Newton-Cartan Gravity in Action

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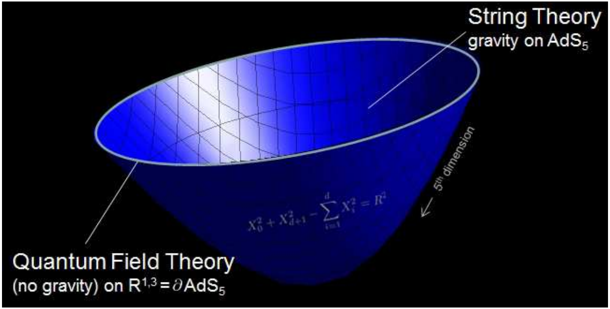
rijksuniversiteit groningen

Motivation

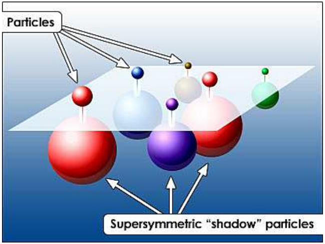
Holography



Gravity is not only used to describe the gravitational force!

Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

Supersymmetry



supersymmetry allows to apply powerful localization techniques to exactly calculate partition functions of (non-relativistic) supersymmetric field theories

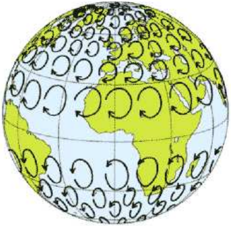
Pestun (2007); Festuccia, Seiberg (2011), Pestun, Zabzine (2016)

Condensed Matter

Effective Field Theory (EFT) coupled to NC background fields

serve as response functions and lead to restrictions on EFT

compare to



Coriolis force

Luttinger (1964), Greiter, Wilczek, Witten (1989), Son (2005, 2012), Can, Laskin, Wiegmann (2014)

Jensen (2014), Gromov, Abanov (2015), Gromov, Bradlyn (2017)

Outline

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Newton-Cartan Geometry

Newton-Cartan Gravity

Non-Relativistic Matter

Outline

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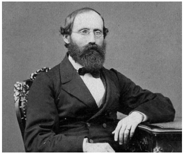
NC Geometry in a Nutshell

- Inertial frames: Galilean symmetries

- Constant acceleration: Newtonian gravity/Newton potential $\Phi(x)$

- no frame-independent formulation

  (needs geometry!)



Riemann (1867)

Galilei Symmetries

- time translations : $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!

- space translations : $\delta x^i = \xi^i$ $\quad i = 1, 2, 3$

- spatial rotations : $\delta x^i = \lambda^i{}_j \, x^j$

- Galilean boosts : $\delta x^i = \lambda^i t$

'Gauging' Galilei

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\tau_{\mu\nu} = \partial_{[\mu}\tau_{\nu]}$
space translations	P^a	${e_\mu}^a$	${R_{\mu\nu}}^a(P)$
Galilean boosts	G^a	${\omega_\mu}^a$	${R_{\mu\nu}}^a(G)$
spatial rotations	J^{ab}	${\omega_\mu}^{ab}$	${R_{\mu\nu}}^{ab}(J)$

Imposing Constraints

${R_{\mu\nu}}^a(P) = 0$: does only solve for part of ${\omega_\mu}^a, {\omega_\mu}^{ab}$

Absolute Time

$$\tau_{\mu\nu} \equiv \partial_{[\mu}\tau_{\nu]} = 0 \qquad \rightarrow \qquad \tau_\mu = \partial_\mu \tau$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\tau \quad \text{is path-independent}$$

From Galilei to Bargmann

the zero commutator

$$[G_a, P_b] = 0$$

implies that a massive particle with non-zero spatial momentum P_b cannot by any boost transformation G_a be brought to a rest frame $\Rightarrow$

$$[G_a, P_b] = \delta_{ab} M \quad \rightarrow \quad \text{extra gauge field } m_\mu$$

The NC Transformation Rules

The independent NC fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\delta\tau_\mu = 0\,,$$
$$\delta e_\mu{}^a = \lambda^a{}_b\, e_\mu{}^b + \lambda^a\tau_\mu\,,$$
$$\delta m_\mu = \partial_\mu\sigma + \lambda_a\, e_\mu{}^a$$

The spin-connection fields $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$ are functions of τ_μ , $e_\mu{}^a$ and m_μ

What about the dynamics ?

Outline

From General Relativity to NC gravity

Poincare $\otimes$ U(1)	$\overset{\text{`gauging'}}{\Longrightarrow}$	GR plus $\partial_\mu M_\nu - \partial_\nu M_\mu = 0$
contraction $\Downarrow$		$\Downarrow$ the NC limit
Bargmann	$\overset{\text{`gauging'}}{\Longrightarrow}$	Newton-Cartan gravity

Contraction Poincare

$$\left[P_A, M_{BC}\right] = 2\,\eta_{A[B}\,P_{C]}\,, \qquad \left[M_{AB}, M_{CD}\right] = 4\,\eta_{[A[C}\,M_{D]B]}$$

$$P_0 = \frac{1}{2\omega} H\,, \qquad P_a = P_a\,, \qquad A = (0, a)$$

$$M_{ab} = J_{ab}\,, \qquad M_{a0} = \omega\, G_a$$

Taking the limit $\omega \rightarrow \infty$ gives the Galilei algebra:

$$\left[P_a, M_{b0}\right] = \delta_{ab} P_0 \quad \Rightarrow \quad \left[P_a, G_b\right] = 0$$

Contraction Poincare ⊗ U(1)

$$[P_A, M_{BC}] = 2\,\eta_{A[B}\,P_{C]}\,, \qquad [M_{AB}, M_{CD}] = 4\,\eta_{[A[C}\,M_{D]B]} \qquad \text{plus} \qquad \mathcal{Z}$$

$$P_0 = \frac{1}{2\omega}H + \omega\,Z\,, \qquad \mathcal{Z} = \frac{1}{2\omega}H - \omega\,Z\,, \qquad A = (0, a)$$

$$P_a = P_a\,, \qquad M_{ab} = J_{ab}\,, \qquad M_{a0} = \omega\,G_a$$

Taking the limit $\omega \to \infty$ gives the Bargmann algebra including Z:

$$[P_a, M_{b0}] = \delta_{ab}P_0 \qquad \Rightarrow \qquad [P_a, G_b] = \delta_{ab}\,Z$$

The NC Limit I

Dautcourt (1964); Rosseel, Zojer + E.B. (2015)

STEP I: express relativistic fields $\{E_\mu{}^A, M_\mu\}$ in terms of non-relativistic fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$

$$E_\mu{}^0 = \omega\,\tau_\mu + \frac{1}{\omega}\,m_\mu\,, \qquad M_\mu = \omega\,\tau_\mu\,, \qquad E_\mu{}^a = e_\mu{}^a$$

$$\text{constraint :} \qquad \partial_{[\mu}\tau_{\nu]} = 0$$

N.B. PN approximation uses $E_\mu{}^0 = \omega\,\tau_\mu + \frac{1}{\omega}\,n_\mu$ and <u>no</u> M_μ

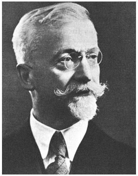
The NC Limit II

STEP II: substitute the expressions into the transformation rules and the e.o.m. and take the limit $\omega \to \infty \quad \Rightarrow$

- the NC transformation rules are obtained and agree with the gauging procedure
- the NC equations of motion are obtained

Note: the standard textbook limit gives Newton gravity

The NC Equations of Motion



Élie Cartan 1923

The NC equations of motion are given by

$$\mathcal{R}_{0c}{}^{c}(G) \;=\; \mathcal{R}_{0c}{}^{ca}(J) \;=\; \mathcal{R}^{(a}{}_{c}{}^{cb)}(J) \;=\; 0$$

1 **a** **(ab)**

- there is no known action that gives rise to these equations of motion
- after gauge-fixing $\tau_\mu = \delta_{\mu,0}$, $e_\mu{}^a = \delta_\mu{}^a$ and $m_0 = \Phi$ the 4D NC e.o.m. reduce to $\triangle\Phi = 0$

what about non-relativistic matter?

Outline

Motivation

special feature FQH Effect: existence of a gapped collective non-rel. parity non-invariant helicity-2 excitation, known as the GMP mode

Girvin, MacDonald and Platzman (1985)

recent proposal for a non-relativistic spatially covariant bimetric EFT describing non-linear dynamics of this massive spin-2 GMP mode

Haldane (2011), Gromov, Geraedts, Bradlyn (2017), Gromov, Son (2017), Nguyen, Gromov, Son (2017)

in a linearized approximation around a flat background this gives rise to a single spin-2 Planar Schrödinger Equation

$$i\hbar\dot{\Psi} + \frac{\hbar^2}{2m}\nabla^2\Psi = 0$$

Key Question

Rosseel, Townsend + E.B. (2018)

has this single helicity 2

Planar Schrödinger Equation

a (massive) gravity origin?

The 'force limit' of spin 0

$$\frac{1}{c^2}\ddot{\Phi} - \nabla^2\Phi + \left(\frac{mc}{\hbar}\right)^2\Phi = 0$$

Take the non-relativistic limit $c \to \infty$ keeping $\lambda = \hbar/mc$ fixed $\rightarrow$

$$\nabla^2\Phi = \frac{1}{\lambda^2}\Phi$$

no massive spin 0 particle !

N.B. The limit can also be taken in an arbitrary background

The 'particle limit' of complex spin 0

$$\frac{1}{c^2}\ddot{\Phi} - \nabla^2\Phi + \left(\frac{mc}{\hbar}\right)^2\Phi = 0$$

To avoid infinities we redefine

$$\Phi = e^{-\frac{i}{\hbar}(mc^2)t}\Psi$$

so that the Klein-Gordon equation becomes

$$-\frac{1}{2mc^2}\left(i\hbar\frac{d}{dt}\right)^2\Psi - i\hbar\dot{\Psi} - \frac{\hbar^2}{2m}\nabla^2\Psi = 0$$

and the $c \to \infty$ limit yields the Schrödinger equation

$$i\hbar\dot{\Psi} + \frac{\hbar^2}{2m}\nabla^2\Psi = 0$$

General Feature

one complex massive helicity mode $\Leftrightarrow$ one Schrödinger Equation

Alternative Particle Limit of 3D Real Proca

- make time-space decomposition $A_\mu = (A_0, \vec{A})$
- eliminate auxiliary field A_0
- rescale $\vec{A} \to \vec{B}$ and define $B = \frac{1}{\sqrt{2}}(B_1 + iB_2) \quad \Rightarrow$

$$\mathcal{L} = \frac{1}{c^2}\dot{B}^\star \dot{B} + B^\star \nabla^2 B - \left(\frac{mc}{\hbar}\right)^2 B^\star B$$

redefine $\quad B = e^{-\frac{i}{\hbar}(mc^2)t}\Psi[1] \; :$ breaks parity $\quad \Rightarrow$

$$i\hbar\dot{\Psi}[1] \; + \frac{\hbar^2}{2m}\nabla^2\Psi[1] = 0$$

single planar spin-1 Schroedinger equation

From Spin-1 to Spin-2

A_μ: $3 = 1+2$ under spatial SO(2):

$$A_0 \quad \text{and} \quad A_1 + iA_2$$

$f_{\mu\nu}$ with $\eta^{\mu\nu} f_{\mu\nu} = 0$: 5=1+2+2 under spatial SO(2):

$$f_{11} + f_{22}, \quad f_{01} + if_{02} \quad \text{and} \quad \frac{1}{2}\left(f_{11} - f_{22}\right) + if_{12} \quad \Rightarrow$$

single planar spin-2 Schroedinger equation

Same result can be obtained from a SS null-reduction of 4D GR

Towards Interactions: special features of 3D

J. Rosseel, P. Townsend + E.B., work in progress

- 'taking the square-root':

$$\Box - m^2 = O(m)O(-m) \quad \text{with} \quad [O(m)]_\mu{}^\rho = \epsilon_\mu{}^{\tau\rho}\partial_\tau + m\delta_\mu{}^\rho$$

- 'boosting up the derivatives':

$$\partial^\mu A_\mu = 0 \quad \rightarrow \quad A_\mu = \epsilon_\mu{}^{\nu\rho}\partial_\rho B_\sigma$$

- 'CS-like' formulation:

$$L = \tfrac{1}{2}g_{rs}a^r \cdot da^s + \tfrac{1}{6}f_{rst}a_r \cdot \left(a^s \times a^t\right) \qquad r = 1, \cdots, N$$

- take real limit or complex limit followed by self-duality truncation?

Non-relativistic 3D Chern-Simons Like Gravity

- The 3D Galilei and Bargmann algebras do not allow an invariant bilinear form

- Precisely in 3D there exists a so-called Extended Bargmann Algebra with two central extensions and an invariant bilinear from. The second central extension is related to spin

  Jackiw, Nair (2000)

- can one use two such algebras to construct a CS-like bi-metric gravity theory describing the non-linear dynamics of a massive spin 2 particle instead of a massive deformation of Poisson's equation?

Outline

This Talk

- It is not yet clear whether the non-relativistic limit of some 3D relativistic massive gravity model or the direct construction of a CS-like gravity theory based upon some non-relativistic algebra give the boost-covariant completion of the EFT proposal for the GMP mode in the FQE Effect

Gromov and Son (2017)

- If it does, it may lead to interesting connections between 3D gravity and condensed matter concerning
    - higher derivatives
    - higher spins

Golkar, Dung Xuan Nguyen, Roberts, Son (2016)

Take Home Message

Newton-Cartan Geometry leads to fruitful interactions between holography, effective field theory and supersymmetry. It even has connections with engineering!