HYPERBOLIC AND SPHERICAL BLACK HOLES IN HYPERSCALING VIOLATING GEOMETRIES

SIGILLUM UNIVERSITATIS ISLANDIAE

HÁSKÓLI ÍSLANDS

VERKFRÆÐI- OG NÁTTÚRUVÍSINDASVIÐ

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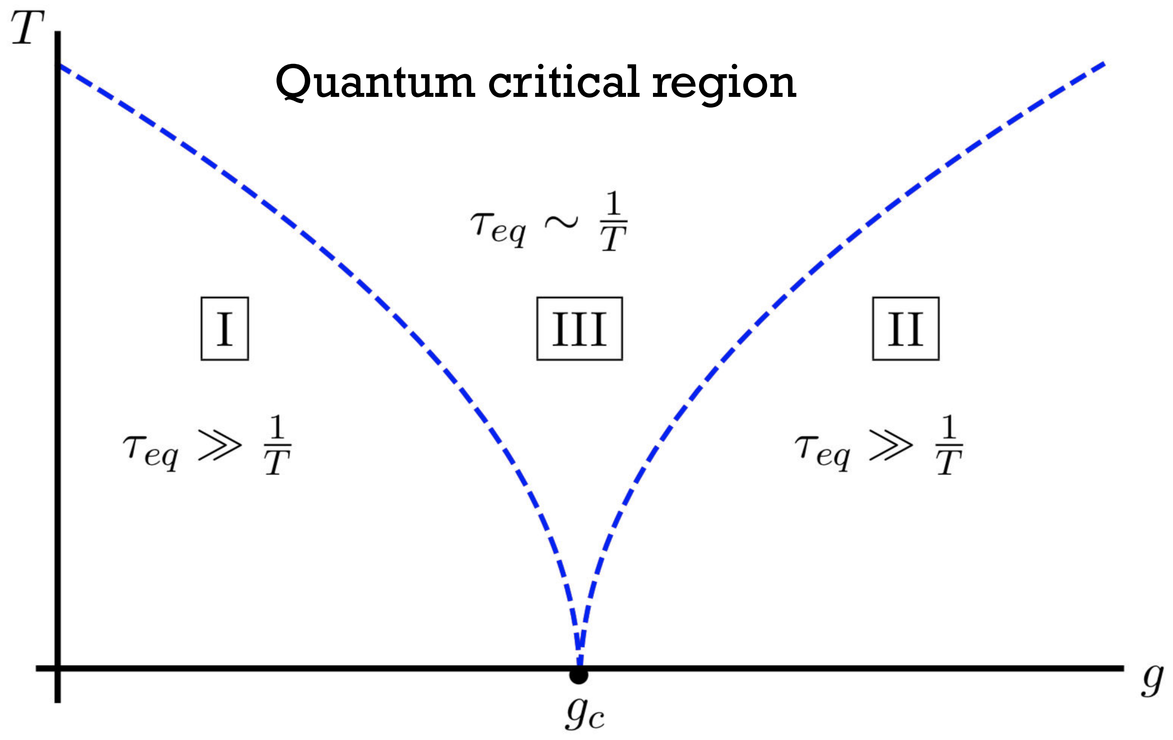
Based on work with Juan Pedraza and Manus Visser

5th of June, Toronto

HYPERSCALING VIOLATION: MOTIVATION



Generating functional

$$Z(J) \equiv \int D\phi e^{iS[\phi]+i\int d^{d+1}x J_j(x)\mathcal{O}_j(x)}$$

Expectation values

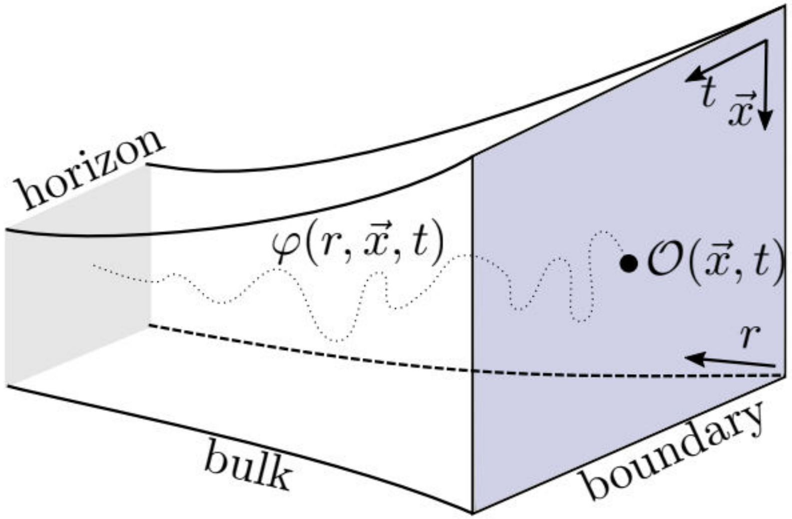
$$\langle \mathcal{O} \rangle_{QFT} \sim \frac{\delta}{\delta J} \log Z(J)$$

Tough when strongly coupled!

HYPERSCALING VIOLATION: MOTIVATION

Expectation values via holography?

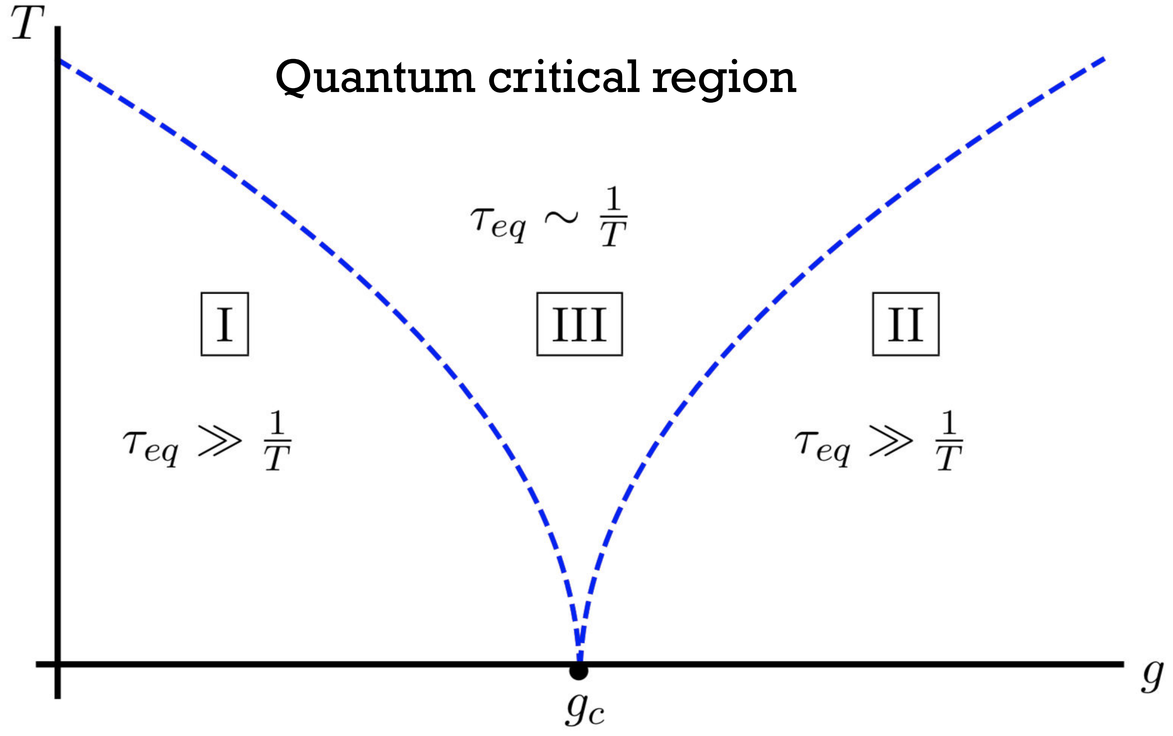
$$\langle \mathcal{O} \rangle_{QFT} \sim \frac{\delta}{\delta J} \log Z(J)$$



Gravitational bulk	Boundary field theory
local symmetries	global symmetries
boundary value field ϕ	source of operator $\mathcal{O}$
field mass m	operator scaling dimension Δ
boundary term of on-shell action	generating functional
black hole temperature T	thermal equilibrium temperature T

[Maldacena'97]
[Witten'98]
[Gubser,Klebanov,Polyakov '98]

HYPERSCALING VIOLATION: MOTIVATION



Lifshitz geometries

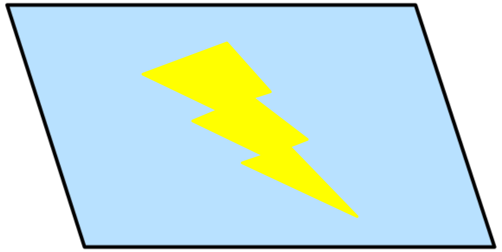
$$\omega \sim |\vec{k}|^z$$

Hyperscaling violating geometries

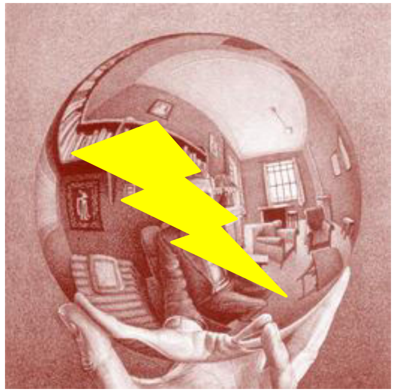
$$S \sim T^{(d-\theta)/z}$$

SOME SPOILERS

- ✓ Anti-de Sitter
- ✓ Lifshitz
- ✓ HSV

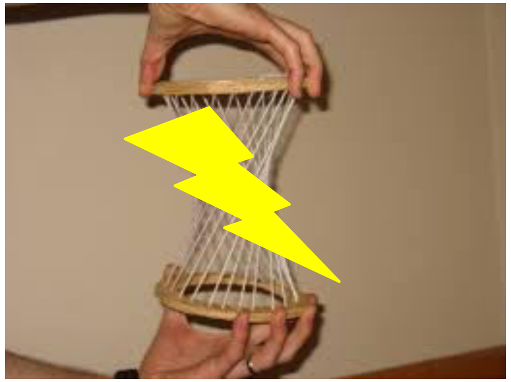


- ✓ Anti-de Sitter
- ✓ Lifshitz
- ✓ HSV



Van der Waals-like
phase transitions

- ✓ Anti-de Sitter
- ✗ Lifshitz
- ✓ HSV



absence tidal divergences

OUTLINE

- *Motivation and spoilers*
- **Engineering the space**
- Phase diagrams
- Puzzles of $\theta = d(z-1)$
- Summary

METRICS DESIRED FOR THIS ENDEAVOR

Anti-de Sitter

$$ds^2 = -\left(\frac{r}{\ell}\right)^2 f(r)dt^2 + \frac{\ell^2}{r^2 f(r)}dr^2 + r^2 d\Omega_{k,d}^2 \quad \Rightarrow \quad \omega \sim |\vec{k}|$$

$$r \to \lambda^{-1} r\,, \quad t \to \lambda t\,, \quad d\Omega_{k,d} \to \lambda d\Omega_{k,d}$$

k=0

k=1

k=-1

Lifshitz

[Kachru, Liu, Mulligan'08]
[Son'08]
[Taylor'08]
[Balasubramanian, McGreevy'09]

$$ds^2 = -\left(\frac{r}{\ell}\right)^{2z} f(r)dt^2 + \frac{\ell^2}{r^2 f(r)}dr^2 + r^2 d\Omega_{k,d}^2 \quad \Rightarrow \quad \omega \sim |\vec{k}|^z$$

$$r \to \lambda^{-1} r\,, \quad t \to \lambda^z t\,, \quad d\Omega_{k,d} \to \lambda d\Omega_{k,d}$$

Hyperscaling violating

[Charmousis, Gouteraux, Kim, Kiritsis, Meyer'10]
[Huijse, Sachdev, Swingle'11]

$$ds^2 = \left(\frac{r}{r_F}\right)^{-2\theta/d}\left[-\left(\frac{r}{\ell}\right)^{2z} f(r)dt^2 + \frac{\ell^2}{r^2 f(r)}dr^2 + r^2 d\Omega_{k,d}^2\right]$$

$$ds \to \lambda^{\theta/d} ds \quad \Rightarrow \quad S \sim T^{(d-\theta)/z}$$

ACTION SATISFYING EINSTEIN EQUATIONS

HSV metric:

$$ds^2 = \left(\frac{r}{r_F}\right)^{-2\theta/d}\left[-\left(\frac{r}{\ell}\right)^{2z} f(r)dt^2 + \frac{\ell^2}{r^2 f(r)}dr^2 + r^2 d\Omega^2_{k,d}\right]$$

HSV | Lifshitz | topology | charge

HSV action:

$$S = -\frac{1}{16\pi G}\int d^{d+2}x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 + V(\phi) - \frac{1}{4}X(\phi)F^2 - \frac{1}{4}Y(\phi)H^2 - \frac{1}{4}Z(\phi)K^2\right]$$

[Taylor'08]
[Tarrio, Vandoren'11]
[Alishahiha, Colgain, Yavartanoo'12]

Ansatz:

$$F_{rt} = a(r)dt\,, \quad H_{rt} = b(r)dt\,, \quad K_{rt} = c(r)dt\,, \quad \phi = \phi(r)\,.$$

$$V = V_0 e^{\lambda_0\phi}, \;\; X = X_0 e^{\lambda_1\phi}, \;\; Y = Y_0 e^{\lambda_2\phi}, \;\; Z = Z_0 e^{\lambda_3\phi}$$

Einstein equations:

$$\begin{aligned}
\nabla^2\phi + \partial_\phi V(\phi) - \frac{1}{4}\partial_\phi X(\phi)F^2 - \frac{1}{4}\partial_\phi Y(\phi)H^2 - \frac{1}{4}\partial_\phi Z(\phi)K^2 &= 0,\\
\nabla_\mu\left(X(\phi)F^{\mu\nu}\right) = 0, \quad \nabla_\mu\left(Y(\phi)H^{\mu\nu}\right) = 0, \quad \nabla_\mu\left(Z(\phi)K^{\mu\nu}\right) &= 0,\\
R_{\mu\nu} + \frac{g_{\mu\nu}}{d}V(\phi) - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}X(\phi)\left(F_{\alpha\mu}F^\alpha{}_\nu - \frac{g_{\mu\nu}}{2d}F^2\right)&\\
-\frac{1}{2}Y(\phi)\left(H_{\alpha\mu}H^\alpha{}_\nu - \frac{g_{\mu\nu}}{2d}H^2\right) - \frac{1}{2}Z(\phi)\left(K_{\alpha\mu}K^\alpha{}_\nu - \frac{g_{\mu\nu}}{2d}K^2\right) &= 0\,.
\end{aligned}$$

ACTION SATISFYING EINSTEIN EQUATIONS

HSV metric:

$$ds^2 = \left(\frac{r}{r_F}\right)^{-2\theta/d}\left[-\left(\frac{r}{\ell}\right)^{2z} f(r)dt^2 + \frac{\ell^2}{r^2 f(r)}dr^2 + r^2 d\Omega_{k,d}^2\right]$$

HSV action:

HSV Lifshitz topology charge

$$S = -\frac{1}{16\pi G}\int d^{d+2}x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 + V(\phi) - \frac{1}{4}X(\phi)F^2 - \frac{1}{4}Y(\phi)H^2 - \frac{1}{4}Z(\phi)K^2\right]$$

Ansatz:

$$F_{rt} = a(r)dt\,, \quad H_{rt} = b(r)dt\,, \quad K_{rt} = c(r)dt\,, \quad \phi = \phi(r)\,.$$

$$V = V_0 e^{\lambda_0\phi}, \;\; X = X_0 e^{\lambda_1\phi}, \;\; Y = Y_0 e^{\lambda_2\phi}, \;\; Z = Z_0 e^{\lambda_3\phi}$$

$$\lambda_0 = \frac{2\theta}{\gamma d}, \;\; \lambda_1 = -\frac{2\,(d-\theta+\theta/d)}{\gamma}, \;\; \lambda_2 = -\frac{2(d-1)(d-\theta)}{\gamma d}, \;\; \lambda_3 = \frac{\gamma}{d-\theta},$$

$$\gamma = \sqrt{2\,(d-\theta)\,(z-1-\theta/d)}$$

Warp function:

$$f = 1 + k\frac{(d-1)^2}{(d-\theta+z-2)^2}\frac{\ell^2}{r^2} - \frac{m}{r^{d-\theta+z}} + \frac{q^2}{r^{2(d-\theta+z-1)}}$$

ALLOWED VALUES FOR EXPONENTS

Null energy condition

$$T_{\mu\nu}\xi^{\mu}\xi^{\nu} \geq 0$$

Resulting restrictions

z	hyperbolic $k=-1$	planar $k=0$	spherical $k=1$
$z<1$	no solution	no solution	no solution
$1 \leq z < 2$	$\theta = d(z-1)$	$\theta \leq d(z-1)$	$\theta \leq d(z-1)$
$z \geq 2$	no solution	$\theta < d$	$\theta < d$

$$\gamma = \sqrt{2\,(d-\theta)\,(z-1-\theta/d)}$$

STRONGLY COUPLED DILATON ACTION

Redefine

$$\phi(r) \to \gamma\tilde{\phi}(r) \qquad \gamma = \sqrt{2\,(d-\theta)\,(z-1-\theta/d)}$$

"New" HSV action

$$S = -\frac{1}{16\pi G}\int d^{d+2}x\sqrt{-g}\left[R - \gamma^2\frac{1}{2}(\nabla_\mu\tilde{\phi})^2 + V(\tilde{\phi}) - \frac{1}{4}X(\tilde{\phi})F^2 - \frac{1}{4}Y(\tilde{\phi})H^2 - \frac{1}{4}Z(\tilde{\phi})K^2\right]$$

$$V = V_0 e^{\tilde{\lambda}_0\tilde{\phi}}, \quad X = X_0 e^{\tilde{\lambda}_1\tilde{\phi}}, \quad Y = Y_0 e^{\tilde{\lambda}_2\tilde{\phi}}, \quad Z = Z_0 e^{\tilde{\lambda}_3\tilde{\phi}}\,,$$

$$\tilde{\lambda}_0 = \frac{2\theta}{d}, \quad \tilde{\lambda}_1 = -2\,(d-\theta+\theta/d)\,, \quad \tilde{\lambda}_2 = -\frac{2(d-1)(d-\theta)}{d}, \quad \tilde{\lambda}_3 = \frac{\gamma^2}{d-\theta}.$$

SUPPORTING THE GEOMETRY

$$\theta = d(z-1)$$
$$1 \leq z < 2$$

HSV Lifshitz charge

$$S = -\frac{1}{16\pi G}\int d^{d+2}x\sqrt{-g}\left[R + V(\tilde{\phi}) - \frac{1}{4}X(\tilde{\phi})F^2 - \frac{1}{4}Z_0 K^2\right]$$

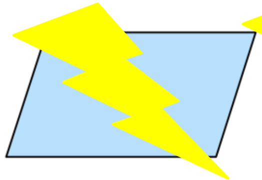
Strongly coupled scalar

[Balasubramanian, McGreevy'09]

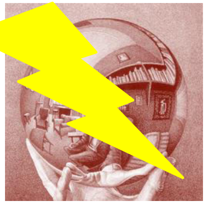
$$\theta < \min[d(z-1), d]$$
$$1 \leq z$$

HSV Lifshitz topology charge

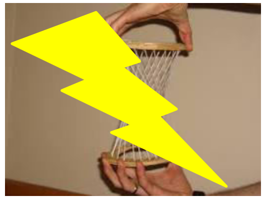
$$S = -\frac{1}{16\pi G}\int d^{d+2}x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 + V(\phi) - \frac{1}{4}X(\phi)F^2 - \frac{1}{4}Y(\phi)H^2 - \frac{1}{4}Z(\phi)K^2\right]$$



k=0



k=1



k=-1

OUTLINE

- *Motivation and spoilers*
- *Engineering the space*
- **Phase diagrams**
- Puzzles of $\theta = d(z-1)$
- Summary

BLACK HOLE THERMODYNAMICS

Warp function

$$f = 1 + k\frac{(d-1)^2}{(d-\theta+z-2)^2}\frac{\ell^2}{r^2} - \frac{m}{r^{d-\theta+z}} + \frac{q^2}{r^{2(d-\theta+z-1)}}$$

Bekenstein-Hawking entropy

$$S = \frac{\omega_{k,d}}{4G} r_h^{d-\theta} r_F^{\theta}$$

Temperature

$$T = \frac{1}{4\pi}\left(\frac{r_h}{\ell}\right)^{z+1}\left|f'(r_h)\right| = \frac{r_h^z}{4\pi\ell^{z+1}}\left[(d-\theta+z) + k\frac{(d-1)^2}{(d-\theta+z-2)}\frac{\ell^2}{r_h^2} - \frac{(d-\theta+z-2)q^2}{r_h^{2(d-\theta+z-1)}}\right]$$

EXTREMALITY

Extremality condition

$$f(r_{ext}) = f'(r_{ext}) = 0$$

Charge

$$q_{ext}^2 = r_{ext}^{2(d+z-\theta-1)} \left[\frac{d-\theta+z}{d-\theta+z-2} + k \frac{\ell^2}{r_{ext}^2} \frac{(d-1)^2}{(d-\theta+z-2)^2} \right]$$

Mass parameter

$$m_{ext} = 2 r_{ext}^{d-\theta+z} \left[\frac{d-\theta+z-1}{d-\theta+z-2} + k \frac{\ell^2}{r_{ext}^2} \frac{(d-1)^2}{(d-\theta+z-2)^2} \right]$$

k=0

k=1

k=-1

ENSEMBLES & BACKGROUND SUBTRACTION

First law and grand canonical (potential fixed)

$$dM = TdS + \Phi dQ$$

$$M = \frac{\omega_{k,d}}{16\pi G}(d-\theta)(m - m_{ground})\ell^{-z-1} r_F^{\theta}$$

[Witten'98]
[Emparan, Johnson, Myers '99]
[Chamblin, Emparan, Johnson, Myers '99]

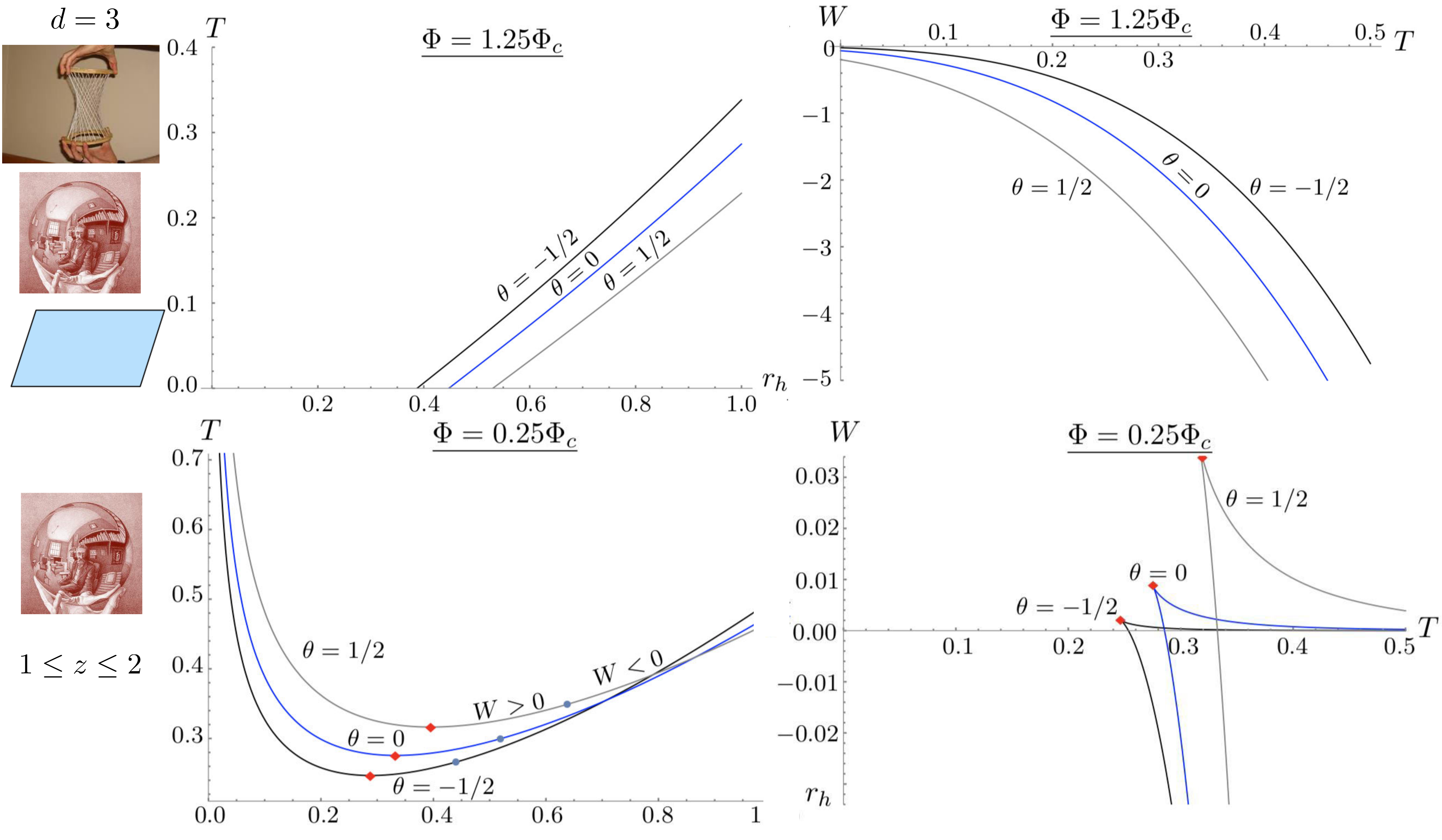
Gibbs $W = M - TS - \Phi Q$

First law and canonical (charge fixed)

$$d(\Delta M) = TdS + (\Phi - \Phi_{ext})dQ$$

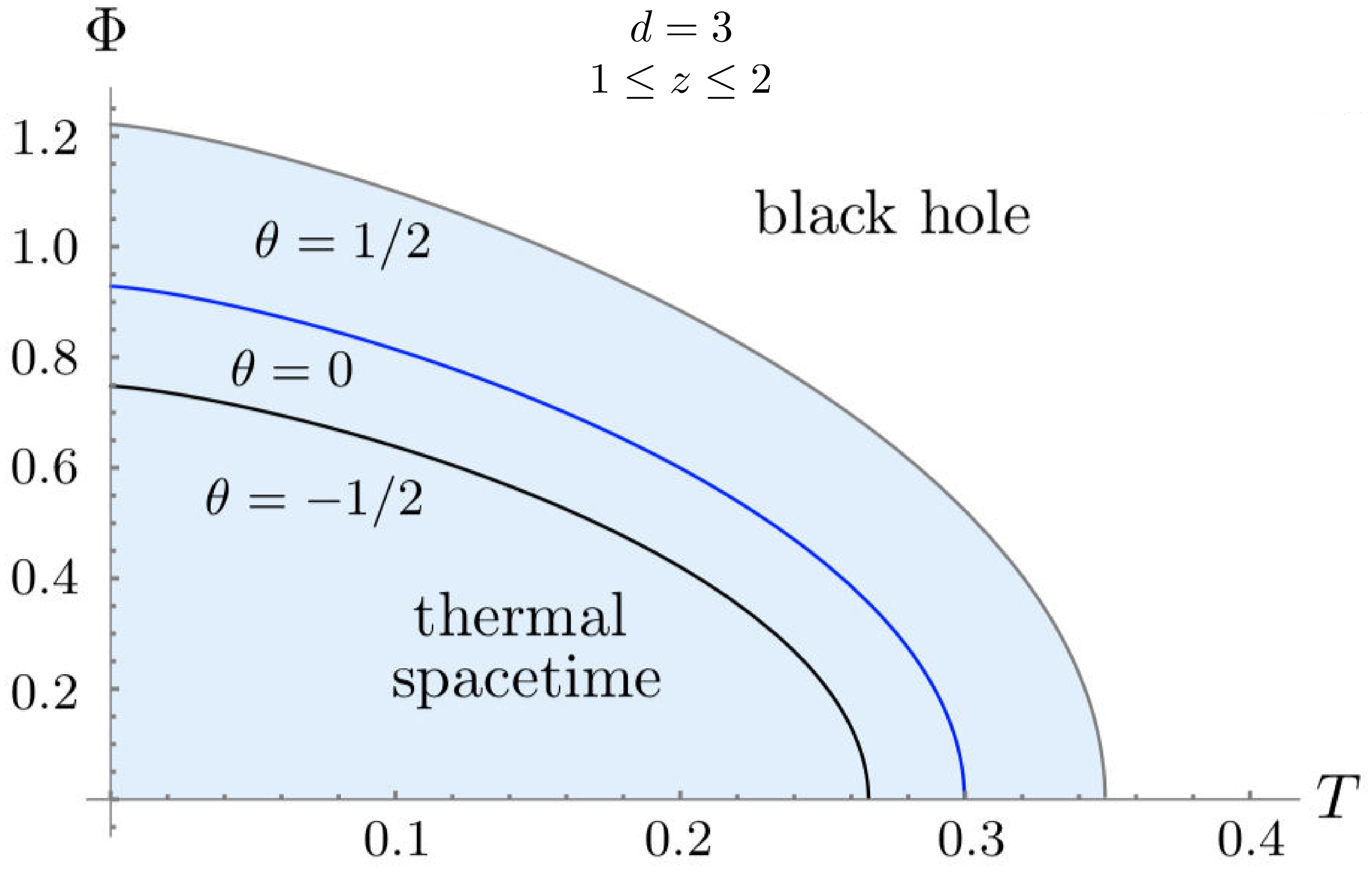
$$\Delta M = \frac{\omega_{k,d}}{16\pi G}(d-\theta)(m - m_{ext})\ell^{-z-1} r_F^{\theta}$$

Helmholtz $F = \Delta M - TS$



$d = 3$
$1 \leq z \leq 2$
$\Phi = 1.25\Phi_c$
T
r_h
$\theta = -1/2$
$\theta = 0$
$\theta = 1/2$
W
$\Phi = 0.25\Phi_c$
$W > 0$
$W < 0$

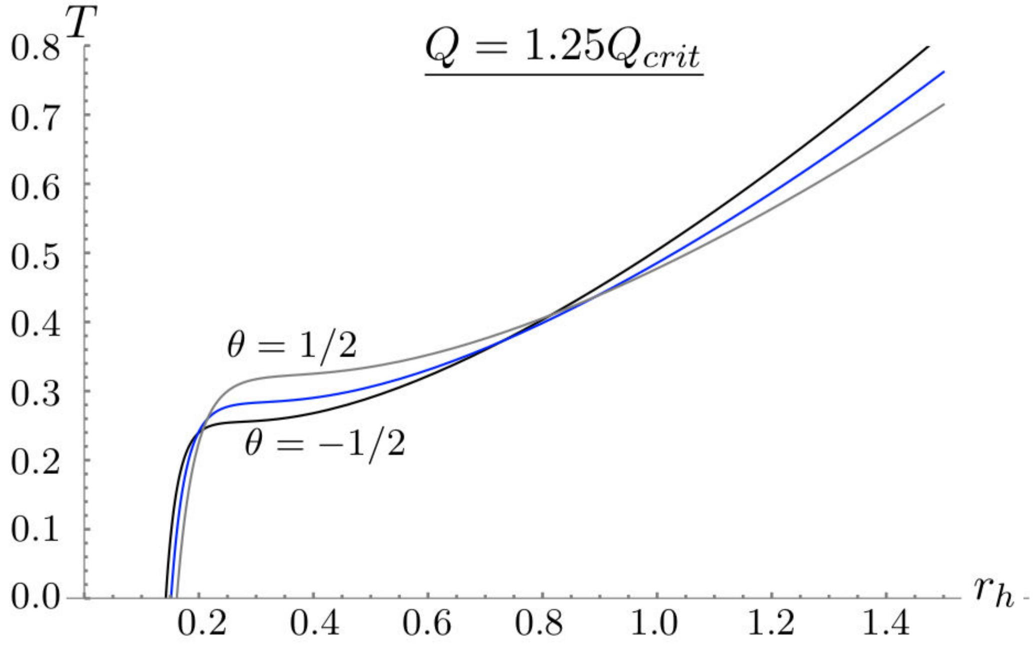
GRAND CANONICAL PHASES

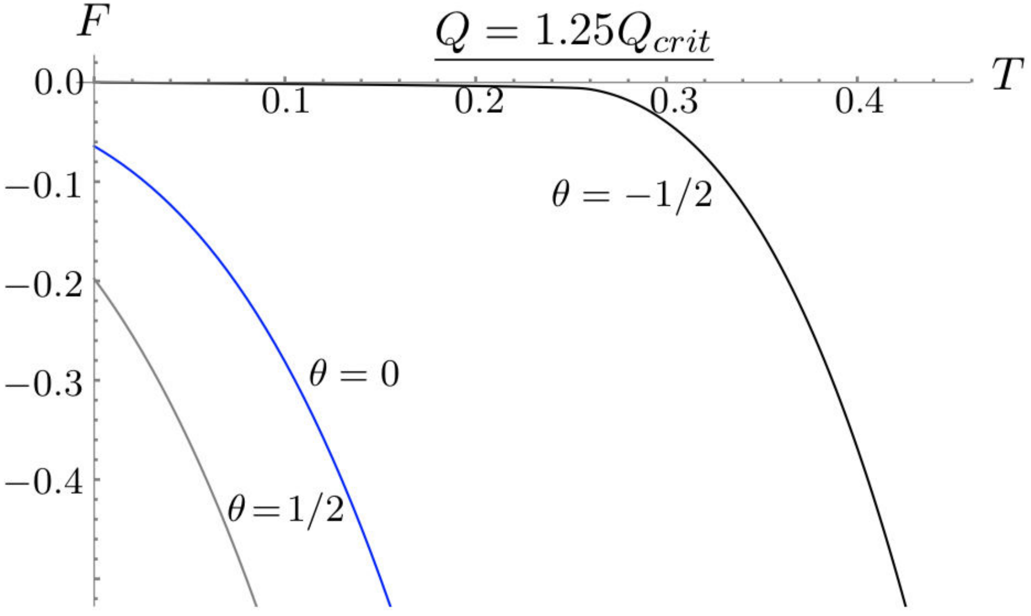


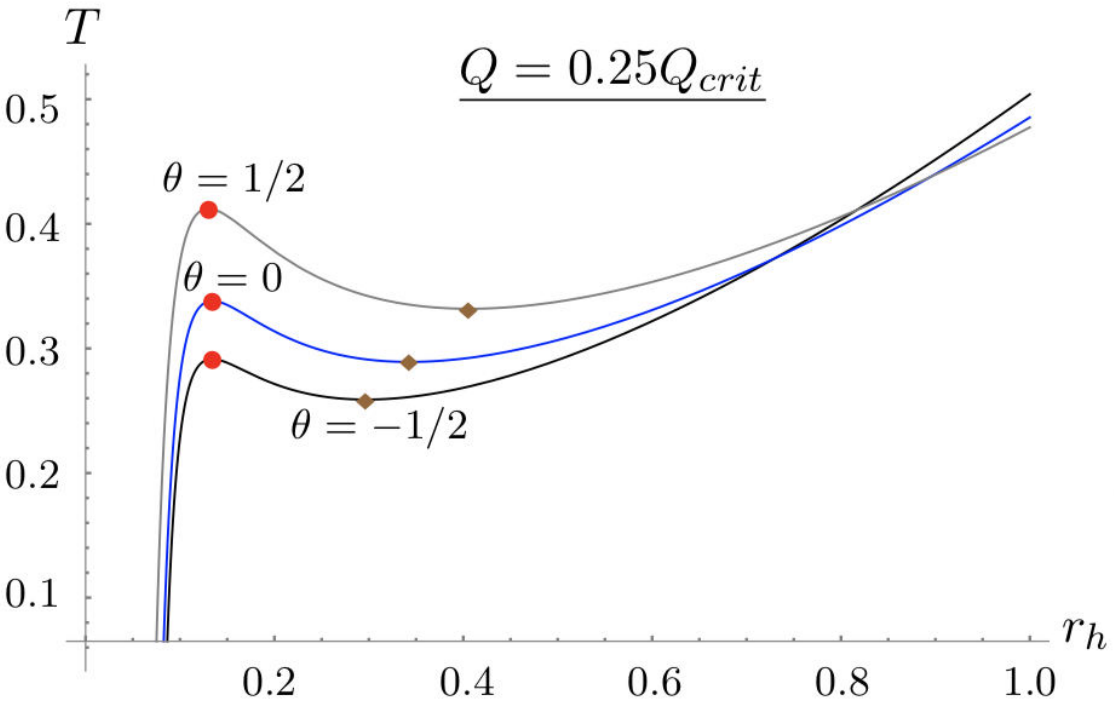
CANONICAL ENSEMBLE

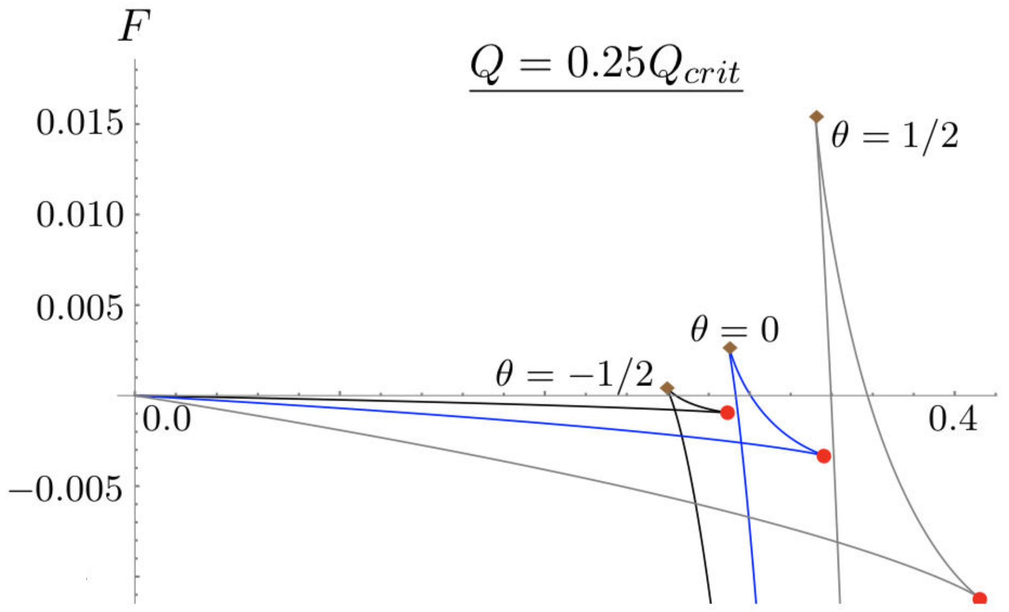
$d = 3$

$1 \leq z \leq 2$







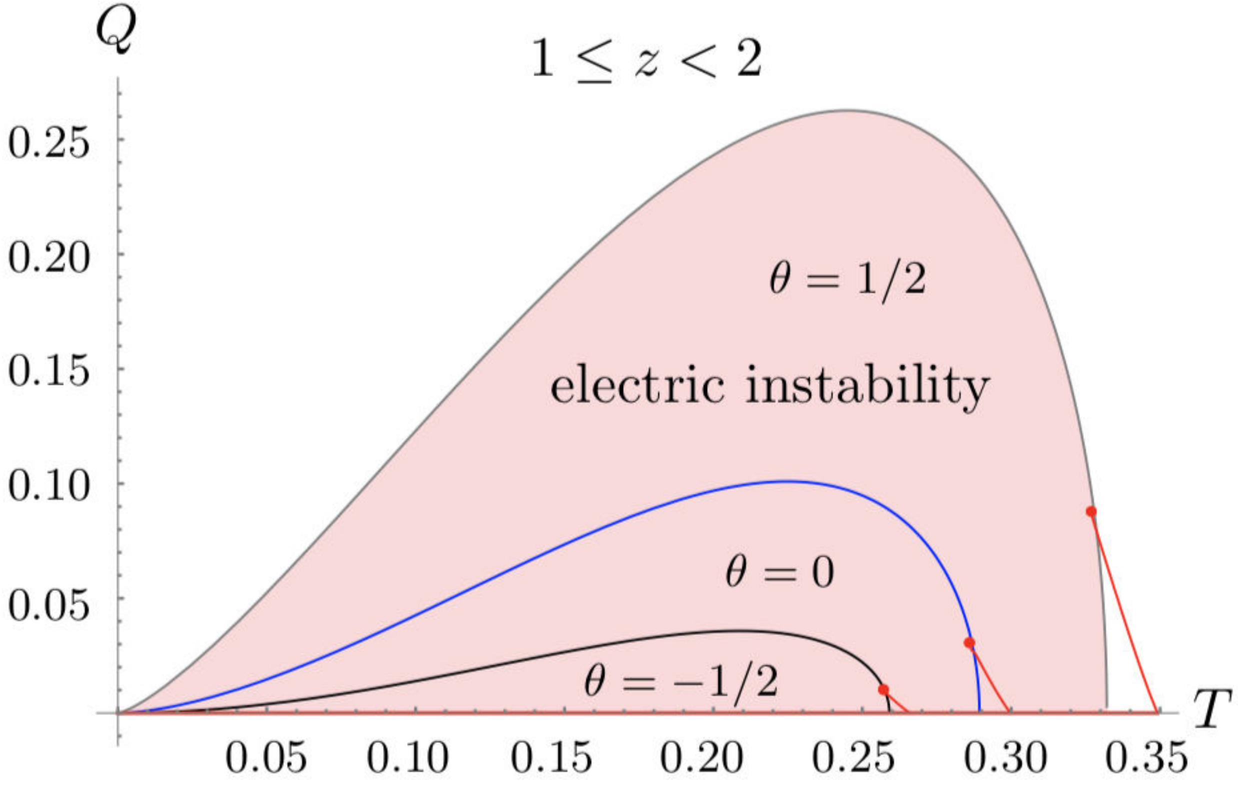


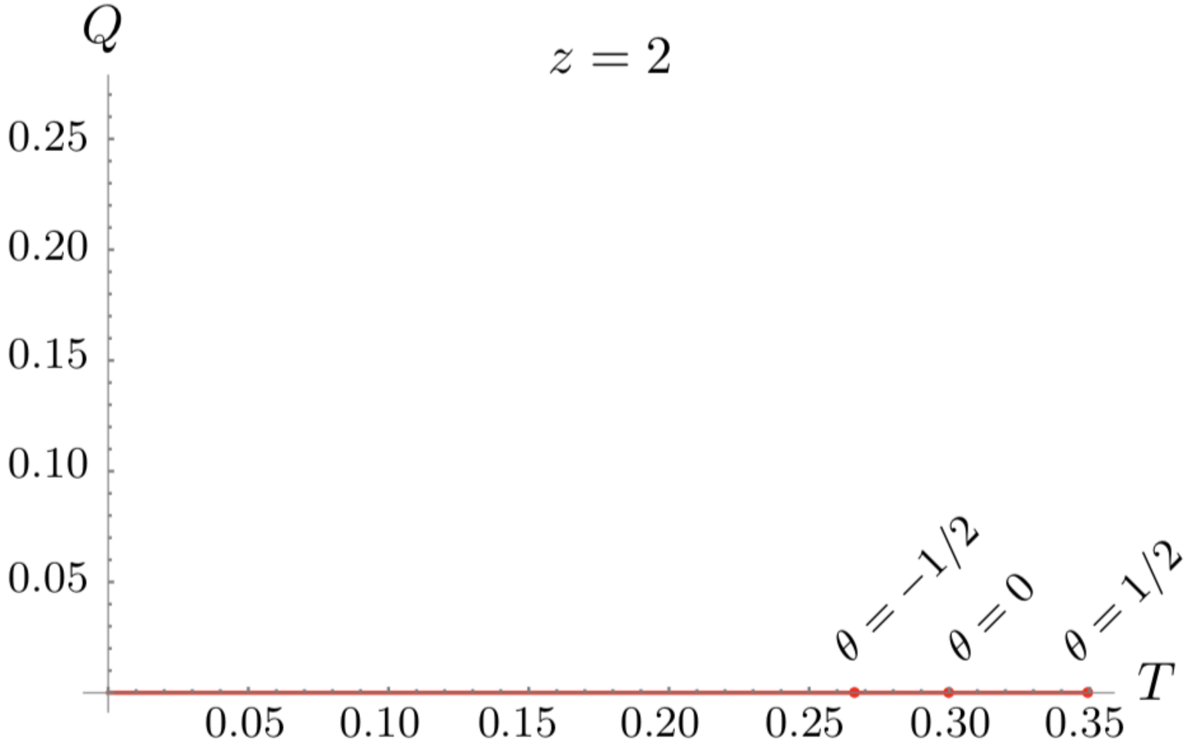
CANONICAL PHASES

$d = 3$

Isothermal susceptibility + electric (in)stability:

$$\chi_T \equiv \left(\frac{\partial Q}{\partial \Phi}\right)_T \geq 0$$





OUTLINE

- *Motivation and spoilers*
- *Engineering the space*
- *Phase diagrams*
- **Puzzles of** $\theta = \mathrm{d}(\mathrm{z} - 1)$
- Summary

MASSLESS HYPERBOLIC BLACK HOLE

$\theta = d(z-1)$
$1 \leq z < 2$

Warp function

$$f = 1 + \frac{k}{(2-z)^2}\frac{\ell^2}{r^2} - \frac{m}{r^{d(2-z)+z}}$$

Special features massless hyperbolic black hole: Rindler

[Emparan'99]
[Casini, Huerta, Myers'11]

Curvature invariants: $R_{\mu\nu}R^{\mu\nu}$ $R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$

General HSV: $$R = \frac{1}{\ell^2}\left(\frac{r}{r_F}\right)^{\frac{2\theta}{d}}\left[a_1 + a_2\frac{\ell^2}{r^2}\right]$$

$\theta = d(z-1)$
$1 \leq z < 2$

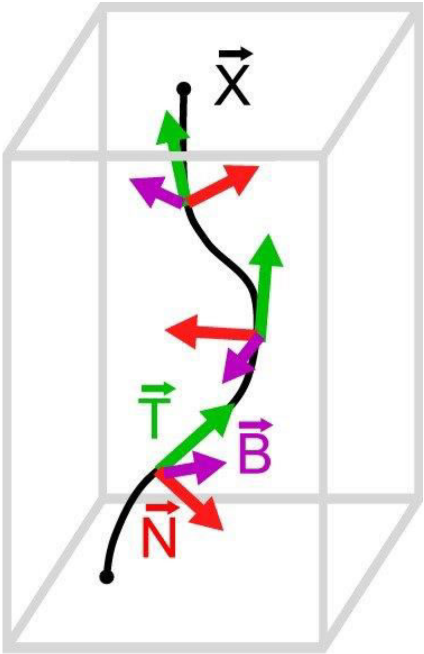
$$R = -\frac{1}{\ell^2}\left(\frac{r}{r_F}\right)^{2(z-1)}$$

Rindler wedge?

ABSENCE OF DIVERGING TIDAL FORCES

Orthonormal frame parallelly propagated along a geodesic

$$\tilde{R}_{mnab} \equiv R^{\mu\nu\alpha\beta}(\tilde{e}_m)^\mu(\tilde{e}_n)^\nu(\tilde{e}_a)^\alpha(\tilde{e}_b)^\beta .$$



General HSV:

$$R_{\hat{1}\hat{1}\hat{1}\hat{1}} = -\frac{d-\theta}{d^2r^2}\left(\frac{r}{r_F}\right)^{2\theta/d}\left[k\frac{(d-1)^2(d(z-1)-\theta)}{(d+z-\theta-2)^2} + \frac{r^2}{\ell^2}\left(dz-\theta-E^2\left(\frac{r}{\ell}\right)^{-2z}\left(\frac{r}{r_F}\right)^{2\theta/d}(d(z-1)-\theta)\right)\right]$$

[Pang'09]
[Copsey, Mann'10]
[Horowitz, Way'11]

$\theta = d(z-1)$
$1 \leq z < 2$

$$R_{0101} = \frac{z}{2-z}R_{\hat{0}\hat{i}\hat{0}\hat{i}} = -\frac{z}{2-z}R_{\hat{1}\hat{i}\hat{1}\hat{i}} = -\frac{z}{(2-z)^2\Theta_k}R_{\hat{i}\hat{j}\hat{i}\hat{j}} = \frac{z}{\ell^2}\left(\frac{r}{r_F}\right)^{2(z-1)}, \quad R_{\hat{0}\hat{i}\hat{1}\hat{i}} = 0 .$$

[Shaghoulian'12]

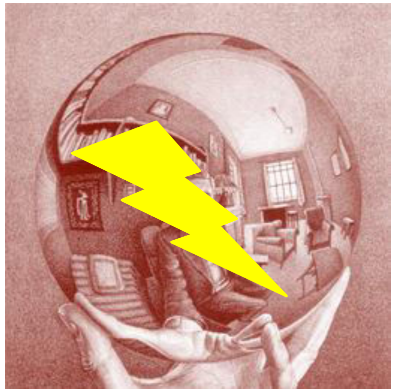
[Blau, Hartong, Rollier'09]

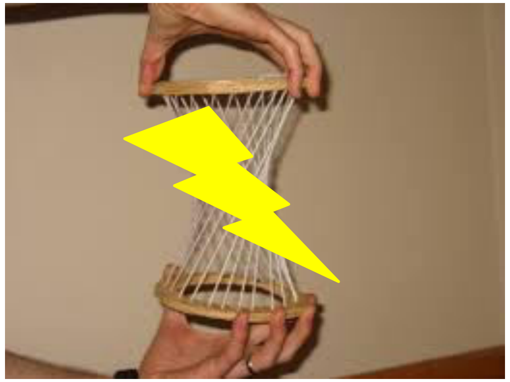
Geodesically complete. Global space?

SUMMARY & OUTLOOK

✓ HSV

✓ HSV





absence tidal divergences

Rindler wedge of …?

Extra symmetries

Holo renormalization

Quasinormal modes

Hydrodynamics

SUGRA embedding

Van der Waals-like phase transitions

$$\theta = d(z-1)$$
$$1 \leq z < 2$$

$$\theta = d(z-1)$$
$$1 \leq z < 2$$

Future

Thank you for listening !!

EXTRA: ADM MASS

$$M_T = -\frac{1}{8\pi G}\int_{S_{k,d}} d^d x\sqrt{\sigma}\, N\Theta\Big|_{r=R}$$

$$M = \lim_{R\to\infty}\left(M_T - \frac{\sqrt{f(R)}}{\sqrt{f_0(R)}}M_0\right) = \frac{\omega_{k,d}}{16\pi G}(d-\theta)m\ell^{-z-1}r_F^{\theta}$$