A VIEW OF THE BULK FROM THE WORLDLINE

[1712.00885]

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AdS/CFT: UV complete, nonperturbative quantum gravity

Problem: describe bulk physics in CFT language

Emergence of bulk spacetime with local physics

Information paradox: breakdown of bulk effective field theory

AdS_3/CFT_2: best use of Virasoro symmetry

Main idea: treat some bulk degree of freedom as a particle rather than a field

Integral over field configurations $\longrightarrow$ particle trajectories

$$\underbrace{\int}_{\text{Field configurations}} \mathcal{D}\phi \, e^{-\int d^{d+1}x \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \cdots\right)} \longrightarrow \underbrace{\int}_{\text{Worldlines}} \mathcal{D}x \, e^{-mL[x]}$$

Worldline action is the length of path.

Why consider worldline formulation?

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Eikonal approximation is worldline classical limit ($\hbar \sim m^{-1}$)

- Geodesics, corrected by m^{-1} worldline loops
- Natural local bulk probes: $\ell_{AdS}^{-1} \ll m \ll \ell_{pl}^{-1}$

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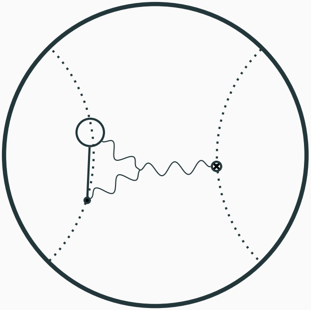
Simple to couple external fields like gravity

- Gives nonlocal operators in external field
- Vertex operator correlators in worldline QM

Four-point function of scalars, quantised in worldline formalism, coupled to 'light', second-quantised bulk fields (gravity)

$$\int \mathcal{D}g\mathcal{D}x \ \exp\left[-\frac{1}{G_N}S_{EH}[g] - mL[x,g]\right]$$



- Conformal blocks emerge naturally
- Multi-graviton exchange $\longrightarrow$ Virasoro blocks (in AdS_3)
- Systematic bulk computation
- Complementary to Witten diagrams

THE WORLDLINE QUANTUM MECHANICS

Want $G(x_0, x_1) = \underset{\text{Worldlines}}{\int} \mathcal{D}x \, e^{-mL[x,g]}$ over worldlines between x_0, x_1.

WORLDLINE PATH INTEGRAL

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- Integrate over functions $x(s)$ from an interval into spacetime.

$$G(x_0, x_1) \quad = \int_{x(0)=x_0, x(T)=x_1} \frac{\mathcal{D}x}{V_{\text{diffs}}} \; e^{-m \int_0^T ds \; \sqrt{g_{ab}(x(s))\dot{x}^a(s)\dot{x}^b(s)}}$$

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$$G(x_0, x_1) = \int_{x(0)=x_0, x(T)=x_1} \frac{\mathcal{D}x\mathcal{D}e}{V_{\text{diffs}}} \, e^{-\frac{m}{2}\int_0^T ds \, e(s)\left(e(s)^{-2} g_{ab}(x(s))\dot{x}^a(s)\dot{x}^b(s)+1\right)}$$

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Compare: worldsheet string theory

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Quantum mechanical sigma-model: $H = -\frac{1}{2m} \nabla^2$

$$G(x_0, x_1) = \frac{1}{2m} \int_0^\infty dT \, e^{-\frac{mT}{2}} \langle x_1 | e^{\frac{T}{2m} \nabla^2} | x_0 \rangle = \langle x_1 | \frac{1}{m^2 - \nabla^2} | x_0 \rangle$$

Allow worldlines to meet at vertices to get perturbative QFT

Integral between points in AdS_{d+1}, at proper distance L:

$$e^{-W_L(m,T)} = \int \mathcal{D}x \, e^{-\frac{m}{2}\int_0^T ds\, g_{ab}(x)\dot{x}^a\dot{x}^b}$$

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$$e^{-W_L(\lambda)} = \int \mathcal{D}x \; e^{-\frac{1}{2\lambda}\int_0^L ds\, g_{ab}(x)\dot{x}^a\dot{x}^b}, \quad \lambda = \frac{T}{mL}$$

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AdS_{d+1}: $$ds^2 = (1+q^2)dt^2 + dq^2 - \frac{(q.dq)^2}{1+q^2} \qquad q=(q^1,\ldots,q^d)$$

Write $t(s) = s + u(s)$, with $u(0) = u(L) = q(0) = q(L) = 0$:

$$S[q,u] = \frac{L}{2\lambda} + \frac{1}{2\lambda}\int_0^L ds \left(\dot{u}^2 + \dot{q}^2 + q^2 + 2\dot{u}q^2 + \dot{u}^2q^2 - \frac{(q\cdot\dot{q})^2}{1+q^2}\right)$$

Leading order: d harmonic oscillators q of frequency ℓ_{AdS}^{-1}

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Correlation functions: take points to boundary of AdS, $L \to \infty$

$W_L(\lambda) \sim L\mathcal{E}(\lambda)$, so modulus integral $\int d\lambda\, e^{-\frac{m^2\lambda}{2}L - W_L(\lambda)}$ localises.

$$m^2 = -2\mathcal{E}'(\lambda), \quad \text{physical parameter } \Delta = \mathcal{E}(\lambda) - \lambda\mathcal{E}'(\lambda)$$

In AdS_{d+1},
$\mathcal{E}(\lambda) = \frac{1}{2\lambda} + \frac{d}{2} + \frac{d^2}{8} \implies \Delta = \frac{1}{\lambda} + \frac{d}{2}, \quad m^2 = \frac{1}{\lambda^2} - \frac{d^2}{4} = \Delta(\Delta - d).$

Want to couple to gravity: perturb metric ($g \longrightarrow g_{\text{AdS}} + h$)

Deform by vertex operator $\mathcal{V}_h = \frac{1}{2\lambda} \int ds\, h_{ab}(x(s))\dot{x}^a(s)\dot{x}^b(s)$

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Order by order in h, get correlation functions of $\mathcal{V}_h$, like

$$\langle \mathcal{V}_h \rangle = \int ds \left(\frac{1}{2\lambda} h_{00} + \frac{1}{8}\partial_i\partial_i h_{00} - \frac{1}{4} h_{ii} + O(\lambda) \right)$$

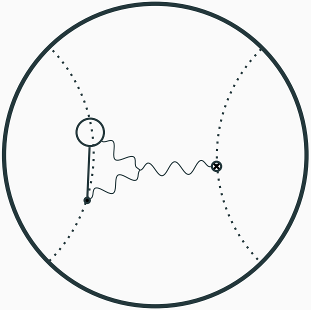
Higher order in h: $\langle \mathcal{V}_h \mathcal{V}_h \rangle$, $\langle \mathcal{V}_h \mathcal{V}_h \mathcal{V}_h \rangle$, ...

Higher order in λ: Taylor expand $h(q) = h + q\partial h + \frac{1}{2} q^2 \partial^2 h + \cdots$: higher derivatives of external field transverse to worldline

Systematic expansion of worldline amplitude in powers of h, λ

THE FOUR-POINT FUNCTION

Four-point function of scalars, quantised in worldline formalism, coupled to second-quantised 'light' bulk fields (gravity)



$$\langle\phi\phi\phi\phi\rangle = \int \mathcal{D}g\mathcal{D}x \ \exp\left[-\frac{1}{G_N}S_{EH}[g] - mL[x,g]\right]$$

Worldline quantum mechanics coupled to bulk theory

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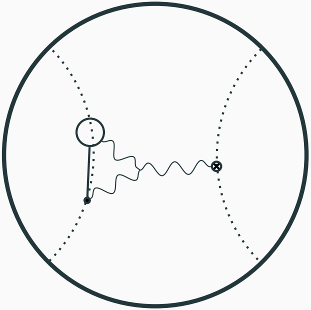

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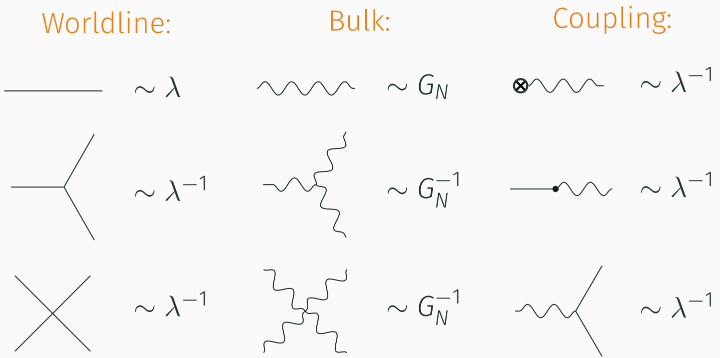
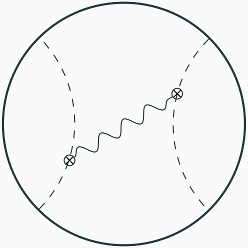
Worldline quantum mechanics coupled to bulk theory

Perturbation theory in $\lambda \sim m^{-1}$ (loops) and $\lambda^{-1}G_N$ (gravity)

$$\frac{\langle \phi\phi\phi\phi \rangle}{\langle \phi\phi \rangle \langle \phi\phi \rangle} = \exp\left[\sum(\text{connected diagrams, coupling worldlines})\right]$$



Order $m^2 G_N$:



$$= \frac{1}{2}\langle\langle\mathcal{V}_h^{(1)}\rangle_{\text{tree}}^{(1)}\langle\mathcal{V}_h^{(2)}\rangle_{\text{tree}}^{(2)}\rangle_{\text{gravity}}$$

$$= \frac{1}{8\lambda_1\lambda_2}\int ds_1\, ds_2 \langle h_{00}(s_1) h_{00}(s_2)\rangle$$

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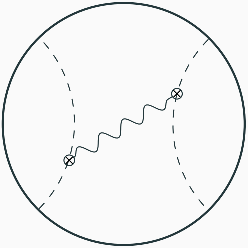

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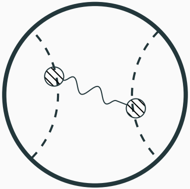
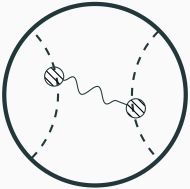
Geodesic Witten diagram *[Hijano,Kraus,Perlmutter,Snively]*

Conformal block $\mathcal{F}_T$



Expand $\langle \mathcal{V}_h \rangle$ in λ: $\langle \mathcal{V}_h \rangle = \int ds\ \left(\frac{1}{2\lambda} h_{00} + \frac{1}{8} \partial_i \partial_i h_{00} - \frac{1}{4} h_{ii} + O(\lambda) \right)$



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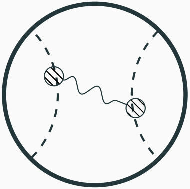

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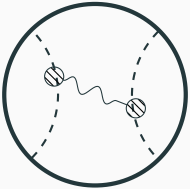
Correct normalisation: $\lambda = \frac{1}{\Delta - \frac{d}{2}} \longrightarrow C_{T\mathcal{O}\mathcal{O}} \propto \sqrt{G_N} \Delta$



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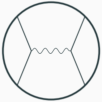
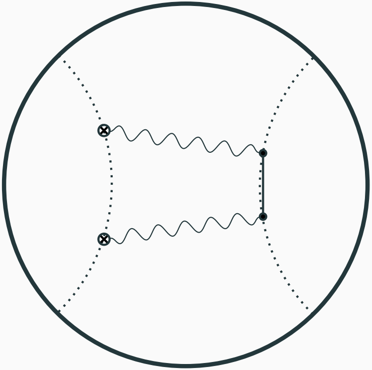
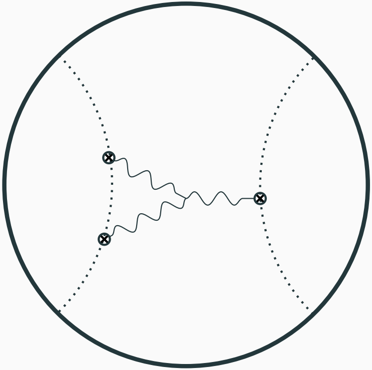
Not equal to Witten diagram  !

Order $\lambda^{-3} G_N^2$:





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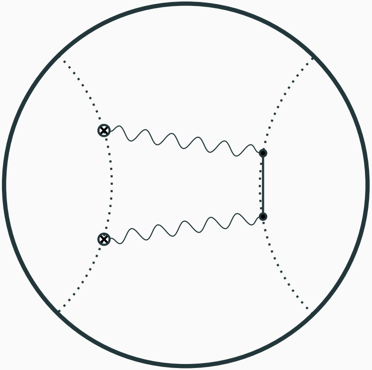
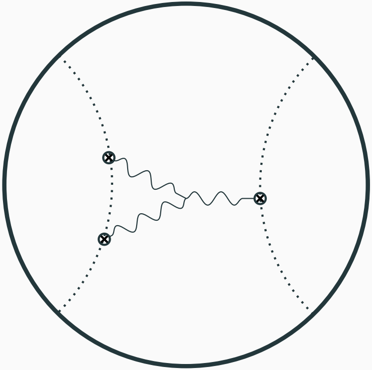




Order $\lambda^{-n-1}G_N^n$ (tree): solves classical physics of particles

CONFORMAL BLOCKS

Important properties of conformal field theories:

Radial quantisation: Use dilatation as 'Hamiltonian'

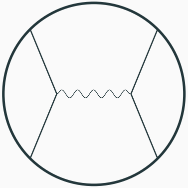
State-operator: Local operators $\longleftrightarrow$ states on S^{d-1}

OPE: $\mathcal{O}_1(0)\mathcal{O}_2(x) = \sum_i C_{12i}|x|^{\Delta_i-\Delta_1-\Delta_2}\mathcal{O}_i$

Leads to conformal block decomposition

$$\langle\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle = \sum_p C_{\mathcal{O}_1\mathcal{O}_2 p}C_{\mathcal{O}_3\mathcal{O}_4 p}\mathcal{F}_p$$

Decompose a Witten diagram as conformal blocks:


$$= \mathcal{F}^{\text{ST}}_{\text{exchange}} + \sum_{n,l} C_{n,l}\mathcal{F}^{\text{DT}}_{n,l}$$

The 'single trace' exchange (e.g. stress tensor) is intuitive

'Double trace' operators $[\mathcal{O}\mathcal{O}]_{n,l} \sim \,:\mathcal{O}\partial^l\Box^n\mathcal{O}:$

Spin l, dimension $\Delta_{n,l} = 2\Delta + 2n + l + \frac{1}{N}\gamma_{n,l} + \cdots$

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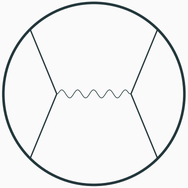

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Non-perturbative in $\frac{1}{m}$: second saddle point, same topology

In CFT_2, have extended Virasoro symmetry.

Some gravitational physics is kinematics!

Sum worldline perturbation theory $\longrightarrow$ Virasoro block

Conformal block: a single saddle-point in worldline formalism

Amplitude is exponential of connected diagrams

Consequence: single-trace blocks exponentiate!

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = \exp\left(G_N \Delta_1 \Delta_2 \mathcal{F}_T + O(G_N^2 \Delta^3) \right)$$

Seen in Virasoro/kinematic limit *[Fitzpatrick,Kaplan,Walters,Wang]*

Summation of Witten diagrams at all loop orders

OUTLOOK

- New approach to compute correlation functions in AdS, using worldline formalism
- Systematic perturbative expansion
- Complementary to Witten diagrams
- Bulk computation of Virasoro conformal blocks
- New perspective on geodesic Witten diagrams

Plenty to explore:

- Computing Virasoro blocks
- One-loop exact (localisation) in AdS_3 *[Duistermaat-Heckman]*
- Higher-point functions
- Other backgrounds *[Kraus,Maloney,HM,Ng,Wu][Anous,Hatman,Rovai,Sonner]*
- Fermions ($\mathcal{N} = 1$ worldline supersymmetry)
- Connect to Chern-Simons formalism *[Fitzpatrick,Kaplan,Li,Wang]*
- Diff invariant bulk observables*[Czech,Lamprou,McCandlinsh,Mosk,Sully]*
- Lorentzian signature, shockwaves
- Bootstrap