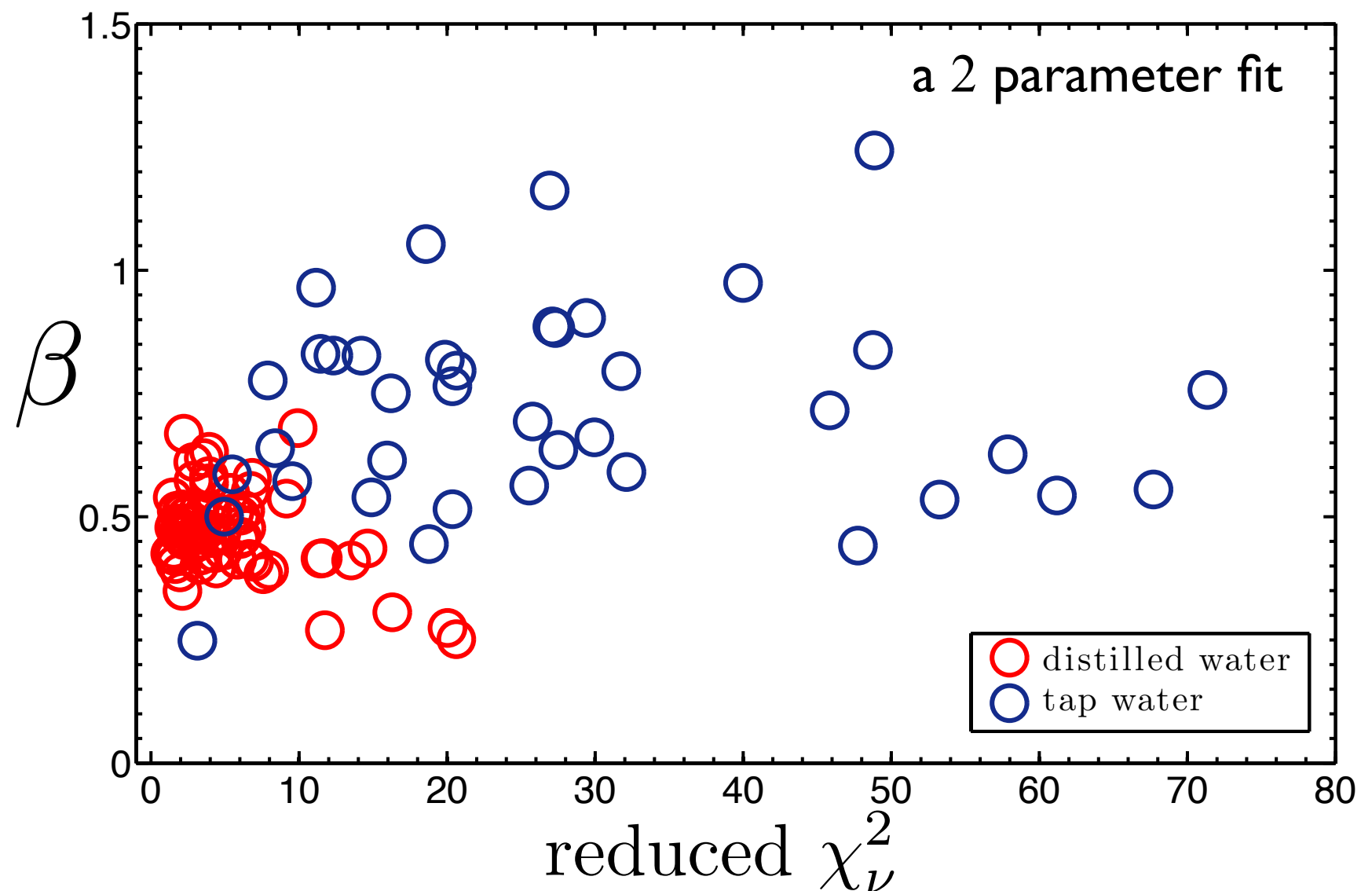


Fit shape to a general power law

$$R = \alpha z^\beta$$

The general power law, being a 2 parameter fit, gives a tighter fit to the data than the one-parameter platonic shape.

But the best fit for the power β clusters nearer to $1/2$ than the expected $3/4$.



tighter scale on this axis than previous

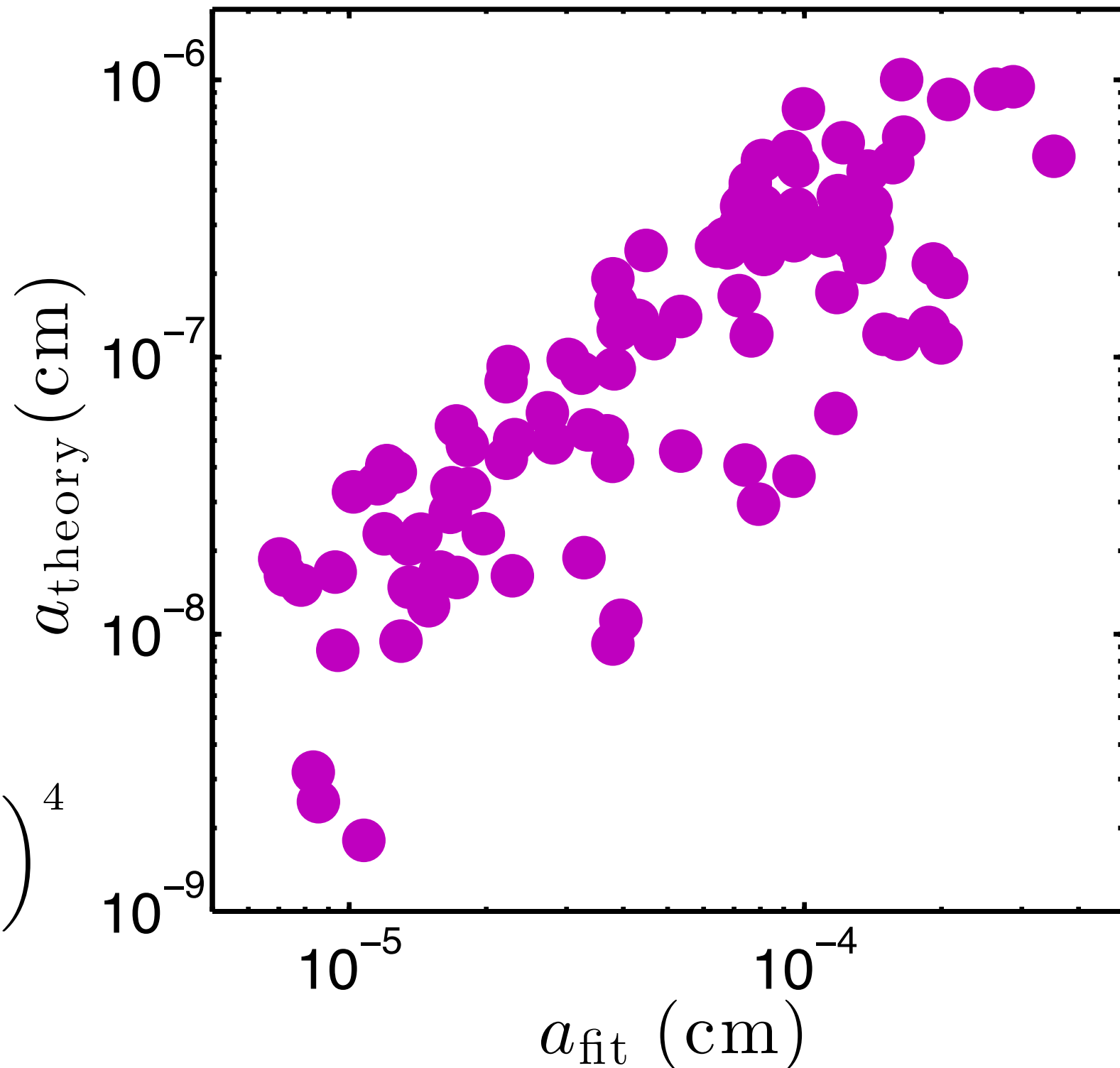
The scale factor a

Direct comparison of a_{theory} and a_{fit}

We find that a_{theory} and a_{fit} are positively correlated but that a_{theory} is 2 or 3 orders of magnitude smaller than a_{fit} , taking $C = 1$.

Since C is raised to the 4th power in the expression for a , this discrepancy could be removed by taking $C = 0.2$.

$$a_{\text{theory}} = \frac{g\beta_a(\Delta T_a)^5}{\nu_a^2} \left(\frac{\Lambda_a}{L\rho C v_{\text{tip}}} \right)^4$$



The scale factor a

Direct test of the scaling of a_{fit}

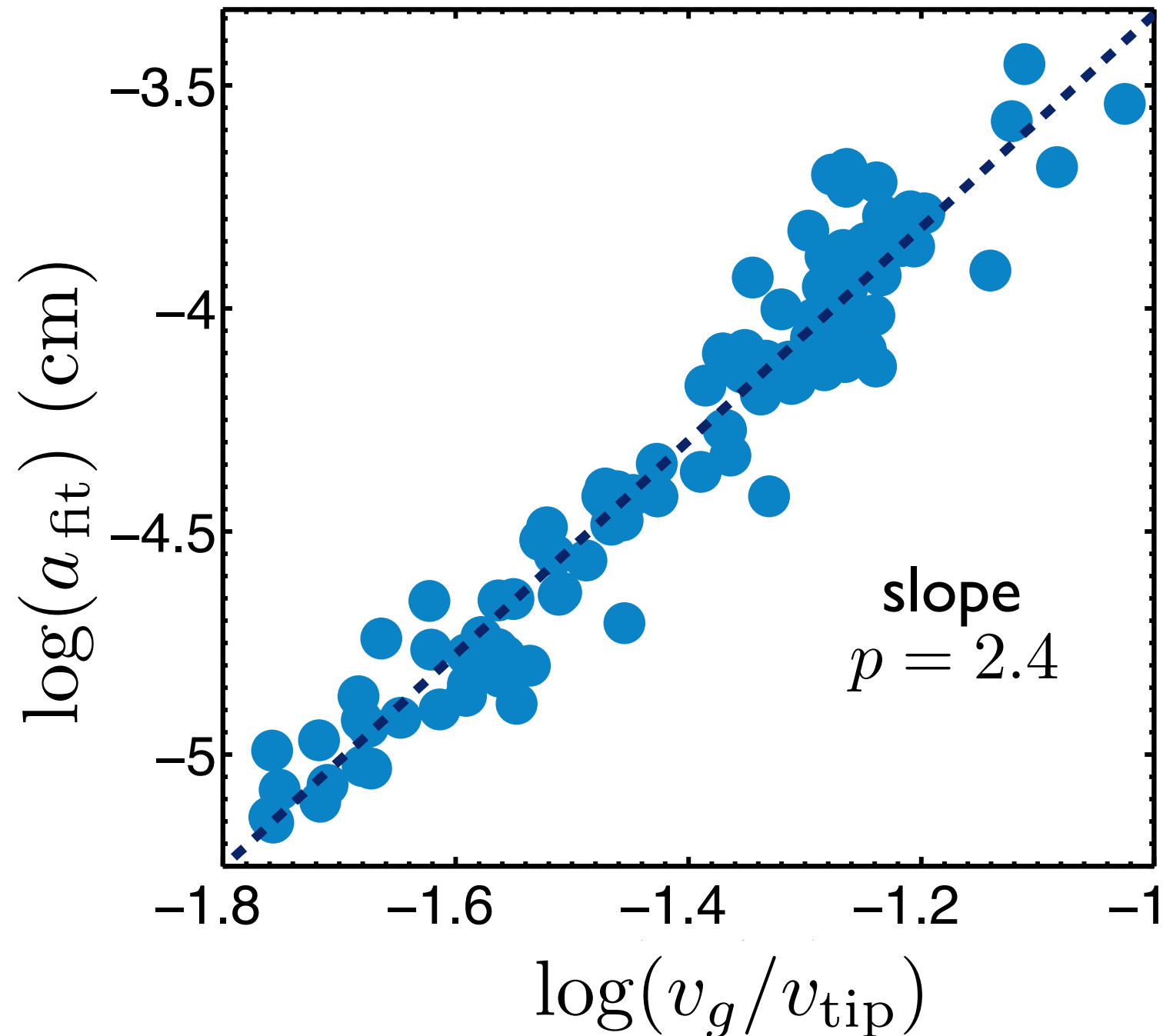
$$a_{\text{theory}} = \ell \left(\frac{v_c}{v_{\text{tip}}} \right)^4$$

$$v_g = v_c \left(\frac{\ell}{z} \right)^{1/4}$$

$$a_{\text{theory}} = z \left(\frac{v_g}{v_{\text{tip}}} \right)^4$$

So we test the power law

$$a_{\text{fit}} = D \left(\frac{v_g}{v_{\text{tip}}} \right)^p$$



We find $D = 0.11$ cm (a constant) and $p = 2.4$,
not $D = z$ (not a constant) and $p = 4$. Theoretical scaling **does not hold**.

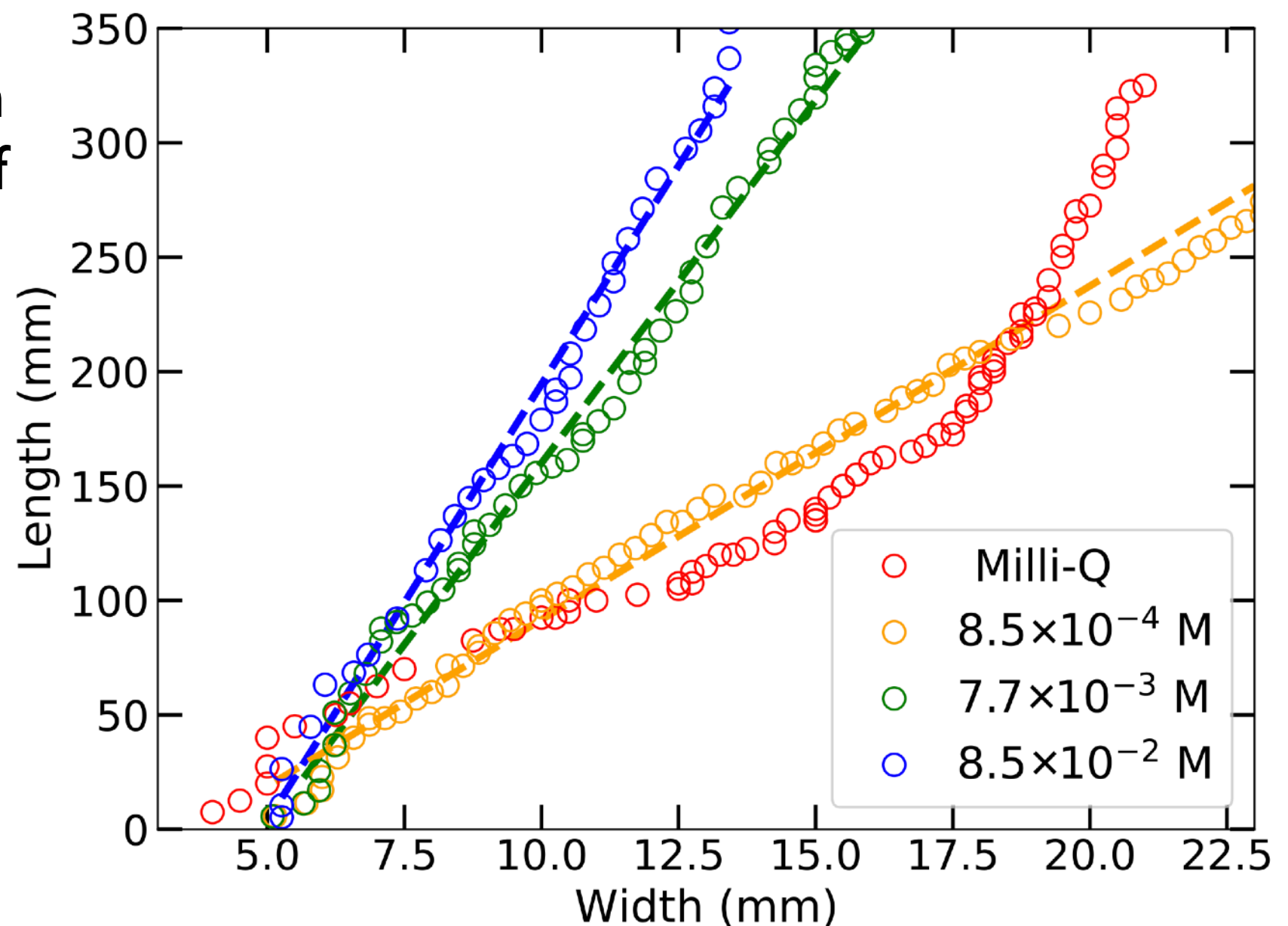
A different experiment

M. Demmenie *et al*, Phys. Rev. Applied 19, 024005 (2023).

Grew icicles with ultra pure “milli-Q” water with impurity concentration of $5.6 \mu\text{M}/\text{kg}$.

about 10x more pure than our distilled water at $(30 - 50) \mu\text{M}/\text{Kg}$.

They fit to a **cone** and saw deviations from platonic-ness for the lowest concentrations.



So it appears that icicles can be platonic if they are pure enough (distilled) but **not too pure** (milli-Q).

Are icicles *platonic*?

Conclusion: qualified success

- PRO:**
- **Some** icicles do fit the platonic form, if the water is pure enough (but not *too* pure).
 - The very small value of a more-or-less agrees with theory (if one takes $C = 0.2$).
- CON:**
- The theory relies heavily on the scaling of a **laminar** boundary layer, but the icicles can be approximately platonic even for turbulent air.
 - Fitting the shape with a 2 parameter power law gives a power of about $1/2$ instead of the expected $3/4$, for small a .
 - a does not scale properly with v_g/v_{tip} . It fits to a pure power law.
 - a is absurdly small and its physical interpretation is obscure.
 - v_{tip} is not modeled physically the way v_g is. It is treated as a geometric parameter to enforce a uniformly translating shape with $v_g = v_{\text{tip}} \sin \phi$. In reality, there is **physics there**.

You decide!



Fin

A melting cylinder of ice

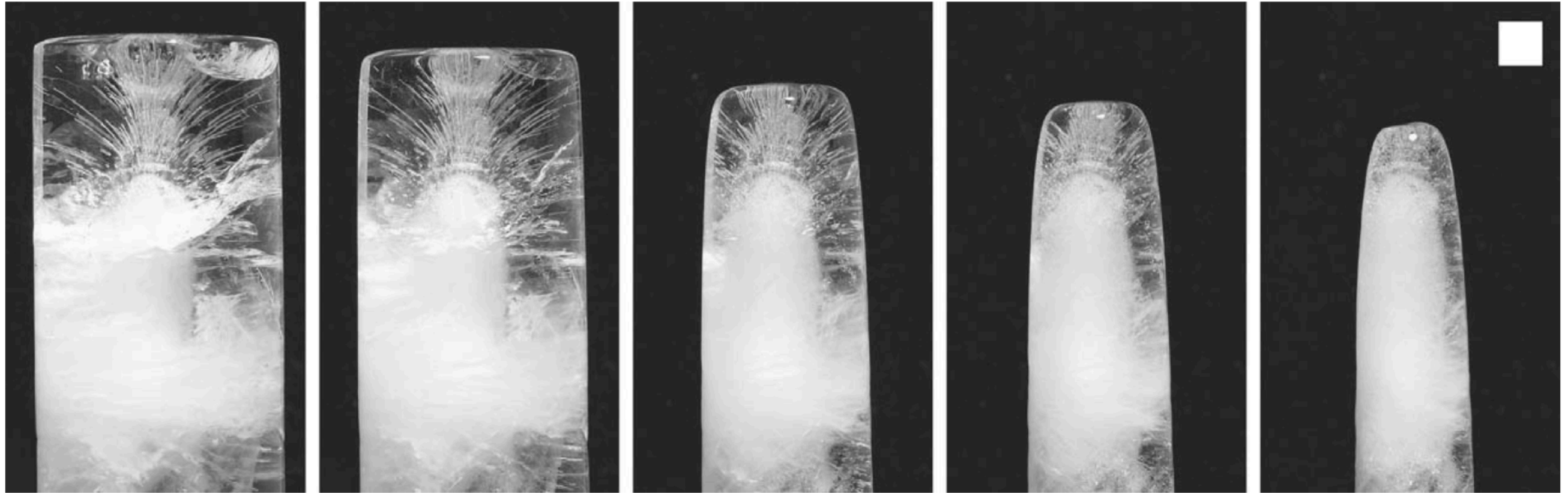


FIGURE 1. A sequence of high-resolution images showing the melting of a cylinder of ice in air of far-field temperature $T_{\infty} = 21.4^{\circ}\text{C}$ taken at times $t = 10, 51, 114, 164, 214$ and 266 min. The white square for scale is 1 cm on a side.

J.A. Neufeld, R. E. Goldstein and M. G. Worster, *J. Fluid Mech.*, **647**, 287 (2010).

Death by icicle

First reported by the roman epigrapher Martial around 100 AD.

Martial, Epigrams. Book 4.

XVIII. ON A YOUTH KILLED BY THE FALL OF A PIECE OF ICE.

Just where the gate near the portico of Agrippa is always dripping with water,¹ and the slippery pavement is wet with constant showers, a mass of water, congealed by winter's cold, fell upon the neck of a youth who was entering the damp temple, and, when it had inflicted a cruel death on the unfortunate boy, the weapon melted in the warm wound it had made. What cruelties does not Fortune permit? Or where is not death to be found, if you, waters, turn cut-throats?



See also **Die Hard 2** (1990),

and Mythbusters: **Can icicles be deadly?**



The image at the left shows the final state of this run.

- **Conditions**

Wall temperature:	-12.30 C
Air temperature:	-9.85 C
Water flow rate:	1.9767 g/min
Water type:	Canadian Springs (TM) distilled water
Salt concentration:	8.00e-03 wt %
Surfactant concentration:	no added surfactant
Input nozzle temperature:	1.02 C
Fan flux:	0.82 m ³ /min per fan
Support rotation:	indexed support rotation with 8 tabs
Support rotation rate:	4.06 min/rot
Relative humidity:	81.6 %
Scale of raw images:	54.640 pixel/cm

- **Movies**

- [Raw movie with rotation](#) (browser friendly Quicktime)
- [Raw movie with rotation](#) (large Quicktime download)
- [Raw movie for each tab separately](#) (browser friendly Quicktime)
- [Raw movie for each tab separately](#) (large Quicktime download)

- **Still images**

- [Browse raw images](#)
- [Zip file](#) of all the raw images for this run (large file)
- [Browse cropped and background subtracted images](#)
- [Zip file](#) of all the cropped and background subtracted images for this run (large file)
- Spacetime images of ripples: [\[tab 1\]](#) [\[tab 2\]](#) [\[tab 3\]](#) [\[tab 4\]](#) [\[tab 5\]](#) [\[tab 6\]](#) [\[tab 7\]](#) [\[tab 8\]](#)

- **Associated files**

- [Zip file](#) of detected edges, ripples and shape.
- [Zip file](#) of various timeseries data files.

An open access
dataset containing 237
icicles measured by
Antony Chen

~230k images
and > 400 movies,
available under
Creative Commons 4.0
license

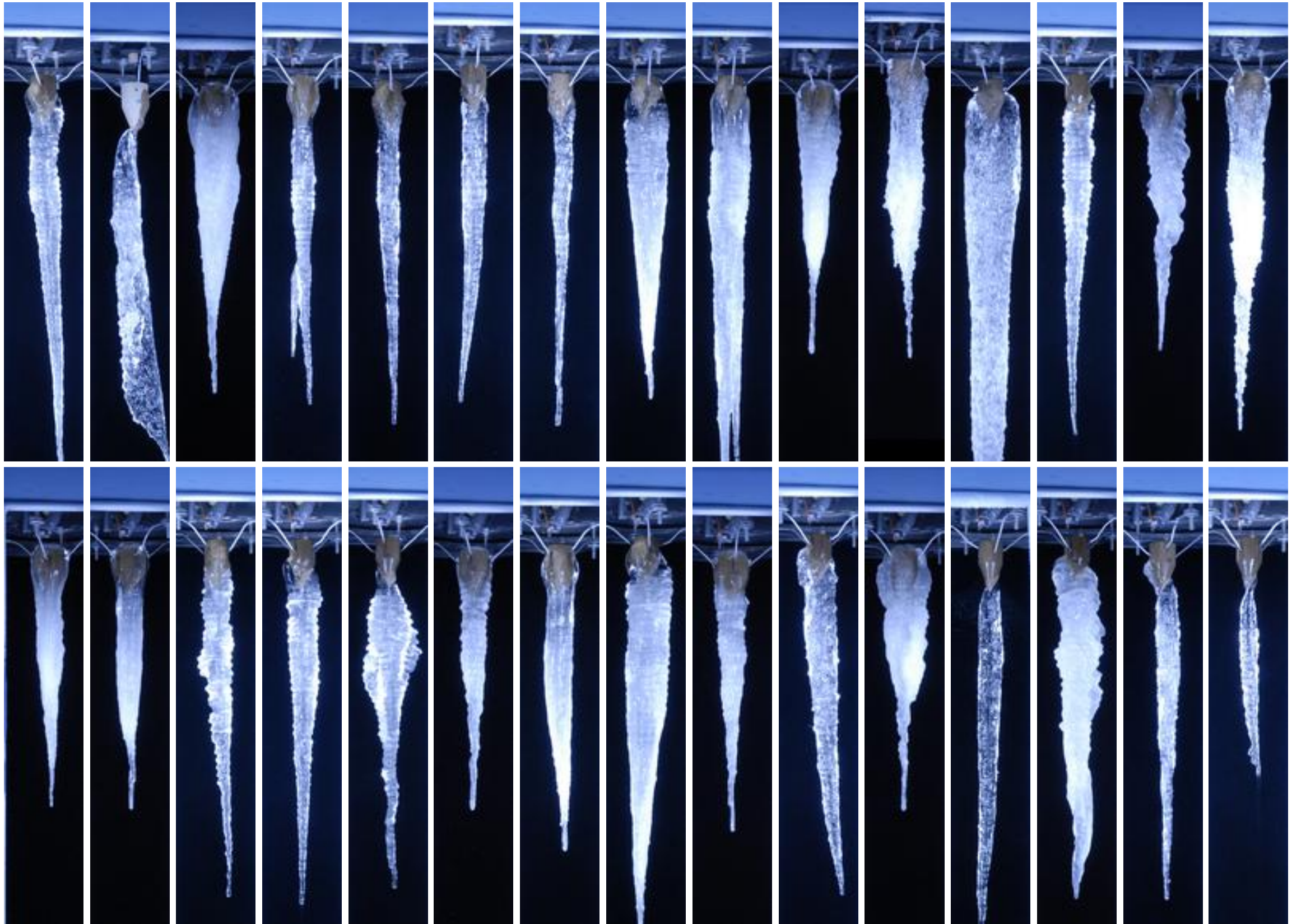
(John Ladan's data is
not yet included)

[[link](#)]

The Icicle Atlas: A rogues gallery of icicles

[[link](#)]

Click on any image to be taken to the homepage of that run.



Fame

The **Icicle Atlas**
made it into
the *New York Times*
on March 16, 2015

[[link](#)]

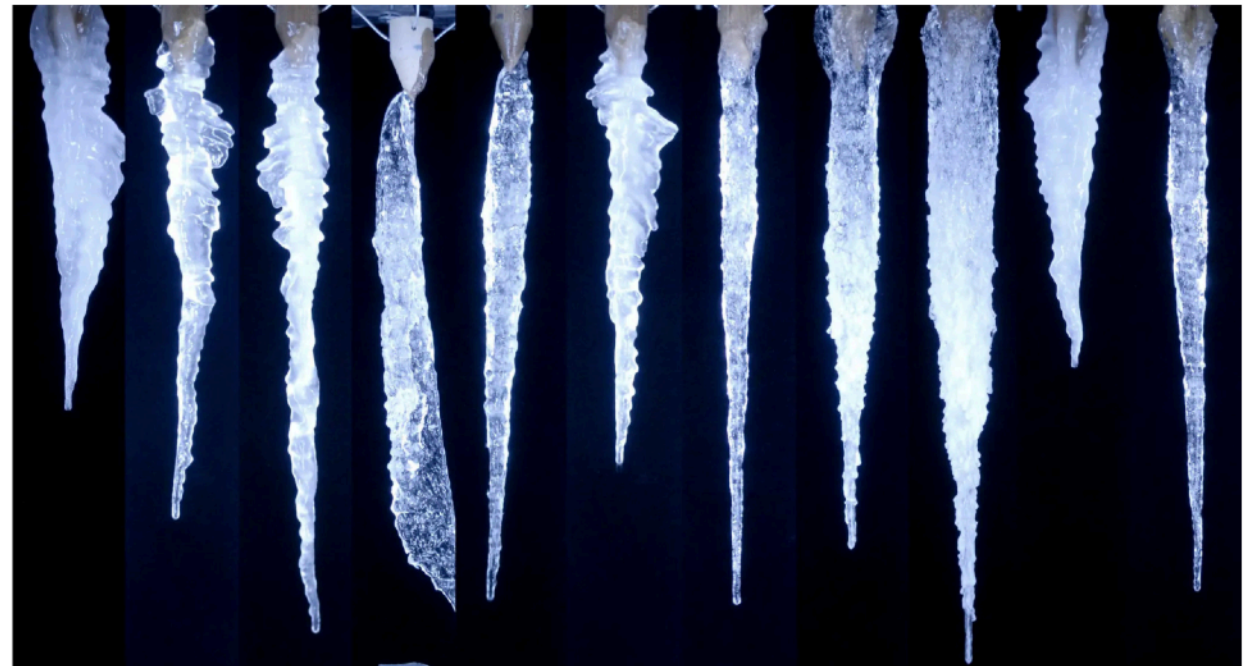
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Why Icicles Look the Way They Do

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A composite of different examples, created artificially in a lab, from the Icicle Atlas. The foggy and rippled ones were formed with small amounts of salt in the water. The clearer ones with fewer or no ripples were formed from distilled water. Antony Chen and Stephen Morris

By **James Gorman**

March 16, 2015

The reason [Stephen Morris](#) studies icicles is simple. They are common and beautiful, and they make you wonder. And they are now documented online in all their chilly, wet glory in the [Icicle Atlas](#).