Structure of Terrestrial Atmospheres

Lecture 2: Horizontal Structure: Hadley Cells and Jets

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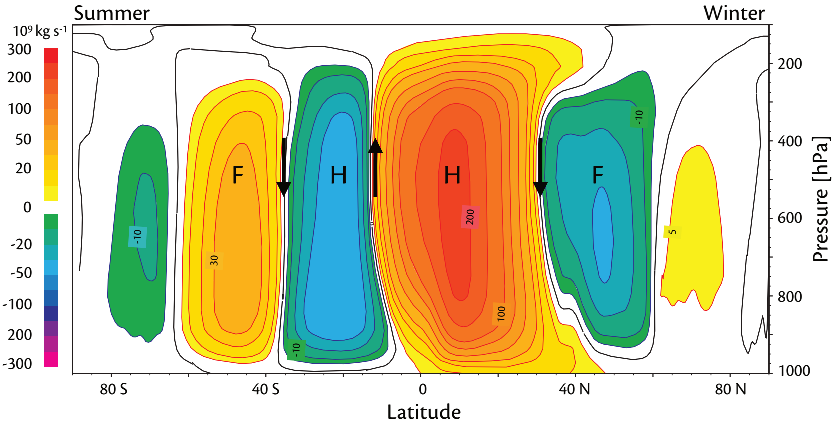
http://tiny.cc/Vallis

The Overturning Circulation

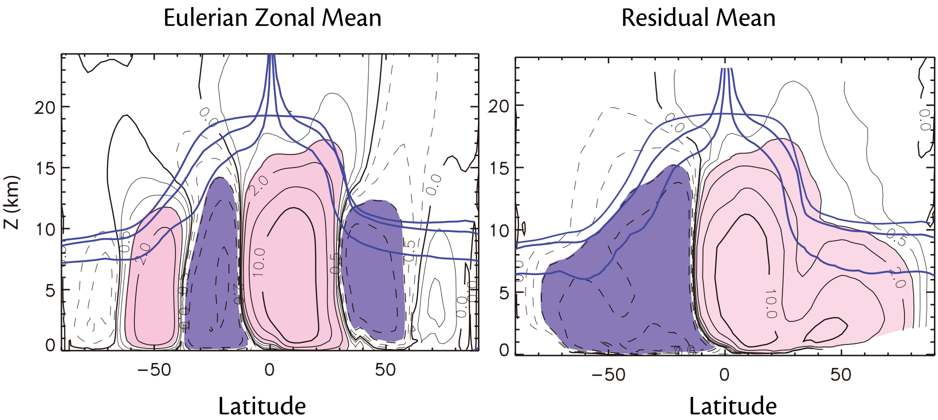
Hadley and Ferrel Cells



ECMWF re-analysis.

The Overturning Circulation

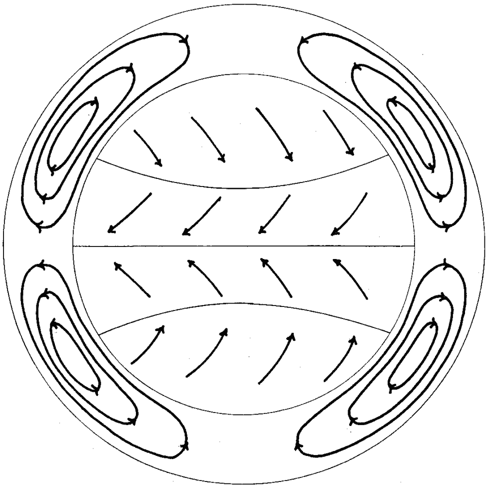
Hadley and Ferrel Cells



Residual mean takes into account eddy motion and better represents the actual path of fluid parcels

(Jucker, 2004, via Vallis, 2017)

In the Beginning
was Hadley



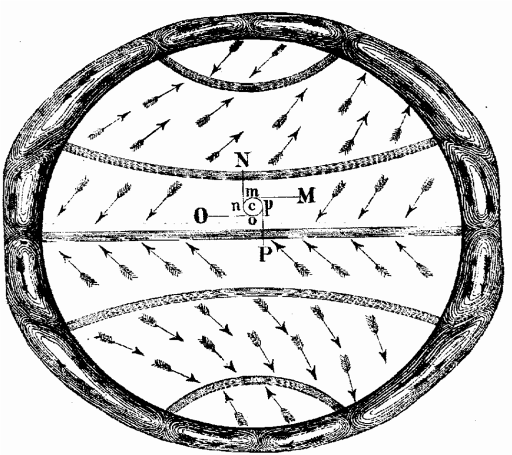
Hadley's vision of the overturning circulation

(from Lorenz 1967. No figures in Hadley's paper!)

Single overturning cell.

FERREL

Evolving views



Ferrel (1856). A distinct Ferrel Cell

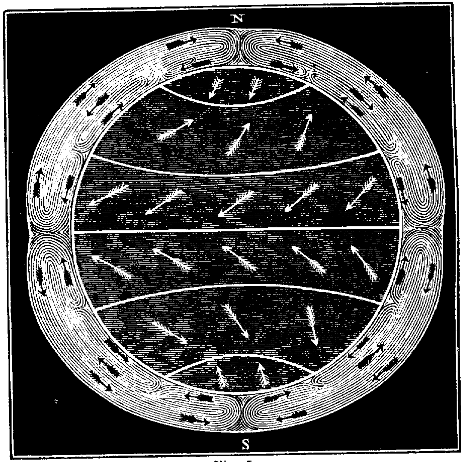
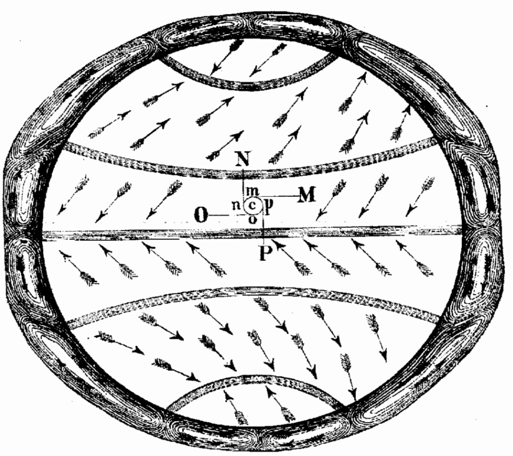


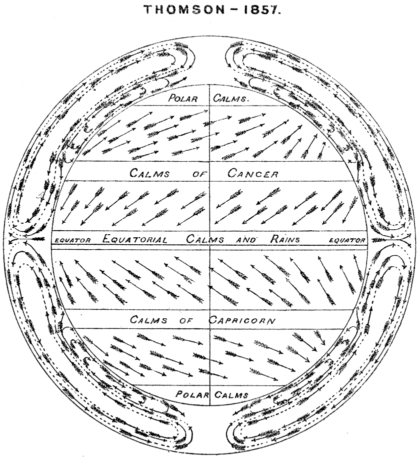
Fig. 5.

Ferrel (1859). Backtracked to a single Hadley Cell with a Ferrel Cell beneath it.

Ferrel
Evolving views



Ferrel (1856). A distinct Ferrel Cell!



Review by Thomson, 1892.

Hadley, Ferrel and Williams (Gareth!)

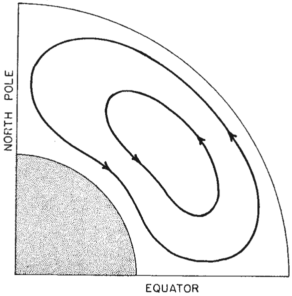


Figure 1. Schematic representation of the mean meridional circulation in one hemisphere, as visualized by Hadley (1735).

Hadley

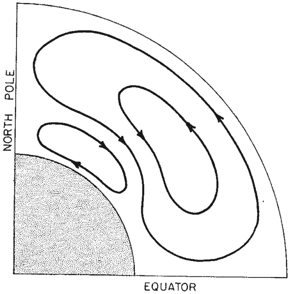
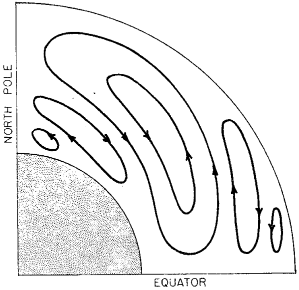


Figure 2. Schematic representation of the mean meridional circulation in one hemisphere, as visualized by Thomson (1857) and Ferrel (1859).

Ferrel



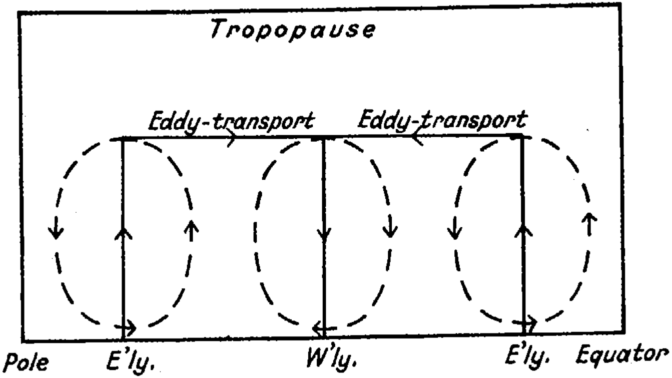
Williams (1968, rotating tank simulation)

(all figures adapted by Lorenz 1967).

Eady and the Ferrel Cell

Ironically, more progress was made in understanding the cause of the Ferrel Cell than the Hadley Cell!

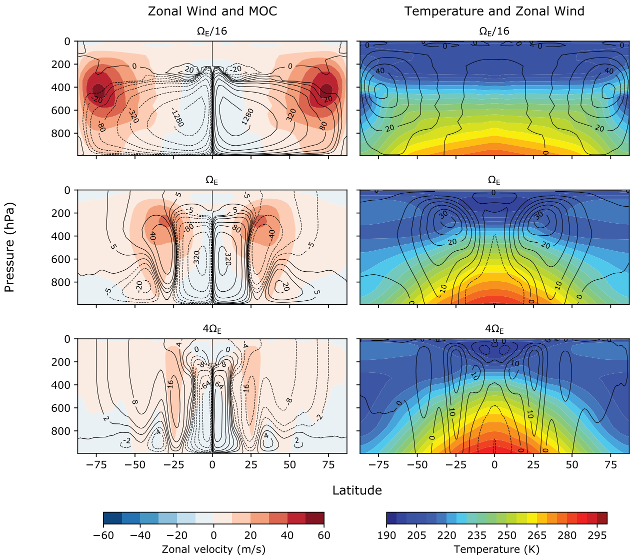
Eady (1950)



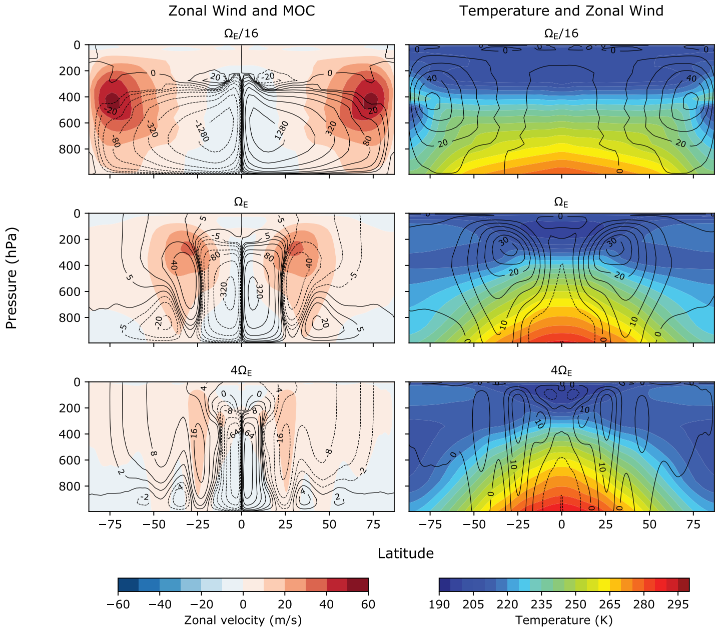
1. Thus, it was understood that the circulation in mid-latitudes was baroclinically unstable, preventing the Hadley Cell from reaching the pole.
2. But what exactly is the circulation (the basic state) that is baroclinically unstable? What is the ideal Hadley circulation?

Hadley Cell Theory for Low Rotation

- How far does the Hadley Cell go?
- What values does zonal wind take? Does it approach a limit?
- Then what happens at zero rotation? Is it a singular limit?
- Are there any discontinuities?
- Venus GCMs are all over the place — they differ from each other and are very sensitive to parameter choices.



Hadley Cell as Rotation Changes



Simulations with Isca, by J. Eager.

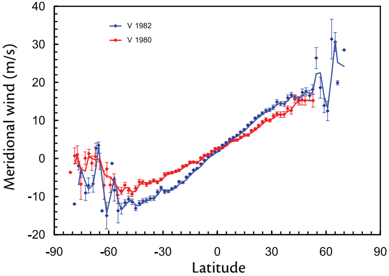
↓ Rotation increasing
↓ downwards

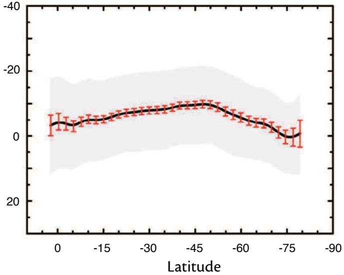
Hadley Cell on Slow Rotating Planets

Venus, cloud-tracked wind

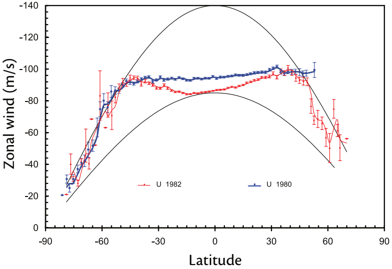
Dayside only

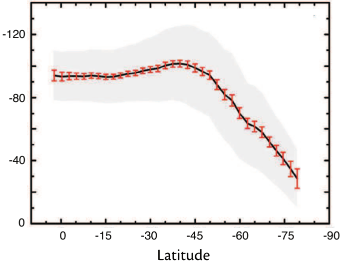
Venus Express orbiter.





Meridional wind





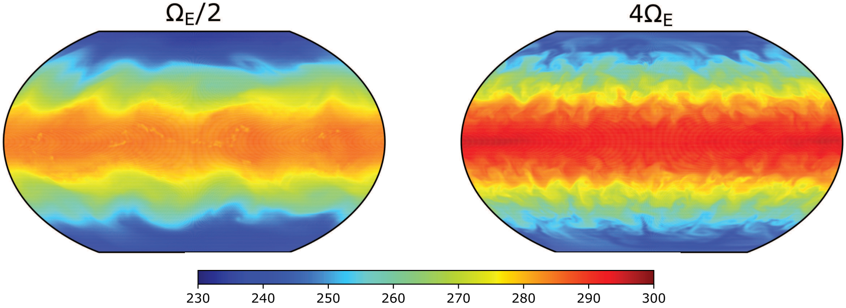
Zonal wind

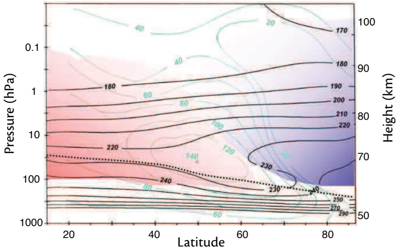
Limaye (2007),

Khatuntsev et al (2013)

Near Surface Temperature as Rotation Changes

GCM Simulation:



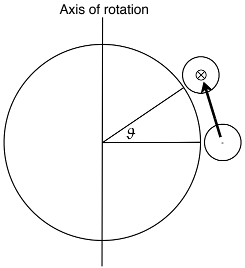


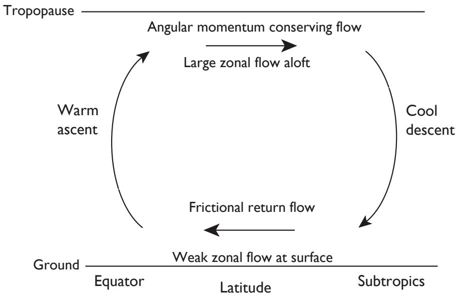
Venus observation (Titov, 2013, orbiting infra-red radiometer)

Meridional temperature gradient evidently smaller at low rotation.

Hadley Cell

Axi-symmetric Theory





(E.K. Schneider, Held and Hou)

Assume flow is axi-symmetric.
Outflow is angular momentum conserving:

$$U = \Omega a \frac{\sin^2 \vartheta}{\cos \vartheta}$$

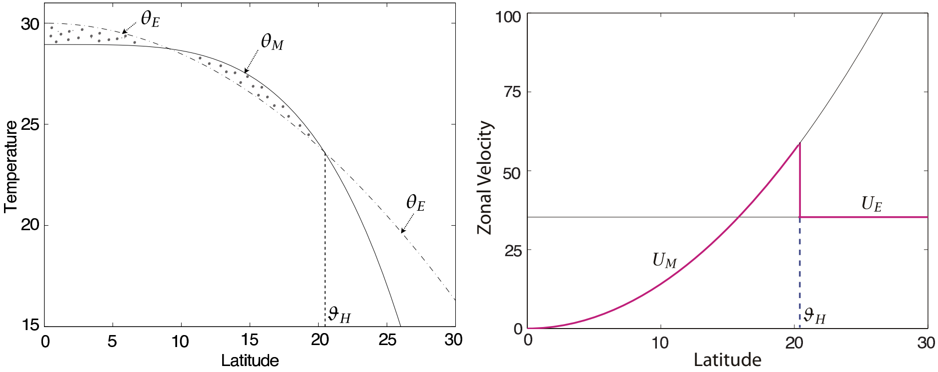
Temperature from thermal wind balance:

$$T = T(0) - \frac{T_0 \Omega^2 a^2 \vartheta^4}{2gH}$$

Temperature falls rapidly with latitude.

Width of Hadley Cell is constrained by thermodynamics:
Air gets too cold and sinks.

Hadley Cell Calculations
Original theory



θ_E = radiative equilibrium temperature,

θ_M = temperature from angular momentum conserving wind (via therml wind)

Discontinuity in zonal wind at edge of Hadley Cell.

Held–Hou model
Equations :(

Steady zonal momentum equation with $w = 0$ and no eddies.

$$v\left(\frac{1}{a}\frac{\partial u}{\partial \vartheta} - f - \frac{u\tan\vartheta}{a}\right) = 0$$

Either:

(i) $v = 0$ (identically). Radiative equilibrium solution.

(ii) Or, angular momentum conserving solution:

$$\frac{1}{a}\frac{\partial u}{\partial \vartheta} - f - \frac{u\tan\vartheta}{a} = \frac{1}{a^2\cos\vartheta}\frac{\partial M}{\partial \vartheta} = 0$$

So that: $M = (u + \Omega a\cos\vartheta)a\cos\vartheta = \text{constant}$

$$u = u_M(\vartheta) + \frac{u_0}{\cos\vartheta} = \frac{\Omega a\sin^2\vartheta}{\cos\vartheta} + \frac{u_0}{\cos\vartheta}$$

- Low latitudes – momentum-conserving solution
- High latitudes – no circulation (radiative equilibrium).

Hadley Cell Width

To satisfy thermodynamic balance Hadley Cell must descend.

Latitude of the edge of Hadley Cell is given by

$$\vartheta_H = \left(\frac{5\Delta\theta_h g H}{3a^2\Omega^2\theta_0}\right)^{1/2} \propto \frac{1}{\Omega}.$$

Goes further poleward as rotation falls.

Latitude related to external Rossby number: $Ro_E = \frac{U}{\Omega a} = \frac{R\ \theta_H}{\Omega^2 a^2}.$

Now consider low rotation limit.

Thermal Wind

More equations (!)

$$f u + \frac{u^2 \tan\vartheta}{a} = -\frac{gH}{a\Theta_0}\frac{\partial\overline{\Theta}}{\partial\vartheta}$$

Angular Momentum Conserving Regime:

With $u = u_M$ and $u_0 = 0$ solution is

$$\frac{\overline{\Theta}(0) - \overline{\Theta}}{\Theta_0} = \frac{u_M^2}{2gH} = \frac{\Omega^2 a^2 \sin^4\vartheta}{2gH\cos^2\vartheta}\,. \tag{10}$$

Radiative Equilibrium Regime:

Given a typical $\Theta_E(\vartheta)$ the solution is

$$u_E = \Omega a \cos\vartheta\left(\sqrt{2R+1} - 1\right), \qquad \text{where} \qquad R \equiv \Delta_H g H / \Omega^2 a^2. \tag{11}$$

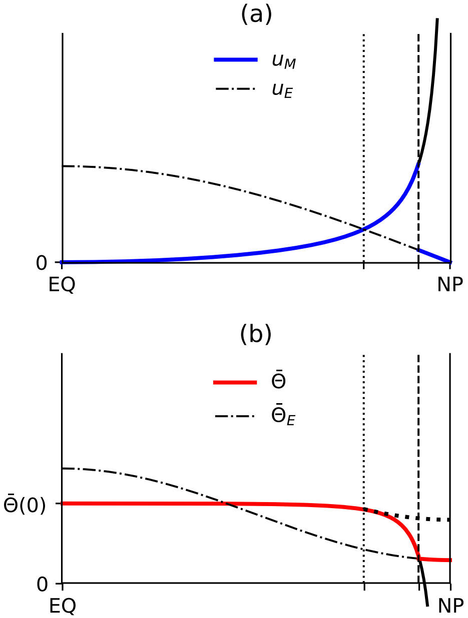
For small Ω:

$$u_E = \sqrt{2\Delta_H g H}\cos\vartheta \approx \sqrt{2\Delta_H g H}\varphi \tag{12}$$

where φ is co-latitude. u_E is independent of Ω.

Solutions

At low rotation



1. Continuity of temperature at θ_H:

$$\overline{\Theta}(\theta_H-) = \overline{\Theta}_E(\theta_H+)$$

2. Closure of the energy budget over the Hadley cell $\theta < \theta_H$:

$$\int_0^{\theta_H} \overline{\Theta} \cos\theta d\theta = \int_0^{\theta_H} \overline{\Theta}_E \cos\theta d\theta.$$

(Equal area construction).

Effects of Rotation Rate

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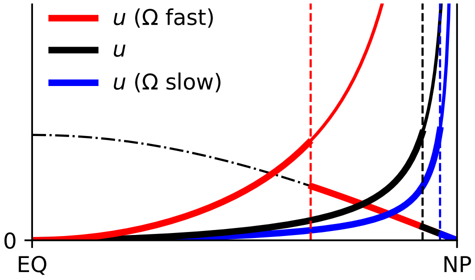
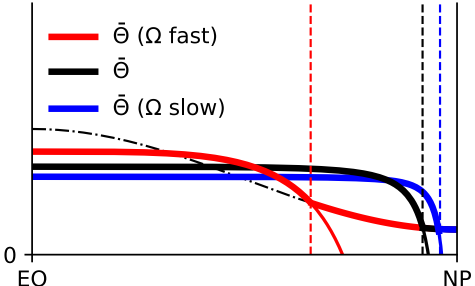


(b)



Low-rotation limit, $\Omega \longrightarrow 0$:

$$\frac{\overline{\Theta}(0) - \overline{\Theta}}{\Theta_0} = \frac{\Omega^2 a^2 \sin^4 \vartheta}{2gH\cos^2\vartheta} \longrightarrow 0$$

$\overline{\Theta} \longrightarrow \overline{\Theta}(0) =$ constant except at v. high latitude.

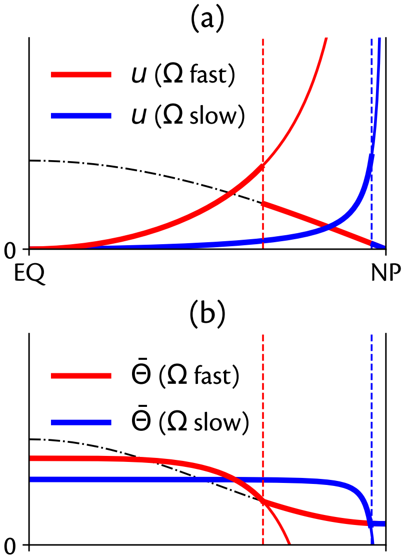
Let $\varphi \equiv \pi/2 - \theta \ll 1$:

$$\frac{\overline{\Theta}(0) - \overline{\Theta}}{\Theta_0} = \frac{\Omega^2 a^2}{2gH\varphi^2}$$

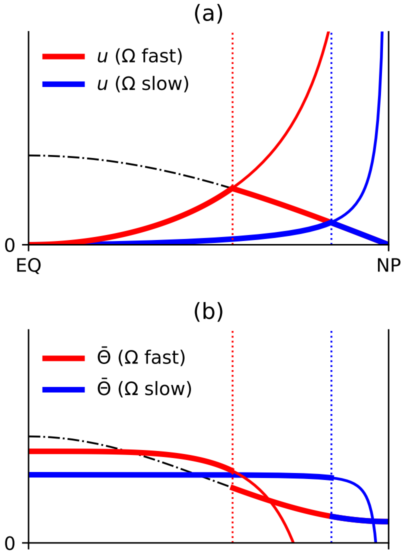
Transition latitude:

$$\varphi_H = \frac{\sqrt{3}\Omega a}{2\sqrt{\Delta_H g H}} = \sqrt{\frac{3}{4R}}$$

Theoretical Prediction

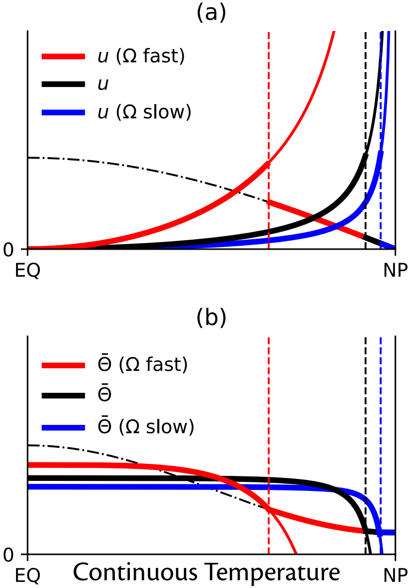


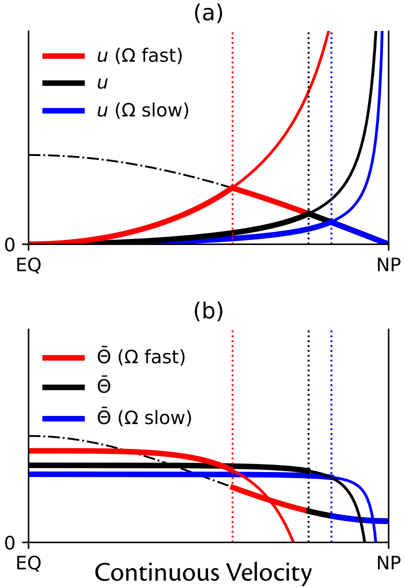
Continuous Temperature



Continuous Velocity

Theoretical Prediction



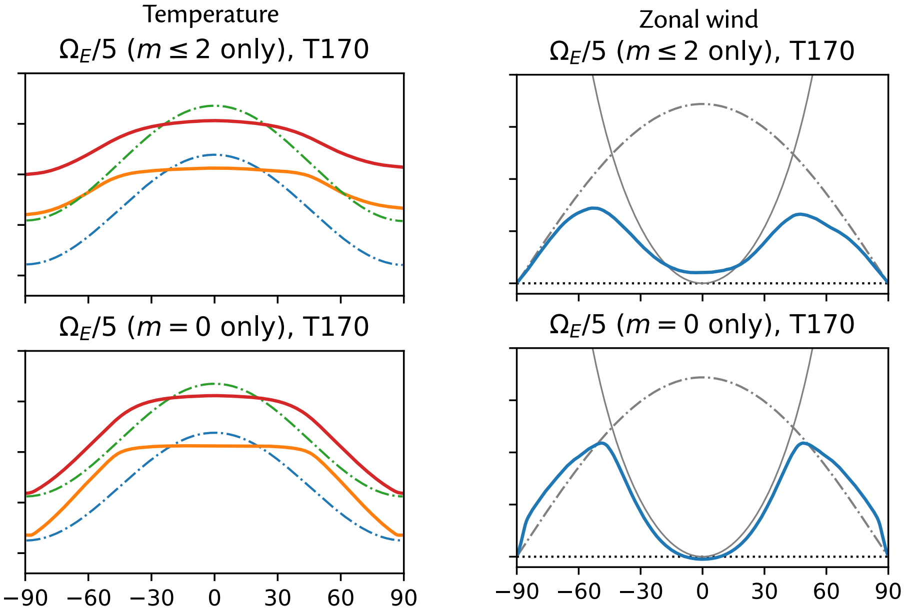


Zonal wind approaches rotation independent wind at low rotation.

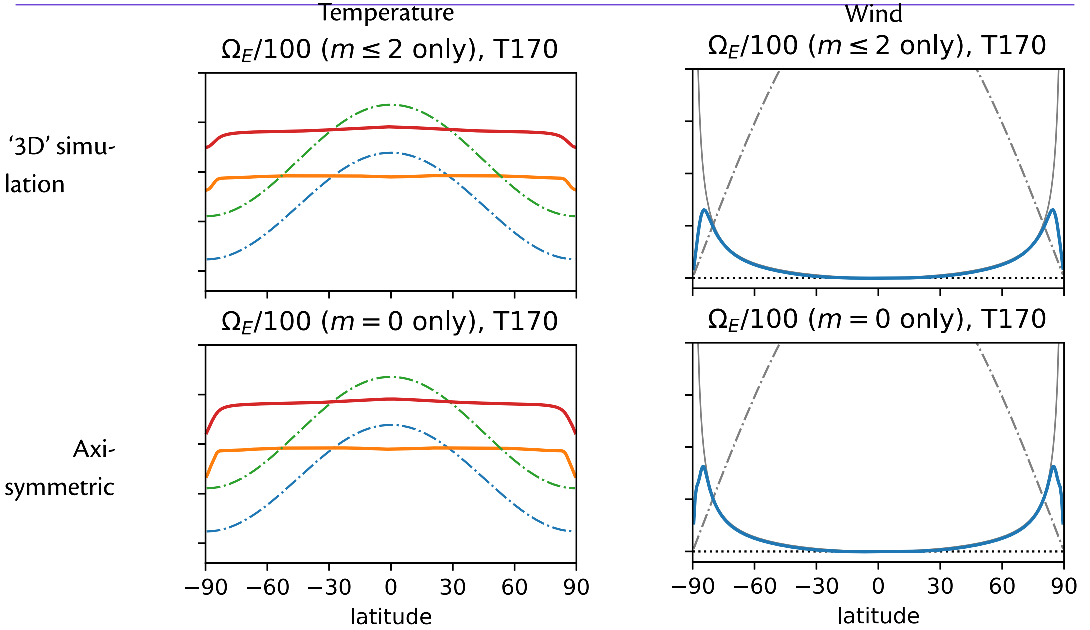
Temperature and Velocity, $\Omega = \Omega_E/5$

'3D' simulation

Axi-symmetric



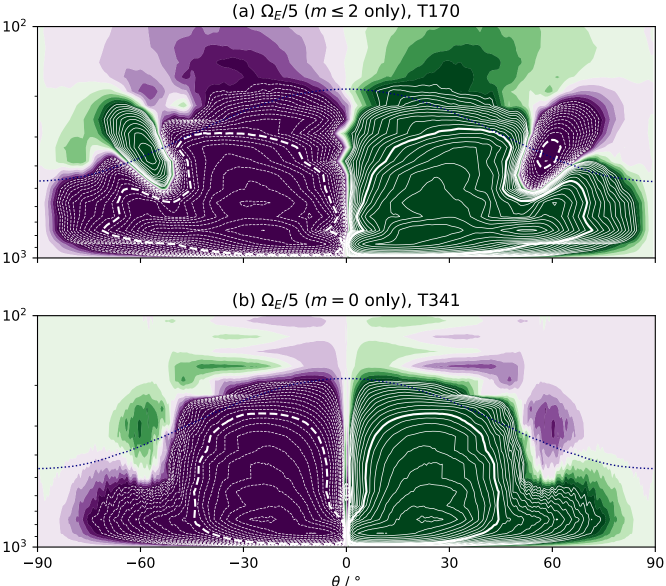
Temperature and Velocity, $\Omega = \Omega_E/100$



Non-Zero MOC at all Latitudes

Keeps everything continuous

Continuity of all fields is maintained by non-zero MOC at high latitudes.



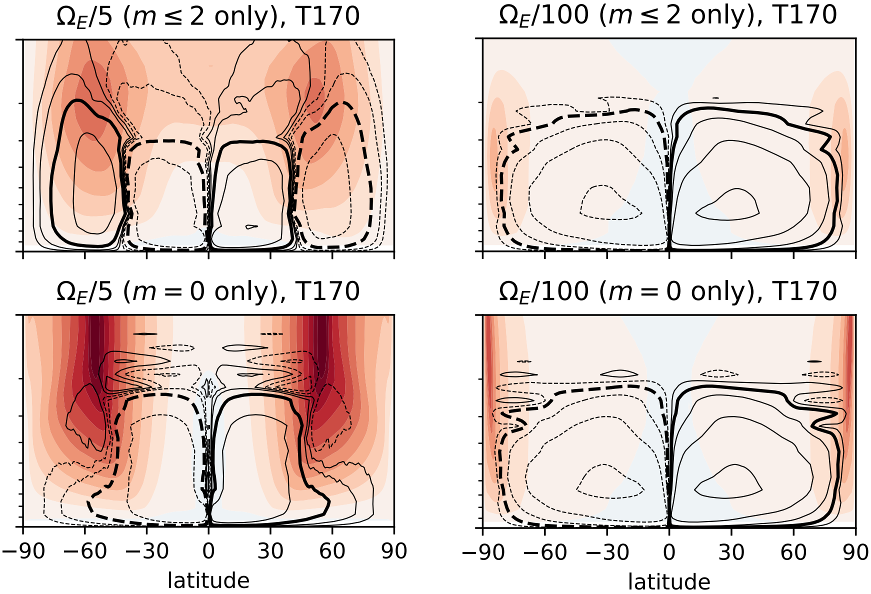
Non-Zero MOC at all Latitudes

Keeps everything continuous

‘3D’ simulation

Axi-symmetric

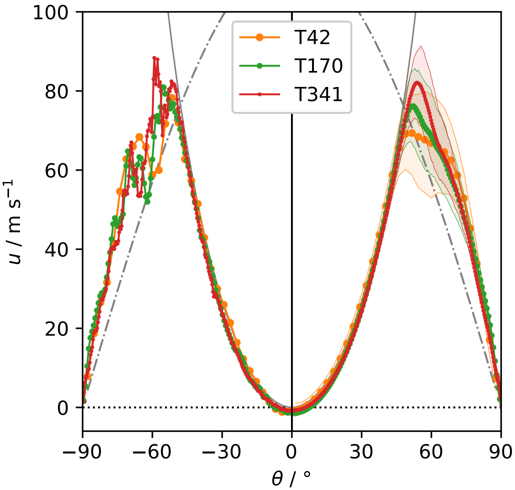
Weak direct Cell at high latitudes



Continuity of all fields is maintained by non-zero MOC at high latitudes.

Zonal wind

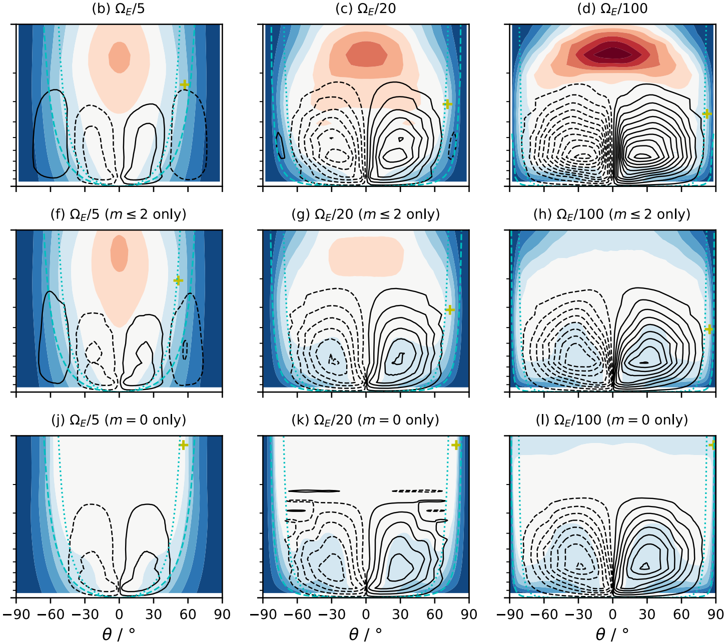
Keeps everything continuous



Zonal wind continuous.

Super-Rotation

Spontaneously appears at low rotation



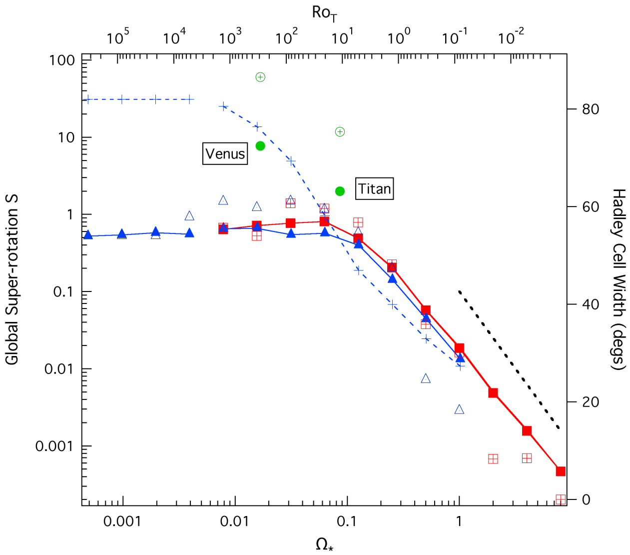
Colors are:

$M/\Omega a^2 - 1$

$$M = (u + \Omega a \cos\vartheta) a \cos\vartheta$$

Super-rotation

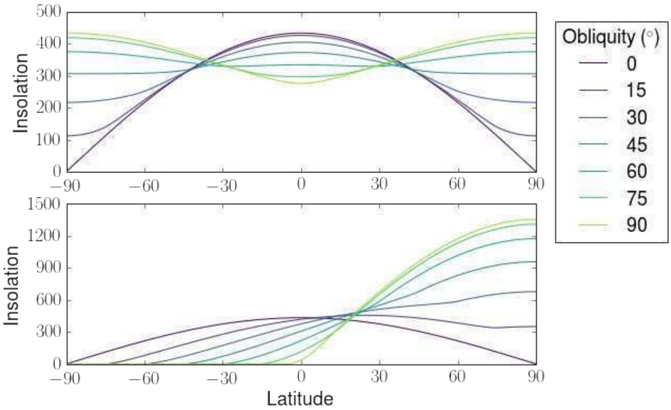
N. Lewis, P. Read, N. Fachreddin-Tabata



Hadley Cell Theory for High Obliquity

Insolation at various obliquities:

Annual average



Solstice

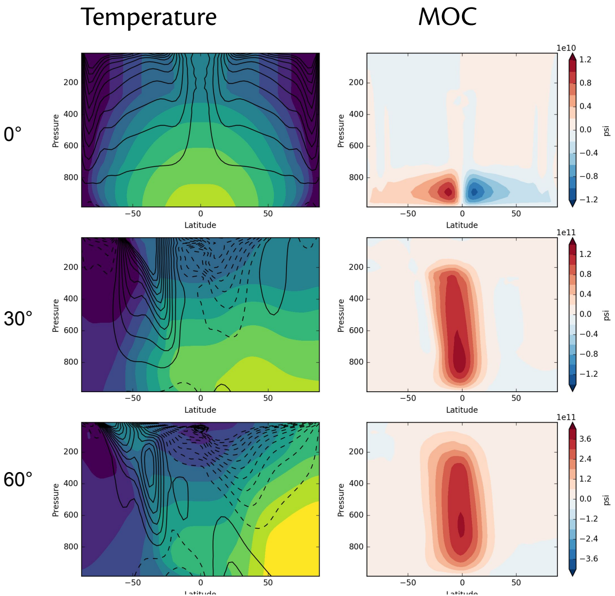
- How far does the Hadley Cell extend?
- Where does it go up?
- Where does it sink?

Circulation at Various Obliquities

Solsticial means, axisymmetric

- Axi-symmetric simulations, solsticial means.
- Weak equinoctial Hadley Cell.
- A strong‘winter Hadley Cell’ dominates at higher obliquties.
- Rising Hadley Cell does not move poleward of about 20° in the summer hemisphere.
- The Hadley Cell circulation is similar at still higher obliquity.

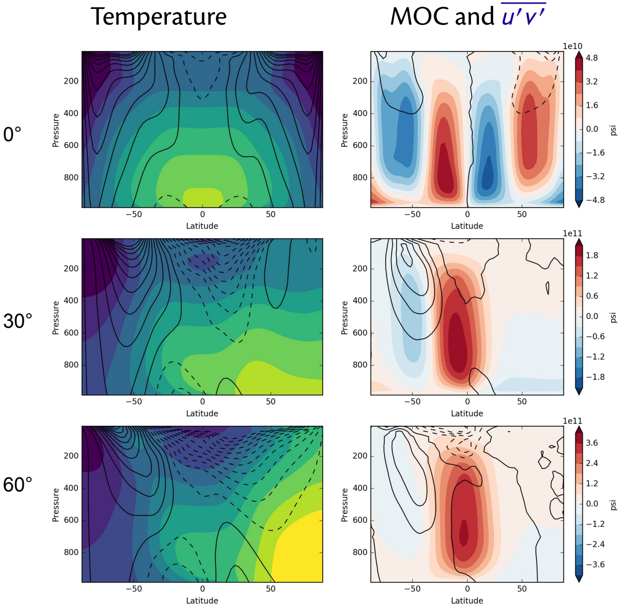
(cf. Faulk et al, Hill et al., Singh)



Circulation at Various Obliquities

Solsticial means, 3D simulation

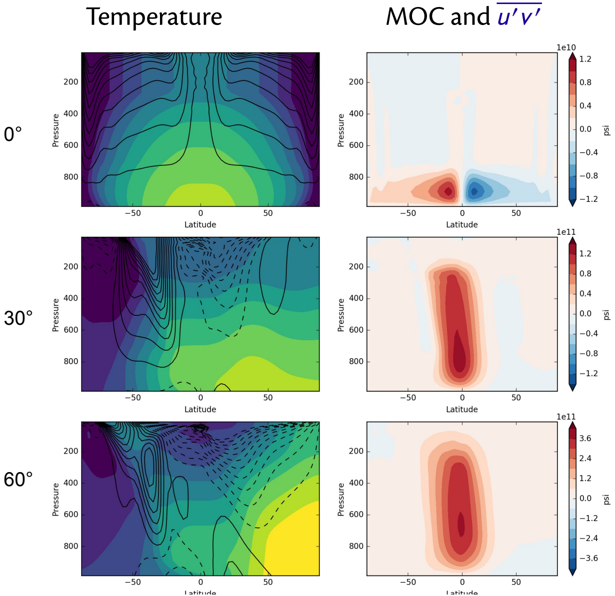
- 3D simulations, solsticial means.
- Eddies greatly strengthen the equinoctial Hadley Cell, but have less effect on solsticial Hadley Cell.
- A stronger'winter Hadley Cell' dominates at higher obliquties.
- Rising Hadley Cell does not move poleward of about 20° in the summer hemisphere.



Location of Hadley Cell

At High Obliquity

- Pole is the hottest place on Earth in high-obliquity summer. So why does the Hadley Cell not go further poleward?
- Proximate answer: Air rises where it is being heated, not where it is hottest.
- Has to satisfy same angular-momentum/ thermal constraints to the usual Hadley Cell.
- Cannot rise at pole and sink at equator! Winds and temperature gradient would be enormous!
- Cannot have radiative equilibrium temperature at low latitudes, because that would be unstable.
- Result: Near-radiative-equilibrium at high latitudes, Hadley Cell at low latitudes.



JUPITER

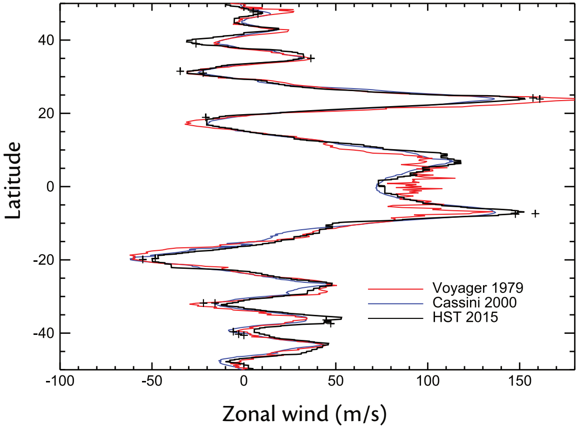
Jupiter from Space

(Enhanced color from 3 images, by K. M. Gill)

Characteristics:

- Weather, clouds
- Jets!
- Global organization.
- Very zonal flow.

Jupiter and its Jets



– Strong, sharp jets. Barotropically unstable $\beta - \partial^2 U/\partial y^2$ changes sign.

– Superrotates.

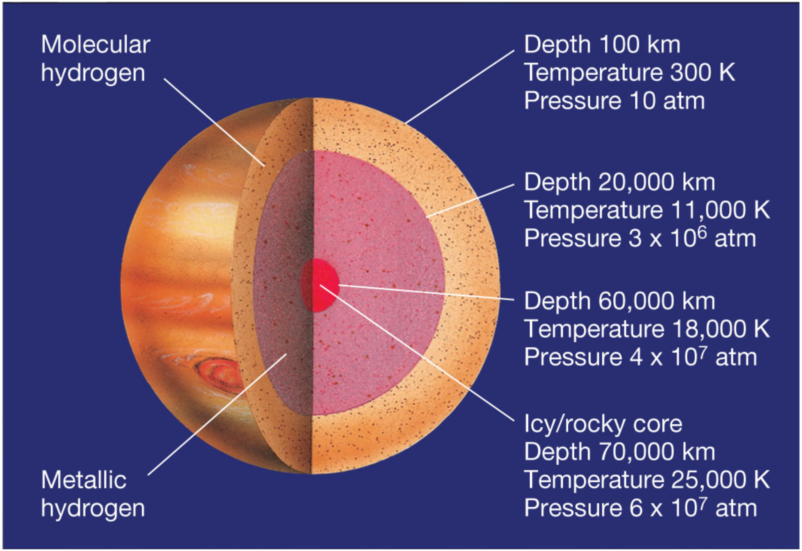
– Multiple super-rotating jets in the tropics!

Problems and Speculations

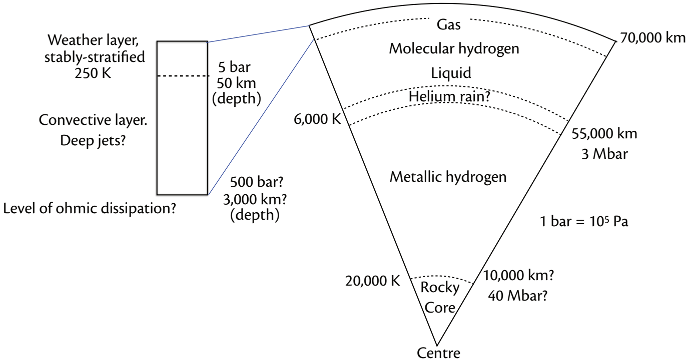
1. What causes the jets?
2. Why is there superrotation?
3. How deep are the jets?
4. Are the weather-layer jets the same as the deep jets?
5. What is the role of moisture?

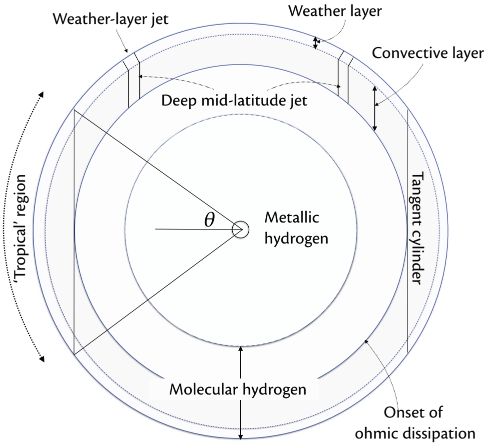
Structure of Jupiter

Popular science book.

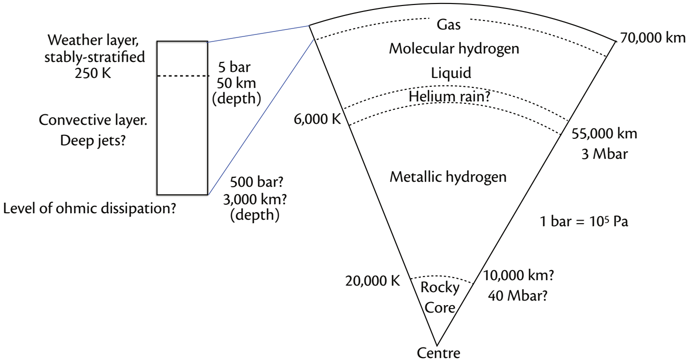


Structure of Jupiter





Structure of Jupiter



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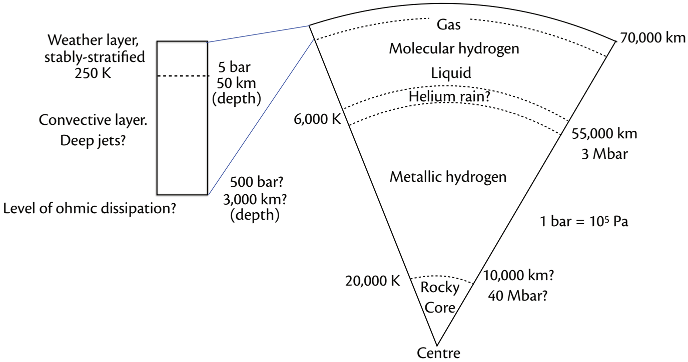
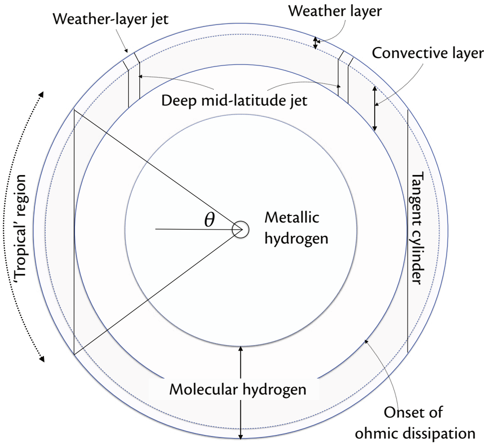
Structure of Jupiter





Weather-layer Jets

Easy!

Two-dimensional beta-plane turbulence! (Rhines, Williams, Vallis & Maltrud)

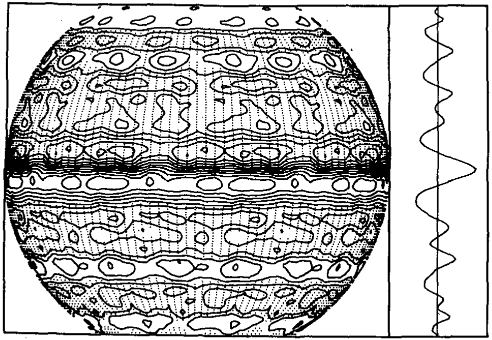
$$\frac{DQ}{Dt} = F - D, \qquad Q = \beta y + \zeta$$

Here, β is dues to differential rotation:
$\beta = \partial f / \partial y = 2\Omega \cos\vartheta / a$.

$$\frac{D\zeta}{Dt} + \beta v = F - D,$$

$$\text{Jet Scale} = \sqrt{\frac{U}{\beta}} \sim \sqrt{\frac{100}{(2\Omega\cos\vartheta/a)}}$$

$$\sim \sqrt{\frac{100}{5\times 10^{-12}}} \sim 5000\,\text{km}$$

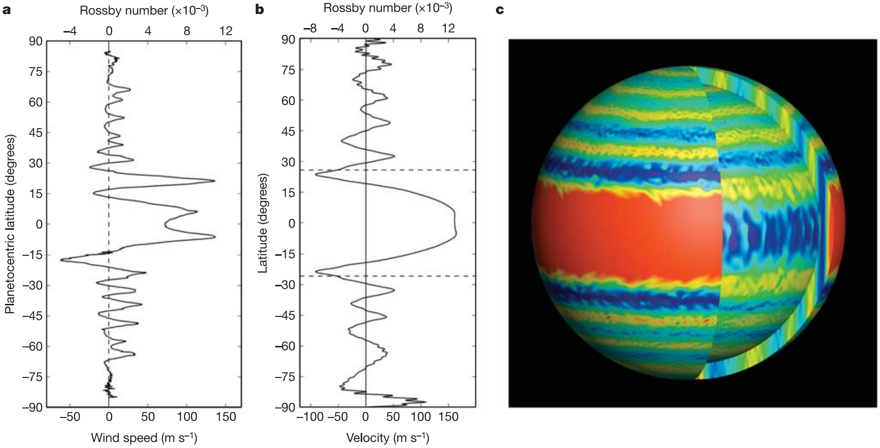


Williams (41 BN) (before now! i.e., 1978)

Deeper Jets

Taylor-columns (Busse, Schubert etc)

Simulation by Heimpel
Zonal velocity
Equatorial jet too broad.

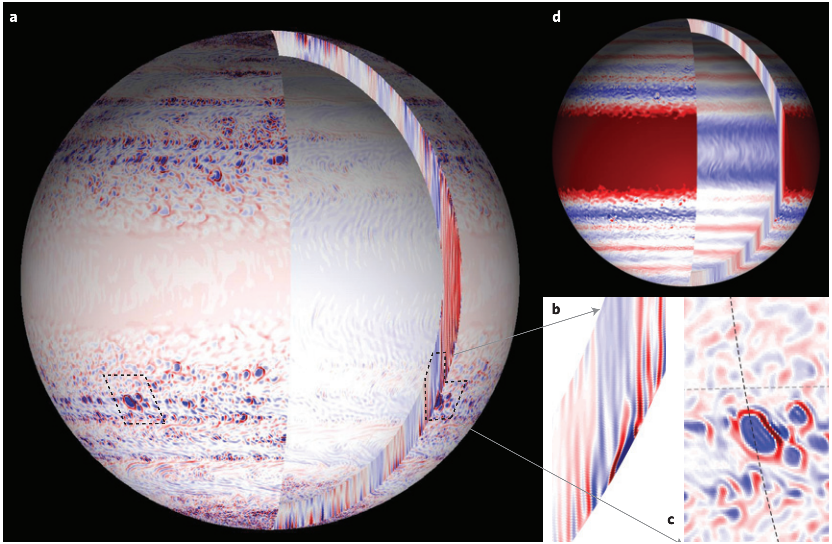


Observed Modelled Modelled

Deeper Jets

Taylor-columns?

Vorticity and velocity:
With stratified upper layer
Heimpel, Arnou et al:



The topographic beta effect for deep jets

Potential vorticity of a column:

$$\frac{DQ}{Dt} = 0, \qquad Q = \left(\frac{\zeta + 2\Omega}{h}\right).$$

$$Q = \left(\frac{\zeta + 2\Omega}{h}\right) \approx \left(\frac{\zeta + 2\Omega}{H}\right),$$

or, approximately,

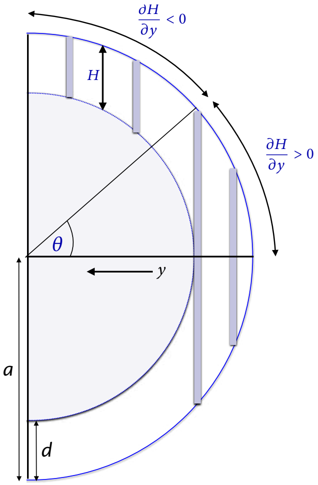
$$\frac{D\zeta}{Dt} + \beta^* v = 0 \qquad \text{where} \qquad \beta^* = -\frac{2\Omega}{H}\frac{\partial H}{\partial y}.$$

Just like flow in the weather layer!

$\beta^* > 0$ region insider the tangent cylinder (the extra-tropics)

$\beta^* < 0$ and outside the tangent cylinder, in the tropics.

$$\cos\theta = \frac{(a-d)}{a}$$



Jovian Superrotation

The topographic beta effect for deep jets

Homogenize the potential vorticity to give:

$$\frac{\partial \overline{u}}{\partial y} = A - |\beta^*| y, \qquad \text{or} \qquad \overline{u} = Ay - \frac{1}{2}|\beta^*| y^2 + B, \quad (13)$$

If $\partial \overline{u}/\partial y = 0$ at $y = 0$ then $A = 0$.

$\overline{u}$ is a *maximum* at $y = 0$.

Width $\sim \sqrt{\overline{u}/\beta^*}$

NB $\beta^* \neq \beta$. — so weather-layer jets might not coincide with deep jets!

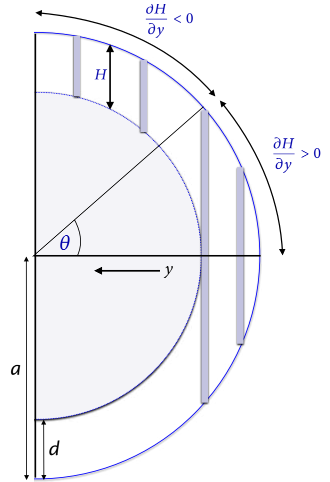
Tropical region is

$$\cos\theta = \frac{(a-d)}{a}$$

where a = planet radius, d = depth of outer zone.

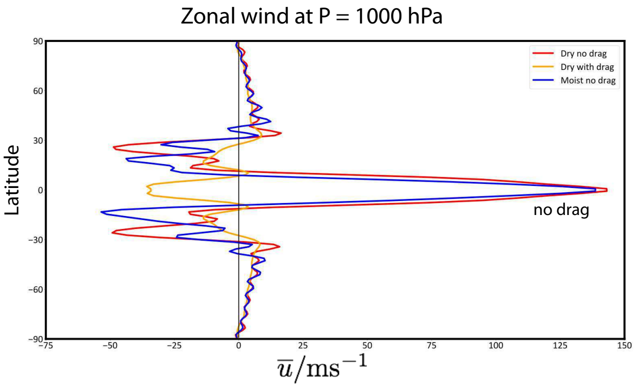
If $d = 3500$ km and $a = 70,000$ km then $\theta = 18°$ (Jupiter).

If $d = 90000$ km and $a = 58,000$ km then $\theta = 32°$ (Saturn).



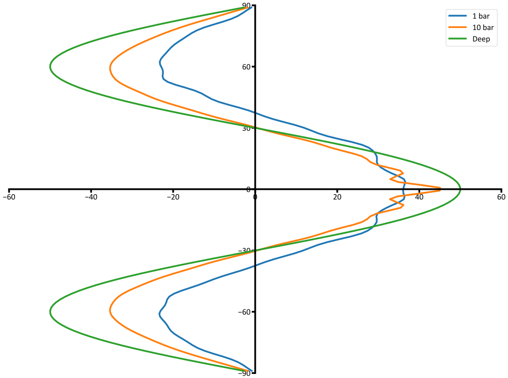
Jupiter Weather-Layer Winds

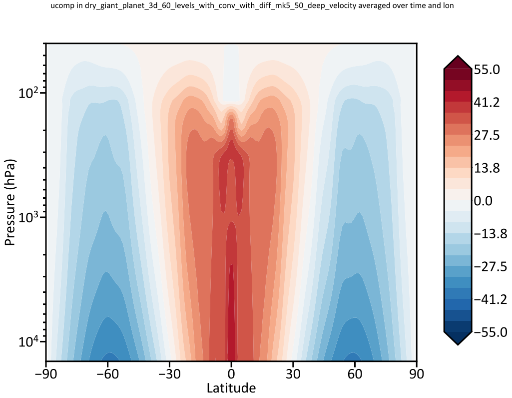
- Weather-layer, primitive equation model (Isca!)
- Jovian parameters: size, rotation rate, hydrogen atmosphere, gravity, etc.
- Forced with both incoming solar radiation and outgoing flux (6 W/m^2).
- Total Depth = 20 bars
- Superrotation only if drag removed at equator (c.f. Schneider & Liu, 2012)



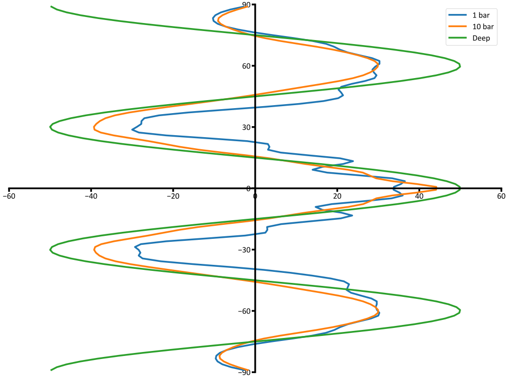
Prescribe a Deep Flow

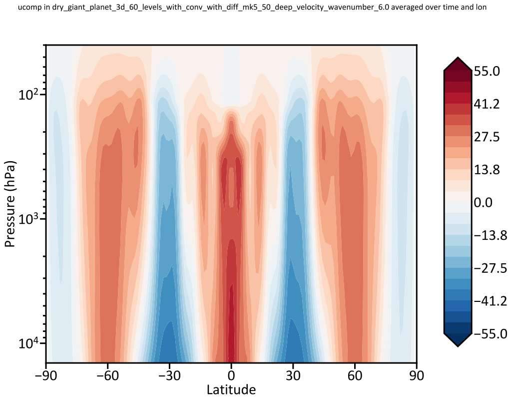
On a Weather-layer Jupiter GCM





Prescribe a Deep Flow
On a Weather-layer Jupiter GCM





At upper levels the jets break down into more jets.

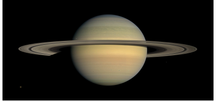
Not satisfactory, but demonstrates some decoupling between deep and shallow jets.

Jovian possibilities

Consistent with observations

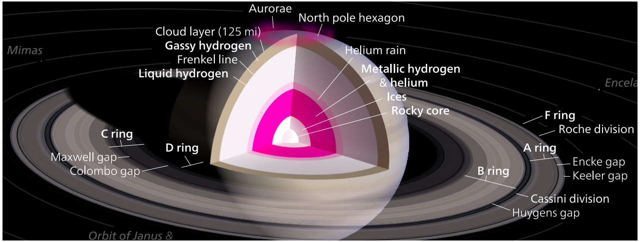
1. Jets descend to about 3000 km at all latitudes (Kaspi et al interpretation of Juno).
   Deep jets coincident with shallow jets.
2. Weather-layer jets are *decoupled* from deep jets at all latitudes.
   Weather-layer jets are shallow, beta-plane turbulence.
3. Mid-latitude jets are shallow, equatorial jets are deep.
4. Evidence that the equatorial and mid-latitude jets have a different character? – *Saturn!*

Saturn



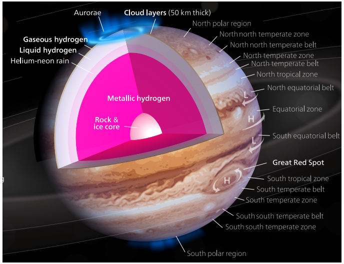
Natural light, Cassini, equinox.

Saturn:



- $a \approx 58,000$ km
- Sidereal day $\approx$ 10.5 hours
- Mass = 95 Earths = 5.7×10^{26} kg.
- Hydrogen becomes metallic at $r \leq 0.5a$? (30,000 km from surface).

Jupiter:



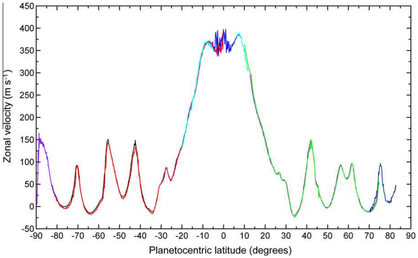
- $a \approx 70,000$ km
- Sidereal day $\approx$ 10 hours
- Mass = 318 Earths = 1.9×10^{27} kg.
- Hydrogen becomes metallic at $r \approx 0.8a$ (15,000 km from surface).

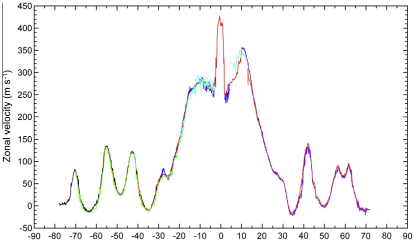
Figures by Kelvinsong, https://commons.wikimedia.org/w/index.php?curid=32219154.

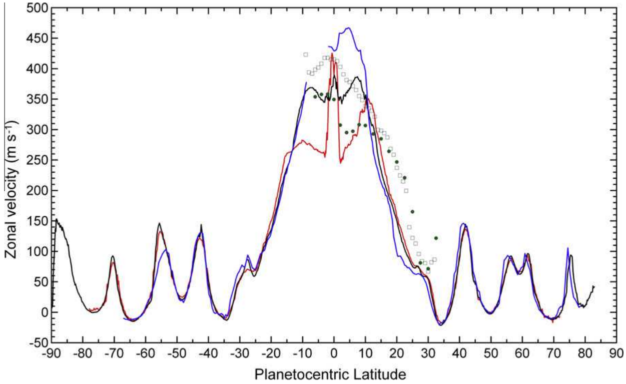
Saturn

Transition to metallic hydrogen is much deeper, because the planet is smaller.

Equatorial winds are stronger and wider than Jupiter! (But maybe not wide enough? Being quantitative is difficult.)





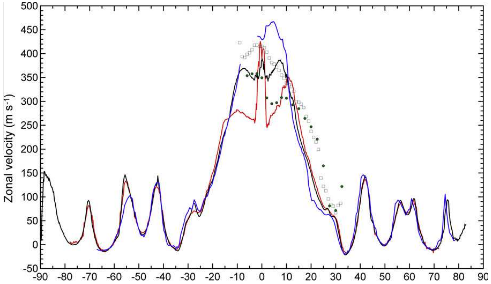


Various estimates from Garcia et al (2011), Cassini mission.

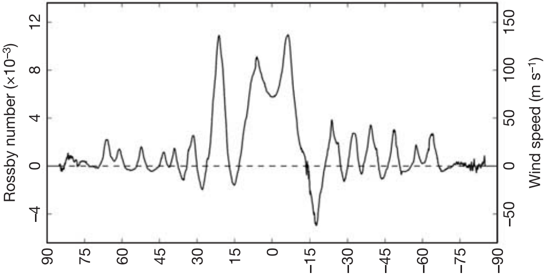
Saturn and Jupiter

Observed Zonal Winds

Saturn



Jupiter



- Wider and stronger equatorial winds on Saturn.
- Suggests equatorial winds are connected to the deep.
- Mid-latitude zonal winds are stronger too…