University of Toronto
ADVANCED PHYSICS LABORATORY

COMP

Measurement of the Compton Total Cross Section

Revisions:
2018 September: David Bailey <dbailey@physics.utoronto.ca >
1994: Derek Paul

Please send any corrections, comments, or suggestions to the professor currently supervising this experiment, the author of the most recent revision above, or the Advanced Physics Lab Coordinator.

Introduction

The goal of this experiment is to measure the inelastic scattering of photons by free electrons and determine the fundamental parameter of Quantum Electrodynamics (QED) with the highest precision you can achieve. This parameter – the Fine Structure Constant α_{QED} – measures the coupling strength of the QED gauge boson (the photon) to a charged spin ½ point fermion (the electron). Precision measurements of α_{QED} are an important way to look for new interactions beyond the Standard Model of Particle Physics.

In this experiment, the absorption of gamma-rays by materials with various atomic numbers is measured. The total cross section for Compton scattering is determined, and a value for the fine-structure constant calculated. Arthur Compton won the 1927 Nobel Prize in Physics for his discovery of the Compton effect.

Theory

Compton[1] showed that the relationship between incident and scattered photon wavelengths is

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta) \tag{1}$$

where λ' is the scattered wavelength, λ is the incident wavelength, θ is the scattering angle of the photon (Figure 1), and m_e is the electron rest mass. The coefficient $\lambda_c = h/m_e c$ is called the Compton wavelength of the electron.

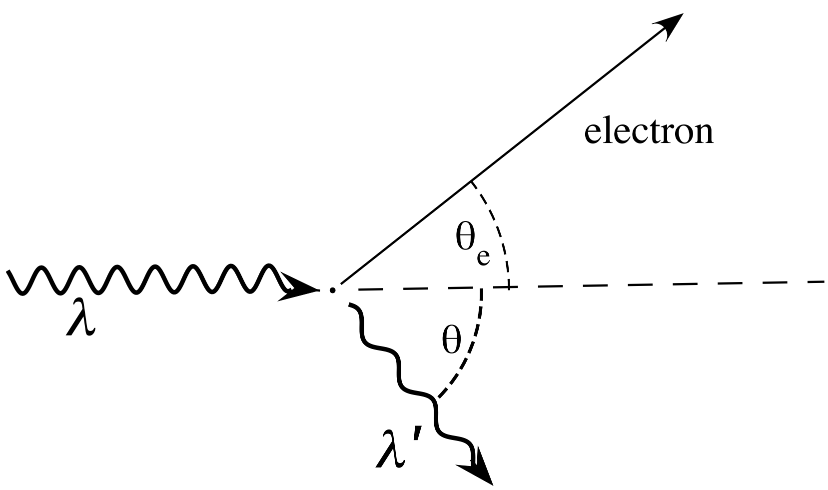

Figure 1. Compton scattering geometry.

The formula is easily derived by assuming a relativistic collision between the gamma ray and an electron initially at rest. A useful form of the formula is

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1 - \cos\theta)} \tag{2}$$

which gives directly the scattered gamma energy hν' terms of the incident energy hν. In terms of energy and momentum conservation, the Compton collision is an elastic collision if the electron was initially free. The collision is inelastic in the sense that one photon is absorbed and another of different frequency and momentum is emitted.

The lowest order Feynman diagram [See, for example, R.B. Leighton[2], §20-12] for Compton scattering is shown in **Figure 2**. The graph is one-dimensional, with the solid line representing the electron moving forward in time, and the wavy paths representing the incident and emitted photons. Because of the uncertainty principle, energy and momentum do not have to be conserved between times 1 and 2.

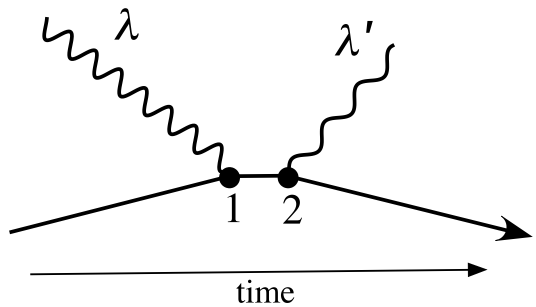

Figure 2. Lowest order Feynman graph for a photon scattering from a free electron, i.e. Compton Scattering.

Every time the photon couples to an electron, we expect a factor of $\alpha_{QED} \approx 1/137.037$ in the rate, so we expect the rate to be proportional to $(\alpha_{QED})^2$ for this process. The probabilities for scattering processes are parameterized by *cross-sections* that have units of area, i.e. length squared. Since neither the photon nor the electron has an intrinsic size, the only length scale in the process is given by the photon centre-of-momentum wavelength λ_{cm}, so we expect the total cross-section, σ, for the process shown in **Figure 2** to be of order $\alpha_{QED}{}^2\lambda_{cm}{}^2$. Using simple relativistic kinematics, we can show that

$$\lambda_{cm}^2 = \frac{h^2}{p_{e,cm}^2} = \frac{4h^2}{\left(1+2\gamma\right)m_e^2c^2}$$

So we expect the total Compton cross-section to be of order

$$\sigma \sim \alpha_{QED}^2\lambda_{cm}^2 = \frac{4\alpha_{QED}^2h^2}{\left(1+2\gamma\right)m_e^2c^2} = \frac{4h^2c^2}{\left(1+2\gamma\right)}r_e^2$$

where $r_e \equiv \dfrac{\hbar c\alpha_{QED}}{m_e c^2} \approx 2.812\times10^{-15}m$ is the *classical electron radius*, and $\gamma = E/m_ec^2$ is the lab frame photon energy in units of the electron mass. So we expect the total Compton cross-section to be approximately constant for photon energies less than the electron mass, and to fall off with photon energy at high energies greater than the electron mass.

Differential cross section for Compton Scattering

The cross section for Compton scattering depends on the spin orientation of the target electron and the polarization of the incident gamma rays, but most targets contain ~100% unpolarized electrons, and most ordinary gamma ray sources are unpolarized. The Compton scattering differential cross section for gamma rays from an unpolarized target was first calculated by Klein and Nishina[3] (also see R.D. Evans[4], Chapter 23 §2, and W. Heitler[5], Chapter V, §22]) to be

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \frac{\gamma'^2}{\gamma^2} \left(\frac{\gamma}{\gamma'} + \frac{\gamma'}{\gamma} - \sin^2\theta \right) \tag{3}$$

where γ and γ' are the energies of the incident and scattered gamma rays in units of $m_e c^2$.

Lastly, we note that the target electrons are not free as has been assumed so far, but bound in atoms, molecules, often in condensed matter. However, the low-Z elements do not have tightly bound electrons. For example, the average K-shell binding energy of oxygen electrons is 0.7 keV, which is very much less than the 662 keV energy of gamma rays from Cs^{137}, a typical source for these experiments. The great majority of the electrons in these light elements have much smaller binding energies, of order 10 eV. The momenta of most target electrons in light elements make little difference to the observed Compton cross sections.

Experiment

Safety Reminders

- You must receive instruction in the safe use of radioactive sources from either a Professor or the Advanced Lab Technologist before using any radioactive sources.
- Radioactive sources must be signed out and signed back in each time they are used.
- When using a radioactive source, always inspect it for damage or corrosion. Report any problems to a professor or the Advanced Lab Technologist; ask if you are in doubt.
- Radioactive sources must never be left unattended in an unlocked room.
- Food or drink are not allowed in any of our Advanced Lab Experiment rooms. Radioactive materials and lead are most dangerous when they are ingested or inhaled.
- Gloves should be worn when handling unpainted or untaped lead shielding.
- Be careful when moving lead shielding. Lead blocks are very heavy and can easily break a finger or a foot if dropped.
- Lead must never be cut, filed, or machined in any way by students.
- Wash your hands when you leave the lab.
- *NOTE: This is not a complete list of every hazard you may encounter. We cannot warn against all possible creative stupidities, e.g. juggling cryostats. Experimenters must use common sense to assess and avoid risks, e.g. never open plugged-in electrical equipment, watch for sharp edges, don't lift too-heavy objects, ... If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. When in doubt, ask! If an accident or incident happens, you must let us know. More safety information is available at http://www.ehs.utoronto.ca/resources.htm.*

In this experiment, the total cross section σ_{Tot}, per electron is measured in various target materials, where

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \tag{4}$$

and the integral is taken over the entire solid angle of 4π steradians about the scattering centre. Usually $d\Omega$ is written

$$d\Omega = 2\pi \sin\theta d\theta \tag{5}$$

where θ is a polar angle, to be identified with the scattering angle in Figure 1.

This experiment can be done using a single scintillation counter and a mono-energetic gamma source such as Cs^{137}. For nuclear detection techniques see Leo[6]. About 5 µCi of Cs^{137} is sufficient. A suitable geometrical arrangement of source, collimator, absorbers (scatterers), scintillator, and photomultiplier tube (PMT) is shown in Figure 3.

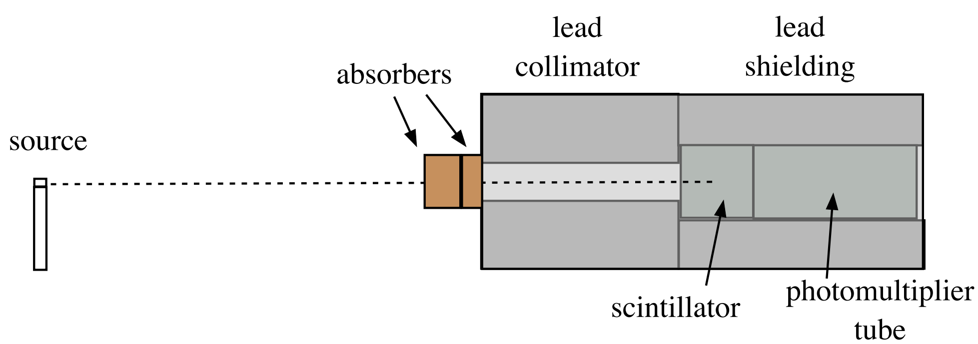

Figure 3. Sketch of Compton experimental set-up

It is easily shown that in an ideal situation, in which no scattered gamma rays pass through the collimator into the detector, the number of gammas reaching the detector through the collimator is given by

$$I(x) = I(0) \exp(-n\sigma_{Tot}x) \tag{6}$$

where n is the atomic density of the absorbers (atoms/m^3), σ_{Tot} is the total cross section per atom for <u>all processes</u> of gamma ray interaction with the absorbers and x is the absorber thickness.

The experiment consists "merely" of measuring $I(x)$ versus x and deducing the absorption coefficient $n\sigma_{Tot}$ from the results, and of making careful thickness and density measurements on the absorbers. This results in a total cross section σ_{Tot} for each of several materials. We know that σ_{TOT} is the sum of several partial cross sections

$$\sigma_{Tot} = \sigma_{pe} + \sigma_{pair} + \sigma_{el} + Z\sigma \tag{7}$$

where

- σ_{pe} is the photo-electric cross section for total gamma ray absorption.
- σ_{pair} is the cross section for the production of electron-positron pairs, and which is forbidden by energy conservation if the energy of the photon is insufficient to produce such a pair, i.e. if $h\nu < 2m_ec^2$.

- σ_{el} is the elastic gamma scattering cross-section which we shall assume to be small[†].
- σ is the Compton scattering cross section per electron as defined in equation (3), and *Z* is the atomic number of the material which is also the number of electrons per atom.

Equation (7) shows that a plot of σ_{Tot} versus Z should yield σ provided the other cross sections are very small over some part of the range. The Klein-Nishina formula (3) can be integrated analytically and yields

$$\sigma = 2\pi\, r_o^2 \left\{ \frac{1+\gamma}{\gamma^2} \left[\frac{2(1+\gamma)}{1+2\gamma} - \frac{1}{\gamma}\,\ell n(1+2\gamma) \right] + \frac{1}{2\gamma}\,\ell n(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2} \right\} \tag{8}$$

Equation (5), and therefore also (8), is the result of a purely quantum electrodynamic (QED) calculation based on the first order Feynman graph of Figure 1. The experiment therefore represents a direct first-order test of modern electromagnetic theory (QED). For high Z absorbers the photoelectric cross section is however usually non-negligible and has been found experimentally to vary approximately as

$$\sigma_{pe} \propto Z^{4.2} \tag{9}$$

It is thus possible to determine σ_{pe} and σ if sufficient care is taken in carrying out and analyzing the experiment, and in interpreting the theory. [Comparison of σ_{pe} with theory requires care, as the cross section is often calculated in separate publications for the different contributions from K-, L-electrons, etc. See W. Heitler[5] §21.]

You should compare your experimental measurement of σ with the QED prediction. From your experimental value of σ and the known mass of the electron, you can calculate the fine structure constant α using equation (8) and the definition of r_e, and compare this to the literature value[7]. Precision measurements of α are the basic method by which QED is tested[*], and any discrepancies generate great interest[†].

Corrections to total cross sections

Forward scattering

The set-up illustrated in **Figure 3** is of course not ideal. Some Compton scattered photons travel in the forward direction and are detected in the scintillator counter as if they were not scattered at all, so the experimental integration of Eq. (4) is not quite over all 4π steradians. This effect is small, but with some effort it is possible to make measurements that are precise enough that this correction is not insignificant. The geometry can be improved slightly by adding a second

---

[†] The elastic (Rayleigh) scattering from the absorbers is very strongly forward peaked in these cases where it contributes significantly to σ_{Tot}. For the geometry of Figure 1 the correction for Rayleigh scattering is small.

[*] https://en.wikipedia.org/wiki/Precision_tests_of_QED gives a non-expert outline of precision tests of QED.

[†] For example, G.W. Bennett et al. (2002) Phys. Rev. Lett. 89:101804 has been cited over 450 times (https://inspirehep.net/record/591756) since it somewhat disagrees with QED.

collimator between source and scatterer, but only at the expense of total solid angle: i.e. the source usually has to be further removed from the detector to do this effectively.

To obtain a reliable value of σ, an estimate of the forward scattering correction should be made by numerically integrating Eq. (4) over the range of angles where scattered photons can be detected. This probability changes with position of the absorber relative to the collimator.

Absorber impurities

Pb absorbers sometimes contain 3% to 5% Sb, which must be allowed for in computations of σ_{pe} for Pb. If your result for the Compton total cross-section is sensitive to σ_{pe} for Pb, then you might try to estimate the Sb content in your Pb samples using X-ray fluorescence or from the density of the samples in comparison with the density of pure Pb.

Effect of electron binding energies

Another correction is in principle necessary because the electrons are initially bound, not free. This correction is known to be small if the electrons are lightly bound in the solid scatterer, but this is not the cause for the inner electron in heavy atoms such as Lead or Bismuth.

Photoelectric cross-section

If you have sufficient data, you may be able to test whether Equation (9) describes the photoelectric cross-section, or whether the power may be different from "4.2".

Hints

- Make sure you know what a "Compton Edge" is. It appears in your spectra, so since the experiment is all about Compton scattering, it is something you might be asked about.
- Analyse your data as you go along. This will help you decide how to most efficiently spend your time to get the most accurate result possible, and will reduce the chances of taking poor quality data.

References

[1] Compton, A.H. (1923). *A quantum theory of the scattering of x-rays by light elements*, Phys. Rev. 21(5), 483-502; doi:10.1103/PhysRev.21.483.

[2] Leighton, R.B. (1959). *Principles of Modern Physics*, New York; McGraw-Hill; https://archive.org/details/PrinciplesOfModernPhysics.

[3] Klein, O. and Nishina, Y. (1929). *The Scattering of Light by Free Electrons according to Dirac's New Relativistic Dynamics*, Nature, 122, 398-399; doi:10.1038/122398b0.

[4] Evans, R.D. (1955). *The Atomic Nucleus*, New York: McGraw-Hill; https://archive.org/details/atomicnucleus032805mbp.

[5] Heitler, W. (1954). *The Quantum Theory of Radiation*, 3rd ed. Oxford: Clarendon Press; https://archive.org/details/in.ernet.dli.2015.177583.

[6] Leo, W.R. (1994). *Techniques for Nuclear and Particle Physics Experiments: a How-To Approach*, 2nd rev. ed. Berlin: Springer-Verlag.

[7] https://physics.nist.gov/cgi-bin/cuu/Value?alphinv gives the current best value.