AdS_3/CFT_2 duality with $\mathcal{N} = (3, 3)$ supersymmetry

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Based on: L. Eberhardt and IGZ [arXiv: 1805.$ijk\ell m$]

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▶ String theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ supports the large $\mathcal{N} = 4$ SCA.

[Boonstra, Peeters, Skenderis; '98; de Boer, Pasquinucci, Skenderis '99]

▶ $\mathcal{S}_\kappa$ CFTs are $\mathcal{N} = 1$ WZW models $\mathfrak{su}(2)^{(1)}_{\kappa+2} \times \mathfrak{u}(1)^{(1)}$ associated to $\mathrm{S}^3 \times \mathrm{S}^1$ and have large $\mathcal{N} = 4$ susy.

[Sevrin, Troost, van Proeyen '88; Gukov, Martinec, Moore, Strominger '04]

▶ Holographic duality: string theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ is dual to the symmetric product orbifold $\mathrm{Sym}^N(\mathcal{S}_\kappa)$.

[Eberhardt, Gaberdiel, Li; Eberhardt, Gaberdiel, Gopakumar, Li '17]

▶ We consider string theory on $\mathrm{AdS}_3\times(\mathrm{S}^3\times\mathrm{S}^3\times\mathrm{S}^1)/\mathbb{Z}_2$. This background supports $\mathcal{N}=3$ or $\mathcal{N}=1$ SCA depending on the action of $\mathbb{Z}_2$. [Yamaguchi, Ishimoto, Sugiyama, '99]

▶ Holographic duality with $\mathcal{N}=(3,3)$ susy: string theory on $\mathrm{AdS}_3\times(\mathrm{S}^3\times\mathrm{S}^3\times\mathrm{S}^1)/\mathbb{Z}_2$ is dual to symmetric orbifold $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$. [Eberhardt, IGZ; to appear]

▶ The BPS spectrum of the dual CFT matches that of the worldsheet string theory. Moreover, the modified elliptic genera of the CFT and string theory match.

- Stringy AdS_3/CFT_2 dualities with $\mathcal{N}$ =(4,4) and (4,4)
- $\mathcal{N}$ =(3,3) duality: world-sheet theory
- $\mathcal{N}$ =(3,3) duality: dual CFT
- Elliptic genus
- Conclusions

D1-D5 brane system

	0	1	2	3	4	5	6	7	8	9
Q_1 D1 branes	—	—	$\sim$	$\sim$	$\sim$	$\sim$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
Q_5 D5 branes	—	—	—	—	—	—	$\cdot$	$\cdot$	$\cdot$	$\cdot$

where $\mathcal{M}$ is $\mathbb{T}^4$ or $K3$. Near horison geometry: $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathcal{M}$.

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Q_1 D1 branes	—	—	$\sim$	$\sim$	$\sim$	$\sim$	·	·	·	·
Q_5 D5 branes	—	—	—	—	—	—	·	·	·	·

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In the limit where the size of $\mathcal{M} \ll$ size of S^1, the worldvolume gauge theory of D branes is a 2d field theory that lives on S^1.

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Q_5 D5 branes	—	—	—	—	—	—	·	·	·	·

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In the limit where the size of $\mathcal{M} \ll$ size of S^1, the worldvolume gauge theory of D branes is a 2d field theory that lives on S^1.

It flows in IR to a CFT described by a sigma model whose target space is a resolution of symmetric product orbifold $\mathrm{Sym}^N(\mathcal{M})$.
[Vafa, '95; Strominger, Vafa '96]

Symmetric product orbifold $\mathrm{Sym}^N(\mathcal{M})$

- 2d SCFT with small $\mathcal{N} = (4,4)$ susy and $SO(4)_R \cong SU(2)_L \times SU(2)_R$ R-symmetry.

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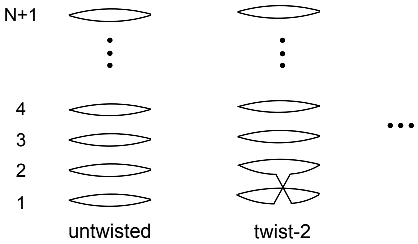
▶ The orbifold group, S_N, acts by permuting N copies of seed theory. It defines a new sector of Hilbert space, the *twisted* sector, through imposing new bc's on the fundamental fields.



$\mathsf{AdS}_3/\mathsf{CFT}_2$ with small $\mathcal{N} = (4, 4)$ susy

- Stringy duality: string theory on $\mathsf{AdS}_3 \times \mathsf{S}^3 \times \mathcal{M}$ is dual to symmetric orbifold $\mathrm{Sym}^N(\mathcal{M})$.

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▶ For $\mathcal{M} = K3$, the elliptic genera of the CFT and supergravity match. [de Boer, '98]

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- 3-point functions of chiral primaries in orbifold CFT, world-sheet theory, and supergravity match.

[Gaberdiel, Kirsch, '07; Dabholkar, Pakman, '07; Taylor, '08]

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▶ Great progress in constructing new classes of 3-charge D1-D5-P black hole microstates with arbitrary finite angular momenta.

[Bena, Giusto, Martinec, Mathur, Peet, Russo, Shigemori, Turton, Warner]

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- Embedding of higher spin/CFT duality in stringy duality.
[Gaberdiel, Gopakumar '14]

Large $\mathcal{N} = (4,4)$ susy

D brane system:

	0	1	2	3	4	5	6	7	8	9
Q_1 D1 branes	—	·	·	·	·	—	$\sim$	$\sim$	$\sim$	$\sim$
Q_5^+ D5 branes	—	·	·	·	·	—	—	—	—	—
Q_5^- D5 fluxes	·	·	·	·	·	·	$\circ$	$\circ$	$\circ$	·

where $\mathcal{M} = S^3 \times S^1$. Near horison geometry: $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$.

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Q_5^- D5 fluxes	·	·	·	·	·	·	$\circ$	$\circ$	$\circ$	·

where $\mathcal{M} = S^3 \times S^1$. Near horison geometry: $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$.

D1-branes are instantons in D5-brane worldvolume theory. In IR, the theory is described by a sigma model on moduli space of instantons $\mathcal{M}_{Q_1, Q_5^+ = 1, Q_5^-} \cong \mathrm{Sym}^{Q_1}(S^3_{Q_5^- - 1} \times S_1)$.

[Eberhardt, Gaberdiel, Li; Eberhardt, Gaberdiel, Gopakumar, Li '17]

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Q_5^+ D5 branes	—	·	·	·	·	—	—	—	—	—
Q_5^- D5 fluxes	·	·	·	·	·	·	$\circ$	$\circ$	$\circ$	·

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[Eberhardt, Gaberdiel, Li; Eberhardt, Gaberdiel, Gopakumar, Li '17]

$\mathcal{S}_\kappa$ is CFT on $\mathrm{S}^3 \times \mathrm{S}^1$: an $\mathcal{N} = 1$ WZW model $\mathfrak{su}(2)^{(1)}_{\kappa+2} \oplus \mathfrak{u}(1)^{(1)}$, $\kappa = Q_5^- - 1$. It has large $\mathcal{N} = (4,4)$ linear A_γ SCA.

[Gukov, Martinec, Moore, Strominger '04]

Large $\mathcal{N} = 4$ SCA:

- $\mathcal{S}_\kappa$ is a 2d SCFT with large $\mathcal{N} = (4,4)$ susy, $\mathfrak{su}(2)_{k^+} \oplus \mathfrak{su}(2)_{k^-}$ R-symmetry algebra, and levels $k^+ = 1$, $k^- = \kappa + 1$.

[Sevrin, Troost, van Proeyen '88; Gukov, Martinec, Moore, Strominger '04]

- Generators of left-moving SCA: L_n, $G_r^{\mu\nu}$, A_n^{+i}, A_n^{-i}, U_n, $Q^{\mu\nu}$.

- $c = \frac{6k^+k^-}{k^++k^-}$, $\gamma = \frac{k^-}{k^++k^-}$.

- Global wedge algebra: $\mathfrak{d}(2,1|\alpha)$, $\alpha = \frac{\gamma}{1-\gamma}$, spanned by L_0, $L_{\pm1}$, $G^{\mu\nu}_{-\frac{1}{2}}$, $A_0^{\pm i}$.

- BPS bound of A_γ SCA:

$$h_{\rm BPS}(j^+, j^-, u) = \frac{1}{k^++k^-}\left(k^+j^- + k^-j^+ + (j^+ - j^-)^2 + u^2\right) ,$$
$$k^\pm \in \{0, \tfrac{1}{2}, 1, \cdots , \tfrac{k^\pm-1}{2}\} .$$

[Gunaydin, Petersen, Taormina, van Proeyen '89, Petersen, Taormina '90]

$\mathcal{S}_\kappa$:

▶ Each BPS multiplet of the large $\mathcal{N}=4$ SCA with $j^+ = j^-$ has two states which are chiral primaries of the $\mathcal{N}=2$ subalgebra.

▶ Hodge diamond of $\mathcal{S}_\kappa$:
$$\begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array},$$

4 cp-cp: $|0\rangle_{\rm NS}$, $\psi^{++}_{-1/2}|0\rangle_{\rm NS}$, $\tilde\psi^{++}_{-1/2}|0\rangle_{\rm NS}$, $\psi^{++}_{-1/2}\tilde\psi^{++}_{-1/2}|0\rangle_{\rm NS}$.

$\mathcal{S}_0$:

▶ Theory of 1 free boson and 4 free fermions, $k^+ = k^- = 1$, $c = 3$, $j^+ = j^- = u = 0$.

$\mathrm{Sym}^{Q_1}(\mathcal{S}_\kappa)$:

- Low-lying single-particle BPS spectrum:

[Eberhardt, Gaberdiel, Li '17]

$$\bigoplus_{j\in\frac{1}{2}\mathbb{Z}_{\geq 0}}^{\frac{c}{12}} [j,j,u=0]_S \otimes \overline{[j,j,u=0]_S} \ .$$

- Hodge diamonds:

$$\begin{array}{ccc} \vdots & \vdots & \vdots \\ 1 & & 1 \\ & 2 & \\ 1 & & 1 \\ & 2 & \\ 1 & & 1 \\ & 1 & \end{array} \ .$$

$\text{AdS}_3/\text{CFT}_2$ with large $\mathcal{N}=(4,4)$ susy

▶ Stringy duality: string theory on $\text{AdS}_3\times\text{S}^3\times\text{S}^3\times\text{S}^1$ is dual to symmetric orbifold $\text{Sym}^{Q_1 Q_5^+}(\mathcal{S}_\kappa)$.

[Eberhardt, Gaberdiel, Li '17]

▶ World-sheet theory is described by a WZW model based on $\mathfrak{sl}(2,\mathbb{R})^{(1)}_{k=\frac{k^+k^-}{k^++k^-}} \oplus \mathfrak{su}(2)^{(1)}_{k^+} \oplus \mathfrak{su}(2)^{(1)}_{k^-} \oplus \mathfrak{u}(1)^{(1)}$ for pure NS-NS background. [Elitzur, Feinerman, Giveon, Tsabar '99]

▶ BPS spectrum matches the CFT spectrum:

[Eberhardt, Gaberdiel, Li; Baggio, Ohlsson Sax, Sfondrini, Stefanski, Torielli '17]

$$\bigoplus_{j\in\frac{1}{2}\mathbb{Z}_{\geq 0}}^{\frac{c}{12}} [j,j,u=0]_S \otimes \overline{[j,j,u=0]}_S \ .$$

▶ Supergravity limit corresponds to $k\to\infty$.

▶ BPS spectrum is also derived using integrability techniques. [Ohlsson Sax, Sfondrini, Stefanski, Torielli '17]

$\mathrm{AdS}_3/\mathrm{CFT}_2$ with large $\mathcal{N} = (4,4)$ susy

▶ Stringy duality: string theory on $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{S}^3\times\mathrm{S}^1$ is dual to symmetric orbifold $\mathrm{Sym}^{Q_1 Q_5^+}(\mathcal{S}_\kappa)$.

[Eberhardt, Gaberdiel, Li '17]

▶ BPS bound of the supergravity symmetry algebra $\mathfrak{d}(2,1|\alpha)$:

$$h_{\mathrm{BPS}}(j^+, j^-) = \frac{k^+ j^- + k^- j^+}{k^+ + k^-} \ .$$

Only agrees with A_γ BPS bound for $j^+ = j^-$ and $u = 0$.

▶ BPS spectrum of supergravity is computed through compactifying 9d sugra on $\mathrm{S}^3\times\mathrm{S}^3$. The spectrum perfectly matches that of string theory and CFT: [Eberhardt, Gaberdiel, Gopakumar, Li '17]

$$\bigoplus_{j\in\frac{1}{2}\mathbb{Z}_{\geq 0}}^{\infty} [j, j, u = 0]_S \otimes \overline{[j, j, u = 0]}_S \ .$$

- Stringy AdS_3/CFT_2 dualities with $\mathcal{N}$ =(4,4) and (4,4)
- $\mathcal{N}$ =(3,3) duality: world-sheet theory
- $\mathcal{N}$ =(3,3) duality: dual CFT
- Elliptic genus
- Conclusions

World-sheet theory: action of $\mathbb{Z}_2$

▶ String theory on $\mathrm{AdS}_3 \times (\mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1)/\mathbb{Z}_2$: the $\mathbb{Z}_2$ acts by exchanging the two spheres and changing the signature of fields on S^1 .
[Yamaguchi, Ishimoto, Sugiyama, '99]

▶ This amounts to exchanging the two affine $\mathfrak{su}(2)^{(1)}_{k^\pm}$ SCAs: imposes the constraint $Q_5^+ = Q_5^-$, $\kappa = 0$.

▶ The diagonal part of the $\mathfrak{su}(2)^{(1)}_{k^+} \oplus \mathfrak{su}(2)^{(1)}_{k^-}$ algebra of the large $\mathcal{N} = 4$ SCA survives the $\mathbb{Z}_2$ action: $\mathfrak{su}(2)^{(1)}_{k^d}$, $k^d = k^+ + k^- = 2k^+$.

▶ Supercharges originally transformed in the $(\mathbf{2}, \mathbf{2})$ of $\mathfrak{su}(2)_+ \oplus \mathfrak{su}(2)_-$. Under the $\mathbb{Z}_2$, they transform in $\mathbf{3} \oplus \mathbf{1}$ of diagonal $\mathfrak{su}(2)$. $\mathbb{Z}_2$ actions which project out the singlet/triplet yield spacetime $\mathcal{N} = 3/\mathcal{N} = 1$, respectively.

World-sheet theory: $\mathbb{Z}_2$ untwisted sector

▶ BPS bound of $\mathcal{N} = 3$ SCA: $h_{\rm BPS} = \frac{\ell}{2}$, ℓ is $\mathfrak{su}(2)_d$ spin, $\ell \leq \frac{k_d}{2}$.
[Miki, '90]

▶ Global algebra of $\mathcal{N} = 3$ SCA is $\mathfrak{osp}(3|2)$ with bound $h_{\rm BPS} = \frac{\ell}{2}$.

▶ Taking into account the NS and R sectors, and orbifold unprojected and projected contributions, BPS spectrum in supergravity limit, $k_d \to \infty$, is organised into representations of the $\mathcal{N} = 3$ SCA:

$$\bigoplus_{\ell \in \mathbb{Z}_{\geq 0}} [\ell]_S \otimes \overline{[\ell]}_S \ \oplus \ [\ell+1]_S \otimes \overline{[\ell+1]}_S \ ,$$

$$\begin{array}{ccc} \vdots & \vdots & \vdots \\ 0 & & 0 \\ & 2 & \\ 0 & & 0 \\ & 2 & \\ 0 & & 0 \\ & 1 & \end{array} \quad .$$

World-sheet theory: $\mathbb{Z}_2$ twist 2 sector

- NS sector does not contribute to BPS states; R sector does:

$$\bigoplus_{\ell \in \frac{1}{2}\mathbb{Z}_{>0}} 2\ [\ell]_{\mathrm{S}} \otimes [\ell]_{\mathrm{S}}\ ,$$

$$\begin{matrix} \vdots \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 0 \end{matrix} \quad .$$

- Multiplicity: $\mathbb{Z}_2$ orbifold action on S^1 has two fixed points.

World-sheet theory: full BPS spectrum in $k_d \to \infty$

$$\bigoplus_{\ell \in \mathbb{Z}_{\geq 0}} \left([\ell]_{\rm S} \otimes [\ell]_{\rm S}\right) \oplus 2\left([\ell + \tfrac{1}{2}]_{\rm S} \otimes [\ell + \tfrac{1}{2}]_{\rm S}\right) \oplus \left([\ell + 1]_{\rm S} \otimes [\ell + 1]_{\rm S}\right) \ \oplus$$

$$\bigoplus_{\ell \in \mathbb{Z}_{\geq 0} + \frac{1}{2}} 2\Big([\ell + \tfrac{1}{2}]_{\rm S} \otimes [\ell + \tfrac{1}{2}]_{\rm S}\Big) \ ,$$

$$\begin{array}{ccc} & \vdots & \\ 0 & & 0 \\ & 2 & \\ 0 & & 0 \\ & 2 & \\ 0 & & 0 \\ & 1 & \end{array} \quad \oplus \quad \begin{array}{c} \vdots \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 0 \end{array} \quad = \quad \begin{array}{ccc} & \vdots & \\ 0 & 2 & 0 \\ & 4 & \\ 0 & 2 & 0 \\ & 4 & \\ 0 & 2 & 0 \\ & 1 & \end{array} \ .$$

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Dual CFT: action of $\mathbb{Z}_2$

- $\mathcal{S}_0$ is the theory of one free boson and four free fermions.
- Action of $\mathbb{Z}_2$ is realised in two ways:

$$(i) \quad \partial X \mapsto -\partial X \ , \quad \psi^{++} \mapsto -\psi^{++} \ , \quad \psi^{--} \mapsto -\psi^{--} \ ,$$
$$\psi^{+-} \mapsto \psi^{-+} \ , \qquad \psi^{-+} \mapsto \psi^{+-} \ ,$$
$$G^{++} \mapsto G^{++}, \quad G^{--} \mapsto G^{--}, \quad G^{+-} \mapsto G^{-+}, \quad G^{-+} \mapsto G^{+-} \ .$$

$$(ii) \quad \partial X \mapsto -\partial X \ , \quad \psi^{++} \mapsto \psi^{++} \ , \quad \psi^{--} \mapsto \psi^{--} \ ,$$
$$\psi^{+-} \mapsto -\psi^{-+} \ , \qquad \psi^{-+} \mapsto -\psi^{+-} \ ,$$
$$G^{++} \mapsto -G^{++}, \quad G^{--} \mapsto -G^{--}, \quad G^{+-} \mapsto -G^{-+}, \quad G^{-+} \mapsto -G^{+-} \ .$$

- The large $\mathcal{N} = 4$ SCA reduces to:

$$(i) \ \mathcal{N} = 3 \ , \quad G^{++}, \quad G^{--}, \quad \frac{1}{2}\left(G^{+-} + G^{-+}\right) \ ,$$
$$(ii) \ \mathcal{N} = 1 \ , \qquad \frac{1}{2}\left(G^{+-} - G^{-+}\right) \ .$$

BPS spectrum of $\mathcal{S}_0/\mathbb{Z}_2$

- We recall that the BPS spectrum of $\mathcal{S}_0$ is

$$\begin{matrix} & 1 & \\ 1 & & 1 \\ & 1 & \end{matrix}\ , \quad \text{4 cp-cp: } |0\rangle_{\rm NS},\, \psi^{++}_{-1/2}|0\rangle_{\rm NS},\, \tilde{\psi}^{++}_{-1/2}|0\rangle_{\rm NS},\, \psi^{++}_{-1/2}\tilde{\psi}^{++}_{-1/2}|0\rangle_{\rm NS}\ .$$

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$$\begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array}, \quad \text{4 cp-cp: } |0\rangle_{\rm NS},\, \psi^{++}_{-1/2}|0\rangle_{\rm NS},\, \tilde{\psi}^{++}_{-1/2}|0\rangle_{\rm NS},\, \psi^{++}_{-1/2}\tilde{\psi}^{++}_{-1/2}|0\rangle_{\rm NS}\,.$$

- For $\mathcal{N} = (3,3)$ susy, $\psi^{++}_{-1/2}|0\rangle_{\rm NS}$ and $\tilde{\psi}^{++}_{-1/2}|0\rangle_{\rm NS}$ do not survive $\mathbb{Z}_2$ action. In $\mathbb{Z}_2$ twist-2 sector, S^1 has two fixed points. The BPS spectrum is then:

$$\begin{array}{ccc} & 1 & \\ 0 & 2 & 0 \\ & 1 & \end{array}\,.$$

BPS spectrum of $\mathcal{S}_0/\mathbb{Z}_2$

- Similarly, for theories with $(\mathcal{N}, \widetilde{\mathcal{N}}) = (3,1)$, $(1,3)$ and $(1,1)$ supersymmetry we can derive the reduced spectrum of the BPS states of $\mathcal{S}_0$ which survive the $\mathbb{Z}_2$ projection. In the untwisted sector we have:

	$\mathcal{N} = 3$	$\mathcal{N} = 1$
$\mathcal{N} = 3$	1 0 0 1	0 0 1 1
$\mathcal{N} = 1$	0 1 0 1	1 1 1 1

Dual CFT: BPS spectrum of $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$, S_N odd twisted sector

$$
\begin{array}{ccc}
 & \vdots & \\
0 & & 0 \\
 & 2 & \\
0 & & 0 \\
 & 2 & \\
0 & & 0 \\
 & 1 &
\end{array}
\qquad \oplus \qquad
\begin{array}{c}
\vdots \\
2 \\
0 \\
2 \\
0 \\
2 \\
0
\end{array}
$$

$\mathbb{Z}_2$ untwisted sector	$\mathbb{Z}_2$ twist-2 sector
$h = \bar{h} = \frac{n-1}{4}$	$h = \bar{h} = \frac{n}{4}$

Dual CFT: BPS spectrum of $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$, S_N even twisted sector

	$\vdots$
	0
	2
	0
	2
	0
	0
$\mathbb{Z}_2$ untwisted sector	$\mathbb{Z}_2$ twist-2 sector
no contribution	$h = \bar{h} = \frac{n}{4}$

Dual CFT: full BPS spectrum of $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$

$$
\begin{array}{ccc} & \vdots & \\ 0 & & 0 \\ & 2 & \\ 0 & & 0 \\ & 2 & \\ 0 & & 0 \\ & 1 & \end{array}
\oplus
\begin{array}{c} \vdots \\ 2 \\ 0 \\ 2 \\ 0 \\ 2 \\ 0 \end{array}
\oplus
\begin{array}{c} \vdots \\ 0 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \end{array}
=
\begin{array}{ccc} & \vdots & \\ 0 & 2 & 0 \\ & 4 & \\ 0 & 2 & 0 \\ & 4 & \\ 0 & 2 & 0 \\ & 1 & \end{array} .
$$

- BPS spectrum of $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$ matches the that of the world-sheet theory.

- Stringy AdS_3/CFT_2 dualities with $\mathcal{N}$ =(4,4) and (4,4)
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Dual CFT: modified elliptic genus

▶ Elliptic genus is defined as the trace over the RR sector of the Hilbert space of the theory [Witten, '93]

$$\mathcal{Z}_{\mathrm{R}}(z,\tau;0,\bar{\tau}) = \mathrm{tr}_{\mathrm{RR}}\big((-1)^F q^{L_0-\frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0-\frac{c}{24}}\big) \ ,$$

J is the $U(1)$ R-current, $q = e^{2\pi i\tau}$, $y = e^{2\pi i z}$.

▶ Elliptic genus of any CFT with $\mathcal{N}=3$ symmetry vanishes because $\mathcal{N}=3$ SCA contains a free fermion which is a singlet of the R-symmetry $\mathfrak{su}(2)_k$. [Goddard and A. Schwimmer, '88]

▶ We consider the index $\mathcal{Z}_{\mathrm{NS}}$, which is non-vanishing, and define a quantity composed of chiral primaries in the right-moving sector and arbitrary excited states in the left-moving sector:

$$\mathcal{Z}_{\mathrm{NS}}(z,\tau) \equiv \tilde{Z}_{\mathrm{NS}}(z,\tau;\bar{z},\bar{\tau})\Big|_{\bar{h}=\frac{\bar{\ell}}{2}} \ .$$

Dual CFT: modified elliptic genus

- Odd twisted sector of $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$:

$\mathbb{Z}_2$ untwisted sector:

$$\mathcal{Z}^{\mathrm{U}_{\mathbb{Z}_2}}_{\mathrm{NS}}(z,\tau) = \frac{2}{1-yq^{\frac{1}{2}}} + \frac{2}{1-y^{-1}q^{\frac{1}{2}}} - 3 \ .$$

$\mathbb{Z}_2$ twist-2 sector:

$$\mathcal{Z}^{\mathrm{T}_{\mathbb{Z}_2}}_{\mathrm{NS}}(z,\tau) = \frac{2y^{\frac{1}{2}}q^{\frac{1}{4}}}{1-yq^{\frac{1}{2}}} + \frac{2y^{-\frac{1}{2}}q^{\frac{1}{4}}}{1-y^{-1}q^{\frac{1}{2}}} \ .$$

- Even twisted sector of $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$: modified genus vanishes.

- Single-particle contribution to the modified elliptic genus of the CFT matches the that of string theory in $k_d \to \infty$ limit.

- Stringy AdS_3/CFT_2 dualities with $\mathcal{N}$ =(4,4) and (4,4)
- $\mathcal{N}$ =(3,3) duality: world-sheet theory
- $\mathcal{N}$ =(3,3) duality: dual CFT
- Elliptic genus
- Conclusions

$\mathrm{AdS}_3/\mathrm{CFT}_2$ duality with $\mathcal{N} = (3, 3)$ susy

- String theory on $\mathrm{AdS}_3 \times (\mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1)/\mathbb{Z}_2$ is dual to symmetric product orbifold $\mathrm{Sym}^N(\mathcal{S}_0/\mathbb{Z}_2)$.

- BPS spectrum of the dual CFT matches the BPS spectrum of the world-sheet theory.

- Modified elliptic genera of the CFT and string theory match.