Holography as a probe of quantum gravity

Author: Ian T. Jardine

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Outline

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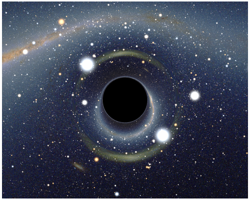
- AdS/CFT duality
- Holographic conformal field theories (CFTs)
- String theory and black hole microstates
- D1D5 system
- Component twist method, based on work in 1704.03401
- Deformed D1D5 CFT operator mixing, based on work in 1703.04744
- Summary

Black holes and the information paradox

Black holes and the information problem



source: BH article on wikipedia

- Hawking 1975: radiation from black hole (BH) depends only on ADM data (M, J_i) & Noether charges Q_i. Mechanism: pair creation.
- Problem: setup leads to information loss, non-unitary evolution.

- Why study quantum gravity? Semiclassical gravity has black hole (BH) information paradox, which must have a solution.
- Mathur 2009: subleading quantum corrections to classical gravity cannot resolve the information paradox [M 2009].
- Simple assumptions led to conclusion: only $\mathcal{O}(1)$ corrections can restore unitarity.

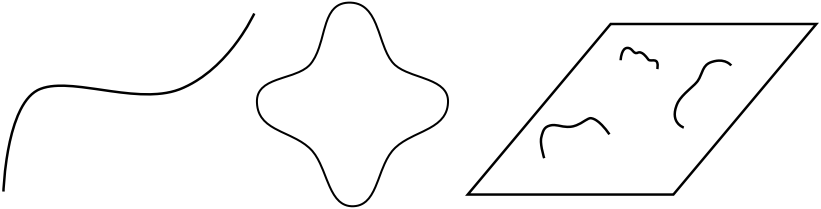
Firewalls

- Black hole complementarity ['t Hooft 1985], [STU 1993]: Infalling observer is both reflected and transmitted through stretched horizon.
- Linearity of quantum mechanics: cannot clone quantum states (no-cloning theorem). Black hole geometry doesn't allow experiments to measure both elements of an EPR pair, so cannot detect violation of no-cloning theorem [HP 2007].
- [AMPS 2012]: complementarity has another flaw: unavoidable excitation of high-frequency modes in the infalling frame. "Fire!"
- AMPS strikes at the heart of the question of figuring out the nature of quantum gravity. When is effective field theory effective?
- Our approach: turn to string theory and the AdS/CFT duality.

AdS/CFT duality

String Theory Introduction

- Parameters of string theory: string length $\ell_s = \alpha'^{1/2}$ & coupling g_s.
- Planck scale $\ell_p \neq \ell_s$ (eg in its native $d = 10$, $\ell_p \sim g_s^{1/4} \ell_s$).
- Having large numbers of strings/Dp-branes N can give parametric enhancement of fundamental length scales. e.g. D-brane metric $\sim g_s N$.
- Dualities connect superstring theories (IIA, IIB, I, HE, HO):
  - e.g. an S-duality connects IIA to $d = 11$ M theory.
  - T-duality swaps momentum/winding modes & large/small radius.



AdS/CFT duality

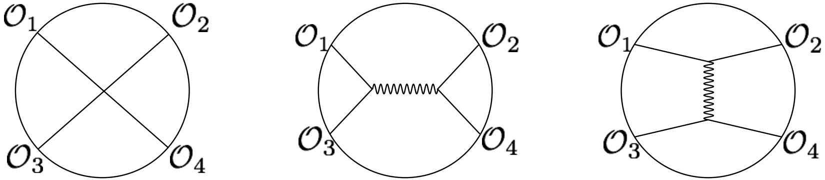
- AdS/CFT duality: Dynamics of asymptotic anti-de Sitter (AdS) gravity is dual to gauge theory in one fewer dimension [M 1997].
- What is AdS? Solution of Einstein's equations with negative cosmological constant.
- Relationship between couplings for gauge, (λ, N), and for gravity, (α', g_s). E.g.

$$\lambda = R^2/\alpha'^2 \qquad N/\lambda = 1/(4\pi g_s)$$

  where R is the scale of the bulk.
- Therefore have strong/weak duality i.e. weakly coupled gravity $\Leftrightarrow$ strongly coupled dual and vice versa.
- Since AdS_{d+1} has $SO(2,d)$ symmetry the dual field theory will have this symmetry as well: conformal field theory.

AdS/CFT holographic dictionary

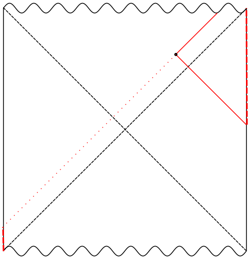
- Holographic dictionary includes boundary operators $\leftrightarrow$ bulk fields, e.g. boundary energy-momentum tensor dual to bulk graviton.
- Can also compute quantities on one side using another. e.g. correlators in CFT can be computed by Witten diagrams.



- Can also study less symmetric field theories with more complicated bulk geometries (AdS/CMT, AdS/QCD).
- Now how can we reconstruct things in the bulk from the CFT?

CFT operators and bulk reconstruction

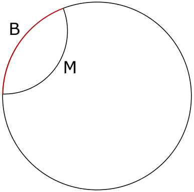
- Bulk reconstruction is a complicated issue: radial dimension is emergent. How can we see this from the CFT?
- Also, how can we reconstruct the local bulk fields from just the boundary? How much knowledge of the CFT is required to do so?
- A long series of papers, beginning with [HKLL 2005], found that a bulk field is expressed in the CFT as a non-local smeared operator.
- More recent works have found refinements on this construction but still non-local.
- Reconstructing interior of black hole using this method is very non-local. Explicit reconstructions in two sided black hole required smearing over both boundaries.



Entanglement and bulk reconstruction

- Another approach of reconstructing bulk: Ryu-Takayanagi (RT) [RT 2006] to geometrize entanglement.

$$S(B) = \frac{A(M)_{\partial B = \partial M}}{4 G_N}$$



- Time-dependent formalism by Hubeney, Ranganmani, and Taykayangai (HRT) [HRT 2007].
- Entanglement entropy not the only quantum information quantity used in holography c.f. mutual information, error correcting codes, modular hamiltonians, etc.
- Can RT or HRT probe inside black hole horizons? Generically no, sometimes cannot probe horizons (entanglement shadows).
- Reconstructing bulk directly to examine horizon and information problem is difficult. Can we probe it without referencing the bulk?

Holographic CFTs

Introduction to CFT

- Suppose we start with a 1+1d CFT on a cylinder. This CFT has the symmetry group $Vir \times \tilde{Vir}$. We can map the cylinder to the plane and see the Fourier modes in powers of $z, \bar{z}$.
- Operators are classified by how they transform under this symmetry. Primary field with weights h and $\bar{h}$ transform as

$$O(z,\bar{z}) \to O'(z,\bar{z}) = \left(\frac{\partial z'}{\partial z}\right)^{h} \left(\frac{\partial \bar{z}'}{\partial \bar{z}}\right)^{\bar{h}} O(z',\bar{z}').$$

- We can also define quasi-primary fields which transform as tensors only under the global subgroup of the full symmetry.
- Virasoro operators, L_n, are modes of stress-energy tensor $T(z)$. Satisfy $[L_n, L_m] = \frac{c}{12}(n^3 - n)\delta_{m+n,0} + (n-m)L_{m+n}$.
- Here c is the central charge, which is a function of N. So large N holography (dual to small g_s) $\sim$ large c.

Conformal Invariance

- Suppose we have primary operator O. Can define descendant operators, $L_{-k_1}...L_{-k_n}O$. Primary and descendants are called conformal family.
- Low point correlators are fixed by conformal symmetry.

$$\langle O_i(z_1)O_j(z_2)\rangle = \frac{\delta_{ij}}{z_{12}^{2h_i}}$$

$$\langle O_i(z_1)O_j(z_2)O_k(z_3)\rangle = \frac{C_{ijk}}{z_{12}^{h_i+h_j-h_k}\, z_{13}^{h_i+h_k-h_j}\, z_{23}^{h_k+h_j-h_i}}$$

- Higher point ones are not fully fixed, depend on cross ratios. eg.

$$\langle O_1(z_1)O_2(z_2)O_3(z_3)O_4(z_4)\rangle = \frac{z_{13}^{h_2+h_4}\, z_{24}^{h_1+h_3}}{z_{12}^{h_1+h_2}\, z_{23}^{h_2+h_3}\, z_{34}^{h_3+h_4}\, z_{14}^{h_1+h_4}} f\left(\frac{z_{12}z_{34}}{z_{13}z_{24}}\right)$$

- C_{ijk} and spectrum of primaries' weights fully define CFT. So how can we reconstruct higher point correlators?

Conformal blocks and bootstrap

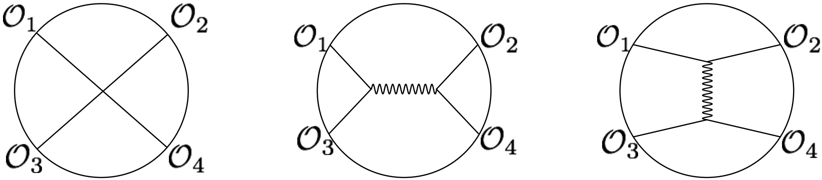
- Higher point functions can be obtained by inserting sets of complete states and summing. This gives products of three point functions, which are fixed by conformal symmetry.
- e.g. four point function of scalars: $\langle\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle =$

$$\sum_{\mathcal{O}} C_{\phi_1\phi_2\mathcal{O}} C_{\phi_3\phi_4\mathcal{O}} \left(\frac{x_{14}^2}{x_{13}^2}\right)^{\Delta_{34}/2} \left(\frac{x_{24}^2}{x_{14}^2}\right)^{\Delta_{12}/2} \frac{g_{\mathcal{O}}(u,v)}{x_{12}^{\Delta_1+\Delta_2} x_{34}^{\Delta_3+\Delta_4}}$$

- Conformal blocks, $g_{\mathcal{O}}$, are functions of cross-ratios $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$. Determined entirely by operator $\mathcal{O}$.
- Can also insert states between ϕ_1 and ϕ_3, gives same 4pf. This is crossing symmetry and places major constraints on CFT.
- Solving crossing equations called conformal bootstrap and is done both numerically and analytically.

Semiclassical holographic CFTs

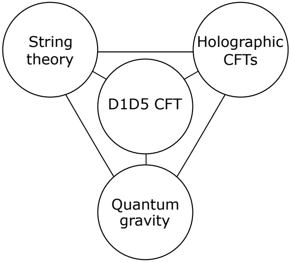
- Overall idea: use suitable CFT to define quantum gravity.
- Holographic CFTs can be defined without string theory, only need sparse spectrum and large c. [HPPS 2009]. This will gives large N and hierarchy between AdS/string scales in the bulk.
- Can translate CFT structures into bulk objects, e.g. conformal blocks are dual to geodesic Witten diagrams. [HKPS 2015] [HKPS 2015a]

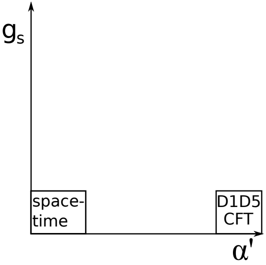


- Can investigate aspects of quantum gravity semiclassically using well expansions in $1/c$.

Conformal field theories as quantum gravity

- One can setup a "CFT information problem" using 2 heavy and 2 light operators and study $1/c$ corrections to conformal blocks of the four point function [FK 2015] + many others.
- One can consider a setup of a null infalling shell of dust in CFT and examine $1/c$ corrections to correlators [AHRS 2016] .
- Our approach is different, not semiclassical. Consider holographic D1D5 CFT. Start from string UV, probing down to IR semiclassical gravity with deformation.





String theory and black hole microstates

Brane construction of mircostates

- Consider Dp-brane solution. Metric looks like

$$dS^2 = H_p^{-1/2}(-dt^2 + dx_{||}^2) + H_p^{1/2} dx_{\perp}^2,$$

  Here $H_p = 1 + \frac{(2\sqrt{\pi})^{5-p}\Gamma[(7-p)]g_s N_p l_s^{7-p}}{2r^{7-p}}$ and r is the radius perpendicular to the brane worldvolume.
- Can construct supersymmetric solutions with multiple brane ingredients. Duality allows these solutions to be studied in different parameter ranges with different ingredients.
- Want low dimensional black holes, not 10d ones. Compactify branes on cycles and dimensionally reduce.
- Classic examples: D1D5P for 3-charge, 5d black hole, D2D6NS5P for 4-charge, 4d black hole.

String and supergravity microstate construction

- What about solutions for astrophysical black holes? Very hard! Supersymmetry gives a lot of theoretical control.
- Huge body of work (e.g. Bena-Warner) on solutions generating and classification:
  - Non-supersymmetric: JMaRT solutions [JMRT 2005]
  - A neutral, *t*-dependent solution [MT 2013]
  - Solutions parameterized by arbitrary functions of two variables e.g. [BGRSW 2015]
  - Solutions with arbitrarily small angular momentum [BFMRSTW 2016]
- General properties: asymptotic infinity, finite AdS throat, horizon sized cap and no black hole singularity.

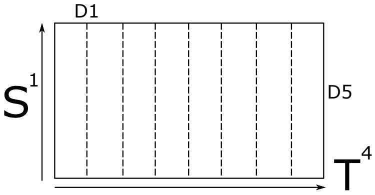
source: S.D. Mathur 1201.2079

Black hole microstates

- How do we know we truly have microstates of black holes?
- Can match entropy from area of horizon from microscopic degrees of freedom. First done by Strominger-Vafa [SV 1996] but matching has been done for a number of different constructions.
- Can go further for non-supersymmetric geometries. Reproduce emission spectrum for non-extremal black holes [CV 2010].
- Can we form these in nature? Rough argument states exponentially small tunnelling compensated by exponentially large phase space of geometries [M 2010].
- Could we see any observational signatures? Some works suggested possibilities but still unclear [P 2014] [HH 2017].

D1D5 system

D1D5 system



- We focus on prototype D1D5 system.
- Constructed by compactifying N_1 D1 branes, N_5 D5 branes on $S^1 \times T^4$. In near horizon limit, geometry is $AdS_3 \times S^3 \times T^4$.
- D1D5 system has size $R_{D1D5} \sim (N_1 N_5)^{1/6} \ell_p$.
- Since the throat has AdS_3, expect CFT_2 dual theory.
- Supersymmetry implies it will be a 2d $\mathcal{N} = (4,4)$ SCFT.
- System has 20 dimensional moduli space, corresponding to 20 marginal deformation operators.
- At one point, described by supergravity, another by a dual symmetric orbifold CFT.

D1D5 CFT

- Symmetric orbifold point is a free (1+1)-dimensional $(T^4)^N/S_N$ superconformal field theory, where $N = N_1 N_5$.
- Each copy has fundamental fields $X^{\dot{A}A}$, $\psi^{\alpha\dot{A}}$, $\tilde{\psi}^{\dot{\alpha}\dot{A}}$. Here $\alpha, \dot{\alpha}$ are for the R-symmetry $SU(2)_L \times SU(2)_R$ and $A, \dot{A}$ are for global symmetry $SU(2)_1 \times SU(2)_2$ from torus isometries
- Orbifold introduces operators with twisted boundary conditions between copies of T^4.
- Implemented by bare twist operators σ_n with conformal weight $\frac{c}{24}(n - \frac{1}{n})$.
- e.g. $\sigma_{(12...n)}$ implements $X_{(1)} \to X_{(2)} \to ... \to X_{(n)} \to X_{(1)}$
- So operators break into untwisted sector and twisted sector, which can further be classified by the length of the permutation.

D1D5 operators

- Algebra consists of modes of fundamental fields together with L_m, modes of the Virasoro algebra, as well as J_q^a and $G_r^{\alpha A}$, modes of the R-symmetry current and the supercharges. Similarly for the anti-holomorphic side.
- Can also defined fractional modes using twist operators,

$$\mathcal{O}_{-m/n}\sigma_n = \oint \frac{dz}{2\pi i} \sum_{k=1}^{n} \mathcal{O}_{-m,(k)} e^{-2\pi i m(k-1)/n} z^{h-m/n-1} \sigma_n$$

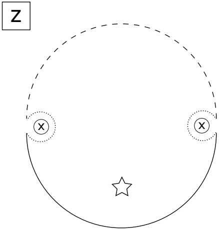
- We focus on deforming with an operator $\mathcal{O}_D$, one of the 20 marginal deformations.
- It is dual to a blow up mode in the bulk and takes the form in the CFT of

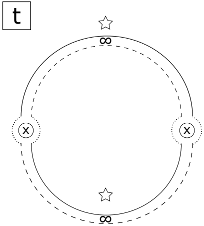
$$\mathcal{O}_D = \epsilon_{AB}\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} G^{\alpha A}_{-1/2} \tilde{G}^{\dot{\alpha}B}_{-1/2} \sigma_2^{\beta\dot{\beta}}$$

Twist operators and Lunin-Mathur

- Difficult to compute correlators in with twisted operators so we use Lunin and Mathur method [LM 2000][LM 2001].
- Lift operators from target space to covering space using map where twists insertions are ramified to identity.

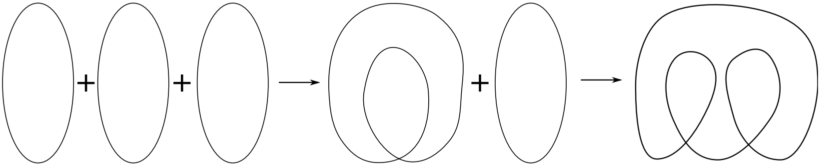
$$z(t) = \frac{t^2}{2t - 1}$$





- Key fact: non-zero central charge ($c = 6N_1N_5$), and so have a Weyl anomaly. Track transformation using Liouville action.
- Twist insertions and infinities regulated, giving non-trivial contributions.

Component twist method



Review of previous work

- Deformation by $\mathcal{O}_D$ is important in studying thermalization of the free CFT. Breaking of symmetric product introduces interactions between the copies of the CFT needed for thermalization.
- Previously: calculations investigated a twist operator acting on two copies and twisting them together [ACM2010] [CHMT2014] [CMT2014] [CMT2014a] [BMPZ2014]. Calculations furthered to second order in twist in [CHM2015,2016,2016a].
- Twist operators yield a squeezed state, e.g. for a single twist,

$$\sigma_2|0\rangle_{(1)}|0\rangle_{(2)} \sim e^{\sum_{s,s'}\gamma^B_{ss'}a^\dagger_s a^\dagger_{s'}}|0\rangle$$

$$a^{(1)\dagger}_q|0\rangle_{(1)}|0\rangle_{(2)} \to \sum_s f^B_{qs}a^\dagger_s e^{\sum_{s,s'}\gamma^B_{ss'}a^\dagger_s a^\dagger_{s'}}|0\rangle$$

- Interested in calculating the Bogoliubov coefficients $\gamma^B_{ss'}$ of the state after the twist has been applied and the transition amplitudes f^B_{qs}.

Twists in action

- Consider component CFT copy (i) before the twist insertions and copy (j') after the twist insertions. One can expand scalar field of a copy as $X^{(i)}(x) = \sum_m \left(h_m^{(i)}(x) a_m^{(i)} + h_m^{(i)*}(x) a_m^{(i)\dagger}\right)$.
- Twists will only twists copies together, so one should be able to relate mode expansion of scalar fields before and after twist. This matching will lead us to the relation,

$$a_m^{(i)} = \sum_n \left(\alpha_{mn}^{(i)(j')} a_n^{(j')} + \beta_{mn}^{(i)(j')} a_n^{(j')\dagger}\right)$$

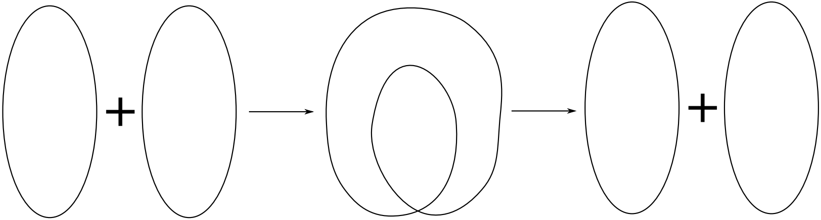
- Using this, we can also find $f = \left(\alpha^{-1}\right)^T$, $\gamma = \alpha^{-1}\beta = f^T \beta$.
- Can then use standard inner product $(h, g) \equiv -i \int_\Sigma d\Sigma^\mu (f \partial_\mu g^* - g^* \partial_\mu f)$ to find

$$\alpha_{mn}^{(i)(j')} = \left(h_m^{(i)}, h_n^{(j')}\right) \qquad \beta_{mn}^{(i)(j')} = \left(h_m^{(i)*}, h_n^{(j')}\right)$$

Method for Computation

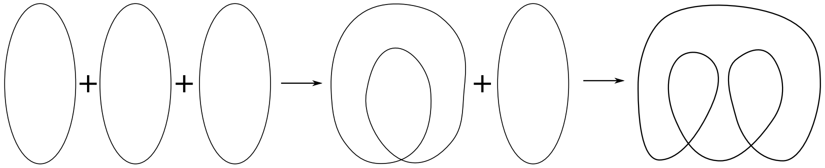
- Purpose of our paper [CJP 2017]: further the results to multiple twists in the continuum limit.
- Continuum limit has expected behaviour of BHs, parametrically small gap by momentum modes scaling as $1/(N_1 N_5 R)$.
- Results can be obtained with Lunin-Mathur, but this generally will run into inverting quintic or higher polynomials.
- Challenge: α is an infinite matrix so inverting to obtain α^{-1} directly is generally not possible.
- Previous work: α^{-1} is related to the transition amplitude f. Our idea: break twists into components twist 2s and build α^{-1} from multiplying transition amplitudes from each component twist.

Confirmation of method



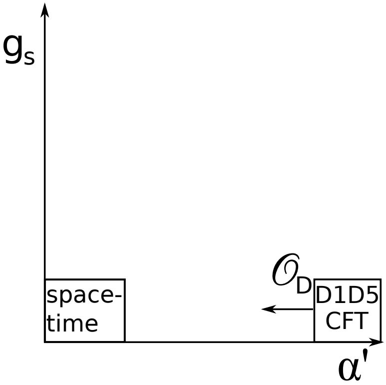
- To confirm the method, we considered the setup $\sigma_{(12)}\sigma_{(21)}$. This twists and then untwists two copies. It was studied with Lunin-Mathur method in previous papers [CHM2015,2016,2016a].
- General results of this method involve infinite sums, so we have numerical calculations to compare (this is also what happens in Lunin-Mathur cases in continuum limit as well).
- Matched previous results with large enough cutoff.

New results



- Turn to new configuration $\sigma_{(12)}\sigma_{(23)}$, which will act as a twist 3 when the two twists are brought together.
- $f^{(1)}$ and $f^{(3)}$ have simple results, agreeing with a single twist. $f^{(2)}$ exhibited new behaviour.
- Confirms previously conjectured general form of the γ and α^{-1}. Power counting w/ this method suggests this is full generic.

Deformed D1D5 CFT operator mixing



Motivation and introduction

- How does the deformation $\mathcal{O}_D$ change the physics from the orbifold point? One approach, examining anomalous dimensions and structure constants C_{ijk} defining CFT.
- Orbifold point has a closed subsector described by massless higher spin fields [GG 2014]. Corresponds to a tensionless limit, where strings are large and floppy.
- Deformation turns on string tension, gives mass to the higher spin fields. Reaches down from $\alpha' \to \infty$ to finite α'. Anomalous dimensions of the higher spin currents were studied in [GPZ 2015].
- We continued the work of [BPZ 2012,2012a] in our paper [BJP2017]. Use conformal perturbation theory to examine mixing.

Conformal Perturbation Theory

- Consider two point function (2pf) in free CFT for quasi-primary ϕ_i: $\langle\phi_i(z_1,\bar z_1)\phi_j(z_2,\bar z_2)\rangle_0 = \delta_{ij} z_{12}^{-h_i}\bar z_{12}^{-\tilde h_i}$, where h_i is conformal dim.
- Compute 2pf with perturbation $\delta S = \lambda\int d^2z\mathcal{O}_D(z,\bar z)$.

$$\langle\phi_i(z_1,\bar z_1)\,\phi_j(z_2,\bar z_2)\rangle_\lambda = \frac{\int d[X,\psi]e^{-S_{\text{free}}+\lambda\int\, d^2z\mathcal{O}_D(z,\bar z)}\phi_i(z_1,\bar z_1)\,\phi_j(z_2,\bar z_2)}{\int d[X,\psi]e^{-S_{\text{free}}+\lambda\int d^2z\mathcal{O}_D(z,\bar z)}}$$

- So to first order:

$$\frac{\partial}{\partial\lambda}\langle\phi_i(z_1,\bar z_1)\phi_j(z_2,\bar z_2)\rangle_\lambda = \int d^2z\langle\phi_i(z_1,\bar z_1)\mathcal{O}_D(z,\bar z)\phi_j(z_2,\bar z_2)\rangle$$

- Recall: three point function is fixed by conformal invariance.
- Regularize and renormalize the field. Anomalous dimension is then

$$\frac{\partial h_i}{\partial\lambda} = -\pi C_{iDi}, \quad \frac{\partial\tilde h_i}{\partial\lambda} = -\pi C_{iDi}$$

- Must diagonalize C_{iDk} over fields with same conformal dimension and so must identify all ϕ_k that mix with ϕ_i and iterate.

OPEs versus correlators

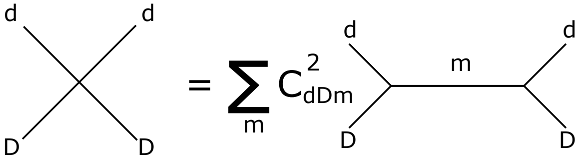
- C_{iDj} computed by three point functions (3pfs), $\langle \mathcal{O}_i \mathcal{O}_D \mathcal{O}_j \rangle$.
- Can be computed by hand but involves hundreds of correlators, most of which are zero. Wrote a package for Mathematica to help.
- However, operator product expansions (OPEs) also involve the structure constants,

$$\mathcal{O}_i(z)\mathcal{O}_D(w) = \sum_k (z-w)^{h_k - h_i - h_D} C_{iDk} \mathcal{O}_k(w).$$

- Idea: if we can lift $\mathcal{O}_i \mathcal{O}_D$ to cover, where twists are ramified, we could extract structure constants and mixing operators directly.
- OPE contains both quasi-primaries and descendants.
- Descendants do not contribute to anomalous dimensions, so we came up with a procedure to project descendants out.

Protected results

- Warm up OPE with operator dual to supersymmetric protected operators, $\mathcal{O}_S(z, \bar{z})\mathcal{O}_D(0, 0)$. Showed no term for anomalous dimension, as expected.
- Next, we specialized to the operator dual to the dilaton $\mathcal{O}_{dil}(z, \bar{z})$.
- $\mathcal{O}_{dil}(z, \bar{z})\mathcal{O}_D(0, 0)$ has leading coefficient 2^{-2}.
- Coefficient matched the results from [BPZ2012a], which found the leading singularity of the coincident limit of the four point function.



$$\langle \mathcal{O}_{dil}(a_1, \bar{a}_1)\mathcal{O}_D(b, \bar{b})\mathcal{O}_D(0, 0)\mathcal{O}_{dil}(a_2, \bar{a}_2)\rangle \sim \frac{2^{-4}}{|a_1|^3|a_2 - b|^3|b|^2}$$

Unprotected results

- Now do an unprotected operator, $\mathcal{O}_C$, and OPE $\mathcal{O}_C(z,\bar{z})\mathcal{O}_D(0,0)$.
- Leading singularity of OPE indicated mixing with deformation that contributes to renormalization; matched results from [BPZ2012a].
- Looked for term indicating mixing that contributes to anomalous dimension, shows up at $z\bar{z} = |z|^2$ order.
- Through much effort, found the result

$$\frac{3}{2^6|z|^2}\mathcal{O},$$

  where $\mathcal{O}$ is a complicated combination of operators. Coefficient matched with coincidence limit of $\langle\mathcal{O}_C\mathcal{O}_D\mathcal{O}_D\mathcal{O}_C\rangle$ from [BPZ2012a].
- Compared this with results obtained from considering the 34 possible mixing operators and computing all the 3pfs with our Mathematica package. They matched!

Summary

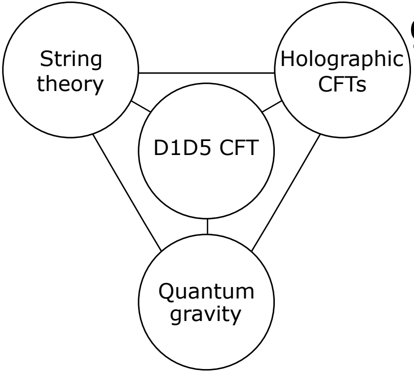
Summary

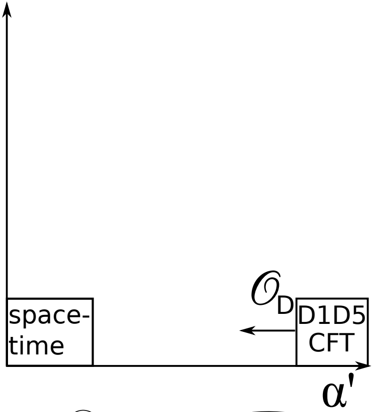
- Quantum gravity must be considered for solution to black hole information paradox.
- Use holography to probe (and maybe even define?) quantum gravity.
- Can consider semiclassical and even some non-perturbative corrections with 1/c.
- Instead consider from a string theoretic point and probe down to semiclassical gravity.
- String theory can be used to construct microstates for black holes, one such system: D1D5.

Summary

- Start with orbifold point of D1D5 CFT and perturbatively deform to semiclassical gravity.
- To examine thermalization, we considered the effect of twists on states in the D1D5 CFT with new composite twist method.
- Our method allows us to examine excitations on a single strand spreading through the twist interactions to higher orders.
- To examine the deformation more closely, we looked at the mixing of operators by lifting the OPE to the cover.
- Could consider continuing work the OPE to find full anomalous dimension. OPE method is less clear with twisted operators.

The End


String theory
Holographic CFTs
D1D5 CFT
Quantum gravity


g_s
$\mathcal{O}_D$
D1D5 CFT
space-time
α'