Dark Quarkonium Formation in the Early Universe University of Toronto, Toronto, CAN

Gabriel Lee

Cornell University/Korea University 1802.xxxxx with M. Geller, S. Iwamoto, Y. Shadmi, O. Telem

Feb 15, 2018

Gabriel Lee (Cornell/Korea)

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Gravitational evidence

- galactic
- cluster
- cosmological

Catch Me If You Can





Catch Me If You Can



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WIMP Miracle



If DM was in thermal contact with the SM in the early universe, the observed DM relic density parameter is

$$0.11 \sim \Omega h^2 = \frac{3 \times 10^{-27} \text{ cm}^3/\text{s}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle}$$
$$\sim 2.5 \times 10^{-10} \text{ GeV}^{-2} \frac{m^2}{\alpha^2},$$

if *s*-wave scattering dominates annihilation cross section.

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if *s*-wave scattering dominates annihilation cross section.

For weak-scale masses and couplings $m\sim 10^3$ GeV, $\alpha\sim 0.6^2/(4\pi)\sim 0.03,$

$$2.5 \times 10^{-10} \frac{10^6}{10^{-3}} \sim 0.25$$
.

DM Mass Parameter Space



from B. Safdi

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DM Mass Parameter Space



- Hidden photons
- Ultra-light scalar fields
- Axions
- Q-balls

- Neutralinos
- Gravitinos
- Sterile neutrinos
- Dark hadrons

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DM Mass Parameter Space



Q-balls

Dark hadrons

No shortage of either bosonic or fermionic candidates.

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Tim Tait's DM Venn Diagram



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Beyond One-Particle Dark Sectors

Motivations:

- asymmetric DM (connecting DM to baryon asymmetry),
- excited or inelastic DM (novel methods for direct detection),
- different cosmology (more possibilities for obtaining present relic density).

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Confining dynamics give us nontrivial spectra!

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Confining dynamics give us nontrivial spectra!

- Generic in dark matter models with nonabelian hidden sectors.
- Below some confinement scale, "coloured" particles must combine to form singlets.
- The hadrons can now have qualitatively different interactions than the constituents (at the very least, nonperturbative).
- Generic in most BSM model building: strings, SUSY, strong dynamics, etc.

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"Colour"-Singlet States in Nonabelian Dark Matter

►	Glueballs (pure Yang-Mills)	Forestell, Morrissey, Sigurdson 1710.06447
	Quirks (heavy fundamentals)	Kribs, Roy, Terning, Zurek 0909.2034
►	Mesons and baryons (light fundamentals)	Kang, Luty, Nasri 0611322; Appelquist et al. 1503.04203; Harigaya et al.
	1606.00159	
Þ	R-hadrons (heavy adjoints)	e.g., gluinos in Arvanitaki et al. 0812.2075; Feng & Shadmi 1102.0282
Þ	Heavy adjoint bound states	De Luca et al. 1801.01135
Þ	Nuclear DM e.g., Hardy et al. 1504.05419; Wise & Zhang 1407.4121 + An 1604.01776; Gresham, Hou, Zurek 1707.02313	
	Sexaquarks	Farrar, 1708.08951
►	Atomic DM Kaplan et al	. 0909.0753; Cyr-Racine & Sigurdson 1209.5752; Boddy et al. 1609.03592

Parameter space in (coupling, mass) varies widely!

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Cosmology deviates from standard freeze-out scenario.

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Second-Stage Annihilation

Cosmology deviates from standard freeze-out scenario.

- Below confinement scale, bound states have finite size dictated by the light coloured states ("brown muck").
- Hadrons can undergo a second stage of annihilation, reducing the relic density (and therefore allowing heavier dark matter masses).
- Argument: no symmetry dictates that the cross section should be smaller than the naïve geometric cross section.

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Which bound state formation (BSF) processes allow for a geometric cross section?

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Griest & Kamionkowski, Phys.Rev.Lett. 64, 615 (1990)

Upper limit on mass of DM once in thermal equilibrium with SM from partial-wave unitarity:

$$(\sigma_J) < \frac{4\pi (2J+1)}{m_X^2 v_{\rm rel}^2}.$$

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$$\begin{split} \Omega_X h^2 &\sim 0.1 \gtrsim \frac{2.5 \times 10^{-10} \,\mathrm{GeV}^{-2}}{4\pi} m_X^2 v_{\mathrm{rel}}^2 \,, \\ \Rightarrow \left(\frac{m_X}{\mathrm{TeV}}\right)^2 \lesssim 10^4 \end{split}$$

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Can this bound be exceeded in the framework above?

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Outline

Toy Model







Comments on Cosmology and Phenomenology

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Matter Content and Symmetries

Field	$\mathrm{SU}(N)$	U(1)	Mass
X, \bar{X}	N, \bar{N}	1, -1	$m_X \gg \Lambda_D$
q,ar q	N, \bar{N}	0	$m_q \sim \Lambda_D$

- Add a flavour symmetry to make X stable.
- The nonabelian gauge group is confining at a scale Λ_D .
- Below confinement temperature, X, \overline{X} will form heavy-light mesons

$$H_X \equiv X\bar{q}, \quad \bar{H}_X \equiv \bar{X}q.$$

Also possible to have an adjoint X without q: "R-hadron"-like colour-singlet states Xg.

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Scales in Confined QCD

$$\begin{array}{c} m_{\pi}, f_{\pi}, m_{q} \\ f_{\pi}^{2} = \frac{V^{3}}{f_{\pi}^{2}}(m_{u} + m_{d}) \\ T_{c} \sim 165 \text{-} 195 \text{ MeV} \text{ (lattice)} & V \sim 230 \pm 30 \text{ MeV} \\ \hline \\ \frac{T_{c}}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N_{c}^{2}} \\ \sigma \sim 0.16 \text{-} 0.19 \text{ GeV}^{2} \underbrace{\longleftarrow}_{\sigma} \sim \frac{C_{F} \Lambda_{\text{QCD}}^{2}}{2\beta_{0}} \xrightarrow{\Lambda_{\text{QCD}}} \Lambda_{\text{QCD}} \sim 332 \pm 17 \text{ MeV} \\ \text{(Regge, pot'n model)} & \sigma \sim \frac{C_{F} \Lambda_{\text{QCD}}^{2}}{2\beta_{0}} \xrightarrow{\Lambda_{\text{QCD}}} \text{(4L \overline{\text{MS}} \text{ RG running})} \end{array}$$

Teper 0812.0085, Brambilla et al. 1010.5827, Simolo 0807.1501, RPP 2016

- ▶ In our toy model, Λ_D is determined by $\alpha_D(m_X), m_X$, both of which are free parameters.
- We will be concerned with the two ratios

$$\frac{T}{\Lambda_D} \lesssim \frac{T_c}{\Lambda_D} \lesssim 1 \,,$$
$$\frac{m_X}{\Lambda_D} \gtrsim \mathcal{O}(10^2) \,.$$

Spectrum of Bound States

We can calculate the spectrum of $X-\bar{X}$ bound states using QM, modelling the interaction using a Cornell potential in analogy to quarkonium.



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With light coloured states, the nonabelian force has finite range in the confined phase. To enforce this, we add a cutoff to the Cornell potential:

$$V(R) = \begin{cases} -\bar{\alpha}_D \left(\frac{1}{R} - \frac{1}{R_c}\right) + \Lambda_D^2(R - R_c) + V_0 & R < R_c \,, \\ V_0 & R > R_c \,. \end{cases}$$

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How to determine R_c ?

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Heavy-Light Mesons

Consider D and B mesons in QCD.

Q	m_Q	$m_{\rm meson}$
c	$1.3{ m GeV}$	$1865{ m MeV}$
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Heavy-Light Mesons

Consider D and B mesons in QCD.

Q	m_Q	$m_{\rm meson}$
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In both cases, we have

 $m_{\rm meson} - m_Q \sim 600 \,{\rm MeV}\,,$

or about twice the "constituent mass" of a light quark.

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In both cases, we have

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or about twice the "constituent mass" of a light quark. Writing this parametrically as

$$m_{\rm meson} - m_Q \sim \kappa_\Lambda \Lambda$$
,

with $\Lambda \sim \sqrt{\sigma} \sim 400 \,\mathrm{MeV} \Rightarrow \kappa_{\Lambda} \sim 1.5$.

Bound States and the Potential Cutoff

A natural way to define the cutoff: the threshold for open production of two H_X hadrons. If this occurs in the linear regime of the potential, then

$$\begin{split} E_b^{\max} &= \Lambda_D^2 r_c = 2 \kappa_\Lambda \Lambda_D \,, \\ R_c &= 2 \frac{\kappa_\Lambda}{\Lambda_D} \sim \frac{3}{\Lambda_D} \,. \end{split}$$

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Spectrum Snapshot



e.g., Quigg & Rosner, Phys.Rept. 56, 167 (1979); Hall & Saad 1411.2023

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Spectrum Snapshot



- hydrogenic: $E_{nl} \sim -\frac{\bar{\alpha}_D^2 \mu}{2n^2}$
- linear: $E_{nl} \propto \Lambda_D \left(\frac{\Lambda_D}{2\mu}\right)^{1/3} \left[\frac{3}{2}\pi \left(n + \frac{l}{2} \frac{1}{4}\right)\right]^{2/3}$

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Thermal History

 $---- m_X$ X, g, q $---- T_{f,X} \sim m_X/20$

g,q

 $---- \Lambda_D \sim T_c$

hadrons

 $---- m_q$

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Thermal History



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Outline

Toy Model



3 Radiative Emission

4 Comments on Cosmology and Phenomenology

Gabriel Lee (Cornell/Korea)

Dark Quarkonium Formation in the Early Universe

Feb 15, 2018 19 / 45

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Setup

 $H_X + \bar{H}_X \rightarrow (X\bar{X}) + (q\bar{q})$

Feb 15, 2018 20 / 45

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$H_X + \bar{H}_X \to (X\bar{X}) + (q\bar{q})$

- ▶ Rigorous calculation for $m_q \lesssim \Lambda_D$ requires pNRQCD machinery (quarkonium).
- Regime with $m_q \gtrsim \Lambda_D$ can be treated using scattering theory in nonrelativistic QM.
- Here, calculation is analogous to H-H
 rearrangement at low temperatures (e.g., for CPT tests)

$$\mathrm{H}(1s) + \bar{\mathrm{H}}(1s) \to \mathrm{Pn}(NLM) + \mathrm{Ps}(nlm) \,.$$

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From QM, the cross section in COM frame is

$$\frac{d\sigma}{d\Omega} = (2\pi)^2 \frac{k_f}{k_i} m_X m_q |\mathcal{M}|^2 \,,$$

with matrix element

$$\mathcal{M} = 2\pi \left\langle \Psi_f \left(\mathbf{R}, \mathbf{r}_q, \mathbf{r}_{\bar{q}} \right) \right| \mathcal{H}_{\mathrm{tr}} \left| \Psi_i \left(\mathbf{R}, \mathbf{r}_q, \mathbf{r}_{\bar{q}} \right) \right\rangle \,.$$

Semiclassical Arguments

Kang, Luty, Nasri hep-ph/0611322

For $m_q > \Lambda_D$, distance b/w X and \bar{q} in H_X and the average force between them are

$$a_q \sim \frac{1}{\alpha_D m_q}, \ F \sim \frac{\bar{\alpha}_D}{a_q^2} \sim \alpha_D^3 m_q^2.$$

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- The velocity is set by the temperature, which is assumed to be of order the binding energy $v \sim \sqrt{T/m_X} \sim \bar{\alpha}_D \sqrt{m_q/m_X}$.
- ▶ When a free X with initial velocity v comes with a distance a_q of a \bar{q} , its velocity changes by

$$\frac{\Delta v}{v} \sim \frac{1}{v} \frac{F}{m_X} \Delta t \sim \frac{1}{v} \frac{F}{m_X} \frac{a_q}{v} \sim \frac{\alpha_D^2 m_q}{m_X v^2} \sim 1 \,.$$

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The change in the position of X due to the force is

$$\frac{\Delta r}{a_q} \sim \frac{1}{a_q} \frac{F \Delta t^2}{m_X} \sim \frac{\alpha_D^2 m_q}{m_X v^2} \sim 1 \,.$$

Rearrangement Hamiltonian

The full interacting Hamiltonian includes terms that couple all heavy and light dof:

$$\begin{split} \mathcal{H}_{\text{tree}} &= -\frac{1}{m_X} \nabla_R^2 - \frac{1}{2m_q} \nabla_{r_q}^2 - \frac{1}{2m_q} \nabla_{r_{\bar{q}}}^2 \,, \\ \mathcal{H}_{\text{int}} &= V_{X\bar{X}} \left(R \right) + V_{q\bar{q}} \left(|\mathbf{r}_q - \mathbf{r}_{\bar{q}}| \right) + \mathcal{H}_{\text{tr}} \,, \\ \mathcal{H}_{\text{tr}} &= V_{q\bar{X}} \left(|\mathbf{r}_q + \mathbf{R}/2| \right) + V_{\bar{q}X} \left(|\mathbf{r}_{\bar{q}} - \mathbf{R}/2| \right) \\ &- V_{\bar{q}\bar{X}} \left(|\mathbf{r}_{\bar{q}} + \mathbf{R}/2| \right) - V_{qX} \left(|\mathbf{r}_q - \mathbf{R}/2| \right) \end{split}$$



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NB: we have taken the states to be in the Coulombic regime, so $V(r) \propto \pm \frac{\bar{\alpha}_D}{r}$.

$$H_X + \bar{H}_X \to (X\bar{X}) + (q\bar{q})$$

- Final: eigenstate of free Hamiltonian and $V_{X\bar{X}} + V_{q\bar{q}}$.
- Initial: eigenstate of free Hamiltonian plus $V_{q\bar{X}} + V_{\bar{q}X}$.

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Easy to see for final state: \u03c8 \u03c4 X \u03c8 and \u03c8 \u03c4 q \u03c9 are standard bound state solutions (for Coulomb potential), multiplied by a plane wave for the translational motion of the \u03c9 \u03c8.

Asymptotically at large R for initial states, residual interaction terms in the Hamiltonian produce a van der Waals-like force since the q, \bar{q} are bound in H_X, \bar{H}_X .

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- First, we solve for light dof with fixed positions for the heavy dof:

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Solve for the heavy dof with distorted potential from light dof:

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- V_{BO}(**R**) should interpolate between
 - at large R (initial state): twice the binding energy of H_X , $\bar{\alpha}_D^2 m_q$,
 - at small R (final state): the q-onium binding energy, $-\bar{\alpha}_D^2 m_q/4$.

Potentials



- Effective potential for $X \bar{X}$: $V_{in} = V_{X\bar{X}} + V_{BO}$.
- ▶ BO potentials taken from results of studies of $H-\bar{H}$ rearrangement. Strasbu

Strasburger, J. Phys. B 35 L435 (2002)

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• Asymptotics determine phase matching of $\psi_i^{X\bar{X}}$ at the screening distance:

$$\sum_{l} i^{l} \sqrt{(2l+1)} e^{i\delta_{l}} \left[\cos \delta_{l} j_{l}(kR) - \sin \delta_{l} n_{l}(kR) \right] Y_{l0}(\theta_{R}).$$

Wavefunctions



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$$\mathcal{M} = \int d^{3}\mathbf{R} \ \psi_{f}^{X\bar{X}*}\left(\mathbf{R}\right) \ \psi_{i}^{X\bar{X}}\left(\mathbf{R}\right) \ T\left(\mathbf{R}\right) ,$$
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- Actually, only the result near $R \approx a_q$ is needed.
- The dependence on the bound state spectrum is encapsulated in E_f, the kinetic energy of the final state.

Results for Partial Waves



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Results for Partial Waves



• Left: $\bar{\alpha}_D \approx 1/137$, cross section shuts off at classical l_{max} .

• Right: cross section dominated by formation of quarkonia states with size $\approx a_q$.

Image: A matrix

Results



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Results



- ▶ blue: total σ_{rearr} for $E_i = (0.6)\bar{\alpha}_D^2 m_q$ is indeed geometric.
- red: unitarity bound $4\pi/k_i^2$.
- greens: partial waves normalized by 2l + 1.

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- Take-away: many partial waves are important in rearrangement process.
- Each partial wave saturates the unitarity bound with

$$\sigma_l \sim \frac{1}{k_i^2} (2l+1) \,.$$

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Since the maximum angular momentum is roughly given by the classical angular momentum l_{max} = k_ia_q, we have

$$\sigma \sim a_q^2 \,,$$

which depends only on the scale of the light dof.
Outline

Toy Model





Comments on Cosmology and Phenomenology

Gabriel Lee (Cornell/Korea)

Dark Quarkonium Formation in the Early Universe

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BSF by Emission of Light Bosons

- There are extensive examples in the literature of bound-state formation via emission of light bosons, e.g., dark photons or (pseudo)-scalars.
- Together with Sommerfeld enhancement from long-range forces, can produce nontrivial effect in halos and structure formation.
 - Dissipation affects the size and morphology of halos.
 - Compton scattering off hidden photons delays kinetic decoupling, modifying small-scale structure.
 - Enhance DM annihilation in halos.

Feng, Kaplinghat, Tu, Yu 0905.3039; Petraki, Pearce, Kusenko 1403.1077; Agrawal, Cyr-Racine, Randall, Scholtz 1610.04611

- It is known that bound states that use a U(1) force have a small effect on present relic density.
- Is this still true if the bound state is governed by a confining force, but formed through perturbative emission of a vector?

Schematically



von Harling & Petraki 1407.7874; Petraki, Postma, Wiechers 1505.00109 Petraki, Postma, de Vries 1611.01394; Cirelli et al. 1612.07295

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Solving for the Wavefunctions

Dimensionless Schrödinger equation and effective potential:

$$-\chi_{nl}''(x) + V_{\text{eff}}(x)\chi_{nl}(x) = \epsilon_{nl}\chi_{nl}(x),$$
$$V_{\text{eff}}(x) = \frac{l(l+1)}{x^2} + \Theta(x-x_c)\left(-a_D\left(\frac{1}{x} - \frac{1}{x_c}\right) + x - x_c\right),$$

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with scalings $\epsilon = E/E_0, x = r/r_0$:

$$r_0 = \frac{1}{\Lambda_D} \left(\frac{\Lambda_D}{m_X}\right)^{1/3}, \quad E_0 = \Lambda_D \left(\frac{\Lambda_D}{m_X}\right)^{1/3}.$$

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- Effective potential has minimum at $x_{\min} = (2l(l+1))^{1/3} \, .$
- Then l_{max} for bound states is governed by condition that

$$x_{\min}(l_{\max}) \le x_c$$
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Wavefunctions





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Near-threshold states (both bound and scattering) have largest overlap.

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Employ standard methods in QFT for computing 2–2 cross section.

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$$v_{\rm rel} \,\sigma_{k\hat{\mathbf{z}}\to nl} = \frac{2e_X^2}{m_X^2} \left(\frac{\Lambda_D}{m_X}\right)^{2/3} \left(\epsilon_k - \epsilon_{nl}\right)^3 \left[(l+1) \left| I_{k,l+1\to nl} \right|^2 + l \left| I_{k,l-1\to nl} \right|^2 \right] \,,$$

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where $\epsilon_k = \frac{E_k}{E_0}$ and I is the radial wavefunction overlap integral

$$I_{k,l\pm 1\to nl} = \int dx \, x \, \chi_{nl}^*(x) \chi_{k,l\pm 1}(x) \, .$$

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Thermal averaging with Maxwell-Boltzmann distribution:

$$\begin{split} f_{\rm MB}(\boldsymbol{p}) &= \left(\frac{2\pi\beta}{m}\right)^{3/2} \exp\left(-\frac{\beta|\boldsymbol{p}|^2}{2m_X}\right) \,,\\ \langle v_{\rm rel}\sigma_{\rm BSF}\rangle &= \sqrt{\frac{16\beta^3}{\pi m_X^3}} \int\!\!dk\,k^2\,e^{-\beta k^2/m_X}\,(v_{\rm rel}\sigma_{\rm BSF})_k \,, \end{split}$$

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Radiative BSF: Overlap Integral



Shallowest bound states give the largest contribution to overlap.



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For large hierarchies, the cross section is not geometric.

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Agrees with semiclassical expectation from Larmor: $m_{X}^{-3/2}$ scaling.

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Summary

- For larger mass hierarchies, the cross section is not geometric.
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- Then $\langle \sigma v \rangle$ will decrease at lower temperatures, so this process will never dominate.

Summary

- For larger mass hierarchies, the cross section is not geometric.
- The cross section approaches an asymptotic value for lower temperatures.
- Then $\langle \sigma v \rangle$ will decrease at lower temperatures, so this process will never dominate.
- Our calculation was largely independent of spin of light state, so should hold for emission of scalars as well.
- Major difference with rearrangement: light dof are spectators in the perturbative process, only entered parametrically through the final-state wavefunctions.

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Outline





Comments on Cosmology and Phenomenology

 Second stage should be over fairly quickly, as depletion is faster nearer to confinement temperature (larger number density).

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- Standard freeze-out relic density parameter:

$$\Omega_X^{\rm ann} h^2 \sim \frac{10^{-9} \,{\rm GeV}^{-2}}{\langle \sigma_X^{\rm ann} v \rangle} \sim \left(\frac{m_X}{10 \,{\rm TeV}}\right)^2 \frac{1}{\alpha_D^2(m_X)} \,.$$

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• With large hierarchy, can dilute the relic density of H_X, \bar{H}_X .

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Other hadronic states in the spectrum with multiple X's, but some amount of light q's are likely to have second-state annihilations with geometric cross sections.

Multicomponent DM

- Other hadronic states in the spectrum with multiple X's, but some amount of light q's are likely to have second-state annihilations with geometric cross sections.
- Chain of processes for N > 2: $H_X H_X$ scattering can also double X baryons, which can continue to interact with other H_X 's in the plasma to form triple X baryons, etc.
- Since these interactions all involve the brown muck, we expect them to have geometric cross sections.

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- Since these interactions all involve the brown muck, we expect them to have geometric cross sections.
- ▶ The X's inside these baryons are also stable, so we can have a multitude of DM states.
- Future work: solve Boltzmann equation with these dynamics to find equilibrium yields.

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Phenomenology

Collider pheno: studies on dark jets at the LHC. 1503.00009 + Mishra-Sharma 1707.05326; Park, Zhang 1712.09279 Schwaller, Stolarski, Weiler 1502.05409; Cohen, Lisanti, Lou

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- Direct detection: couplings to SM are described by effective operators, other energy ranges for DD opened by multitude of final states.
- Indirect detection:
 - Heavy DM can access many hadronic states below confining temperature, could give a boost to signals from dwarf galaxies.
 - Self-interaction with large cross sections can yield interesting halo dynamics (and are constrained by such).

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- There are manifold possibilities for exploring further consequences in DM cosmology and phenomenology.
Conclusion

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- Below the confinement temperature, these bound states can have nonstandard cosmology by undergoing an additional annihilation phase.
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Thank you.

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Partial Waves



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Results from $H-\overline{H}$



Figure 1. The integrand in equation (6) calculated using the full R dependence of the leptonic matrix element (solid line), and using the approximation that the overlap between initial and final is constant leptonic wavefunctions (see equation (9)) (dashed line). We also show their integrals from 0 to R (multiplied by 10) as the dotted line and the dash–dotted line, respectively. The dash–dotted line shows that the constant overlap approximation yields a vanishing T matrix, while the dotted line shows that the small change in the integrand induced by including the R dependence of the overlap results an on-vanishing T matrix.



FIG. 6. Cross sections for the H-H system: elastic cross section obtained from the real part of the phase shift only (solid), elastic cross section including correction for the presence of inelastic scattering (long dashed), rearrangement cross section (dotted), and proton-antiproton annihilation in flight (dashed). At low energies, the elastic cross section is 823 without the correction for inelastic scattering, and 829 including this correction, while the low-energy behavior of σ^{rearr} is $0.09/\sqrt{\epsilon_i}$, and $\sigma_{ij}^{pp} \sim 0.14/\sqrt{\epsilon_i}$.

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Rearrangement at Low Temperatures: H-H



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More Wavefunctions in Radiative BSF



- Bound states with "radial number" n have n-1 nodes.
- Shallower bound states with smaller *l* tend to penetrate into the region beyond $x > x_c$.
- ► Scattering states with larger e and smaller l have larger penetration to x < xc.</p>