Structure of Terrestrial Atmospheres

Lecture 1: Vertical Structure, Tropopause and Gravity

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General Outline

- Vertical Structure 
- Horizontal Structure 
- Equations included but will (where important) always be explained in words.
- Classical results, my old work, and my new work.
  Atmospheres of Earth and other planets, oceans,
  Shaken and stirred into a single cocktail.

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  Shaken and stirred into a single cocktail. 
- Questions and discussion welcomed (in fact required) as we go along.
  - — Not much about GCM results, but GCMs are a key part of what we do — the 'model hierarchy'.
  - — The model hierarchy *is* our theory of climate.

A Note on Equations

Strange as it may sound, the power of mathematics rests on its evasion of all unnecessary thought and on its wonderful saving of mental operations.

Ernst Mach (1838–1916)

1. So mathematics make it easier.
2. But we should always be able to explain in words after the fact.
3. So in these lectures, if you are a trusting soul, you can ignore the equations and just follow the pictures and words!

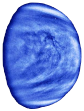
A FEW PLANETS

Three planets in the Solar System span a large parameter space:

1. Earth (of course!)
   Sidereal day = 23 hours 56 minutes. Sol = 24 hours.

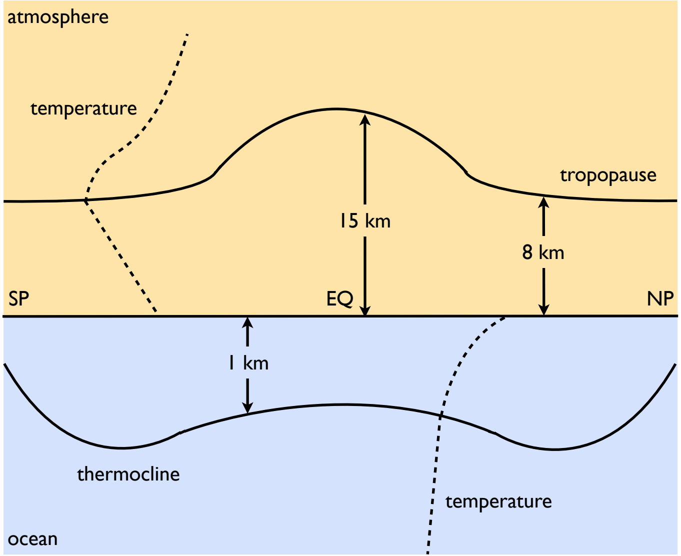
2. Jupiter.
   Sidereal day = 10 hours. Sol = 10 hours.
   Gas giant, hydrogen atmosphere and interior
   (some helium).

3. Venus.
   Sidereal day = 200 Earth days. Sol = 117 days
   Terrestrial, thick CO_2 atmosphere, large greenhouse effect.



Ocean and Atmosphere

Basic structure (Earth, our default planet)



One of our main goals is to understand this structure.

Philosophy:
Think big, research small.

Earth from Space

Characteristics:

- Weather, clouds
- Weather scales (1000 – 10000 km)
- Mid-latitude organization.
- Somewhat zonal flow.

Earth Weather. A Day in Early 2019...

NOAA Global Forecast System (GFS),

Visualization by Martin Jucker:
martinjucker.com

- Weather, clouds
- Weather scales (1000 – 10000 km)
- Mid-latitude organization.
- Somewhat zonal flow.

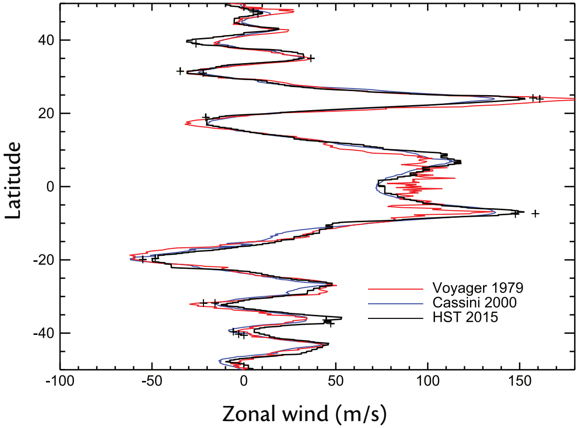
Jupiter from Space

(Enhanced color from 3 images, K. M. Gill)

Characteristics:

- Weather, clouds
- Jets!
- Global organization.
- Very zonal flow, embedded eddies.
- Jets $\sim 50\text{–}100\,\mathrm{m\,s^{-1}}$.
  Eddies $\sim 10\text{–}\ 50\,\mathrm{m\,s^{-1}}$.

Jupiter and its Jets



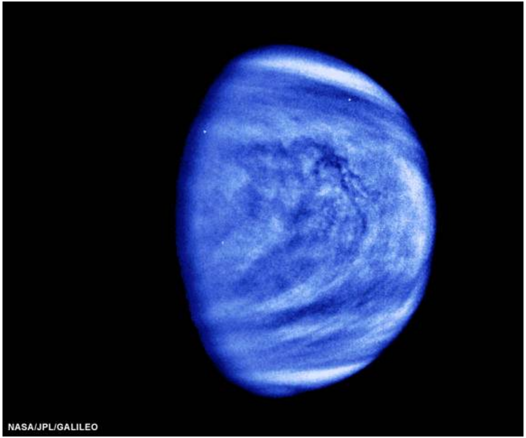
– Strong, sharp jets. Barotropically unstable $\beta - \partial^2 U / \partial y^2$ changes sign.

– Superrotates.

– Multiple super-rotating jets in the tropics!

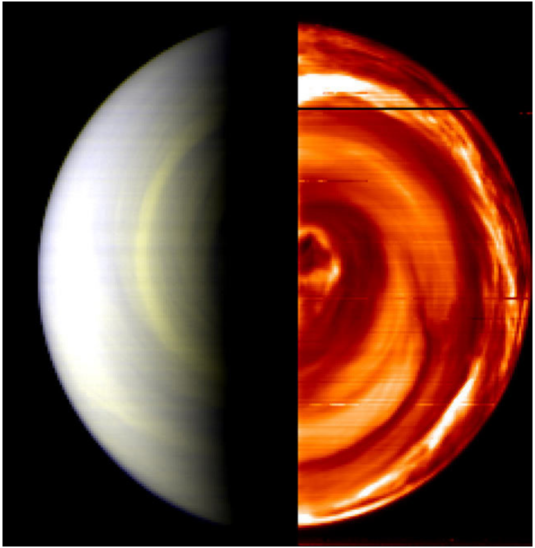
Venus from Space

Clouds (false color). Galileo



Real color. Mariner 10.

Venus – Vortex at South Pole

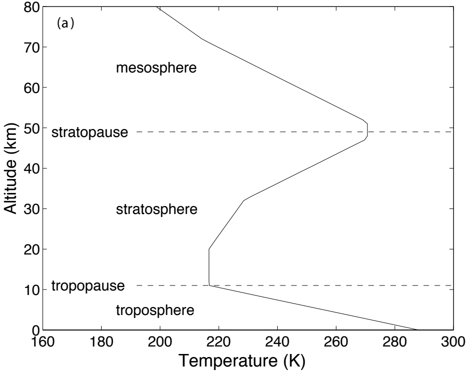


Southern Vortex (false color).

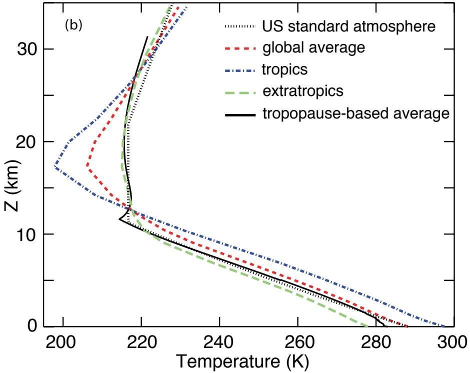
Vertical Structure, Earth

Troposphere, Stratosphere, Tropopause

'US standard atmosphere'



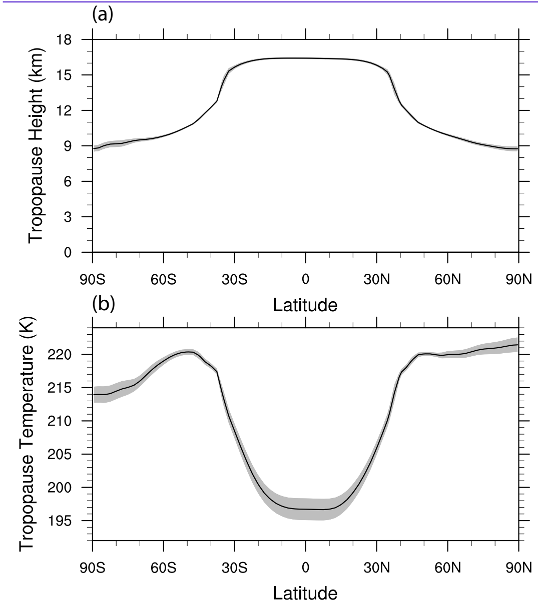
Observed profiles.



Troposphere: A region of fast dynamics in which the stratification is set dynamically.

Stratosphere: The region above that in which stratification is set radiatively

Tropopause Height and Temperature

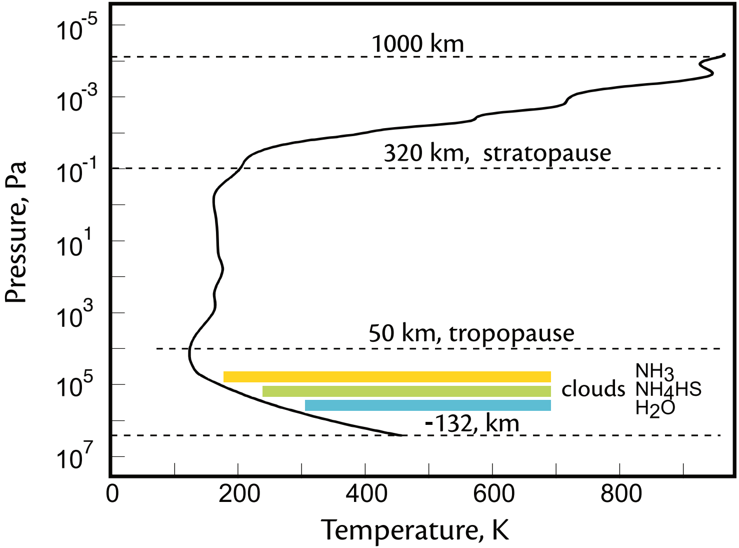


1. Tropical tropopause is higher.

2. Tropical tropopause is cooler.

NCEP2 re-analysis.

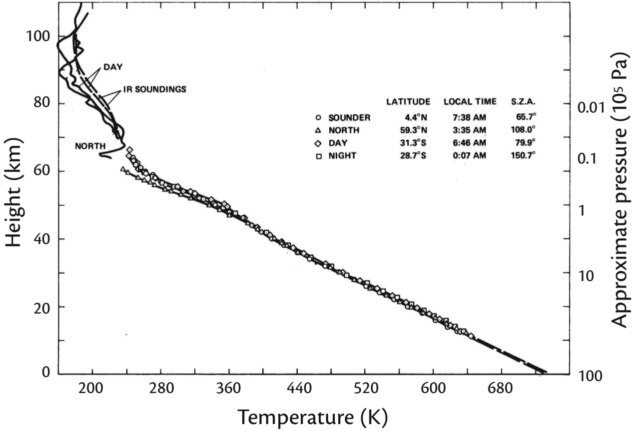
Temperature Profile on Jupiter



Features:

1. There is a troposphere and a stratosphere
2. Troposphere is stably stratified (baroclinic instability? moist convection?)
3. In the deep atmosphere almost neutral stratification.

Temperature Profile on Venus



Four probes from Pioneer. Adapted from Seiff et al (1979)

Features:

1. Almost uniform temperature in the horizontal!
2. Tropopause at about 60 km
3. Closely follows dry adiabatic lapse rate ($g/c_p = 8.8$ K/km).
4. Stratosphere (mesosphere?) weakly stratified.

Radiative Equilibrium

IR radiative transfer (Schwarzschild) equations:

$$\frac{\partial U}{\partial \tau} = U - B \qquad \frac{\partial D}{\partial \tau} = B - D,$$

where $\tau = \tau(z)$ is optical depth, U is upwards irradiance, D is downwards irradiance.

If grey $B = \sigma T^4$.

Boundary conditions at top: U = Incoming solar radiation, $D = 0$

Radiative equilibrium:

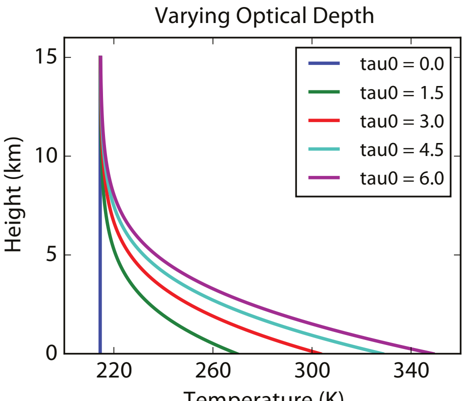
$$D = \frac{\tau}{2} OLR, \quad U = \left(1 + \frac{\tau}{2}\right) OLR, \quad B = \frac{1+\tau}{2} OLR.$$

and if $\tau \ll 1$ (e.g., stratosphere)

$$D = 0, \qquad U = OLR = 2B, \qquad B = \sigma T^4 = OLR/2.$$

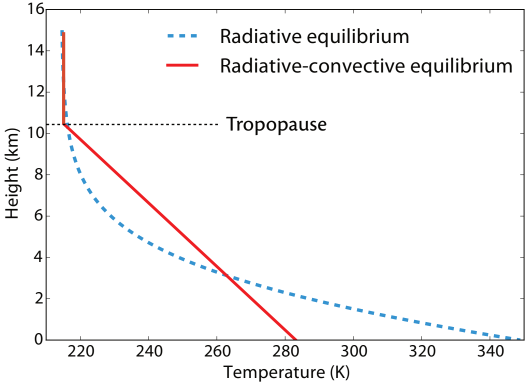
In optically thin limit, stratosphere is isothermal,

$$OLR = 2\sigma T^4_{strat}.$$



Radiative-Dynamical Equilibrium

Atmosphere adjusts to give a specified lapse rate (e.g., by convection or baroclinic instability). (Like an equal-area adjustment.)



- Q. What determines the height to which it adjusts?
- A. Overall radiative balance. The outgoing longwave radiation must balance the incoming solar radiation.
- Tropopause height is function of stratification and optical depth.
- *Tropopause temperature fixed by OLR.* (in a grey atmosphere).

$$\sigma T_{\text{emit}}^4 \equiv OLR = S_0(1-\alpha), \qquad \text{(radiative balance)},$$

$$OLR = 2\sigma T_{\text{strat}}^4 \qquad \text{(as per previous derivation)}.$$

$$T_{\text{trop}} = T_{\text{strat}},$$

$$\therefore \quad T_{\text{trop}} = \frac{T_{\text{emit}}}{2^{1/4}}$$

Tropopause Height

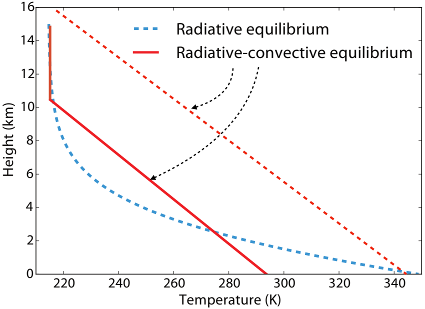
Ball Park Estimate

Tropopause temperature given by

$$2\sigma T_T^4 = S_0(1-\alpha) \equiv \sigma T_e^4 \qquad T_T = \frac{T_e}{2^{1/4}}$$

Surface temperature is:

$$\sigma T_s^4 = S_0(1-\alpha)\left(\frac{1+\tau_0}{2}\right) \qquad \text{or} \qquad T_S = T_T(1+\tau_0)^{1/4}$$



The height of the tropopause, H_T, is then such that $(T_S - T_T)/H_T = \Gamma$ so:

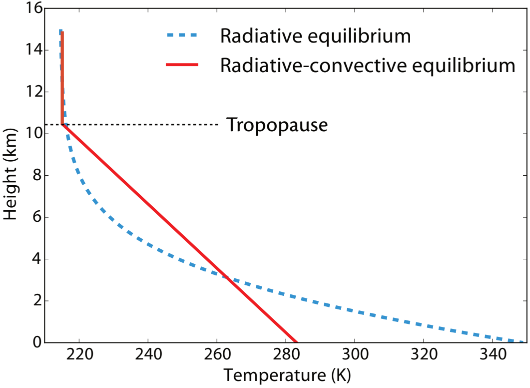
$$H_T = \frac{T_S - T_T}{\Gamma} = \frac{T_T}{\Gamma}\left((1+\tau_0)^{1/4} - 1\right). \qquad \text{or} \qquad \boxed{H_T = \frac{S_N/\sigma^{1/4}}{\Gamma}\left((1+\tau_0)^{1/4} - 1\right).}$$

Venus tropopause height because of very strong greenhouse effect. Surface temperature = 700 K. Tropopause temperature = 200 K. Lapse rate = 10 K/km.

Therefore Venus tropopause height = 80 km. Earth – 15 km

Tropopause Height

Better theory



- Assume 'dynamics' operates to a finite height, and with a specified lapse rate.
- Solve the radiative transfer equations, and demand overall radiative balance, allowing tropospheric height to adjust.
  - *(i)* Outgoing IR at top of atmosphere equals incoming solar
  - *(ii)* Upward IR at surface $= \sigma T_g^4$
- Obtain numerical solution exactly, or analytic solution approximately.

Adjustment to red profile is similar to an 'Equal area construction' (although not exactly).

Tropopause Height
Analytic Expression

$$-\frac{\mathrm{d}U}{\mathrm{d}\tau} = B - U, \qquad \frac{\mathrm{d}D}{\mathrm{d}\tau} = B - D.$$

Suppose that lapse rate, Γ, is *given* up to a height H_T, above which the atmosphere is in radiative equilibrium.

Formal solution:

$$D(\tau') = \mathrm{e}^{-\tau'}\left[D(0) - \int_0^{\tau'} B(\tau)\mathrm{e}^{\tau}\,\mathrm{d}\tau\right], \qquad U(0) = U(\tau')\mathrm{e}^{-\tau'} + \int_0^{\tau'} B(\tau)\mathrm{e}^{-\tau}\,\mathrm{d}\tau$$

Must adjust H_T so that the equations satisfy the boundary conditions (equal area construction, done properly).

After some algebra...

$$\boxed{H_T = \frac{1}{16\Gamma}\left(CT_T + \sqrt{C^2T_T^2 + 32\Gamma\tau_s H_a T_T}\right).}$$

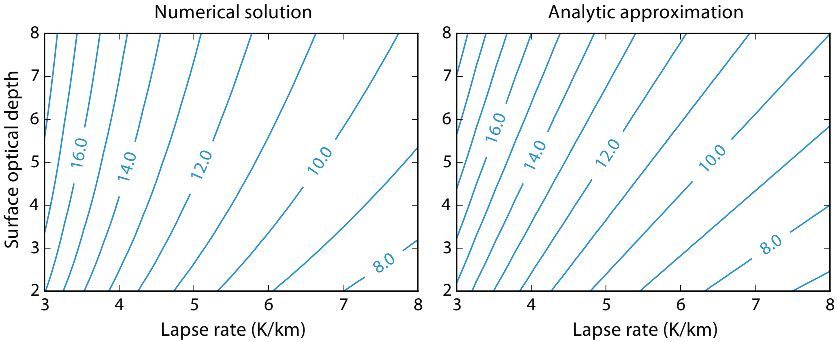
where τ_s is surface optical depth, H_a is the scale height of the main absorber and $C = \log 2$.

Tropopause Height

Analytic vs Numerical

$$H_T = \frac{1}{16\Gamma}\left(CT_T + \sqrt{C^2 T_T^2 + 32\Gamma\tau_s H_a T_T}\right)$$

where $C = 1.4$.



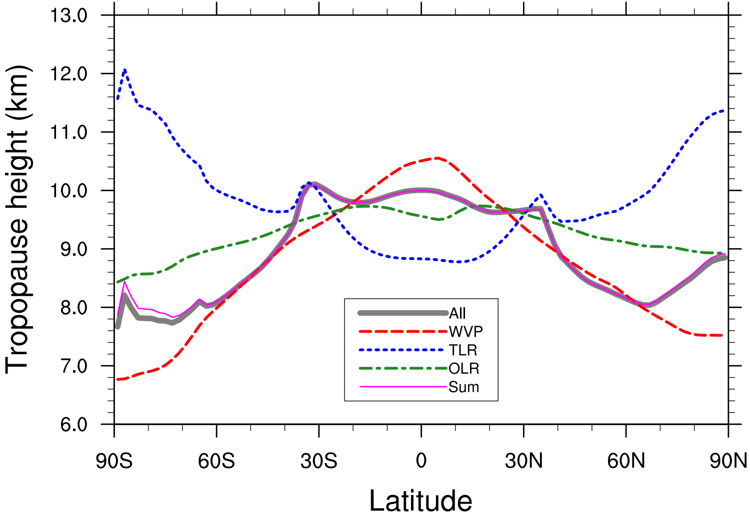
In tropics Γ is approximately moist adiabatic lapse rate, about 6°/km.

Look at other planets – Venus, Mars, Jupiter etc. Also, compare with ‘top-down’ theories by

Tropopause Height

Prediction for Earth

Does the theory reproduced the observed tropopause height? Semi-quantitatively, yes.

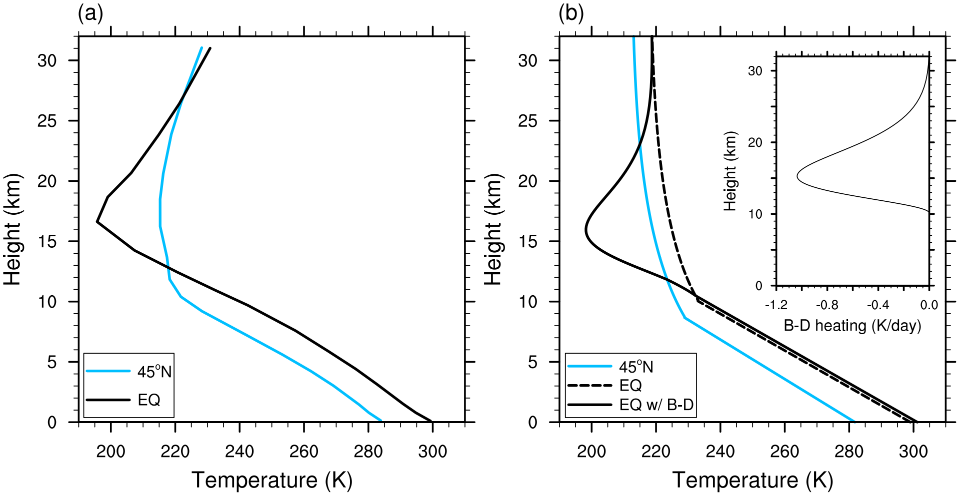


Tropical tropopause is too low

Hu and Vallis 2019.

Effects of the Stratosphere

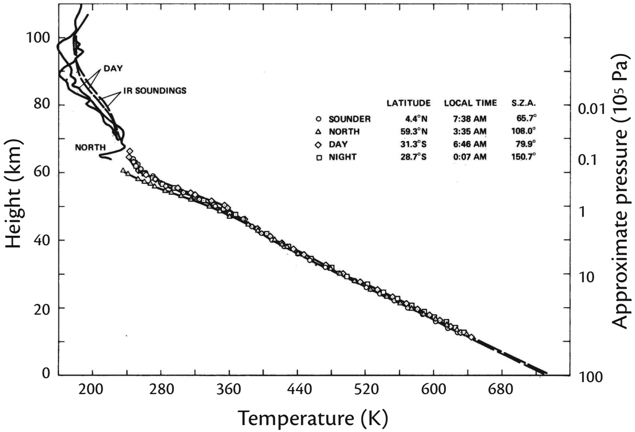
Add a cooling to the stratosphere:



Observations

Theory

Tropopause on Venus



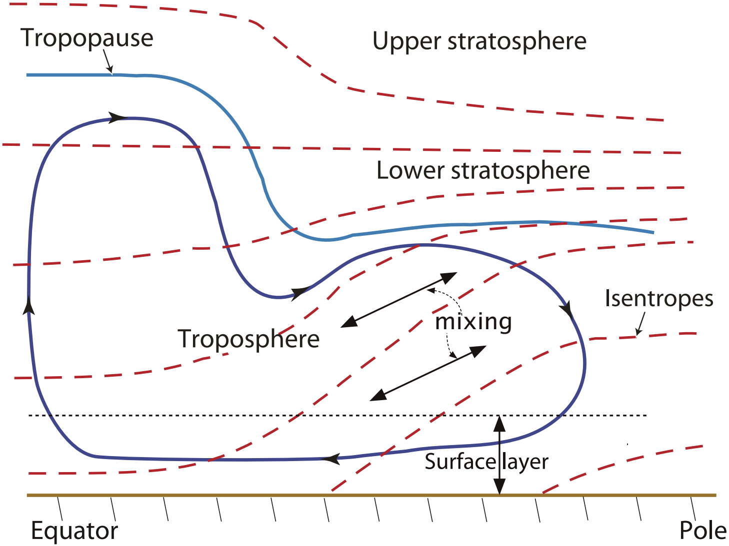
Four probes from Pioneer. Adapted from Seiff et al (1979)

Features:

1. Tropopause about 60 km high!

Tropopause Height, Midlatitudes

Lapse rate now determine by baroclinic eddies:



Take the lapse rate to be such that isotherms pass from surface in midlatitudes to tropopause near the pole (this is ‘baroclinic adjustment’ or ’marginal supercriticality’, Stone (1978).

Baroclinic Adjustment (Marginal Supercriticality)

A Loose Argument

In two-layer model the condition for marginal stability is that the PV gradient just changes sign.

$$\frac{\partial Q}{\partial y} = \beta - \frac{\Delta U}{L_d^2}$$

but, thermal wind:

$$\frac{f \Delta U}{H} = \frac{\Delta_h b}{L}$$

and $N^2 = \Delta_z b / H$. Gives

$$\frac{L\beta}{f} = \frac{\Delta_h b}{\Delta_z b}$$

or, if $\beta \approx f/L$,

$$\text{Isopycnal Slope} \sim \frac{H}{a}$$

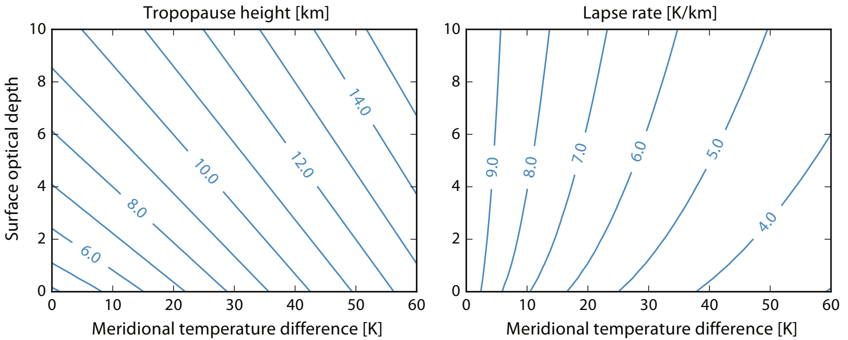
Isotherms pass from surface in tropics to tropopause at pole.

Tropopause Height: Analytic for Midlatitudes

Combine these arguments, predict height *and* stratification.

$$H_T = \frac{1}{16\Gamma_d}\left(A + \sqrt{A^2 + 32\Gamma_d \tau_s H_a T_T}\right), \qquad \Gamma = \Gamma_f + \left(\frac{\partial_y T}{H_T}\right)\frac{f}{\beta},$$

where $A = C\,T_T - 8f\,\partial_y T/\beta$. t T_T = temperature tropopause.

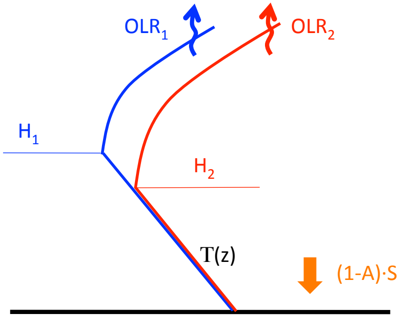


Tropopause Height

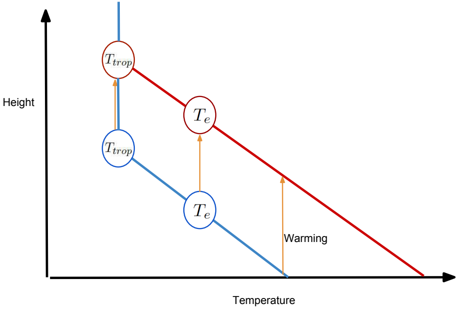
Recap: Increase with Global Warming

Summary of arguments:

- ⟶ Incoming solar radiation = outgoing IR
- ⟶ Stratosphere optically thin, in radiative equilibrium
- ⟶ Uniform tropospheric stratification
- ⟶ Then, outgoing IR radiation can be written *as a function of tropopause temperature only.*



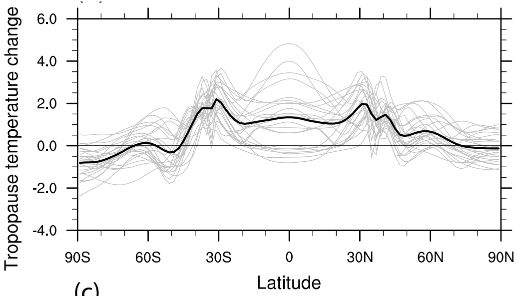
Only one choice of $H(T)$ gives the correct OLR.



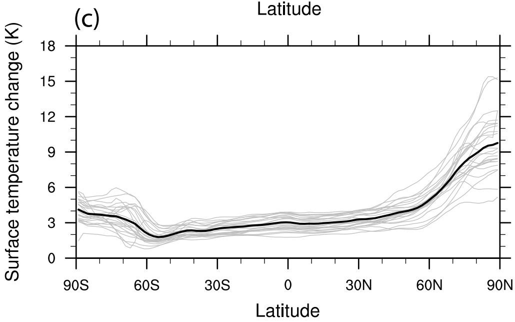
Tropopause height increases with increased COT.

Tropopause Temperature Changes

Global Warming Results from CMIP5



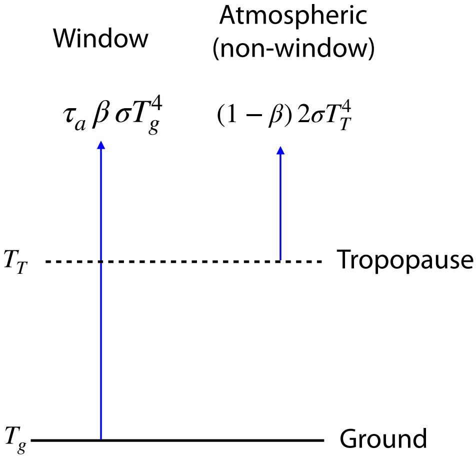
Tropopause temperature change



Surface temperature change

Tropopause temperature change is small – but not zero!

Non-Gray Effects of Radiative Transfer



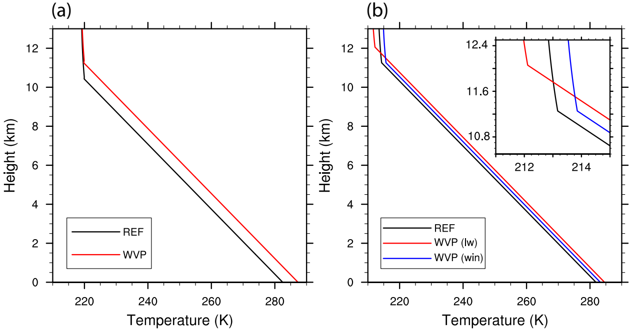
Total OLR remains constant with global warming.

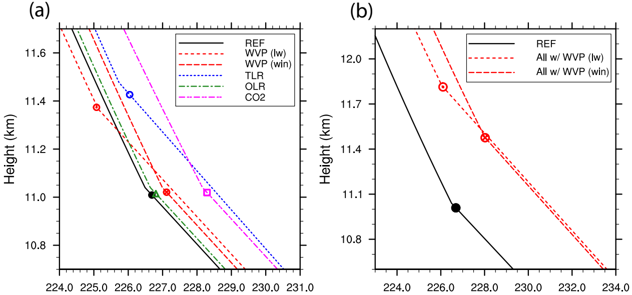
$$\tau_a \beta \sigma T_g^4 + (1-\beta)\,2\sigma T_T^4 = \mathrm{OLR}$$

If τ_a diminishes sufficiently (i.e., the window closes) because of increase greenhouse gases then T_T must increase!

(Needs a detailed calculation to be quantitative.)

A Windowed Calculation





Temperature increase with non-windowed (left) and windowed (right) increase in greenhouse gases.

Tropopause Summary

1. Presented theory for tropopause height.
   - *(i)* Predicts height on Earth, Venus etc reasonably well.
   - *(ii)* Latitudinal structure on Earth.
   - *(iii)* Higher, colder tropical tropopause due to Brewer-Dobson cooling.
2. Height and temperature will both increase with global warming.

Elliptical Exoplanet Orbits!

Test theory in an extreme case

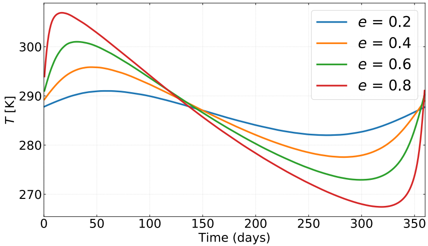
(i) Telluric planet

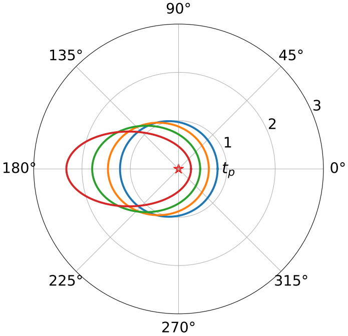
(ii) Various elliptical orbital configurations. Zero obliquity (no tilt).

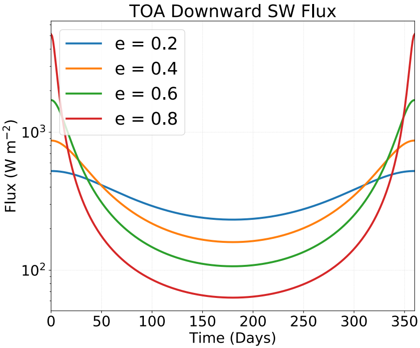
(iii) Kepler's law satisfied by each orbit

(iv) Each orbital configuration receives the same amount of solar radiation.

Temperature over the seasonal cycle



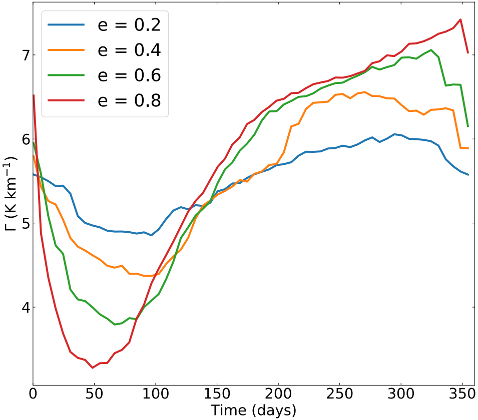




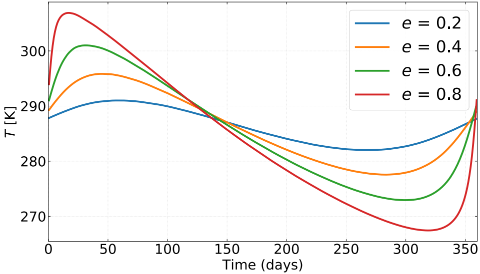
Elliptical Exoplanet Orbits!

Results

Lapse Rate



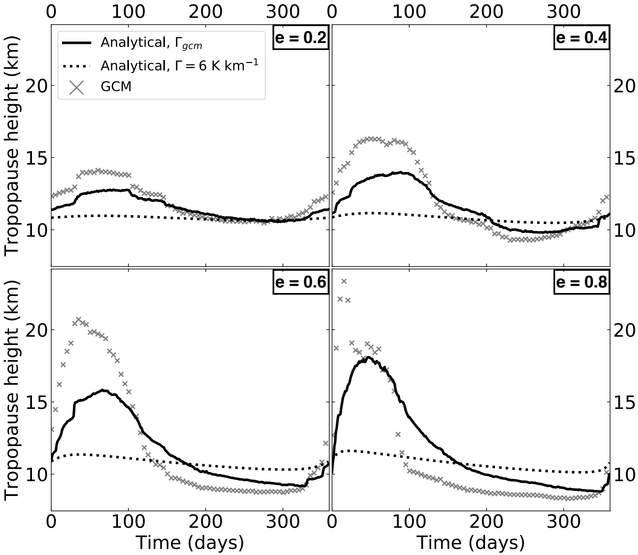
Temperature



High temperature, low lapse rate. (Moist adiabatic.)

Elliptical Exoplanet Orbits!

Tropopause Height

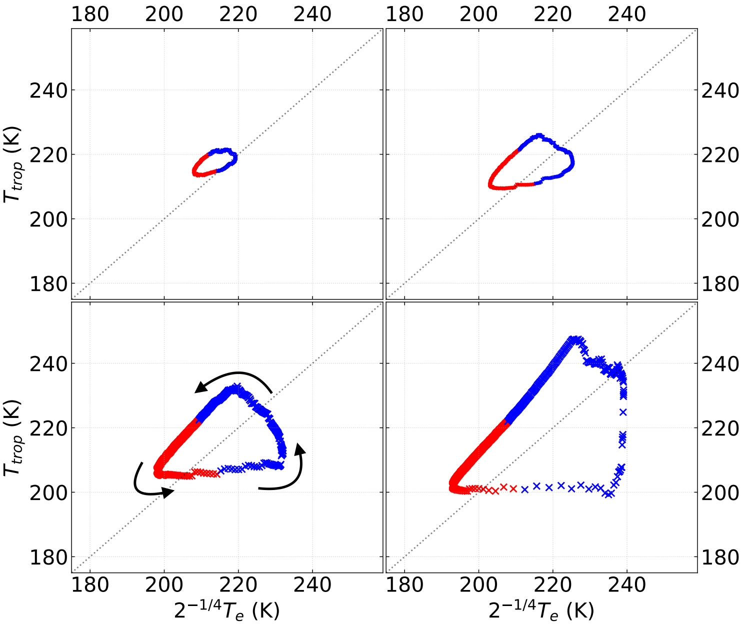


Analytic ——

GCM ×××

Elliptical Exoplanet Orbits!

Tropopause temperature



GRAVITY

Gravity

Suppose we double the gravity (Newtonian acceleration) at Earth's surface, from 9.8 to 19.6 m/s^2. What happens to the circulation?

Gravity

Suppose we double the gravity (Newtonian acceleration) at Earth's surface, from 9.8 to 19.6 m/s^2. What happens to the circulation?

If the atmosphere obeys the dry primitive equations — absolutely nothin'! (to worry about).

Scaling the Equations

The primitive equations of motion are:

$$\frac{Du}{Dt} - 2\Omega v \sin\vartheta + \frac{uv\tan\vartheta}{a} = -\frac{1}{\rho a\cos\vartheta}\frac{\partial p}{\partial\lambda}, \tag{1}$$

$$\frac{Dv}{Dt} + 2\Omega u \sin\vartheta + \frac{u^2\tan\vartheta}{a} = -\frac{1}{\rho a}\frac{\partial p}{\partial\vartheta}, \tag{2}$$

$$\frac{\partial p}{\partial z} = -\rho g. \tag{3}$$

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\boldsymbol{v} = 0, \tag{4}$$

$$c_v\frac{DT}{Dt} + \frac{p}{\rho}\nabla\cdot\boldsymbol{v} = 0, \qquad \text{or} \qquad \frac{D\theta}{Dt} = 0. \tag{5}$$

(Don't worry about the details!)

Change gravity

Change gravity, $g \longrightarrow \alpha g$

$$\boxed{g \longrightarrow \alpha g,} \qquad p \longrightarrow \alpha p, \qquad \rho \longrightarrow \alpha \rho, \qquad (T,\theta) \longrightarrow (T,\theta),$$
$$t \longrightarrow t, \qquad (x,y) \longrightarrow (x,y), \qquad z \longrightarrow z/\alpha, \tag{6}$$
$$(u,v) \longrightarrow (u,v), \qquad w \longrightarrow w/\alpha.$$

Substitute into the equations of motion — *nothing changes!* All factors of α cancel.

Scaling the Equations

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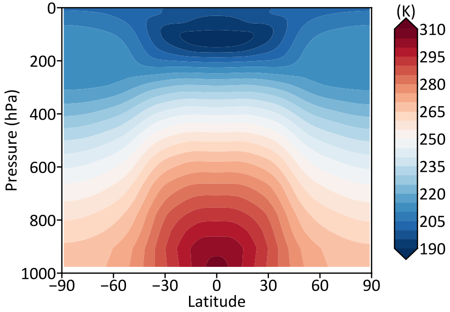
Substitute into the equations of motion — *nothing changes!* All factors of α cancel.

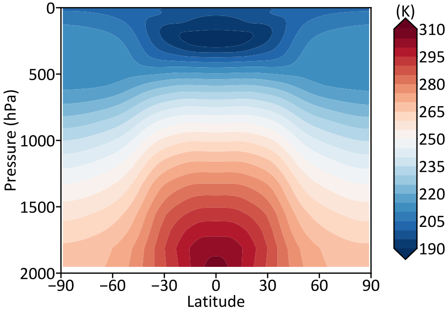
Does not hold if the equations are non-hydrostatic:

$$\frac{\mathrm{D}w}{\mathrm{D}t} - \frac{u^2 + v^2}{r} - 2\Omega u \cos\vartheta = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g. \tag{7}$$

Spot the difference

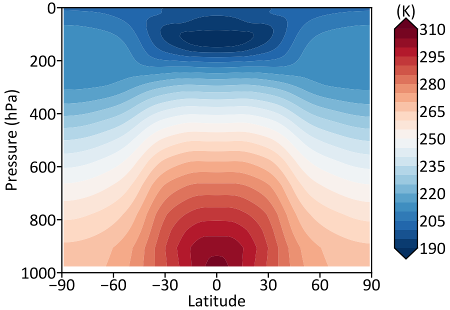


Normal gravity

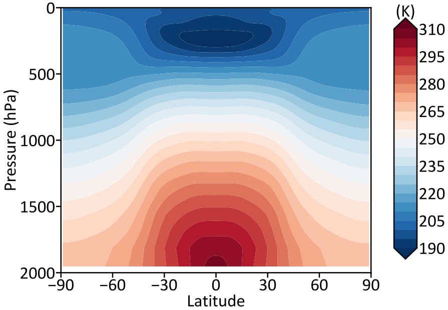


Twice gravity

Spot the difference



Normal gravity



Twice gravity

The only difference is the *y*-axis

Changing the Mass

Change mass, $p \longrightarrow \alpha p$, keep gravity constant.

$$\begin{gathered} g \longrightarrow g, \qquad p \longrightarrow \alpha p, \qquad \rho \longrightarrow \alpha \rho, \qquad (T, \theta) \longrightarrow (T, \theta), \\ t \longrightarrow t, \qquad (x, y) \longrightarrow (x, y), \qquad z \longrightarrow z, \\ (u, v) \longrightarrow (u, v), \qquad w \longrightarrow w. \end{gathered} \tag{8}$$

Substitute into the equations of motion — again nothing changes! All factors of α cancel.

Venus

Surface pressure $=$ 92bars $\approx 100 \times$ Earth surface pressure.

So Venus is not dynamically different from Earth simply because it's atmosphere is more massive.

Rather, a more massive atmosphere gives an enormous greenhouse effect.

Moist Effects
(as gravity changes)

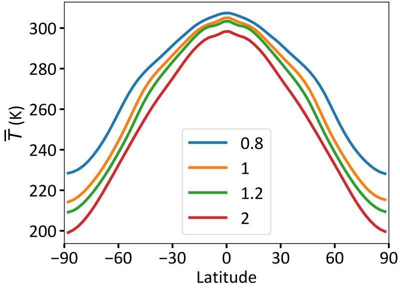
Increase gravity — squashes atmosphere, but no other affect on the dynamics. For example, deformation radius, $H \longrightarrow H/\alpha,\ N(=(g/\theta_0)\partial\theta/\partial z) \longrightarrow N\alpha$ — no change to dynamics!

Therefore $NH/f \longrightarrow NH/f$.

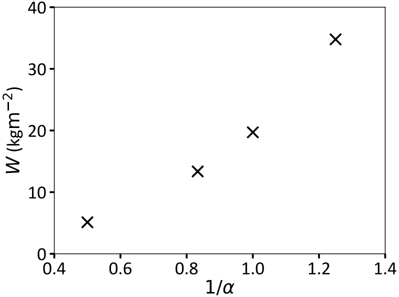
1. But the atmosphere shrinks in height.
2. So overall, less moisture.s
3. Atmosphere gets colder!

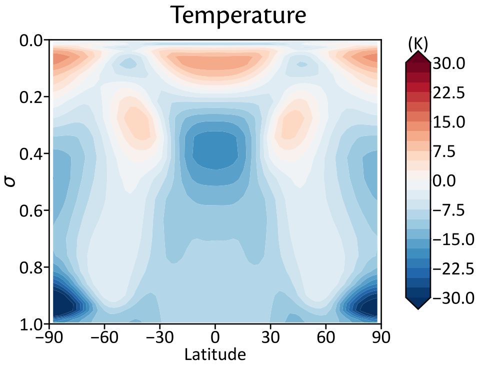
Moisture

Temperature



Water Vapour content .





The opposite of global warming!

Moisture

Specific Humidity

- Temperature stays the same, at lowest order.
- Vapour pressure e is (mainly) a function of temperature.
- $$\text{Specific humidity} = q = \frac{e}{p} \rightarrow \frac{e}{\alpha p} \qquad (9)$$

  So specific humidity falls with increasing gravity.
- Moist adiabatic lapse rate increases, so temperature falls even more at high altitudes in the tropics.
- Lapse rate changes because specific humidity changes:

$$\Gamma_s = -\left.\frac{dT}{dz}\right|_{ad} = \frac{g}{c_p}\frac{1 + Lq_s/(R^d T)}{1 + L^2 q_s/(c_p R^v T^2)}$$