

Notes for the third lecture on “The growth and morphology of icicles”

This pdf contains a few slides that were not shown in the actual lecture, for lack of time.

This pdf contains live links which are underlined like this.

For data and movies, see the *Icicle Atlas*
https://www.physics.utoronto.ca/Icicle_Atlas

All papers, theses, reports etc on this topic are available at:
https://www.physics.utoronto.ca/nonlinear/papers_icicles.html

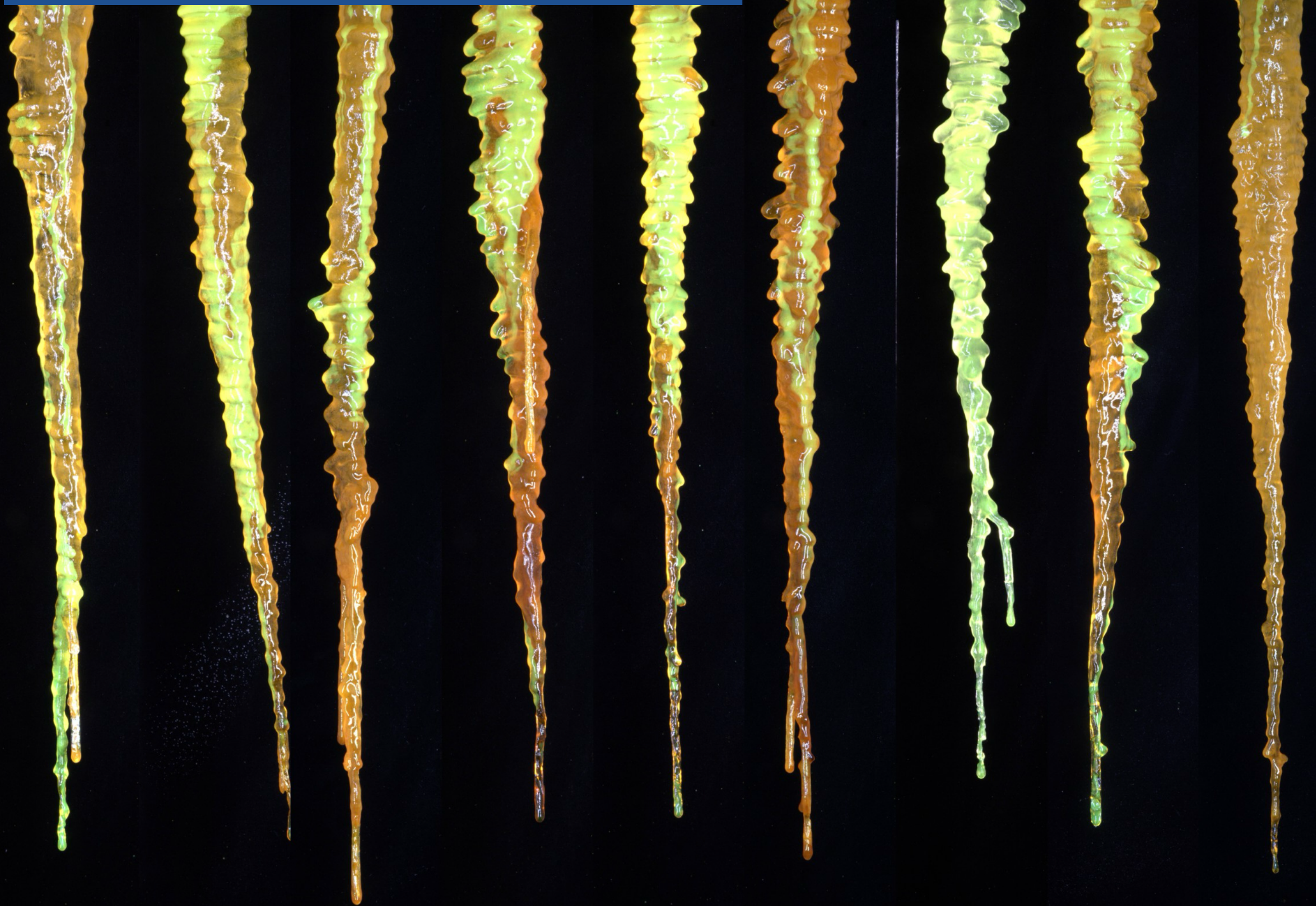
YouTube playlist.

Flickr images and movies of icicles.

Flickr images of the icicle machine.

Do not hesitate to contact me with any questions, ideas or whatever
Stephen Morris patternphysics@gmail.com or sw.morris@utoronto.ca
<https://www.physics.utoronto.ca/nonlinear/>

Part 3: Interior and flow



Toward a comment by Menno Demmenie

Worster considers **pure water** icicles and develops a series of models with increasing effect of advection in the water film.

In all cases, the wavelength of the the most unstable linear mode scales with the **gravity-capillary length**:

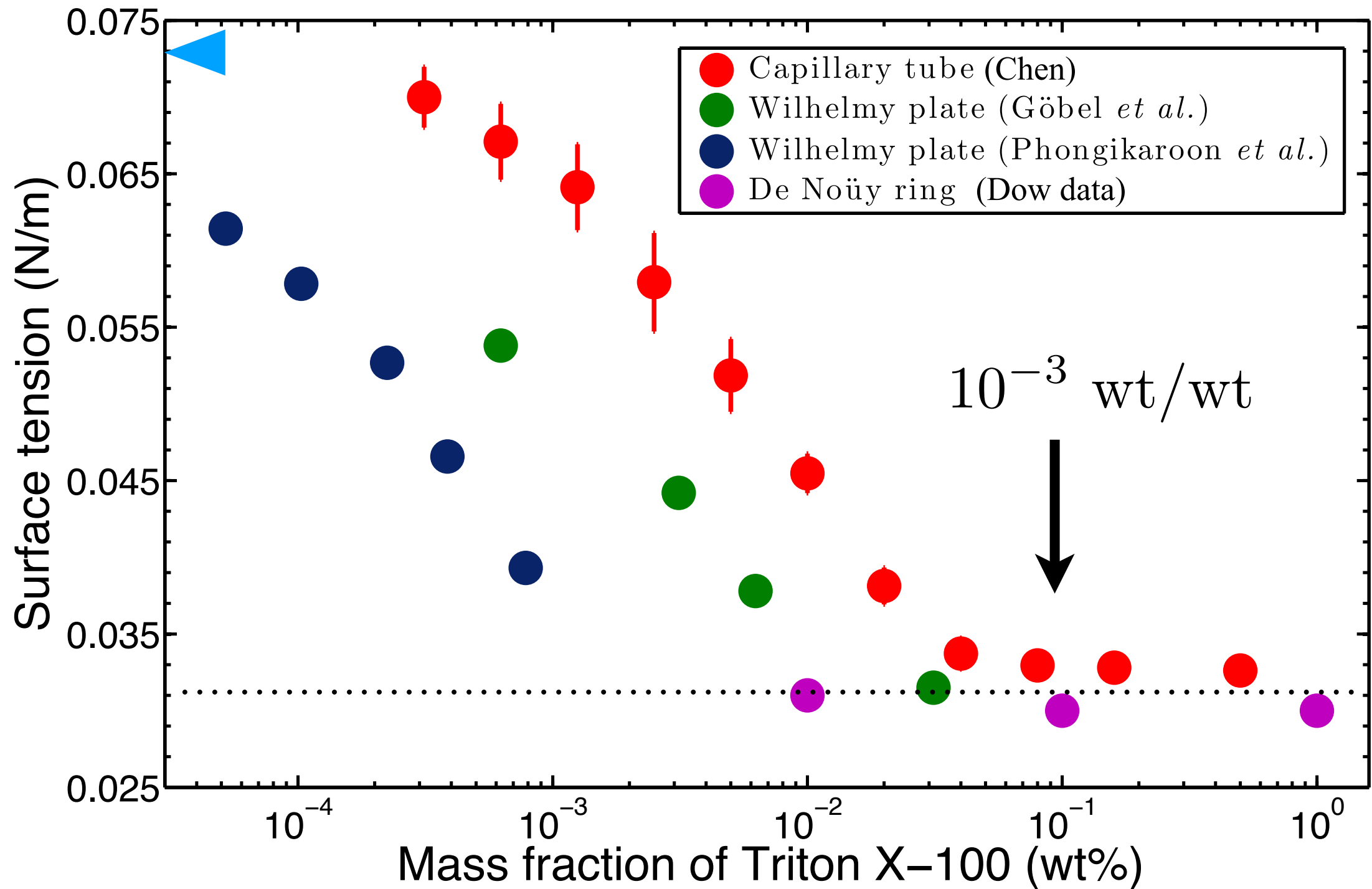
$$\ell_{gc} = \left(\frac{\gamma}{3\rho g} \right)^{1/2}$$

Such that:

$$\lambda \sim \left(\ell_{gc}^2 h \right)^{1/3} \sim \gamma^{1/3}$$

M. G. Worster, *J. Eng. Math*, **145**, 15 (2024).

The effect of TX-100 on surface tension



▶ $\gamma_w = 0.0728 \text{ N/m}$ $\gamma_{TX} = 0.031 \pm 0.003 \text{ N/m}$

The following slide contains a **correction**. The version I presented in the lecture assumed all the error bars for the wavelengths of the TX100/Fluorescein runs were equal. In fact, they vary significantly. The new version uses properly weighted averages and the correctly weighted standard deviation of the mean to get the wavelength and its error bar.

The conclusion is the same.

See also the discussion of surface tension effects in the second lecture.

For reference, here are the raw data:

runid	conc mM/kg	species	other	T_air deg C	flow g/min	wavelength (nm)	wavelength uncertainty (nm)
20180814_3	0.1204	sodium fluorescein	TX100	-13.9611	2.9990	8.5666	0.0968
20180814_1	0.1850	sodium fluorescein	TX100	-14.1973	3.0000	8.7454	0.1309
20180724_1	0.5723	sodium fluorescein	TX100	-13.9597	3.0005	8.8835	0.0757
20180723_1	1.0889	sodium fluorescein	TX100	-13.8637	2.9966	8.7930	0.0679
20180720_1	2.1215	sodium fluorescein	TX100	-13.8815	3.0005	9.0836	0.0743
20180719_1	4.1843	sodium fluorescein	TX100	-13.7210	2.5385	7.5261	0.0278

Toward a comment by Menno Demmenie

From our data for Fluorescein doped icicles with TX-100:

$$\frac{\lambda_{\text{TX}}}{\lambda_0} = \overset{\text{corrected}}{\frac{8.01 \pm 0.02 \text{ mm}}{9.39 \pm 0.06 \text{ mm}}} = 0.853 \pm 0.006 \quad \text{corrected}$$

Predict:
$$\frac{\lambda_{\text{TX}}}{\lambda_0} = \left(\frac{\gamma_{\text{TX}}}{\gamma_{\text{w}}} \right)^{1/3}$$

$$\text{Find: } \left(\frac{\gamma_{\text{TX}}}{\gamma_{\text{w}}} \right)^{1/3} = \left(\frac{0.031 \pm 0.003 \text{ N/m}}{0.0728 \pm 0.0001 \text{ N/m}} \right)^{1/3} = 0.75 \pm 0.02$$

... **not quite in agreement** within the error bars. But of course many of the assumptions of the theory are not true ...

Cross sections of ripply icicles

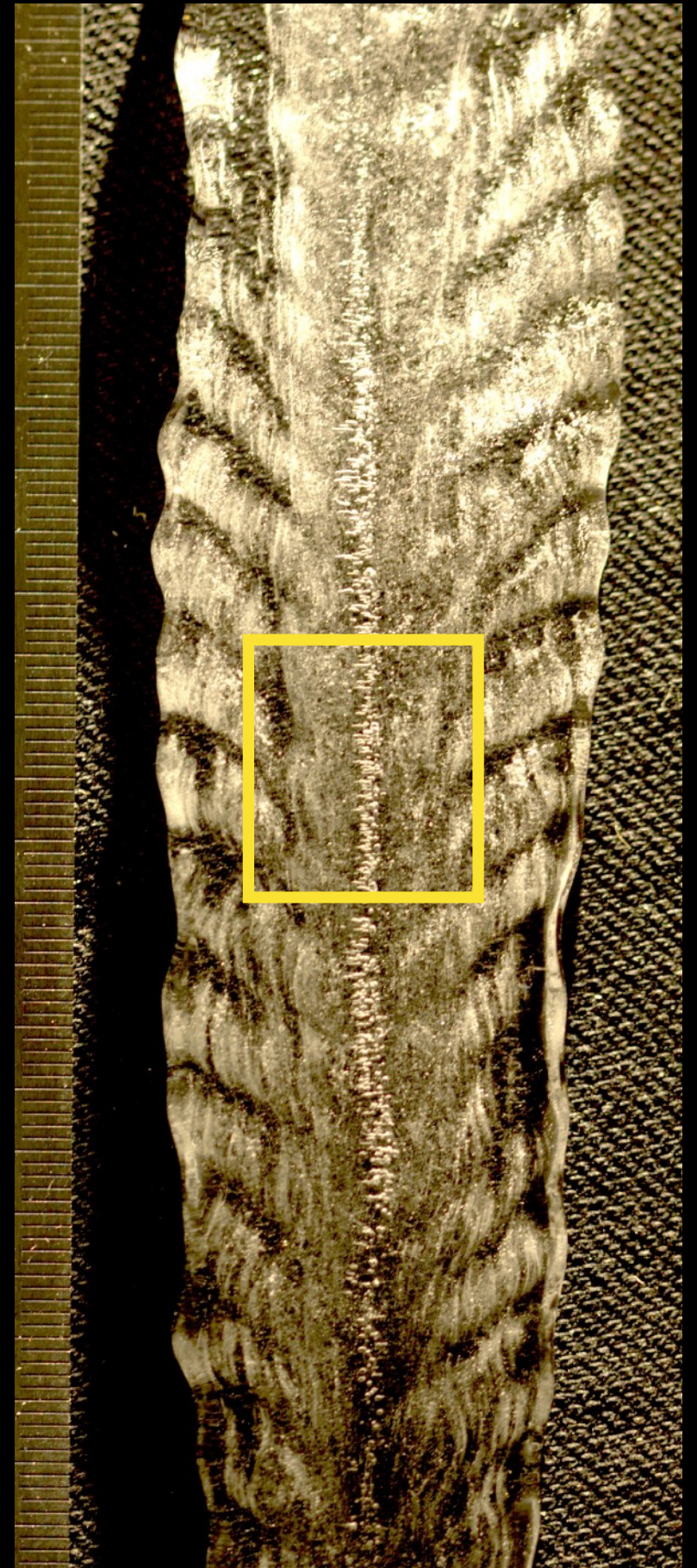
Reveals foggy traces of the ripples in the interior of the icicle.

This fog is **not** air bubbles. Bubbles are only seen along the axis of the icicle due to “mode 3” pipe-filling growth.

First seen on a natural icicle by N. Maeno, L. Makkonen, K. Nishimura, K. Kosugi and T. Takahashi, *J. Glaciol.*, **40**, 319 (1994).



40 ppm NaCl



20 ppm NaCl

Cross sections of ripply icicles

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20 ppm NaCl

Cross sections of ripply icicles

Reveals foggy traces of the ripples in the interior of the icicle.

Will show that these foggy areas are due to small fluid filled **inclusions** containing high concentrations of impurities.

The inclusions are organized into **chevron** structures at the peaks of the ripples.

The troughs of the ripples are clear ice.



40 ppm NaCl

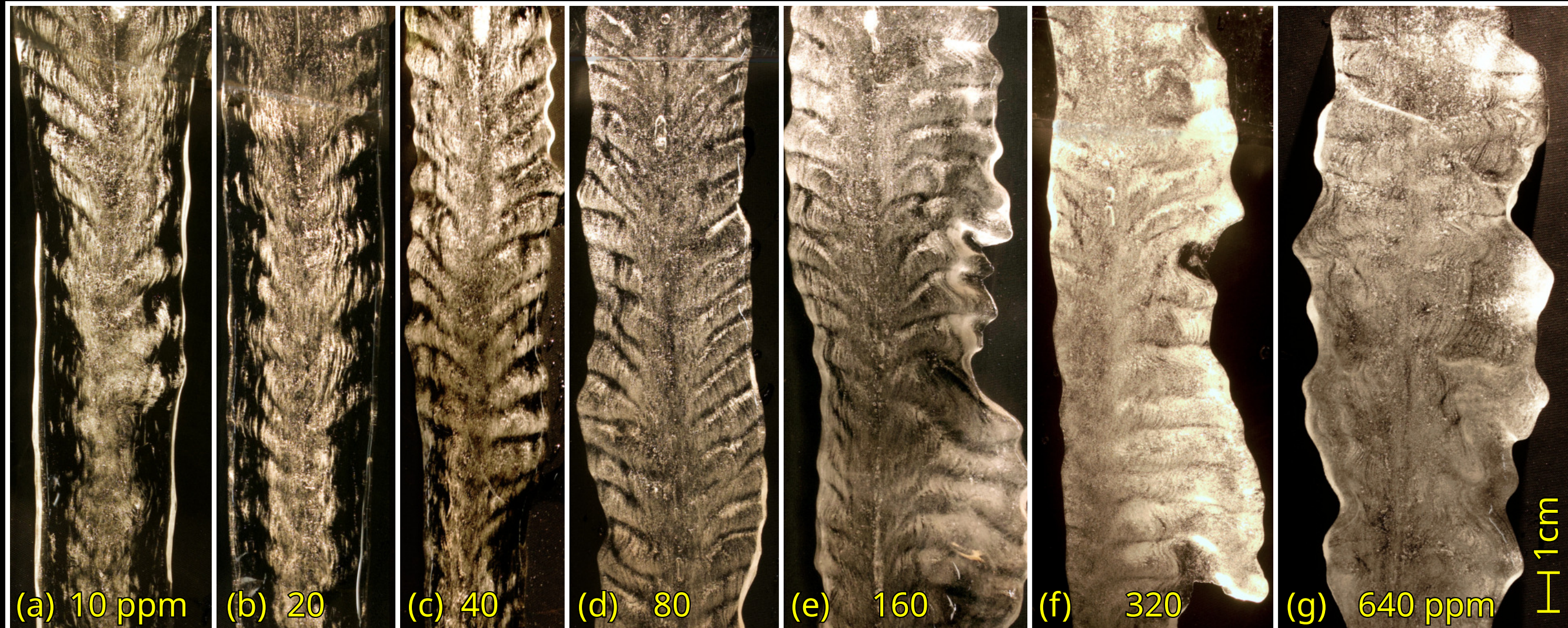


20 ppm NaCl

Cross sections of ripply icicles

Patterns of inclusions as a function of impurity concentration.

NaCl doped icicles



Icicle becomes saturated with multiphase **mushy ice** at the highest concentrations. Chevron patterns fade into general fogginess and ripples become **strongly disordered**.

640 ppm is still much less than sea water which has ~35 000 ppm.

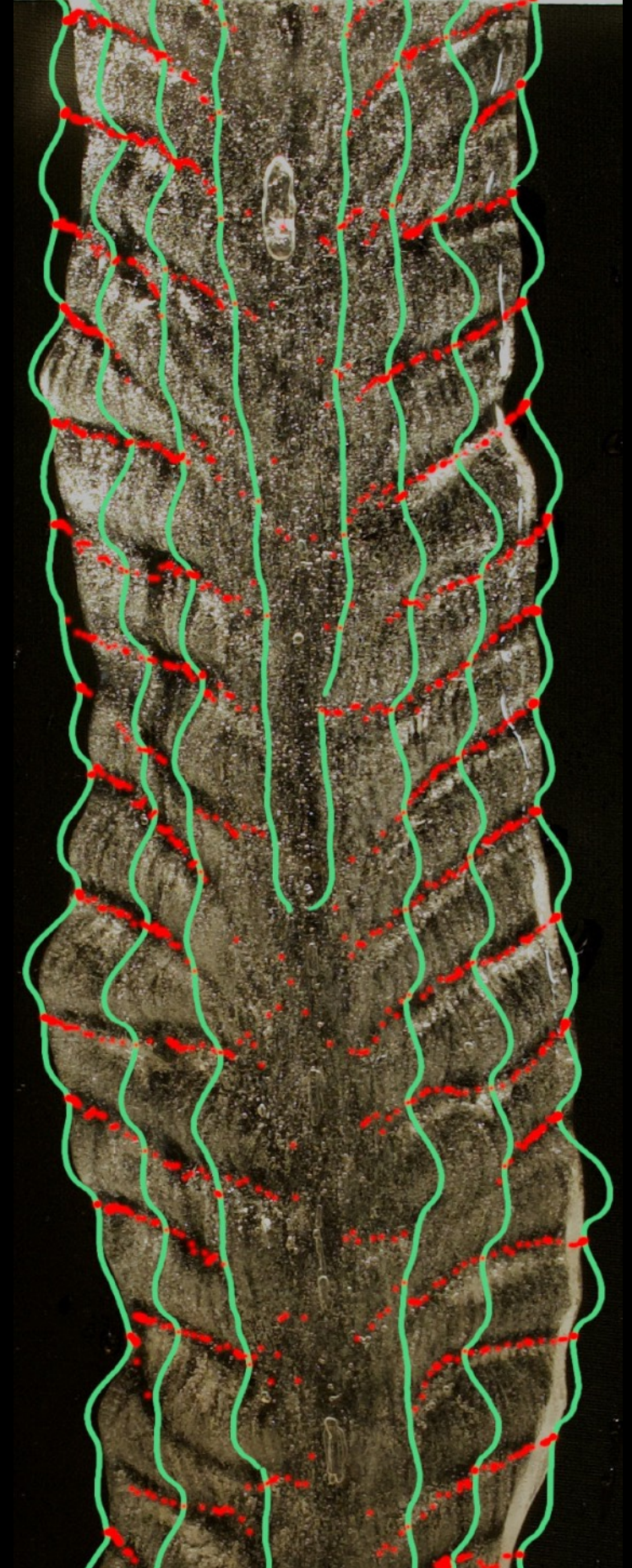
Cross sections of ripply icicles

Track the topographic evolution of lab-grown icicles (**green**) and correlate it with interior structures from cross-section.

The troughs of the ripples found by edge detection (**red**) look dark and are regions of clear ice.

The peaks track the foggy chevrons.

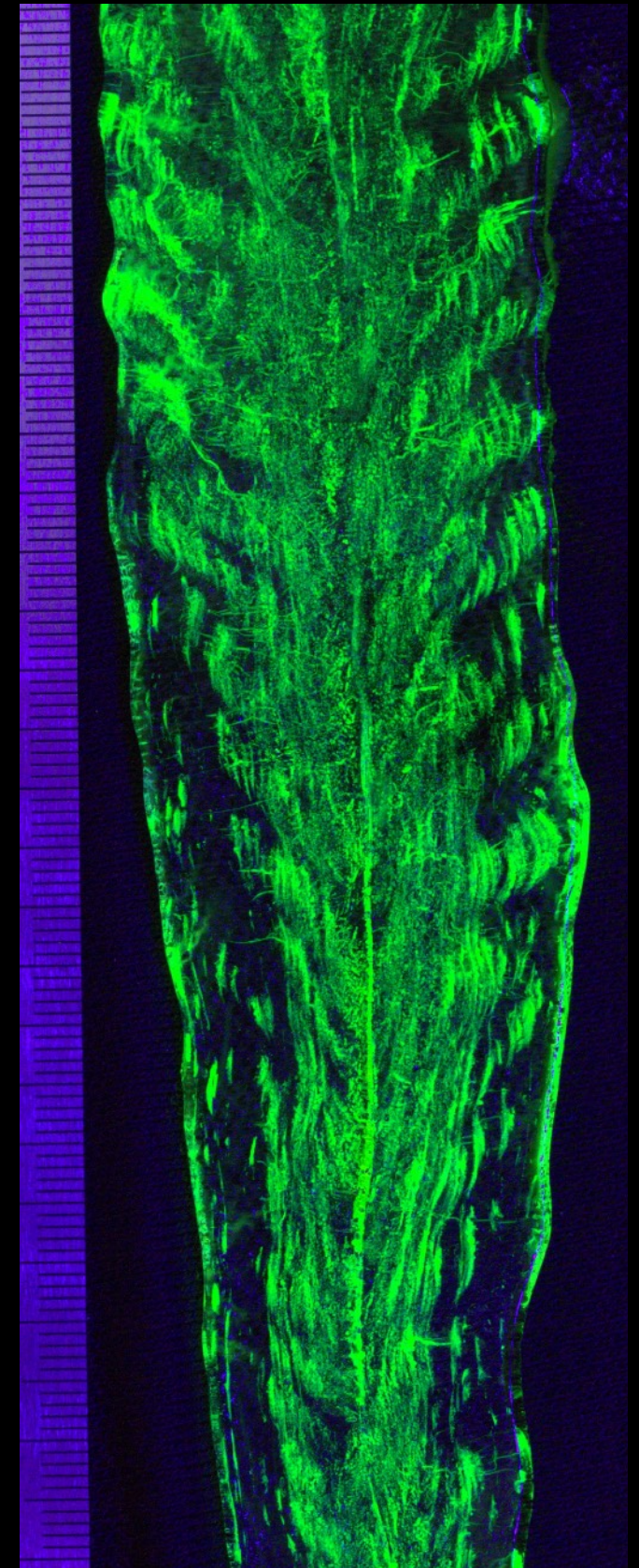
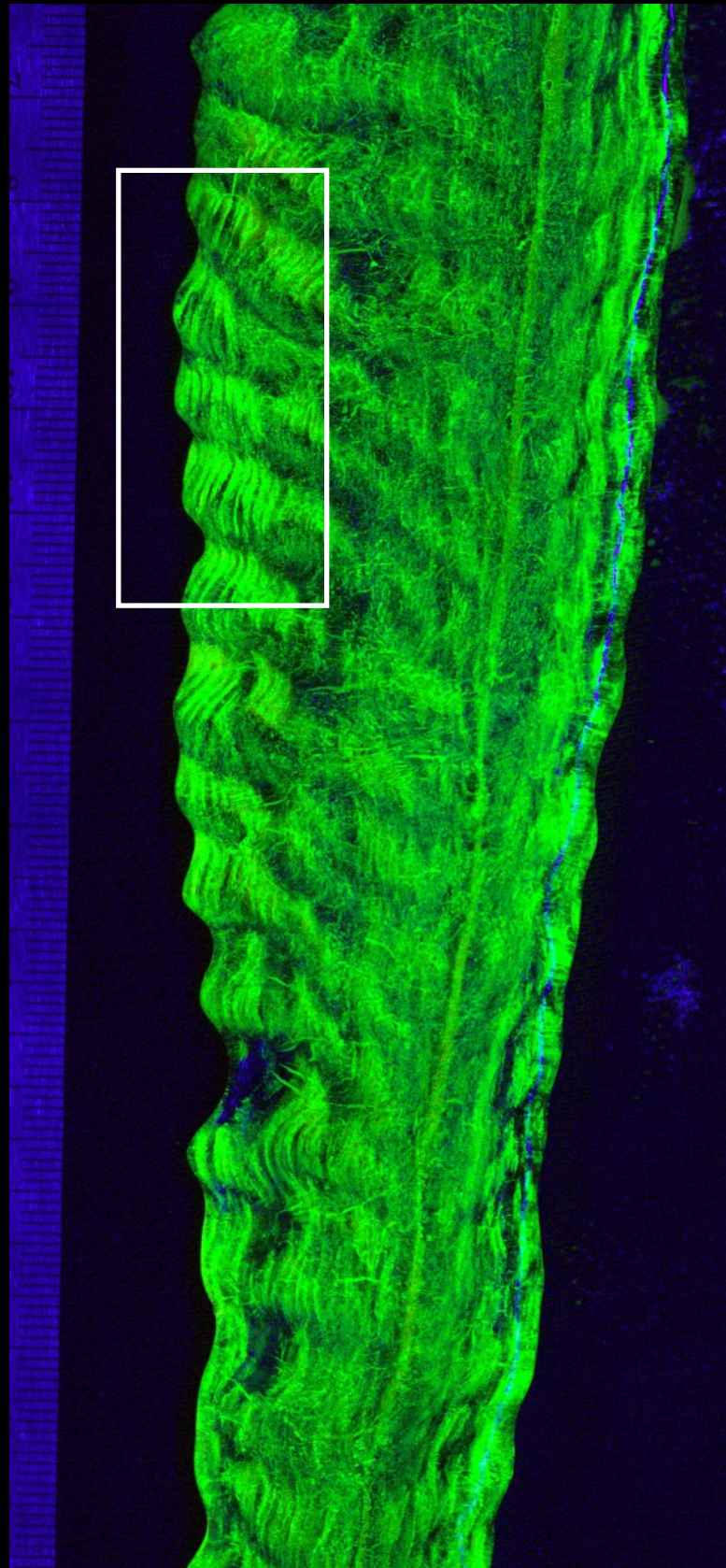
The upward slope of the chevrons matches the wavelength and upward traveling speed of the ripples.



Cross sections of fluorescent icicles

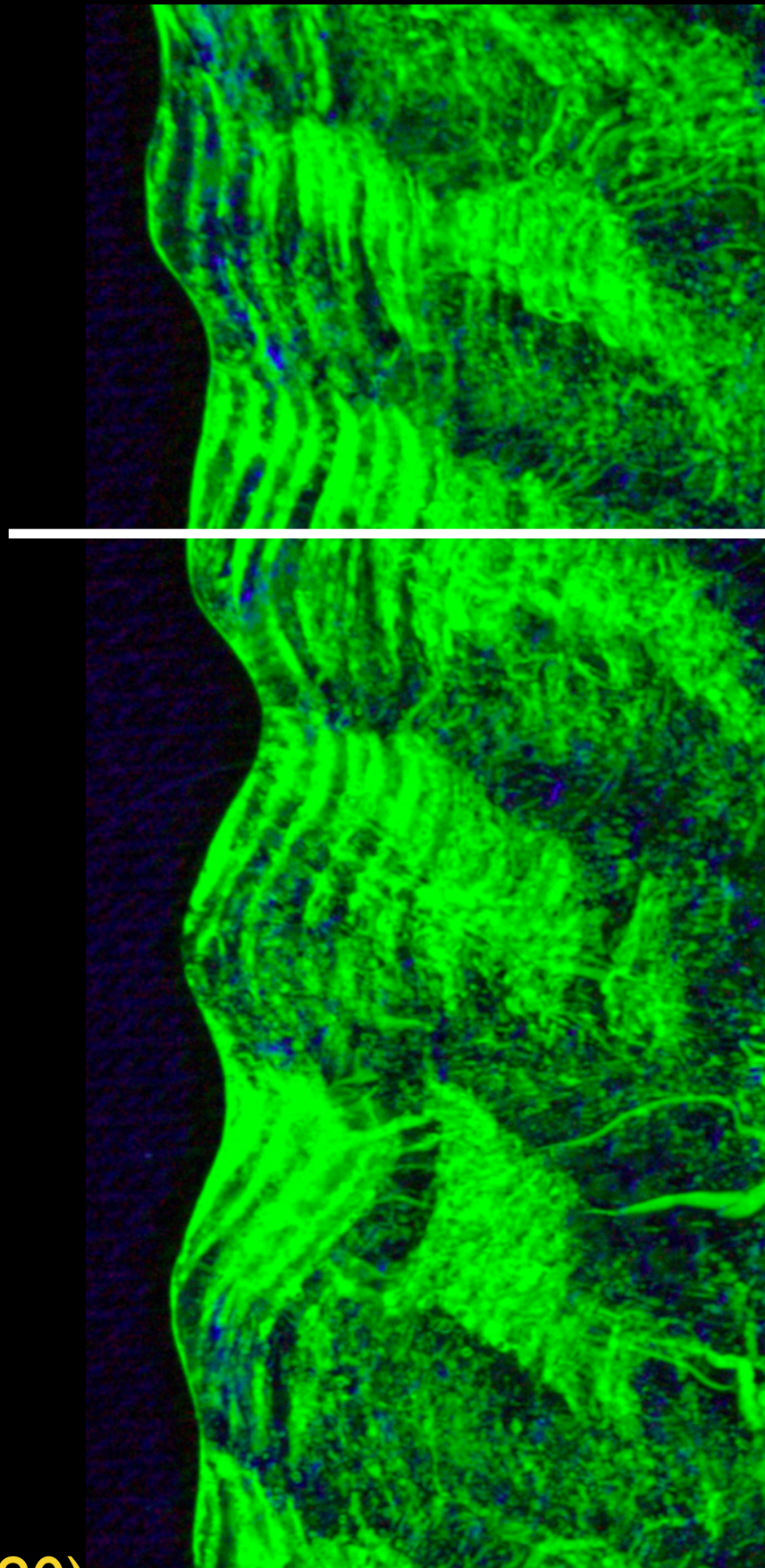
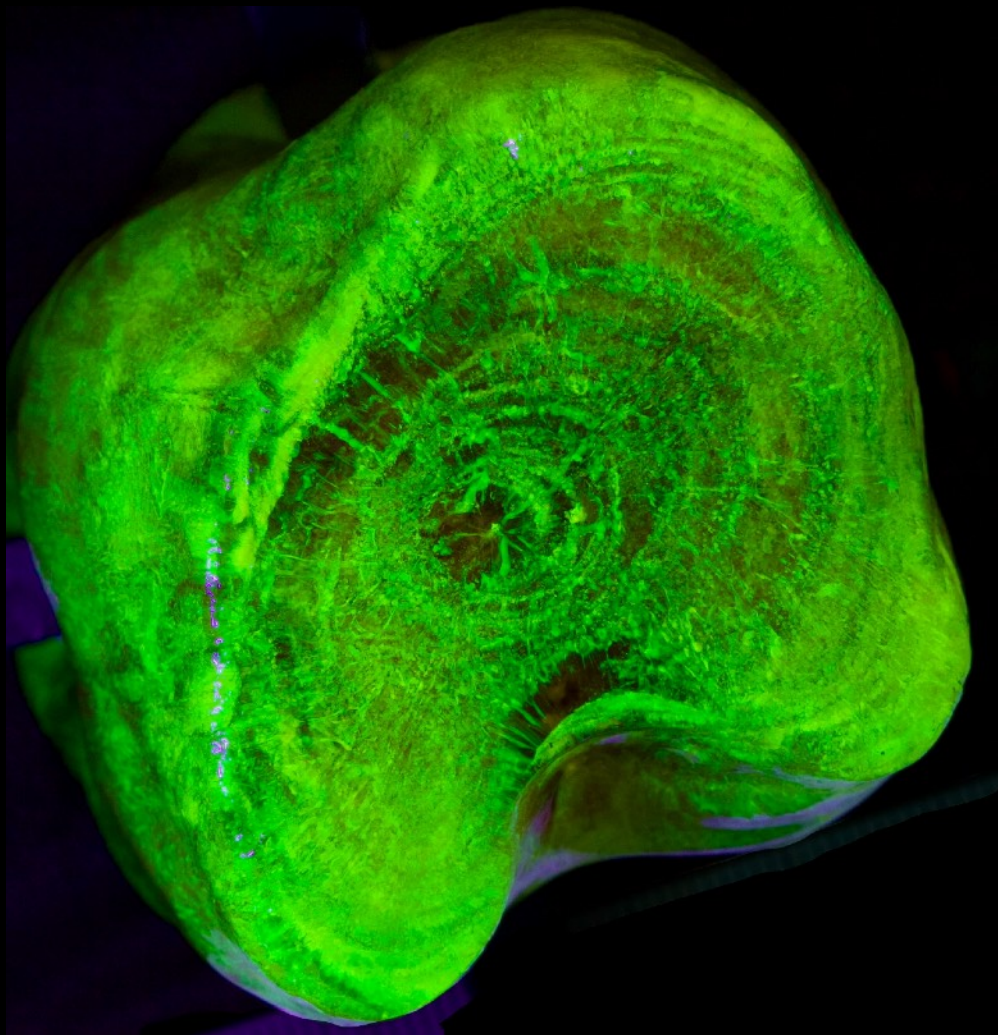
Under UV light, near the
freezing point, the impure
inclusions melt first.

This shows that the
impurities are concentrated
in inclusions near the peaks
of the ripples.



Cross sections of fluorescent icicles

There is a substructure of
crescent shaped clusters
of inclusions that look like
tree rings in cross section.



C.A. Knight, *J. Crystal Growth*, **49**, 193 (1980).

Cross sections of fluorescent icicles

Seen under a microscope,
the tree rings are formed
from dense regions of
spherical, fluid-filled
inclusions (**red**), roughly
100 μm wide (**blue**).



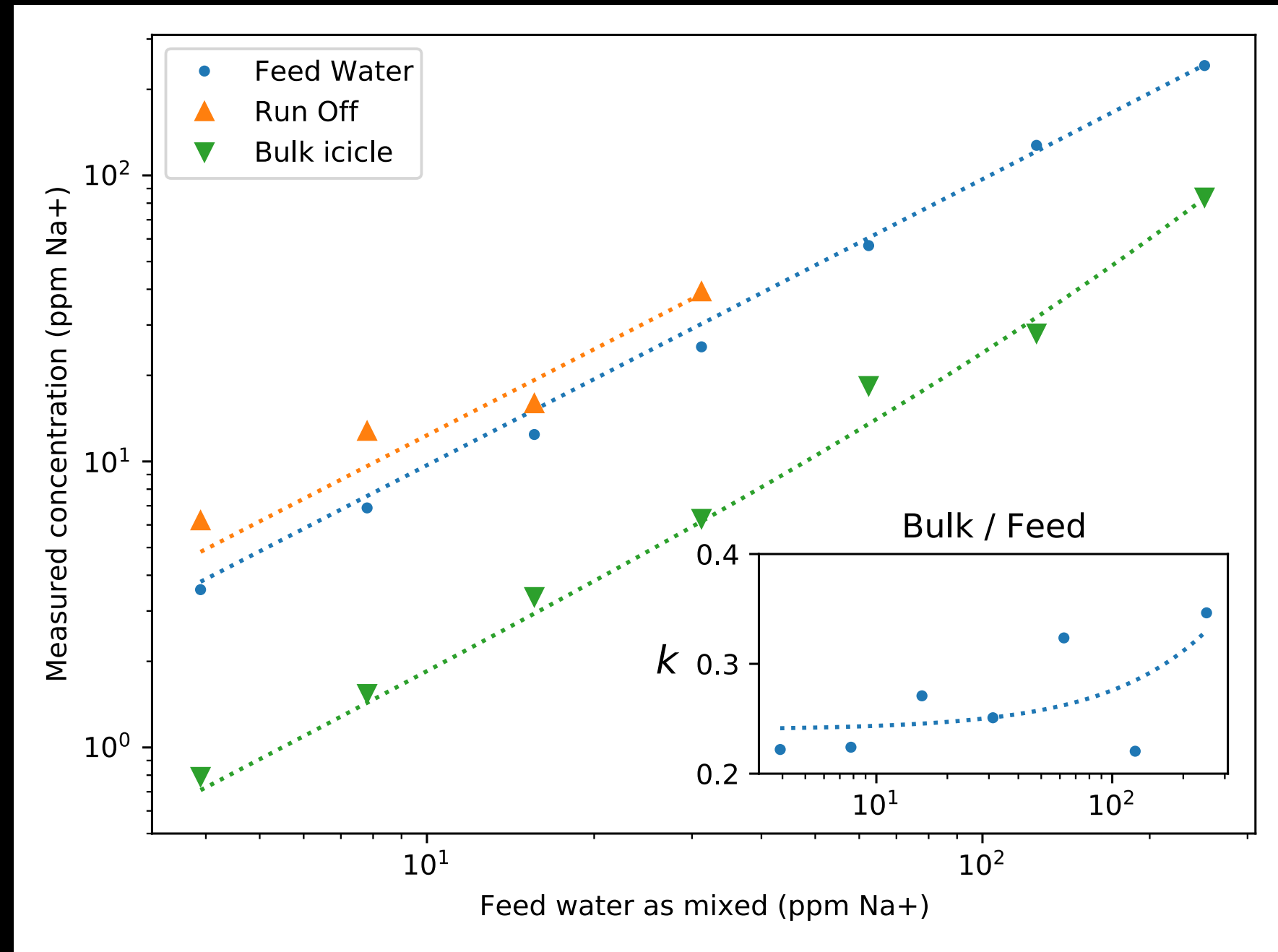
Concentration of impurities trapped in ice

NaCl doped icicles

Measuring the Na^+ concentration in the feed water, the runoff and in the bulk ice, we find that a nearly constant fraction

$$k = 0.27 \pm 0.05$$

of the impurity ends up trapped in the inclusions.



$$\text{fit } (0.05) \log_{10}(c) + 0.19$$

Flow visualization on fluorescent icicles

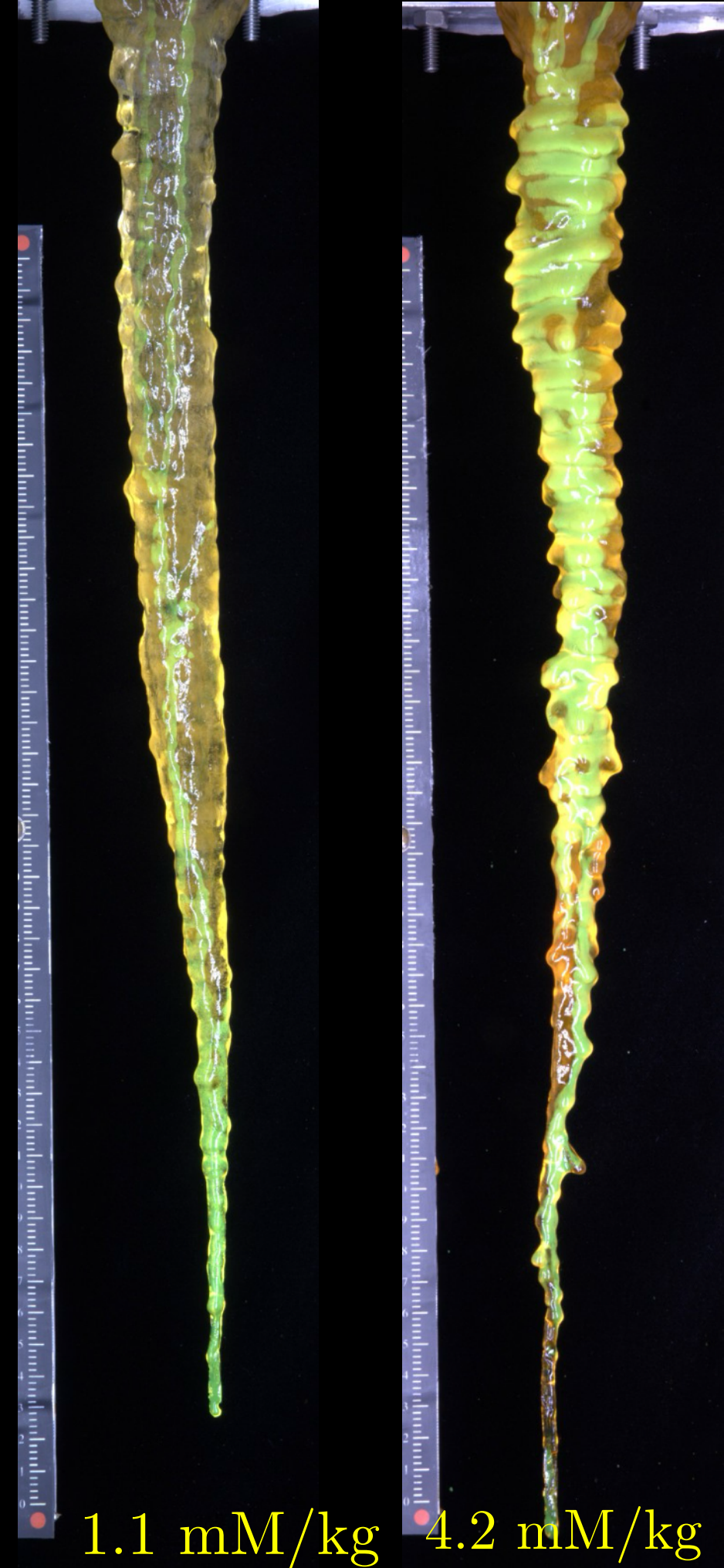
Using fluorescein dye as the impurity, instead of NaCl, allows one to get information about the pattern of the water flow.

Dye in bulk liquid water glows **green**, while dye trapped in solid ice is **orange**.

The surface under the fluorescent regions feels **rough** under your fingernail, while the surrounding regions feel very **smooth**.

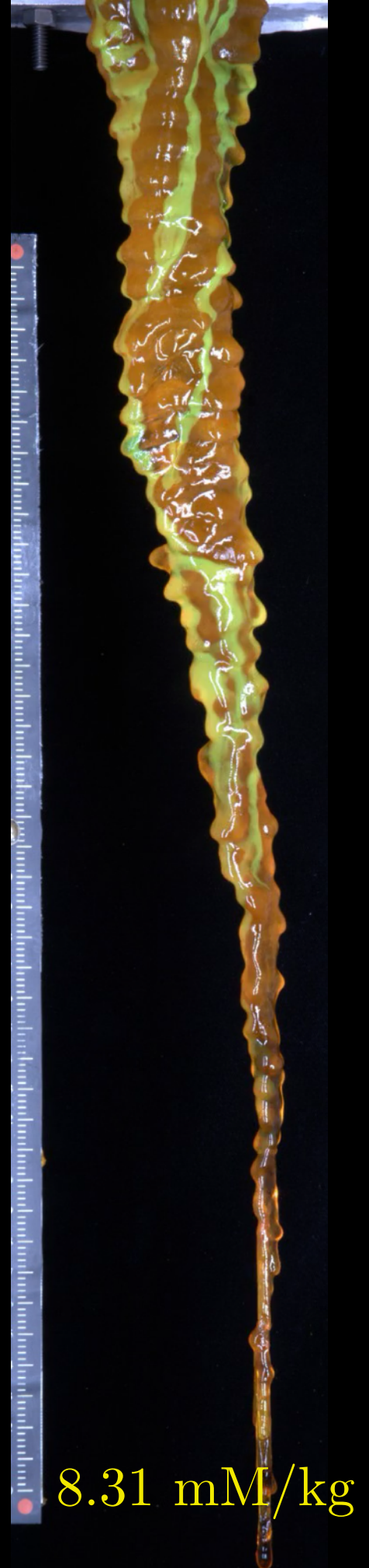
This shows that the surface flow is quite complex and does *not* uniformly cover the whole surface of the icicle.

“Rivulets”



Flow visualization on fluorescent icicles

- The fluorescein dye *only* indicates regions with **macroscopic amounts** of liquid water.
- Only regions with enough dye concentration and thickness of water will give a fluorescent signal.
- If the dye concentration exceeds a certain threshold, the fluorescence is **quenched** (stops glowing) and appears **orange**. Quenching is also temperature dependent.
- We are not just seeing the surface: Dye trapped in inclusions also fluoresces **green** (at low concentration) or appears **orange** (at high concentration).
- It seems clear that the rivulets contain most, if not all of the **macroscopic** surface liquid.

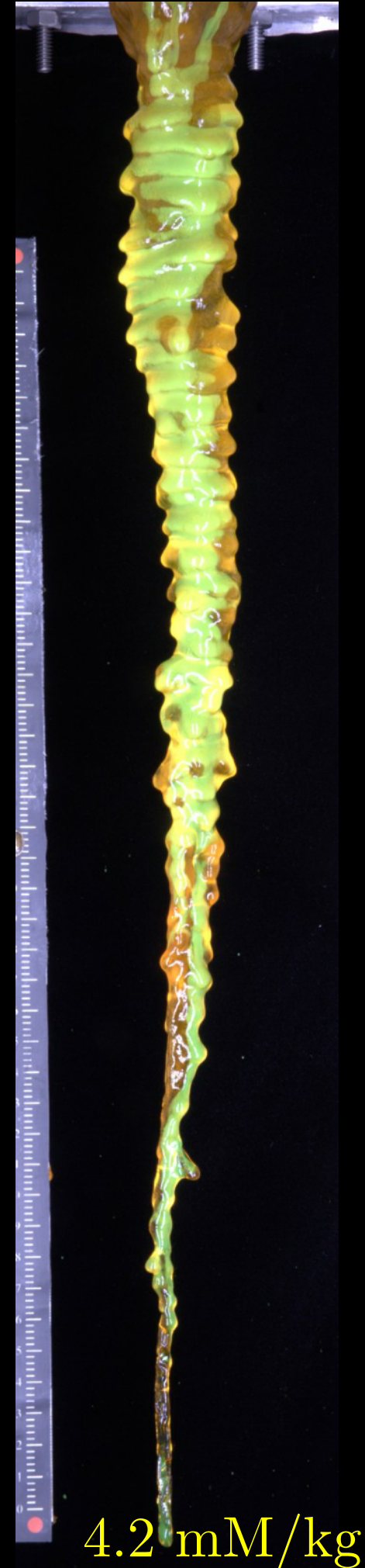


8.31 mM/kg

Flow visualization on fluorescent icicles

The dye signal does **not** show

- If the liquid is flowing, or how fast it is flowing or what its temperature is.
- Whether the surface away from the rivulets is “wet” with a **microscopic** layer of liquid --- a pre-melted film, the QLL *etc.*
- Whether the fluorescent signal on the surface away from the rivulets is **quenched** for some other reason (too cold?).
- Whether the ice is only growing under the rivulets, or also elsewhere.

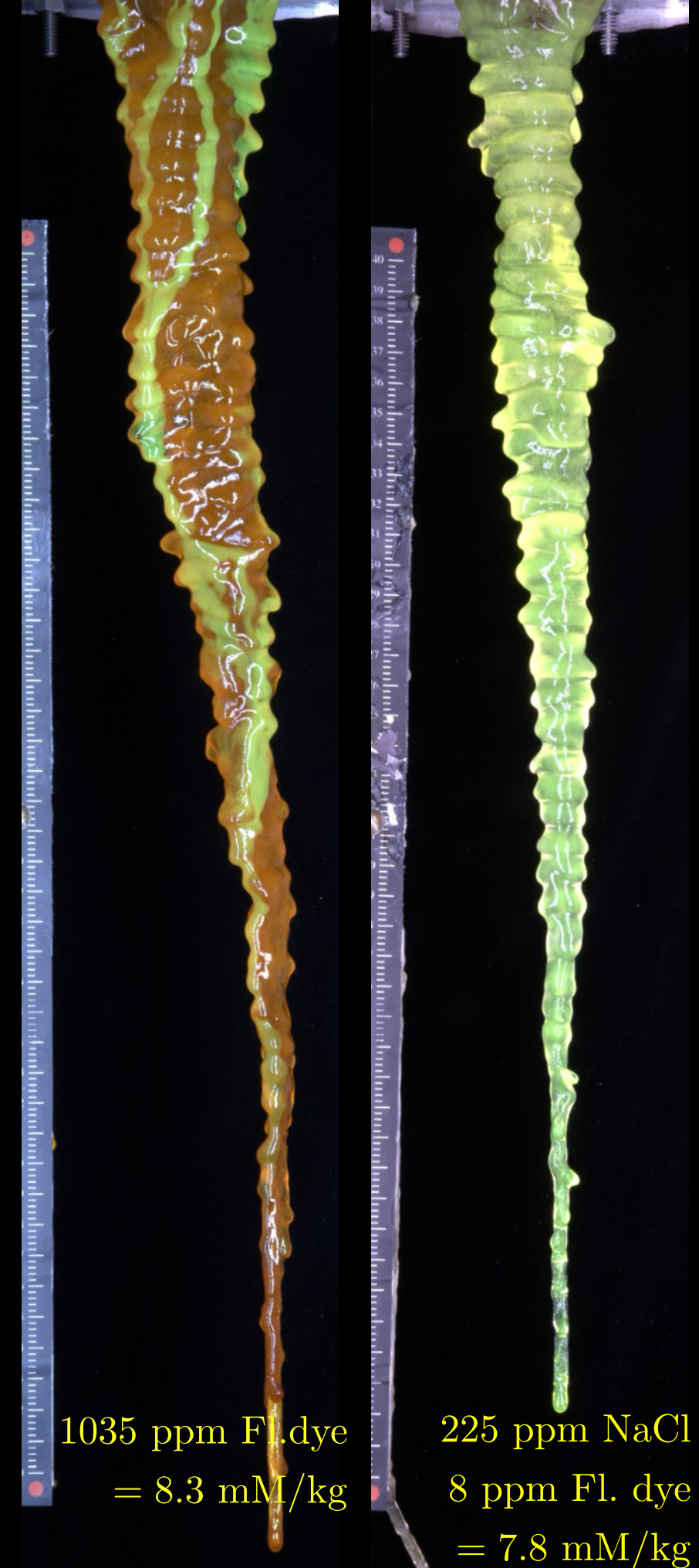


4.2 mM/kg

Flow visualization on fluorescent icicles

If we mix dye and NaCl, we can make icicles for which the concentration of dye remains low enough that it never quenches, even inside the inclusions. Then the whole icicle fluoresces and it is difficult to make out the rivulets.

We did not attempt to map out the C and T conditions for which the dye quenched.



Flow visualization on fluorescent icicles

A mix of dye and NaCl,
shows no obvious rivulets,
if the dye never quenches.



31.1

15.6

7.85

3.96

1.07

0.59

mM/kg

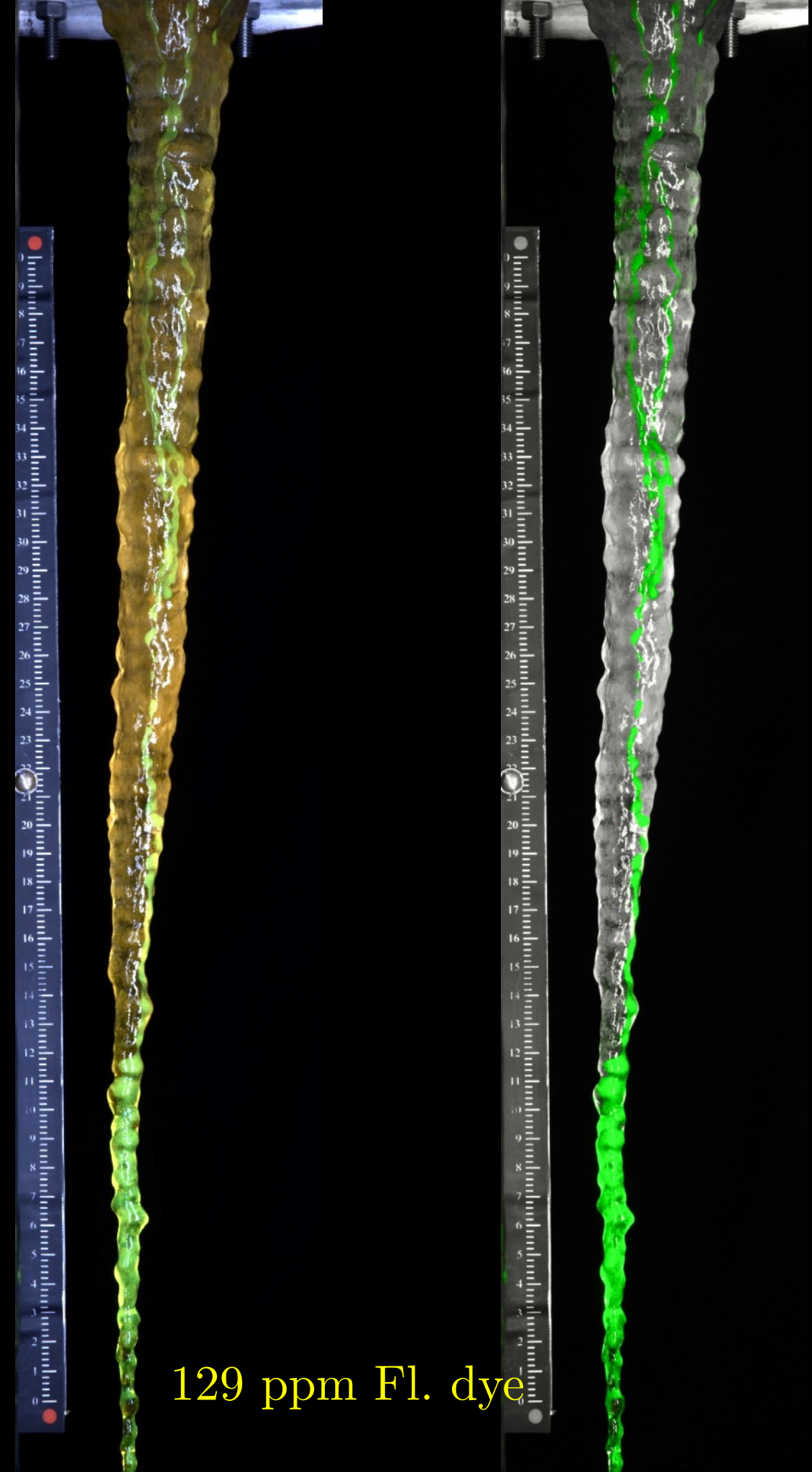
Flow visualization on fluorescent icicles

To analyze the area of the surface covered by rivulets, we use **false colour**.

The **orange** and **green** colours are cleanly separated in a CIE-LAB colourspace.

We use image processing to filter out the **orange**. Then we **unroll** the icicle and average the coverage over one rotation.

J. Ladan and S.W. Morris,
New J. Phys., **23**, 123017 (2021)



Flow visualization on fluorescent icicles

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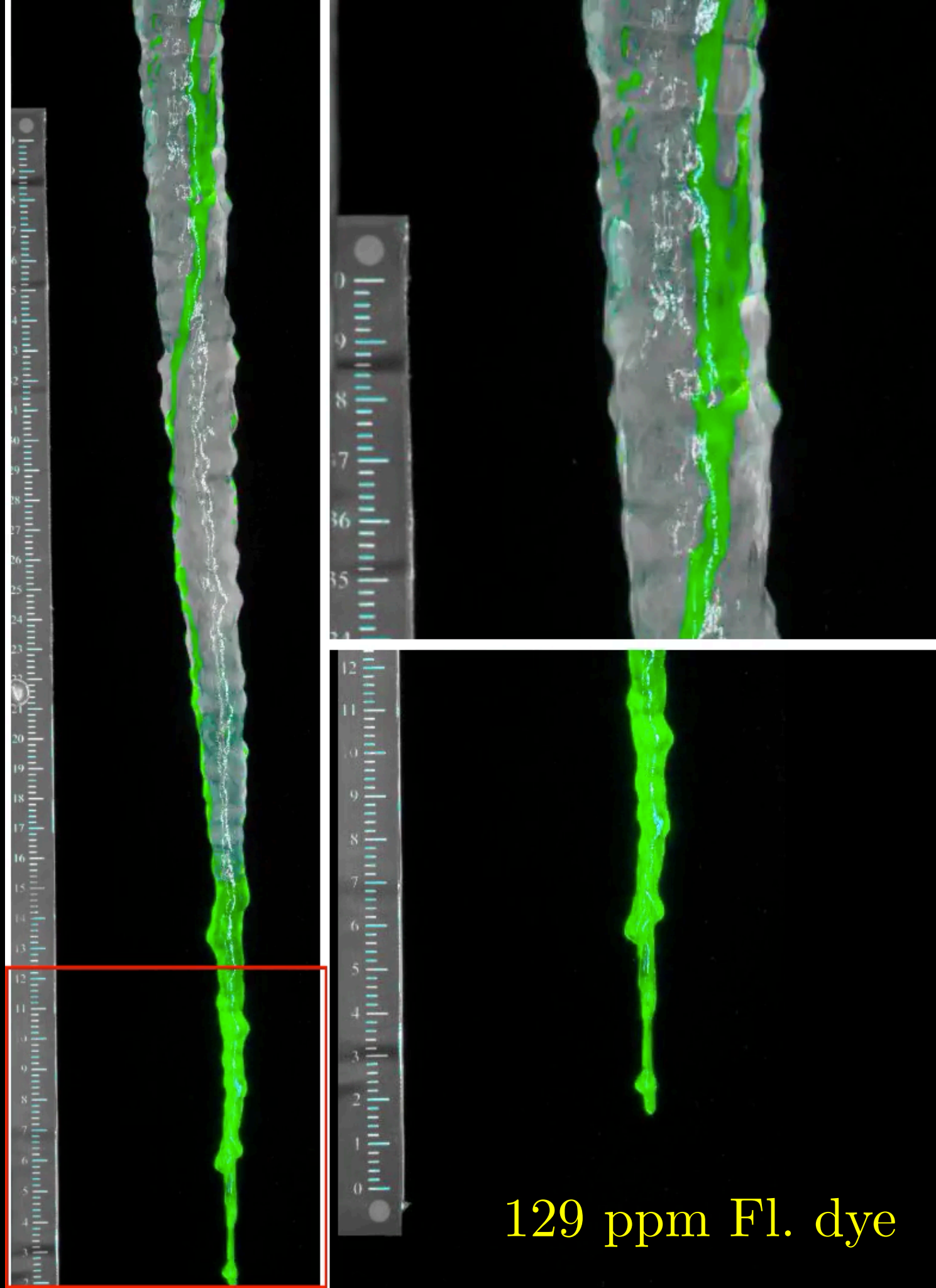
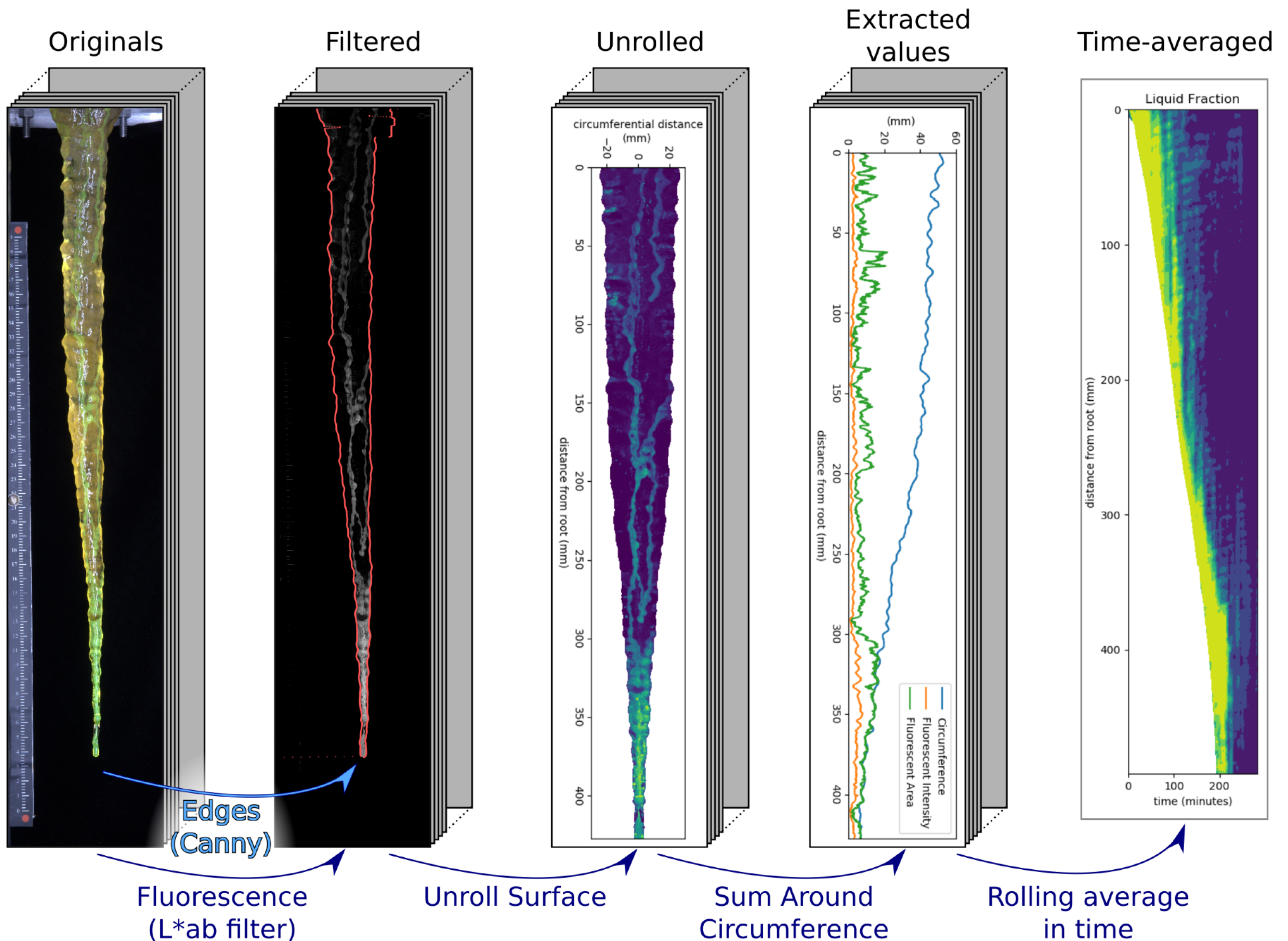
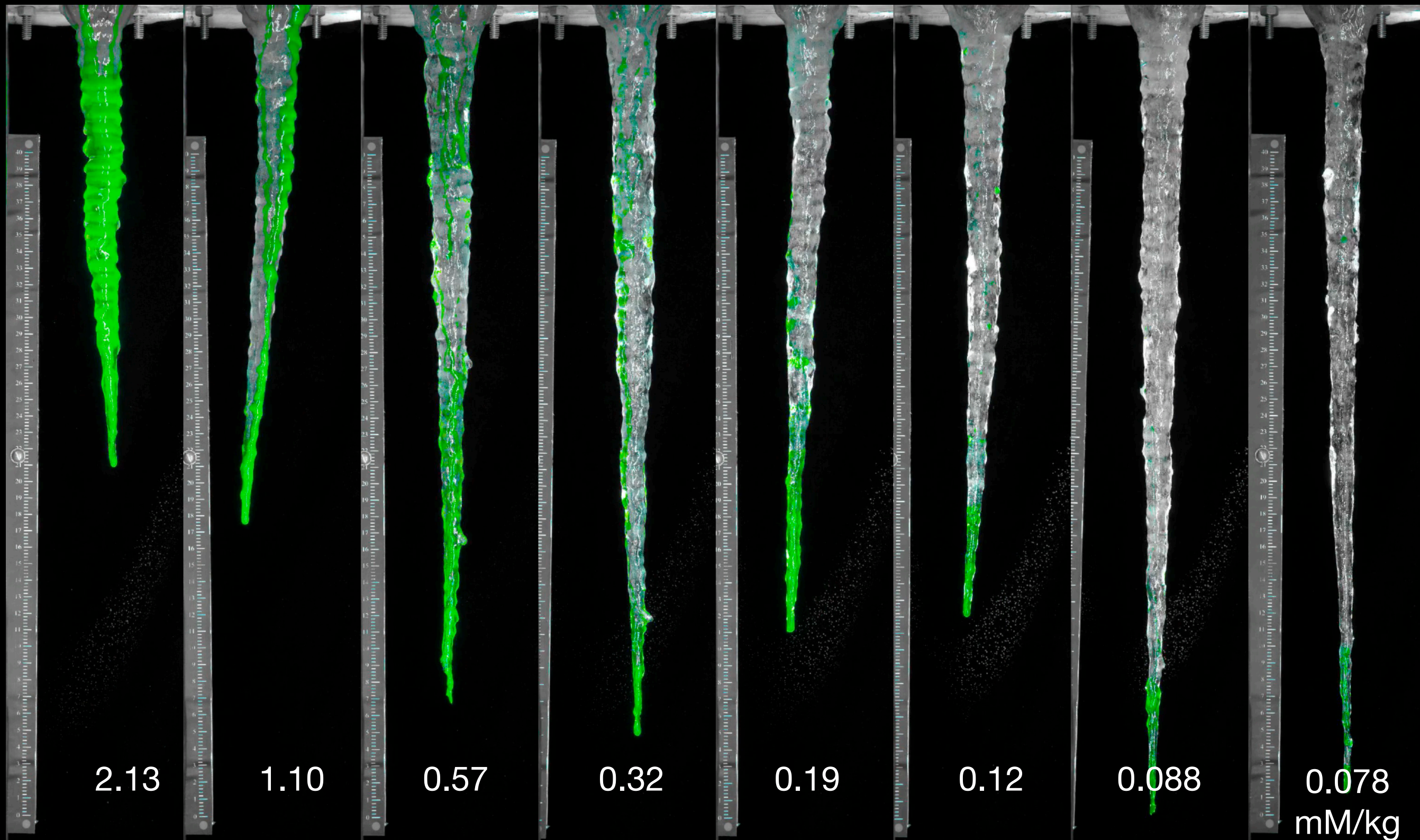


Image analysis for fluorescent icicles



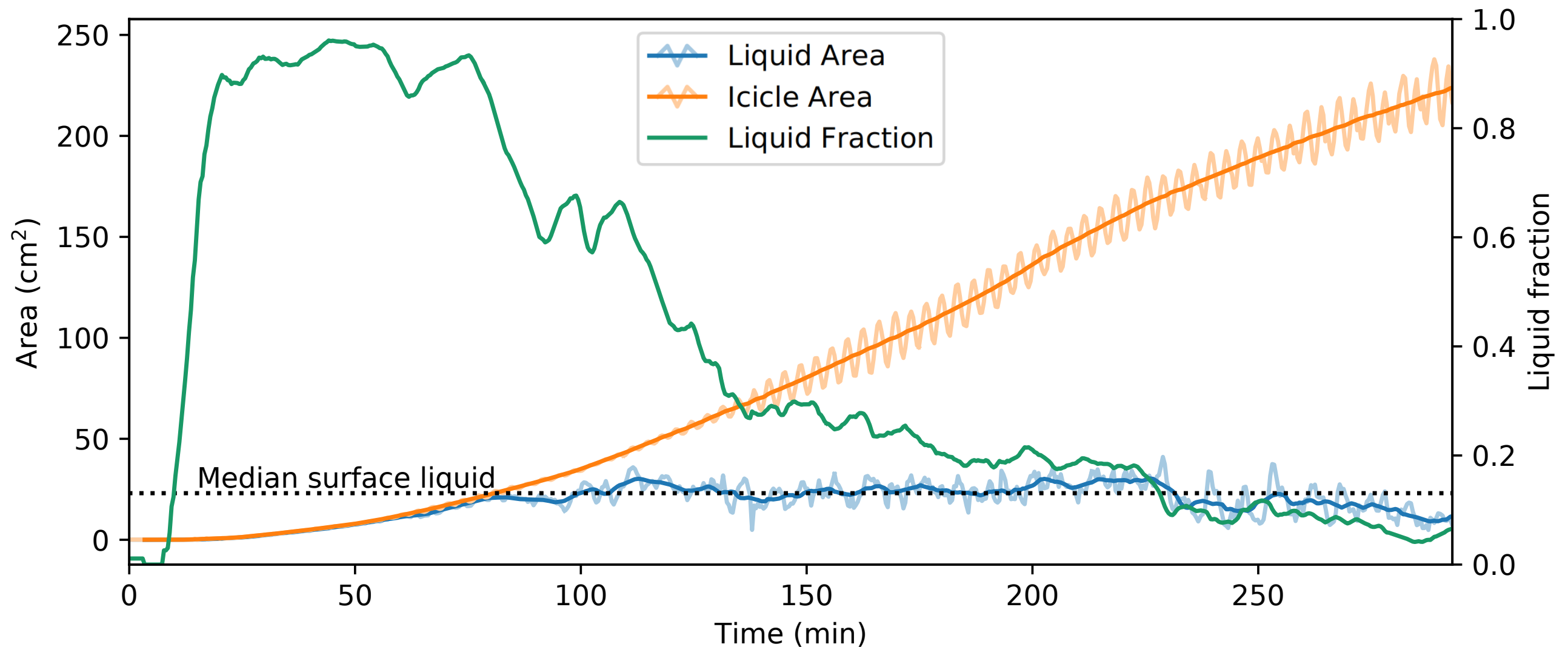
Flow visualization on fluorescent icicles

The amount of liquid on the surface is a
strong function of the impurity
concentration.



Flow visualization on fluorescent icicles

The amount of liquid on the surface is a strong function of the impurity concentration.



All icicles show a common pattern; liquid covers the **entire surface** until a **certain size** is reached, after which the liquid area remains constant through the rest of the icicle growth.

A different experiment

Non-fluorescent dye

M. Demmenie *et al*, Phys. Rev. Applied 19, 024005 (2023).

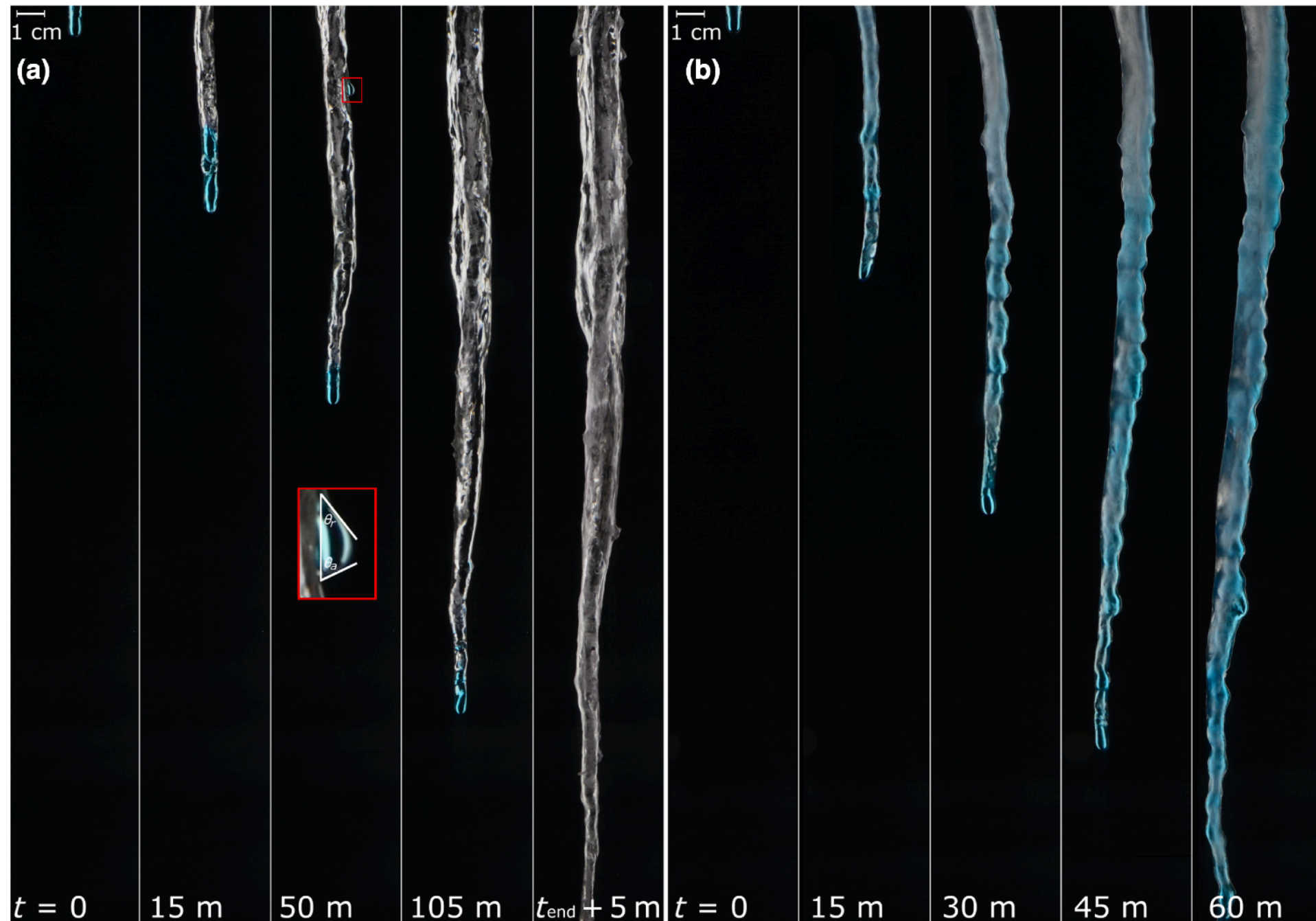


FIG. 8. Presence of liquid water around an icicle during growth at -15°C from highly purified water (a) and water containing $8.6 \times 10^{-2}\text{ M}$ NaCl (b). The red box depicts a magnification of one droplet sliding down, which is used to analyze the advancing (θ_a) and receding (θ_r) contact angle. The water contained $2.5 \times 10^{-5}\text{ M}$ methylene blue in both experiments. This blue dye loses its color when trapped in a solid, therefore it exclusively accentuates the liquid areas on the icicle. Note that figure (a) is displayed on a nonlinear timescale, in contrast to (b). Hence, this emphasizes linear length growth in time for icicles containing solutes. $t_{\text{end}} = 160\text{ m}$.

Milli-Q
water
with
 $25\ \mu\text{M}$
methyl
blue
dye

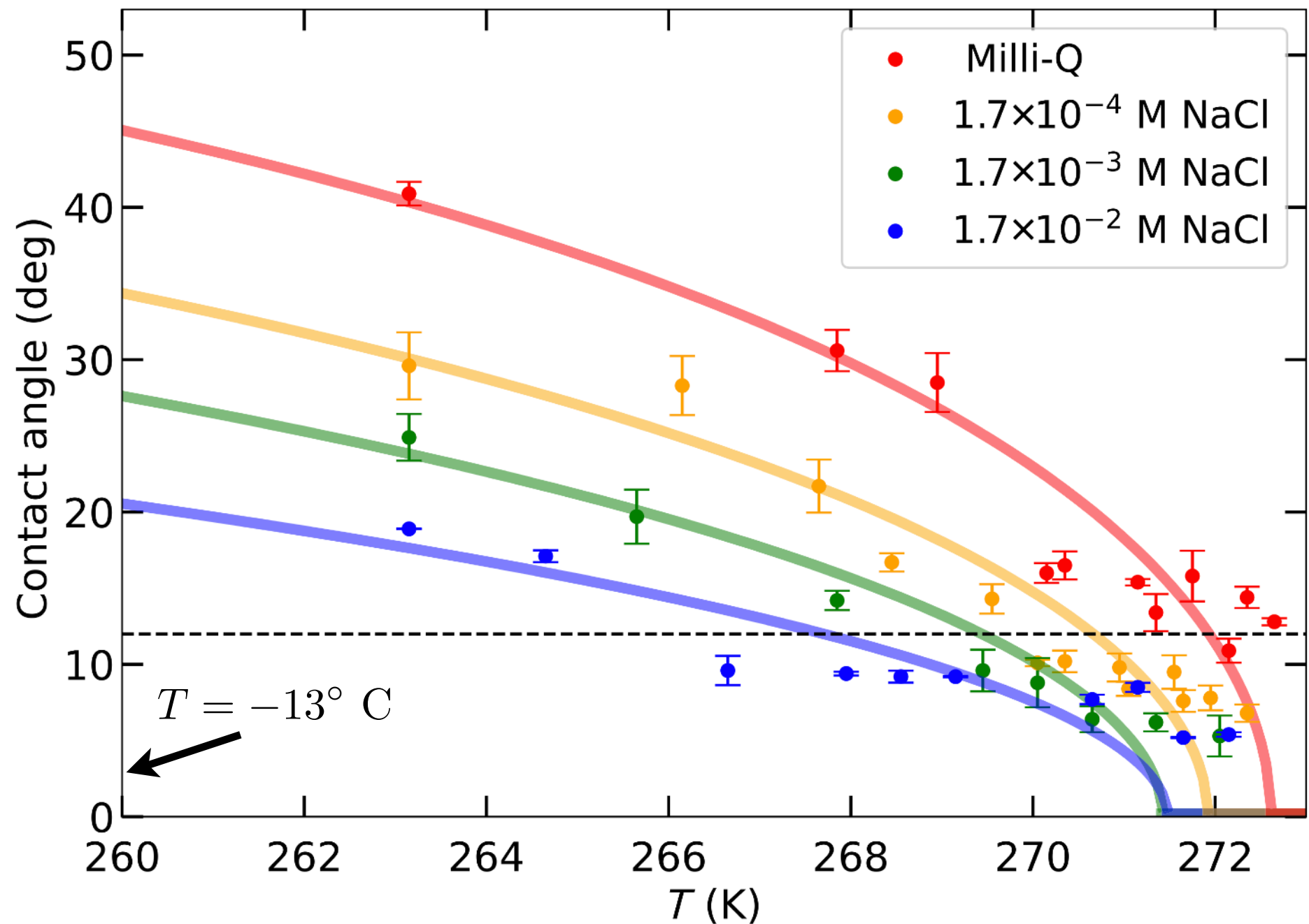
Water
with
 $86\ \text{mM}$
NaCl
and
 $25\ \mu\text{M}$
methyl
blue
dye

A different experiment

The contact angle of impure water on impure ice drops rapidly with impurity concentration.

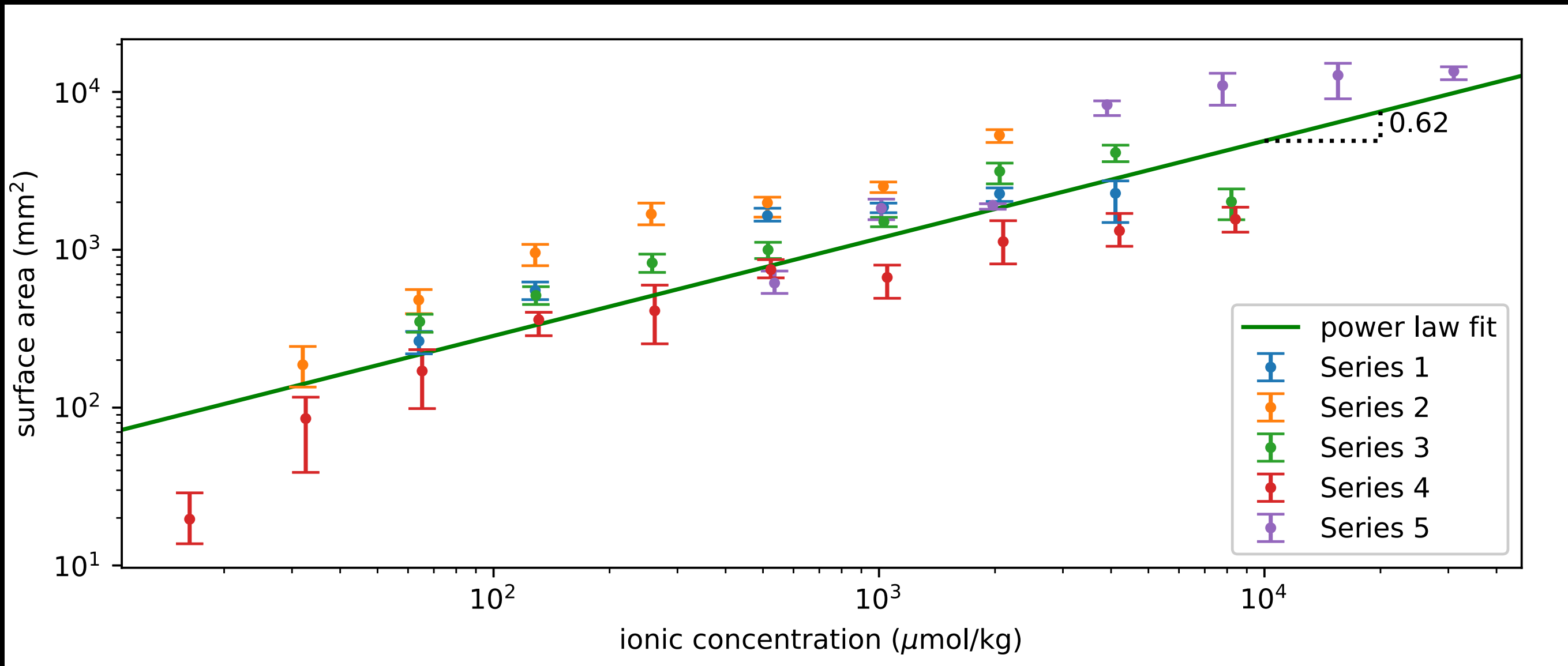
So the impure liquid spreads over a larger area, eventually forming rivulets.

M. Demmenie *et al*, Phys. Rev. Applied 19, 024005 (2023).



Flow visualization on fluorescent icicles

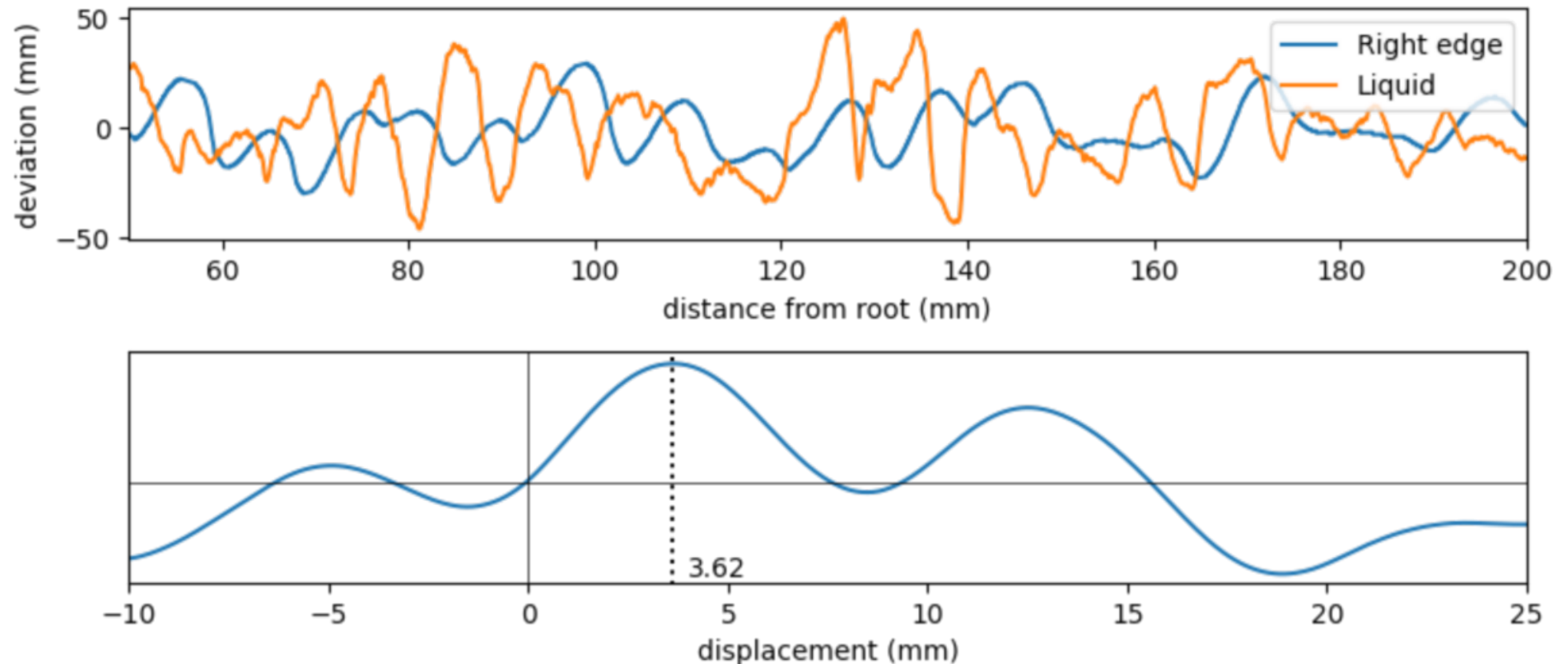
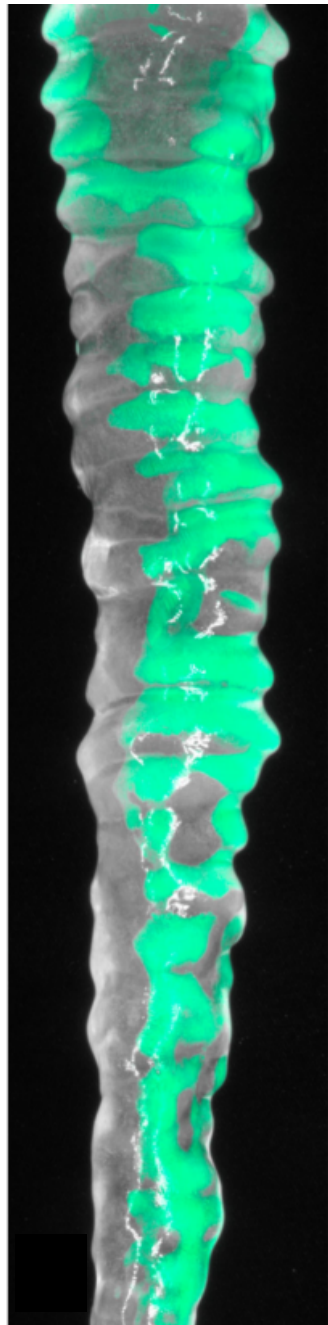
The amount of liquid on the surface is a strong function of the impurity concentration.



The area covered by liquid on the surface increases like a **power law** with the impurity concentration.

Flow visualization on fluorescent icicles

The liquid on the surface is located slightly **on the upper side of the ripples**, on average.



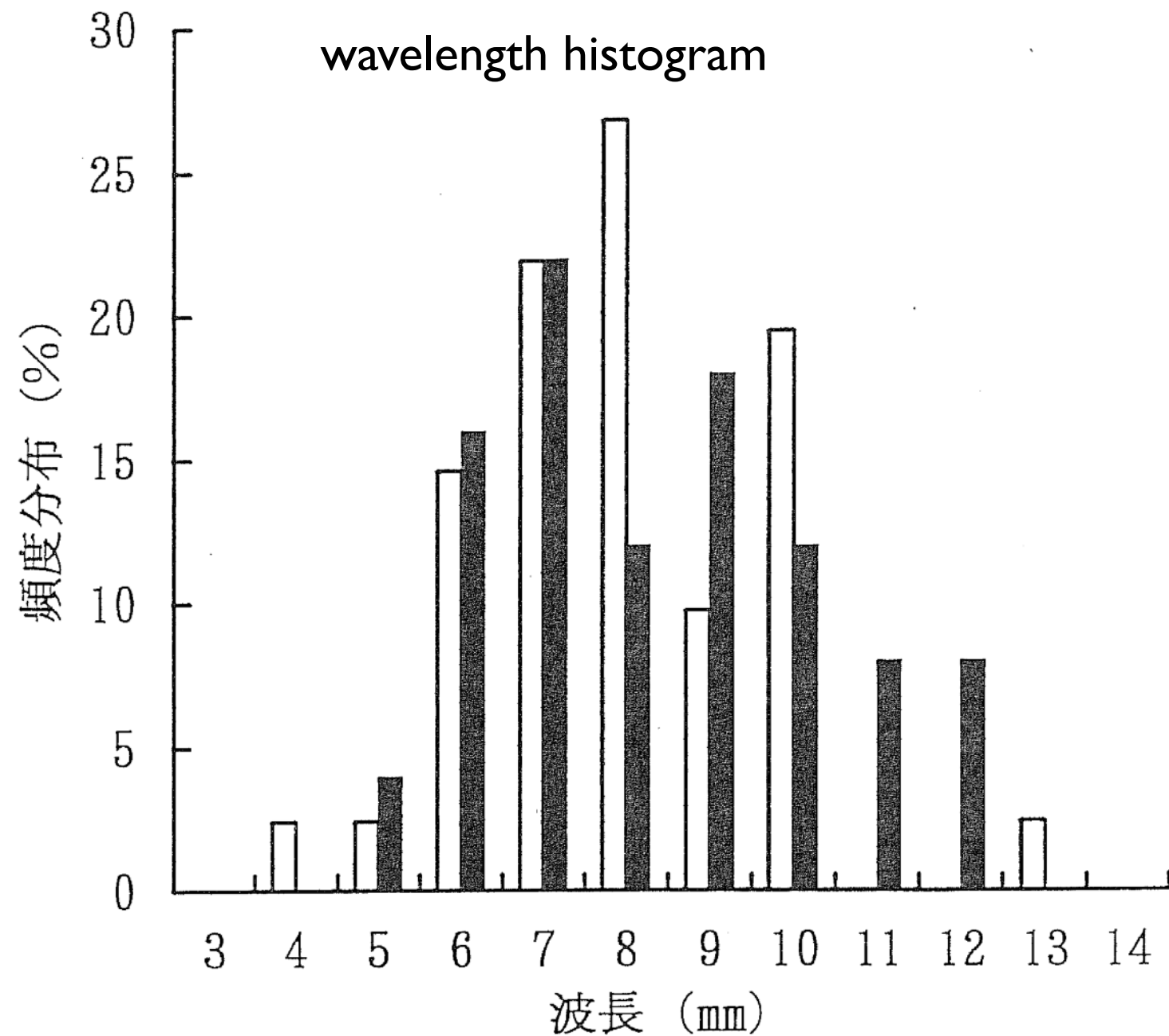
The first correlation peak between the dye signal and the topography is displaced upward by about $1/3$ of a ripple wavelength.

J. Ladan and S.W. Morris, *New J. Phys.*, **23**, 123017 (2021).

This is probably why the ripples move upward.

Ripples on a stick

Matsuda M.Sc. thesis 1997. He used tap water.



■ 自然界のつらら 平均波長 8.5mm, N= 82

□ 槌の表面 (90°) 平均波長 8.2mm, N=100

Ripples on a stick

Ripples on a rough wooden stick, made in the Cambridge cold room in 2006



ice
on a
stick

Ripples on a pipe

Ripples on the exterior
of a large diameter
metal drain pipe.



Cylinder experiments

Start with a pre-formed cylinder made of **distilled water** ice.

Then flow **fluorescein** doped water over it, to form impure ice.

Expect ripples appropriate to this concentration...

Puzzling result:

Impure ice forms and the cylinder thickens, but **no ripples form**.

$$T = -13.7^{\circ}\text{C}$$

$$Q = 3.0 \text{ g/min}$$

$$1.52 \text{ mM/kg Fl. dye}$$

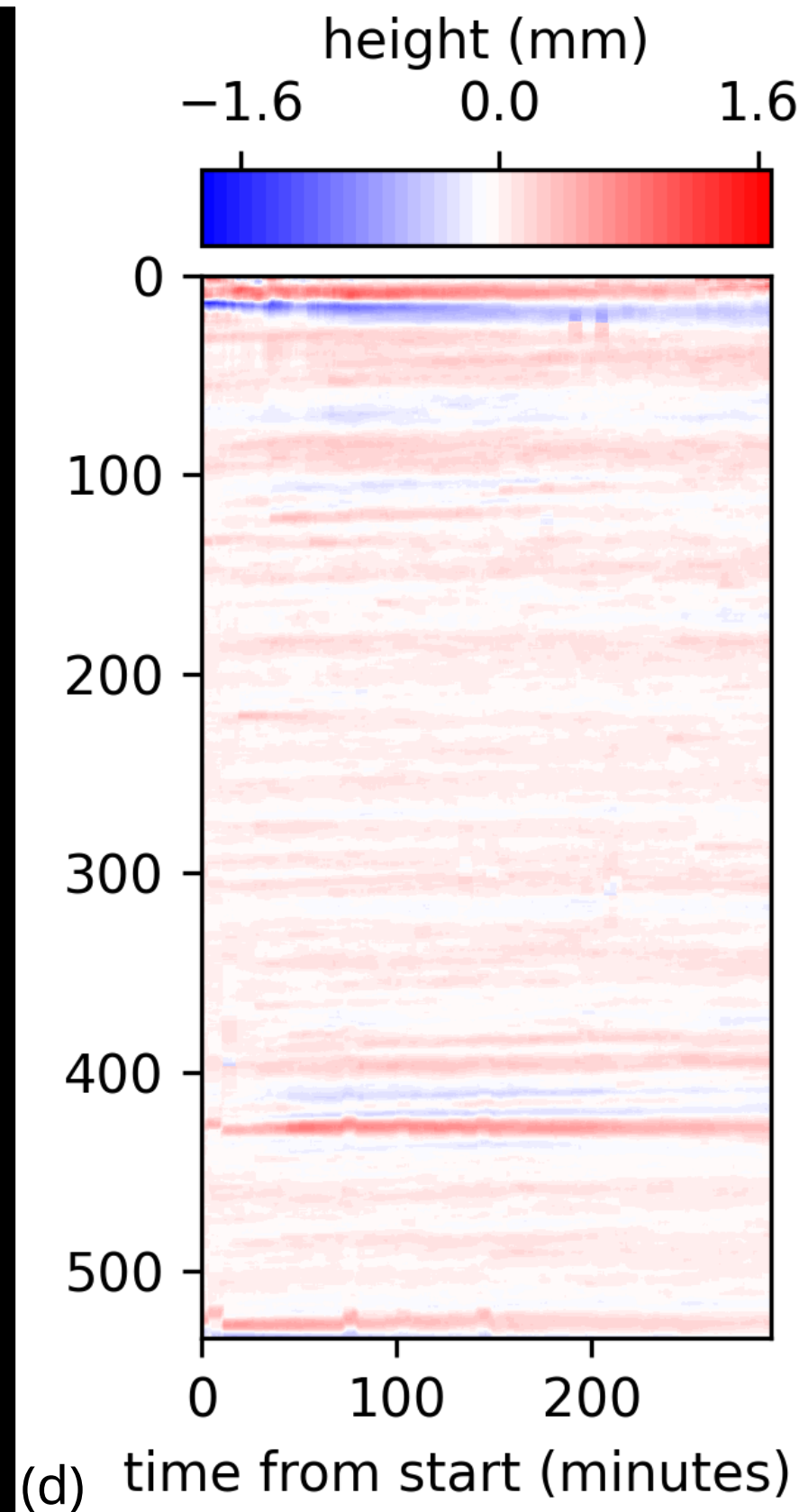
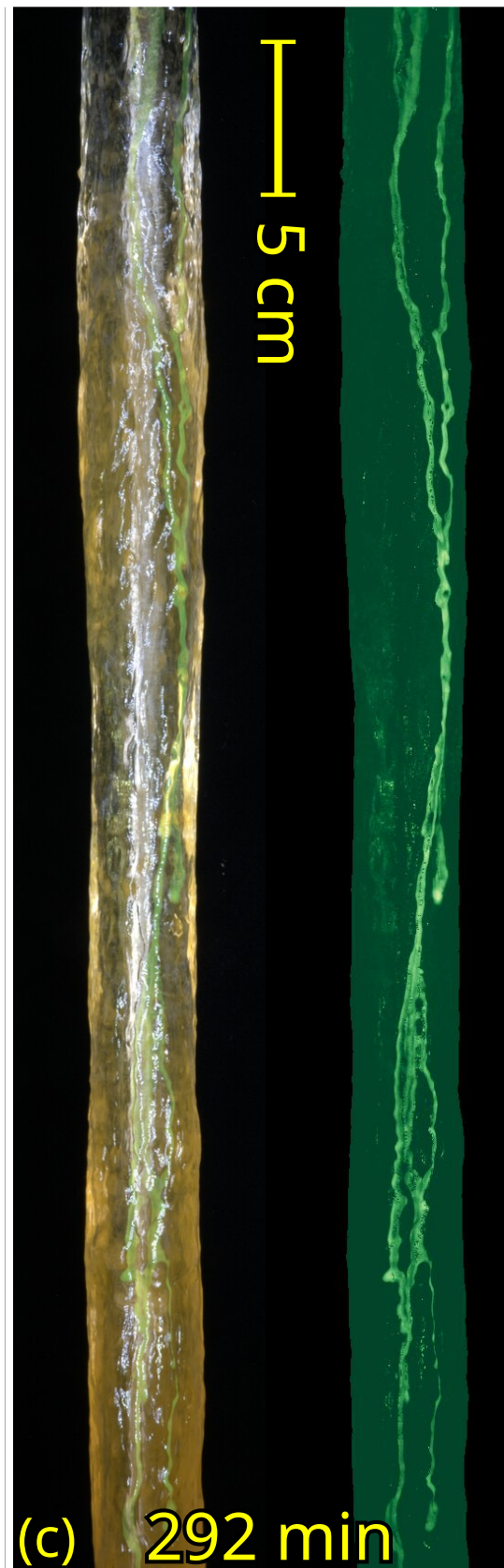


Cylinder experiments

Puzzling result:

Impure ice forms and the cylinder thickens, but **no ripples form**.

We observe rivulets and some dye trapped in the ice, but the cylinder does **not** behave like an icicle made from the same concentration.



Cylinder experiments

Puzzling result:

Impure ice forms and the cylinder thickens, but **no ripples form**.

We observe rivulets and some dye trapped in the ice, but the cylinder does **not** behave like an icicle made from the same concentration.

icicle

$$T = -14.1^{\circ}\text{C}$$

$$Q = 3.0 \text{ g/min}$$

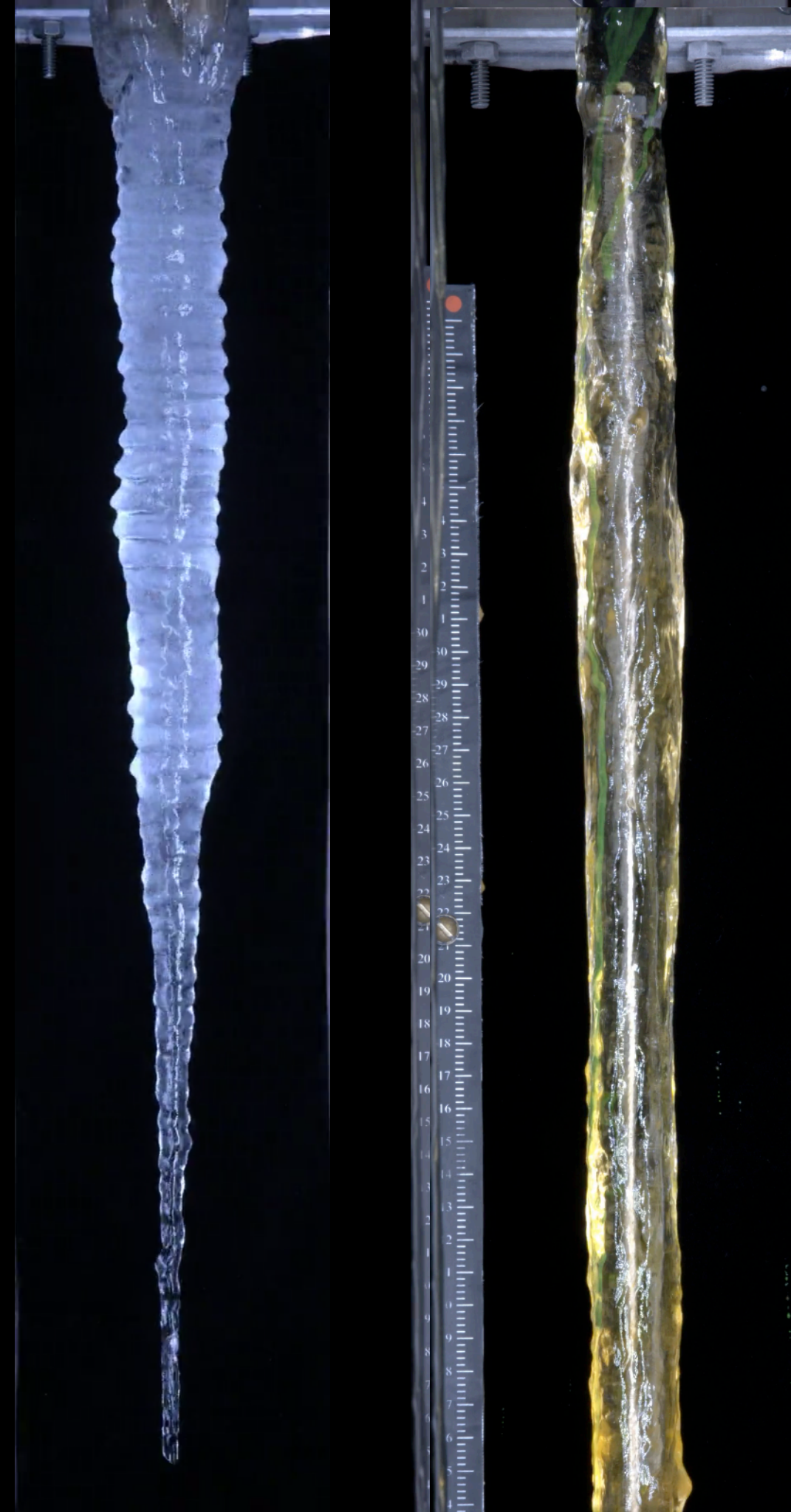
$$1.43 \text{ mM/kg NaCl}$$

cylinder

$$T = -13.7^{\circ}\text{C}$$

$$Q = 3.0 \text{ g/min}$$

$$1.52 \text{ mM/kg Fl. dye}$$



Cylinder experiments

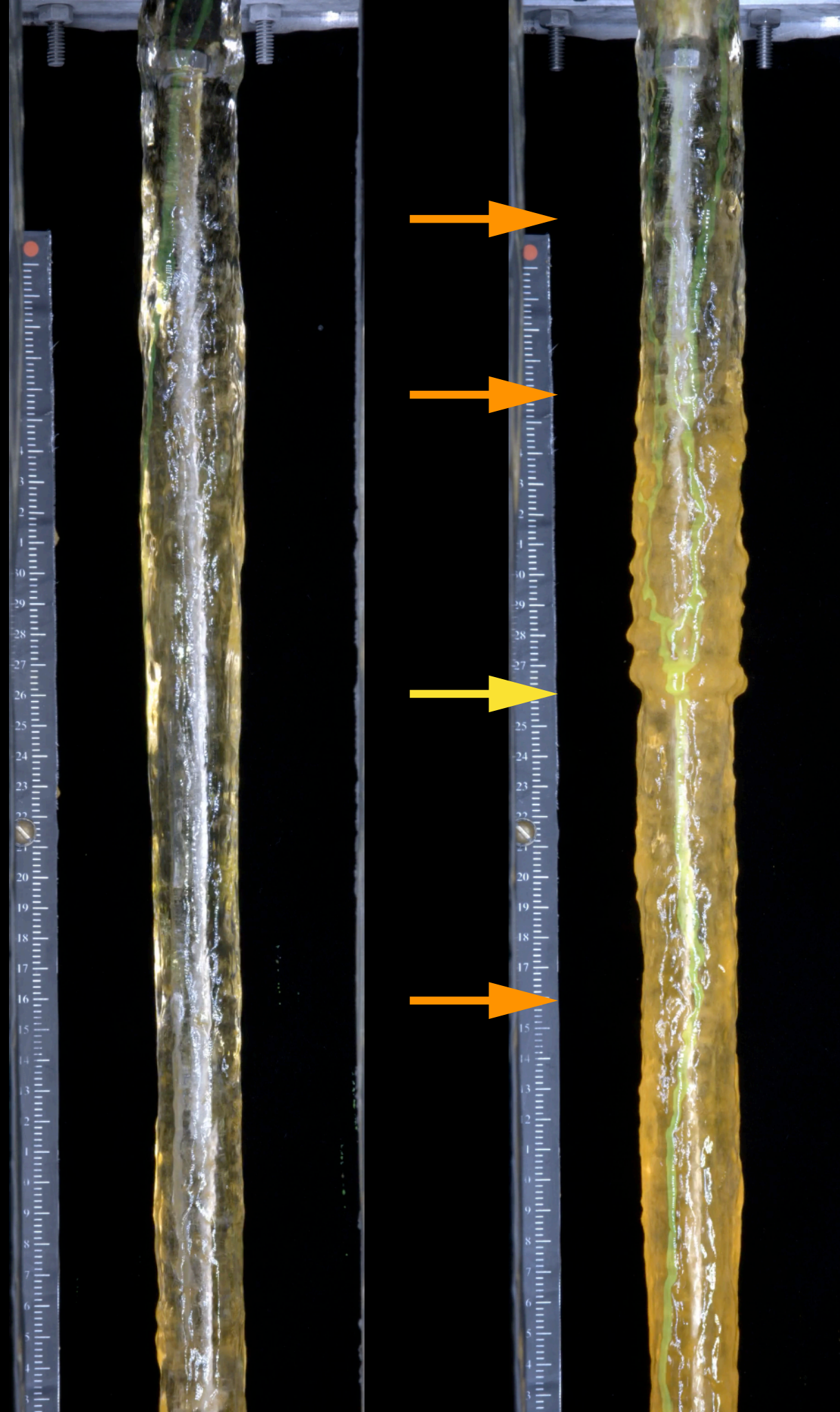
Compare the smooth cylinder to one with **artificial grooves** in it, under otherwise identical conditions.

Deep groove →

Shallow grooves →

Now we see *some* ripples form, mostly above the deep groove, and also observe more trapped impurity (**orange**).

$T = -13.7^{\circ}\text{C}$
 $Q = 3.0 \text{ g/min}$
 $1.52 \text{ mM/kg Fl. dye}$



Cylinder experiments

Start with a **truncated** pre-formed cylinder made of distilled water ice.

Then flow fluorescein doped water over it, that also drips off the end and forms impure ice.

Interesting result:

Impure ice forms and the cylinder thickens, but only slowly, as most of the water drips off. No ripples initially form on the pure substrate.

A typical rippled icicle forms rapidly below.

One ripple eventually grows just **above** the breakpoint.

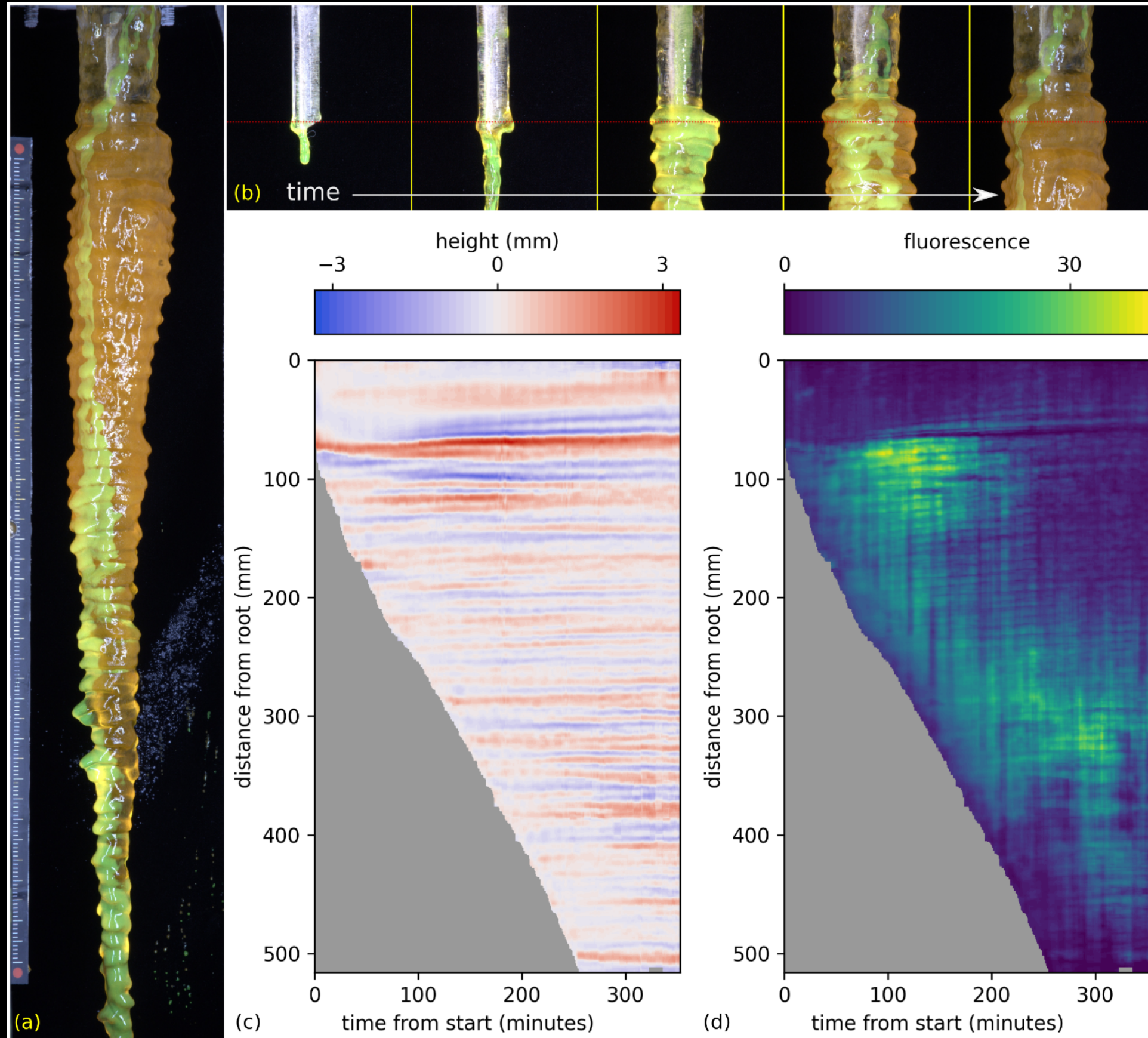
These experiments seem to suggest that ripples require the right ice internal structure, as well as impurities and topography to grow.



Cylinder experiments

Analysis of the truncated cylinder, showing ripple topography and fluorescence.

$T = -11.5^{\circ}\text{C}$
 $Q = 3.0 \text{ g/min}$
3.56 mM/kg Fl. dye



Crystal structure of icicles

Ignored so far

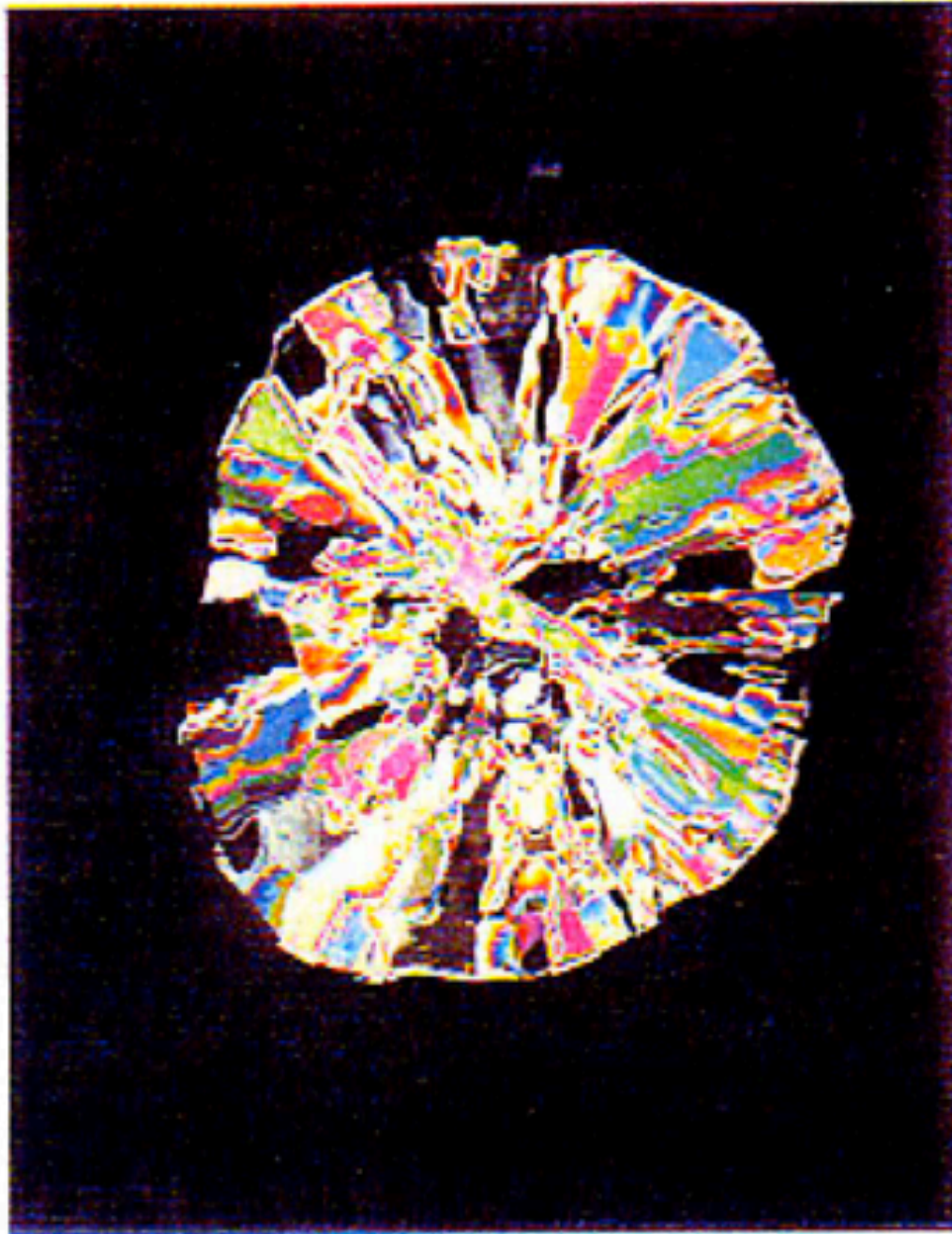


図 4-7-a 天然つらら (横断面)

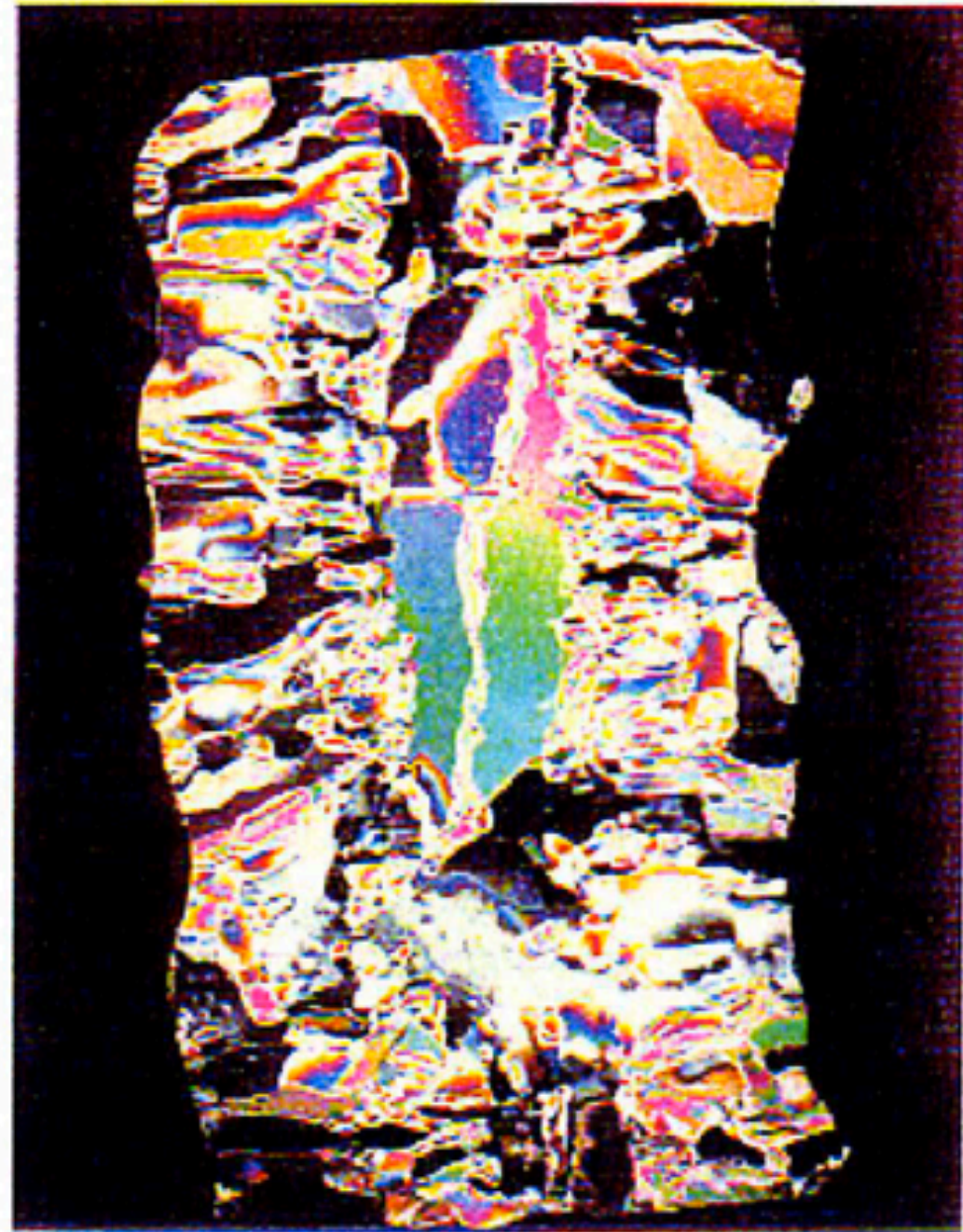
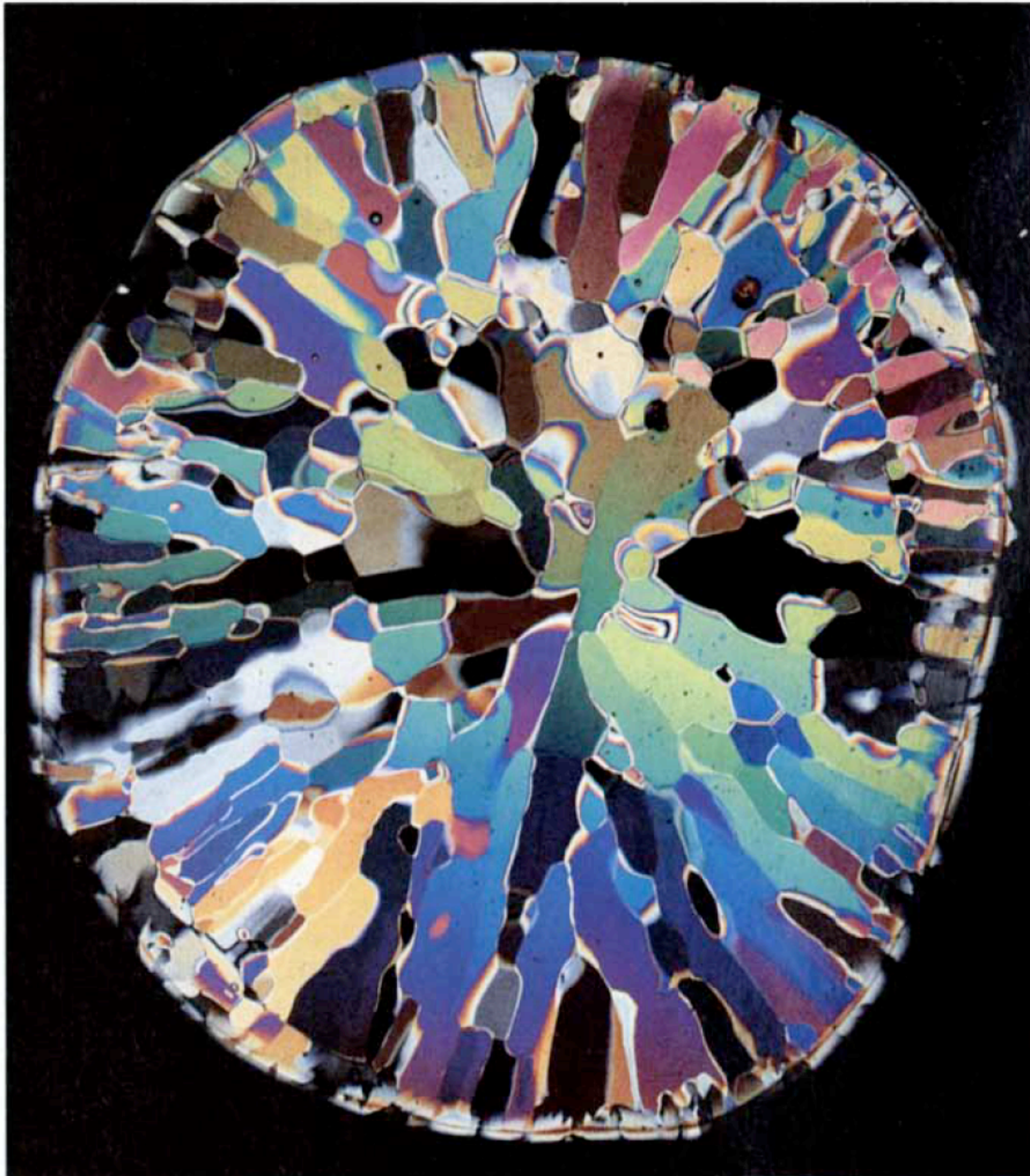


図 4-7-b 天然つらら (縦断面)

S. Matsuda, M.Sc. thesis, Hokkaido University, 1997.

Crystal structure of icicles

Ignored so far



Jearl Walker,
The Amateur Scientist,
Scientific American,
May 1988.

Crystal structure and grain boundaries

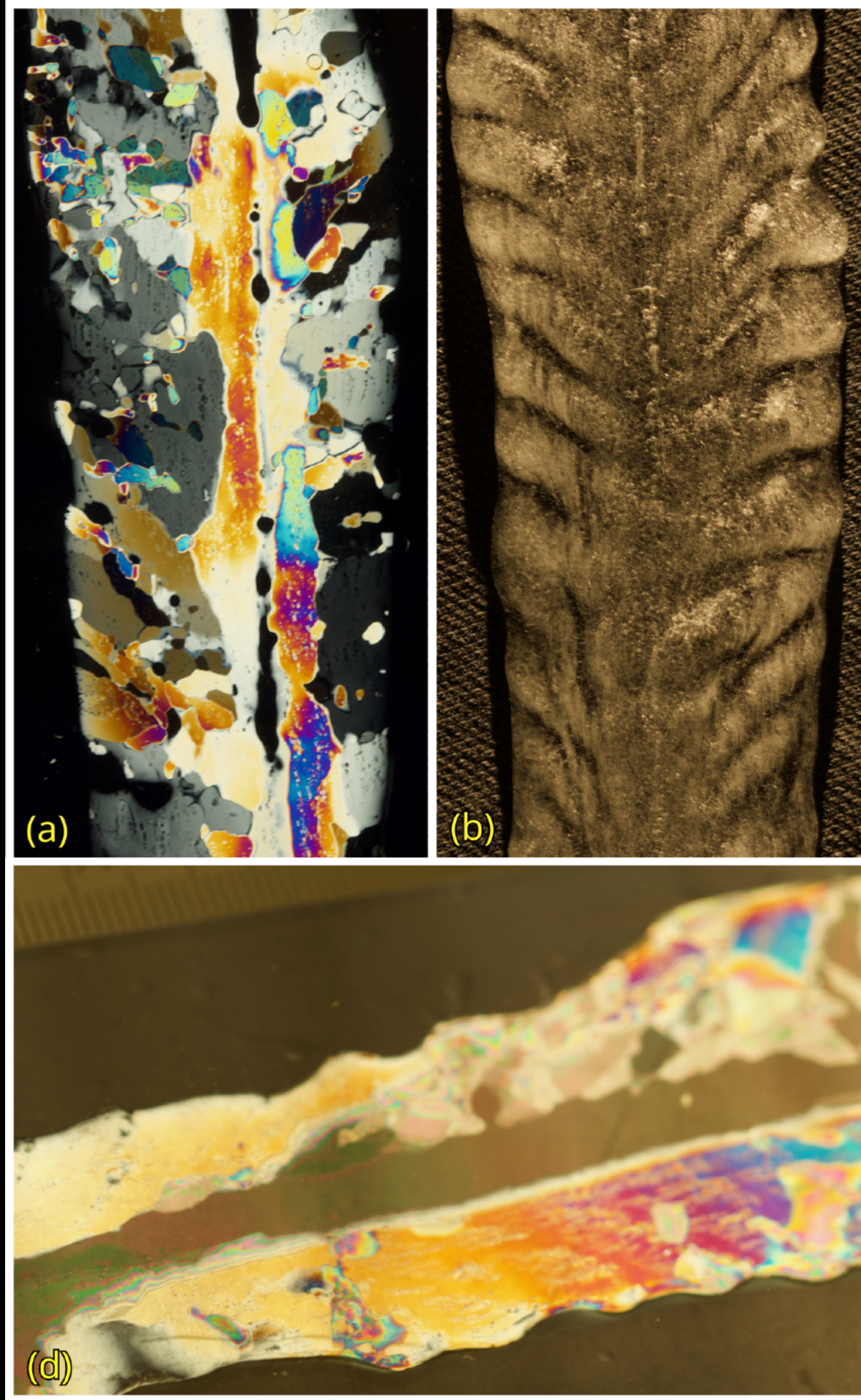
Icicles are **polycrystalline**, with mostly single crystal cores, formed at the advancing tip. Cores contain air bubbles, while other parts of the icicle do not.

Grain shapes are only **weakly correlated** with chevrons and upward ripple growth.

Some grains span several ripples.

C.A. Knight, *J. Crystal Growth*, **49**, 193 (1980).

J. Ladan and S.W. Morris, *New J. Phys.*, **23**, 123017 (2021).



Future work

Homework questions for the student

- **Temperature:** we really only measured T_{air} . What is the temperature inside the ice? On the surface between rivulets? In the rivulets? At the tip?
- **Dye:** Reverse engineer the C, T dependence of dye quenching with mixtures to learn about flow, temperature, inclusions etc.
- **Surface tension:** More experiments with surfactants, anti-freeze proteins, etc.
- **Inclusions:** What is the mechanism of inclusion formation? What determines their density, composition? Why are they clustered radially, like tree rings? Is the radial clustering caused by the successive passage of rivulets?
- **Ripples:** are they really a linear instability? Why are impurities required? Need *single rivulet* experiments (*Monier et al., Phys. Rev. Fluids, 5, 062301 (R) (2020)*).
- **Growth:** Does the ice only grow under rivulets or also elsewhere?
- **Shape:** Can we make a model of mode 1 (tip growth) and mode 3 (pipe filling growth) to account for the ratio v_{tip}/v_g and improve upon the shape theory?

The end

John Ladan



Antony Chen

