A possible pathway beyond the gravitational effective field theory

I) What is wrong with quantum gravity and what did we think was wrong?

Impact of EFT

Including some recent EFT results

II) Trying a field theory treatment for QG beyond the EFT (ongoing)

A. Quartic propagators, negative norms and the physical spectrum

B. Dimensional transmutation and the Einstein action

C. On a possible role of confinement in gravity
- treat spin connection as an independent field
- spin connection asymptotically free → confined?

D. A conformal model of gravity
- incomplete

John Donoghue
Toronto
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I. What is wrong with quantum gravity?

Gravity is different – <u>historically</u>

- SM grew together with QFT techniques
- GR existed far before QFT was developed enough
- Premature attempts to blend with QM/QFT

<u>This lead to a bad reputation for quantum GR:</u>

"Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct."

"From a technical point of view, the problem is that the theory one gets in this way is not renormalizable and therefore cannot be used to make meaningful physical predictions."

This is what we used to think was the problem of quantum gravity

A more modern view:

"A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data."

Frank Wilczek
Physics Today
2002

Progress in Quantum Gravity!

But, EFT points to its own demise

Still need to find better UV theory of gravity

The Effective Field Theory of General Relativity

Can construct GR as a QFT and quantize it naturally

All divergences correspond to local terms in the action – renormalize
- but these are not interesting!

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

EFT tells us how to isolate low energy predictions
- parameters in L are not predictions
- but non-local / non-analytic effects are
- real low energy propagation

Can make real unambiguous calculations at low energy

Power counting

Expansion in the energy/curvature Recall $R \sim \partial^2 g$

Loops create more powers of derivatives/curvatures

No loops ~ order $E^2 \sim R$

One loop ~ order $E^4 \sim R^2$

Two loops ~ order $E^6 \sim R^3$

Good for the EFT

Bad as property of a UV complete theory

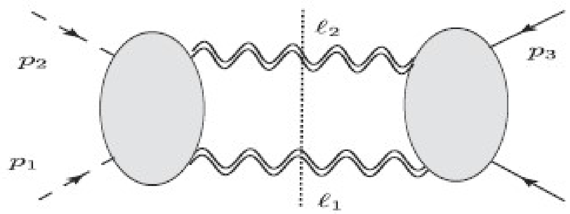
Some results:

Old result: Quantum correction to Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Newer variant: One loop soft theorem
- unitarity techniques
- Compton amplitudes have soft theorems
- leading one loop amplitudes universal

JFD, Bjerrum-Bohr
Vanhove



Light bending at one loop: EP violation
- unitarity plus eikonal

$$\mathrm{bu}^{\varphi} = \frac{371}{120}, \qquad \mathrm{bu}^{\gamma} = \frac{113}{120}.$$

$$\theta_\eta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}.$$

Bjerrum-Bohr, JFD, Holstein
Plante, Vanhove
Bai and Huang

-no longer moving on null geodesics

Non-local effective actions:

Barvinsky, Vilkovisky, Avrimidi

Perturbative running is contained in the R^2 terms

$$S_4 = \int d^4x \sqrt{g}\, \left[c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\right]$$
$$+ \left[\bar{\alpha} R \log(\nabla^2/\mu^2) R + \bar{\beta} C_{\mu\nu\alpha\beta} \log(\nabla^2/\mu^2) C^{\mu\nu\alpha\beta}\right.$$
$$\left. + \bar{\gamma}\left(R_{\mu\nu\alpha\beta} \log(\nabla^2) R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} \log(\nabla^2) R^{\mu\nu} + R \log(\nabla^2) R\right)\right] \quad + \mathcal{O}(R^3)$$

Hints of singularity avoidance:

JFD, El-Menoufi

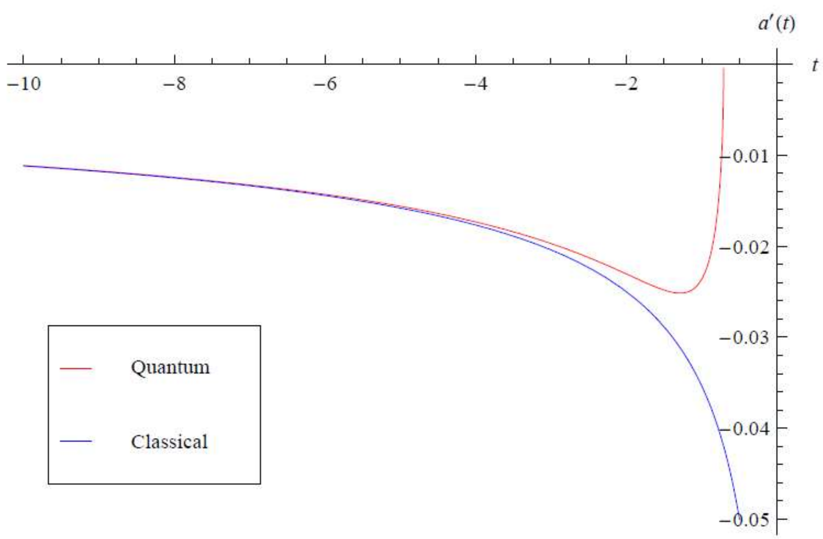


FIG. 12: Collapsing radiation-filled universe with gravitons only considered.

- No free parameters in this result

Limits of the EFT:

EFT points to its demise at M_P

All terms in series become of the same order – **strongly coupled**

Most likely new DOF/interactions at this energy

<u>Modern view:</u>

We have a quantum theory of gravity

It has the form of an effective field theory

We can make predictions at low energy

The effective theory points to the need of a UV completion

We will need to find a more complete theory eventually

This is clear progress!

Gravity fits well with our other interactions in Core Theory

$$W = \overbrace{\int_{k<\Lambda} [Dg][DA][D\psi][D\Phi] \exp}^{\text{quantum mechanics}} \Bigg\{ \overbrace{i \int d^4x \sqrt{-g}}^{\text{spacetime}} \Bigg[\overbrace{\frac{m_p^2}{2} R}^{\text{gravity}}$$

$$\underbrace{- \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{other forces}} + \underbrace{i \bar{\psi}^i \gamma^\mu D_\mu \psi^i + \left(\bar{\psi}^i_L V_{ij} \Phi \psi^j_R + \text{h.c.} \right)}_{\text{matter}} \underbrace{- |D_\mu \Phi|^2 - V(\Phi)}_{\text{Higgs}} \Bigg] \Bigg\}$$

Slide due to
Sean Carroll

What to do?

1) Nothing

- look for other problems that are testable

2) Try to form a non-perturbatively complete theory with GR

Asymptotic safety

3) Drastically change the theory

String theory
Loop quantum gravity
Dynamical triangulations
Gravity as quantum information
Causal sets
Entropic theories

4) Conventional QFT?

Return to the issue of power counting

This is the main barrier to a conventional QFT approach in UV

Starting with EH action, always will generate high curvature terms

Either embrace the infinite number of terms (!)
or don't start from EH action
- must then be induced

Interesting observation:
Matter loops (coupled to gravity) also give R^2 terms at one loop
But, higher order matter loops stay at order R^2

II. What I am trying to accomplish:

Conservative pathway: Gravity as a renormalizeable gauge theory similar to our other theories (but of course we need **some variation**!)

Three ingredients:

1) Dimensional transmutation **inducing** Einstein action
   - from a scale invariant starting point
   - in order to solve the power counting problem

2) Lorentz (spin) connection as independent field producing important dynamics
   - strongly interacting, asymptotically free and possibly **confined**

3) A model based on **conformal symmetry**
   - vierbein and Lorentz connection as basic fields
   - sizeable operator basis may allow one to overcome difficulties

Skating on thin ice:

Warning: Many dangers in what follows

- non-compact groups
- Possible ghosts
- Possible unitarity violation
- Euclidean vs Lorentzian
- ……

May end up being fatal but…..

Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity

Alan D. Sokal
Department of Physics
New York University
4 Washington Place
New York, NY 10003 USA
Internet: SOKAL@NYU.EDU
Telephone: (212) 998-7729
Fax: (212) 995-4016

Whenever you move in some direction in GR, there are always others ahead of you

Important work done by:
Schwinger
Utiyama
Kibble
Mansouri MacDowell
DeWitt
‘t Hooft, Veltman

Stelle
Fradkin Tseytlin
Smilga
Holdom and Ren
Mannhiem
Salvio and Strumia
Lu , Perkins, Pope, Stelle

And many others

A. “Some variation”

Are some “forbidden” QFT aspects OK if they do not appear in the physical spectrum?

For example, quartic propagators

$$\frac{-i}{q^4} \sim \frac{-i}{q^2(q^2-\mu^2)} = \frac{1}{\mu^2}\left(\frac{i}{q^2} - \frac{i}{q^2-\mu^2}\right)$$

looks like a negative norm state – bad news if physica

But if there is no physical state with this property, is this OK?

I will use this reasoning twice in this talk.

Yet, via Path Integrals one can have a fine perturbation theory

Generating functional:

$$Z_0[J] = \int [d\phi] \exp i \int d^4x \frac{1}{2} \Box\phi\Box\phi - J\phi$$

$$Z_0[J] = Z_0[0] \exp -i \int d^4x d^4y \frac{1}{2} J(x)\Delta(x-y)J(y)$$

with propagator:

$$\Delta(x-y) = < x|\frac{1}{\Box^2}|y >= \int d^4q \; \frac{e^{-iq\cdot(x-y)}}{q^4}$$

<u>**Test case:**</u>

Prelim.

Triplet scalar in SU(2) with quartic propagator

SU(2 valued field

$U = e^{i\tau^a \phi^a}$ with $U \to V(x) U V^\dagger(x)$ $V(x)$ in SU(2).

with scale invariant Lagrangian

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} Tr\left[(D^\mu D_\mu U)^\dagger (D^\nu D_\nu U)\right] \\ &+ d_1 \left(Tr\left[D^\mu U^\dagger D_\mu U\right]\right)^2 + d_2 Tr\left[D^\mu U^\dagger D^\nu U\right] Tr\left[D_\mu U^\dagger D_\mu U\right]\end{aligned}$$

Scalar will be confined – usual discussions of g.s. not relevant.

Renormalization is simple:

Expand to quadratic order

$$\mathcal{L} = \frac{1}{2}\phi^a D^2 D^2 \phi^a + ...$$

do the path integral

$$\frac{1}{det(D^2 D^2)} = e^{Tr \log(D^2 D^2)} = e^{\int d^4x <x|Tr \log(D^2 D^2)|x>}$$

Using $\log(D^2 D^2) = 2\log(D^2)$,

the divergences are just twice the usual ones:

$$S_{div} = \int d^4x \, \frac{1}{\epsilon}\frac{2a_2}{16\pi^2} = \int d^4x \, \frac{C_2}{48\pi^2\epsilon} F^i \mu\nu F^{i\mu\nu}$$

Result is asymptotically free

$$\beta(g) = -\left[\frac{11}{3}N - \frac{n_s}{4}C_2\right]\frac{g^3}{16\pi^2}$$

<u>Analogy – Two flavor massless QCD</u>

Theory is weakly coupled in <u>both</u> UV and IR

Massless QCD is classically scale invariant, yet running coupling defines QCD scale

UV story is well known – asymptotic freedom

As we come down in energy – strong coupling region 2 GeV to 0.5 GeV

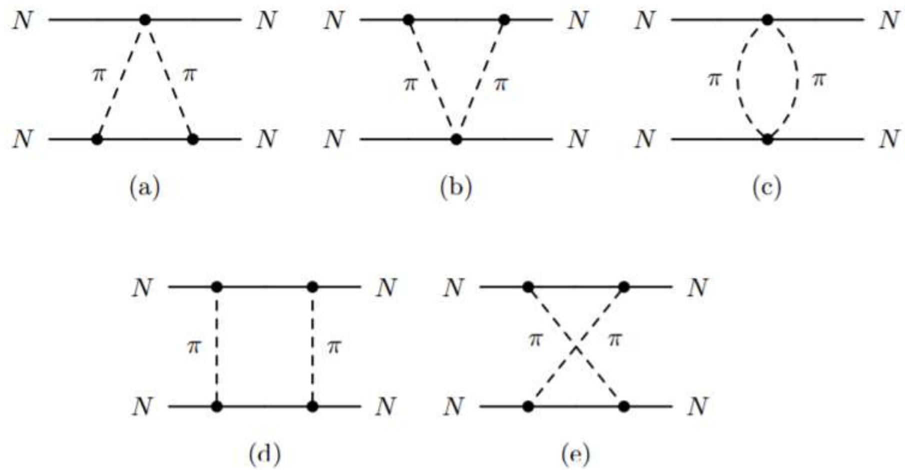
But at low energy, the chiral symmetry requires massless degrees of freedom
- organized as an effective field theory

$$\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \qquad \text{with} \qquad U = exp\left[\frac{i\tau\cdot\pi}{F}\right]$$

This is weakly coupled in the IR
- explicitly depends on QCD scale
- going up in energy enters the strong coupling region

If we had uncovered pionic theory first, we would think that there was an impassable barrier at 1 GeV.

EFT techniques could be applied unambiguously at low energy



Nuclear potentials would be calculable:

$$V_T(r) = \frac{1}{16\pi^4}\frac{g_A^2}{F_\pi^2}\frac{1}{r^3}\left[1 - \frac{c_4}{\pi F_\pi^2}\frac{1}{r^3}\right]$$

and one would uncover a new scale:

$$V_T(r) = -29 \text{ MeV}\left[\left(\frac{r_0}{r}\right)^3 - 0.66\left(\frac{r_0}{r}\right)^6\right] \qquad r_0 = 1 \text{ fm} = (200\text{MeV})^{-1})$$

Dimensional transmutation at work:

Low energy actions are not scale-invariant

Not just logarithmic corrections in running

Scale of QCD features in all low energy physics

Purely perturbative corrections are logarithmic:

$$V_{\mathrm{eff}} = \frac{3e^4}{64\pi^2}\,\phi_c^4\left(\ln\frac{\phi_c^2}{\langle\phi\rangle^2} - \frac{1}{2}\right)$$

Non-perturbative corrections carry power dependence on scale of the theory

$$\Lambda \sim M e^{-\frac{8\pi^2}{g^2(M)}}$$

A simple example with an induced chiral Lagrangian

Not a complete calculation but… Consider:

$$\mathcal{L} = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R$$

$$\psi_L \to L\psi_L, \quad \psi_R \to R\psi_R \qquad \text{with} \quad L, R \ \text{ in } \ SU(2)_{L,R}$$

Factor out chiral directions of the fermion field:

$$\psi_L = \xi\Psi_L, \quad \psi_R = \xi^\dagger\Psi_R \qquad \text{with} \qquad \xi \to L\xi V^\dagger = V\xi R^\dagger$$

The Lagrangian then becomes:

$$\mathcal{L}_D = \bar{\Psi} i D\!\!\!\!/ \Psi$$

with:

$$\begin{aligned}
D_\mu &= \partial_\mu + iV_\mu + iA_\mu\gamma_5 \\
V_\mu &= -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right) \\
A_\mu &= -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right) \quad .
\end{aligned}$$

Induced effects at one loop:

Consider heat kernel evaluation of functional determinant:

$$\det \mathcal{D} = e^{\mathrm{tr}\ \ln \mathcal{D}} = e^{\int d^4x\ \ \mathrm{Tr}\,\langle x|\ln \mathcal{D}|x\rangle}$$

$$\langle x|\ln \mathcal{D}|x\rangle = -\int_0^\infty \frac{d\tau}{\tau}\left\langle x\left|e^{-\tau\mathcal{D}}\right|x\right\rangle + C$$

$$H(x,\tau) \equiv \left\langle x\left|e^{-\tau\mathcal{D}}\right|x\right\rangle$$

$$H(x,\tau) = \frac{i}{(4\pi)^{d/2}}\,\frac{e^{-\tau m^2}}{\tau^{d/2}}\left[a_0(x) + a_1(x)\tau + a_2(x)\tau^2 + \ldots\right]$$

For a single fermion with:

$$\not{D} = \not{\partial} + i\not{V} + i\not{A}\gamma_5$$

we have

$$\begin{aligned} a_0(x) &= -1\,, \qquad\qquad a_1(x) = \sigma\,, \\ a_2(x) &= -\frac{1}{2}\sigma^2 - \frac{1}{12}[d_\mu, d_\nu][d^\mu, d^\nu] - \frac{1}{6}\,[d_\mu, [d^\mu, \sigma]] \quad . \end{aligned}$$

Where

$$\begin{aligned} d_\mu &= \partial_\mu + iV_\mu + \sigma_{\mu\nu}A^\nu\gamma_5 \equiv \partial_\mu + \Gamma_\mu\,, \\ \sigma &= \frac{1}{2}\sigma_{\mu\nu}V^{\mu\nu} - 2A_\mu A^\mu + (i\partial_\mu A^\mu - [V_\mu, A^\mu])\,\gamma_5 \end{aligned}$$

For our purpose, focus on

$$\mathrm{Tr}'\; a_1(x) = -8\mathrm{Tr}\; A_\mu A^\mu = 8\mathrm{Tr}\;(\partial_\mu U \partial^\mu U^\dagger) \qquad \text{with } U = \xi^2.$$

With a proper time cutoff, this would lead to:

$$\Delta\mathcal{L} = \frac{8}{\tau_0}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$$

Implication: Induced quantum effects can generate the chiral Lagrangian

For fun, lets do the same exercise with gravity included

Add the metric to the fermion Lagrangian, and look at the one-loop effect

Contribution to the cosmological constant:

$$a_0 = -1$$

Contribution to G:

$$a_1 = -R$$

Contribution to a Weyl curvature squared Lagrangian:

$$a_2 = \frac{1}{10} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$$

where

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta} - \frac{1}{2}\left(R_{\mu\alpha} g_{\nu\beta} - R_{\nu\alpha} g_{\mu\beta} - R_{\mu\beta} g_{\mu\alpha} + R_{\nu\beta} g_{\mu\alpha}\right) \\ &+ \frac{R(g)}{6}\left(g_{\mu\alpha} g_{\nu\beta} - e_{\nu\alpha} e_{\mu\beta}\right) \end{aligned}$$

B. Dimensional transmutation with gravity?

Start with **scale/conformal** invariant action

Couplings will run

– asymptotically free but strong coupling at low energy

Running coupling defines the Planck scale

Confine/gap the spin connection/metric

Low energy theory is EFT for the metric

- using dimensional transmutation for the scale

$$S = \int d^4x \sqrt{-g} \left[-\Lambda - \frac{2}{\kappa^2} R + ... \right]$$

Weakly coupled in both the UV and IR

Scale invariant gravity - generalities

Einstein Hilbert action is not scale invariant:

$$\frac{2}{\kappa^2}R \sim M_P^2 \partial g \partial g + ...$$

Curvature squared terms are scale invariant

$$c_i R^2 \sim \partial^2 g \partial^2 g + ...$$

R+R^2 theories have ghosts in general
- but disagreements over importance and inevitability

Pure R^2 theories have quartic propagators
- infrared issues

R^2 theories are perturbatively renomalizeable (Stelle)

Power-counting modifications

1) **Difference in power-counting of gauge fields and gravity**

- at one loop both are of order R^2
- but gravity loops keep increasing powers R^n
- further gauge loops stay are order R^2

2) **Pure quadratic gravity also stops at order R^2**

- no scale in theory
- with usual expansion, a bit more subtle

$$\text{propagator} \sim \frac{M_P^2}{k^4} \qquad\qquad \text{vacuum polarization} \sim \kappa k^2 \; (\frac{M_P^2}{k^4})^2 \; \kappa k^2 \;\; \sim \frac{M_P^2}{k^4}$$

QCD plus scale invariant gravity

Could induce Einstein Hilbert action via dimensional transmutation

Divergences from QCD loops proportional to Weyl-squared

But finite induced effects proportional to QCD scale

$$S_{induced} \sim \int d^4x \; [\Lambda_{QCD}^4 \; + \; \Lambda_{QCD}^2 R + ...]$$

Discussions on lattice calc.
Brower, Fleming

Can we repeat this at the Planck scale?

- perhaps with spin connection

SU(N) + gravity

Take the YM theory with the highest energy scale

It should induce largest contribution to M_P

Add scale invariant gravity (or conformal gravity)

Perhaps this is sufficient!?

Intermediate summary #1

Dimensional transmutation may allow a renormalizeable model to lead to Einstein-Hilbert action

C. Spin connection and confinement?

Setting: In construction of GR with fermions, naturally have two fields

-vierbein (tetrad) e_μ^a and spin connection ω_μ^{ab}

- ω_μ^{ab} appears naturally as a gauge field

Recover GR only by extra assumption – metricity for vierbein

$$\nabla_\mu e_\nu^a = 0 = \partial_\mu e_\nu^a + \omega^a{}_{b\mu} e_\nu^b - \Gamma_{\mu\nu}{}^\lambda e_\lambda^a$$

Removes ω_μ^{ab} as independent field

$$\omega_\mu^{ab}(x) = e^{a\nu}(\partial_\mu e_\nu^b - \Gamma_{\mu\nu}{}^\lambda e_\lambda^b)$$

What if we do not assume metricity?

Explorations:

1) With usual gauge action, spin connection is asymptotically free

2) Is the spin connection confined (or condensed, gapped)
   - would yield metric theory without extra assumption

3) In scale invariant theory for ω_μ^{ab} , dimensional transmutation
   will give Einstein-Hilbert action

4) With conformally invariant theory for ω_μ^{ab}, richer set of invariants
   $\rightarrow$ conformal model for gravitons

Quick review: Vierbein and spin connection

From Equivalence Principle one can write the metric in terms of vierbein variables

$$g_{\mu\nu}(x) = \eta_{ab}\; e^a_\mu(x)\; e^b_\nu(x)$$

In addition to general covariance

$$e'^a_\mu = \frac{\partial x^\nu}{\partial x'^\mu} e^a_\nu$$

there is an extra **local Lorentz** symmetry

$$e'^a(x) = \Lambda^a{}_c(x)\; e^c(x) \quad \text{with} \quad \eta_{ab}\; \Lambda^a{}_c(x)\; \Lambda^b{}_d(x) = \eta_{cd}$$

For scalars, this feature is irrelevant. But for fermions, it is important

$$\mathcal{L} = \bar{\psi}[i\gamma^a e^\mu_a(x)\partial_\mu + ...]\psi$$

To include the local Lorentz symmetry

$$\psi \to \psi'(x') = S(x)\psi(x)$$

where

$$S(x) = \exp\left(\frac{-i}{2} J_{ab}\alpha^{ab}(x)\right) \quad , \quad J_{ab} = \frac{\sigma_{ab}}{2} \quad \text{with} \quad \sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b] \quad .$$

To include this, need spin connection and gauge covariant derivative

$$\mathcal{L} = \bar{\psi}[i\gamma^a e_a^\mu(x) D_\mu]\psi$$

$$D_\mu = \partial_\mu - ig\frac{J_{ab}}{2}\omega_\mu^{ab} \equiv \partial_\mu - ig\boldsymbol{\omega}_\mu$$

with gauge transformation

$$\boldsymbol{\omega}'_\mu = S\boldsymbol{\omega}_\mu S^{-1} - \frac{2i}{g}(\partial_\mu S)S^{-1}$$

$$S^{-1}(x)\gamma^a S(x)\Lambda_a^b(x) = \gamma^b$$

$$e_a^{\mu'} = \Lambda_a^b(x) e_b^\mu$$

Relation to GR:

- at this stage we have two fields

- field strength tensor

$$[D_\mu, D_\nu] = -ig\frac{J_{ab}}{2}R^{ab}_{\mu\nu}$$

$$R^{ab}_{\mu\nu} = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + g(\omega^{ac}_\mu \ \omega_{\nu c}{}^{b} - \omega^{ac}_\nu \ \omega_{\mu c}{}^{b})$$

Impose metricity (or first order formalism) (g absorbed here)

$$\nabla_\mu e^a_\nu = 0 = \partial_\mu e^a_\nu + \omega^a{}_{b\mu} e^b_\nu - \Gamma_{\mu\nu}{}^{\lambda} e^a_\lambda$$

Obtain GR with Riemann tensor

$$R_{\mu\nu\alpha\beta} = e_{a\alpha} e_{b\beta} R^{ab}_{\mu\nu}$$

Asymptotic Freedom:

Consider usual gauge Lagrangian

$$\mathcal{L} = -\frac{1}{4} R^{ab}_{\mu\nu} R^{\mu\nu}_{ab}$$

This has SO(3,1) gauge symmetry (**non-compact**)

$$[J_{ab}, J_{cd}] = i\left(\eta_{ad} J_{bc} + \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac}\right)$$

which can be repackaged in more usual gauge notation

$$[J_{ab}, J_{cd}] = 2i f_{[ab][cd][ef]} J^{ef}$$

$$\begin{aligned} f_{[ab][cd][ef]} &= -\frac{1}{4}\left[\eta_{bc}\eta_{de}\eta_{fa} - \eta_{bd}\eta_{ce}\eta_{fa} - \eta_{bc}\eta_{df}\eta_{ea} + \eta_{bd}\eta_{cf}\eta_{ea}\right. \\ &\qquad \left. - \eta_{bca}\eta_{de}\eta_{fb} + \eta_{ad}\eta_{ce}\eta_{fb} + \eta_{ac}\eta_{df}\eta_{eb} - \eta_{ad}\eta_{cf}\eta_{eb}\right] \\ &\equiv 2\eta_{b][c}\eta_{d][e}\eta_{f][a} \end{aligned}$$

and

$$R^{[ab]}_{\mu\nu} = \partial_\mu \omega^{[ab]}_\nu - \partial_\nu \omega^{[ab]}_\mu + g f^{[ab]}{}_{[cd][ef]} \omega^{[cd]}_\mu \omega^{[ef]}_\nu$$

Gauge loops then proceed in the usual way, with substitution

$$f_{imn}f_{jmn} = C_2\delta_{ij} \quad \longrightarrow \quad f_{[ab][cd][ef]}f^{[gh][cd][ef]} = C_2\delta^{[gh]}_{[ab]}$$

with

$$C_2 = 2$$

This then yields the beta function

$$\beta(g) = -\frac{11C_2}{3}\frac{g^3}{16\pi^2} = -\frac{22}{3}\frac{g^3}{16\pi^2}$$

Note: Fermion loops do not contribute to this coupling. **Return to this later**

Confined, condensed, gapped?

Spin connection weakly coupled in UV

Strongly coupled in IR

Running defines a scale – perhaps M_P

Analogies would suggest confinement, but non-compact group?

Singlet channel is attractive, then perhaps condensation

Assume spin connection is not propagating at low energy
- **then symmetry must be realized with metric only**
- explains metric theory without need to assume metricity of vierbein

Should be able to be answered by lattice work

Note: Smilga and Holdom + Ren have suggested confinement for the metric field

What happens at low energy?

Euclidean gravity

For lattice studies transform to Euclidean space

Symmetry:
Lorentzian SO(3,1) goes to Euclidean O(4) (compact)
Beta function is the same
O(4) Yang Mills is confining

But is Euclidean gravity valid?
- most numerical studies assume this

If Euclidean gravity makes sense, the spin connection will be confined with usual action.

Intermediate version – partial Euclidean:

Lattice link variable:

$$U_\mu(n) = \exp\left[\frac{iga}{2} J_{ab}\omega_\mu^{ab}(n)\right]$$

And action defined on a plaquette

$$S = \frac{2}{g^2}\sum_p Re\mathrm{Tr}\left(1 - U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\hat{\nu})U_\mu^\dagger(n)\right)$$

Confinement test would be area law for Wilson loop

Note: cannot include fermions in this partial Euclidean version – only gauge action

Possible to use metricity condition itself as a regulator:

Covariant form:

$$\mathcal{L}_{met} = -\lambda \left[\eta_{ab} g^{\alpha\beta} \nabla_\mu e^a_\alpha \nabla^\mu e^b_\beta\right]^2$$

Contains spin connection – negative definite

$$\mathcal{L}_{met} \sim -\lambda \left[\eta_{ab} g^{\alpha\beta} g^{\mu\nu} \omega_\mu{}^a{}_c e^c_\alpha \omega_\nu{}^b{}_d e^d_\beta\right]^2$$

But, more ingredients needed

This example has flaws:

The a_2 coefficient describes real divergences involving the metric:

$$\Delta\mathcal{L} = \frac{3}{160\pi^2}\frac{2}{d-4}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$$

where the Weyl tensor is

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta} - \frac{1}{2}\left(R_{\mu\alpha}g_{\nu\beta} - R_{\nu\alpha}g_{\mu\beta} - R_{\mu\beta}g_{\mu\alpha} + R_{\nu\beta}g_{\mu\alpha}\right) \\ &+ \frac{R(g)}{6}\left(g_{\mu\alpha}g_{\nu\beta} - e_{\nu\alpha}e_{\mu\beta}\right) \end{aligned}$$

Therefore one needs to include scale (conformal) invariant action for metric also

In addition, fermion loop leads to new divergences (below)

Need a more extensive action for consistency

A scale invariant model:

Include Weyl squared term:

$$S_{s.i.} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g^2} R^{ab}_{\mu\nu} R^{\mu\nu}_{ab} - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

This is peturbatively complete – all perturbative divergences controlled

This theory is doubly asymptotically free:

In general coupled evolution:

$$\beta(g) = f_g(g,\xi) \qquad \beta(\xi) = f_\xi(g,\xi)$$

But at one loop, the beta functions are uncoupled and asymptotically free

$$\begin{aligned}\beta(g) &= -\frac{22}{3\pi^2}g^3 \\ \beta(\xi) &= -\frac{199}{480\pi^2}\xi^3 - \frac{3}{80\pi^2}\xi^3\end{aligned}$$

Fradkin Tseytlin

Induced effects with spin connection and proper time cutoff

$$\Delta \mathcal{L} = \int d^4x \frac{1}{16\pi^2}\left[-\frac{24}{\tau_0^4} - \frac{4}{\tau_0^2}R + ...\right]$$

Again, this is only free vector loop – not a real calculation
 - proper time regularization not appropriate for scale invariant theory

But still, the nature of the corrections is clear.

With dimensional transmutation, Einstein action will appear

Intermediate summary #2

Spin connection provides possible model for dimensional transmutation program

Treated as independent field, spin connection can have strong dynamics

Will it be confined/condensed?

Can this make a useful model of gravity?

D. A conformally invariant model

Weyl term is conformally invariant:

$$\Delta\mathcal{L} = \frac{1}{d-4} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$$

So is the result of the fermion loop.

But, fermion loop with spin connection not captured by above model

Suggests starting with a **conformally invariant** theory

Extra freedom with both vierbein and spin connection

Note: Other ways to construct conformally invariant theories, mostly with extra particles. This involves only vierbein , connection and fermion

Basic conformal symmetry

Local rescaling of the metric:

$$g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu} \qquad\qquad g^{\mu\nu} \to \Omega^{-2}(x) g^{\mu\nu}$$

Vierbein properties then follow:

$$e^a_\mu \to \Omega^{(}x) e^a_\mu \qquad\qquad e^\mu_a \to \Omega^{-1}(x) e^\mu_a \quad .$$

Notation

$$\Omega^2(x) = e^{2\sigma(x)} \qquad \text{such that} \qquad \Omega^{-1}(x)\partial_\mu \Omega(x) = \partial_\mu \sigma$$

Many ways to implement conformal symmetry
- I am trying for minimal version with just vierbein and spin connection

Weyl-squared action is unique conformally invariant if using only metric

Fermions and the spin connection

The fermion action

$$S_D = \int d^4x \sqrt{-g}\bar{\psi} \left[i\gamma^a e_a^\mu (\partial_\mu - i\frac{J_{ab}}{2}\omega_\mu^{ab}) \right] \psi$$

is conformally invariant under

$$\psi \to \Omega^{-3/2}\psi \qquad\qquad e_a^\mu \to \Omega^{-1}(x) e_a^\mu$$

if the spin connection transforms as

$$\omega_\mu^{ab} \to \omega_\mu^{ab} + (e_\mu^a \partial^b \sigma - e_\mu^b \partial^a \sigma)$$

New spin Weyl tensor:

While the Weyl tensor transforms covariantly

$$C_{\mu\nu\alpha\beta} \to \Omega^2 C_{\mu\nu\alpha\beta}$$

the equivalent formed from the spin connection does not

$$\begin{aligned}\delta R^{ab}_{\mu\nu} &= (d_\mu \partial^b \sigma) e^a_\nu - (d_\nu \partial^b \sigma) e^a_\mu \\ &- (d_\mu \partial^a \sigma) e^b_\nu + (d_\nu \partial^a \sigma) e^b_\mu \\ &+ \partial^b \sigma E^a_{\mu\nu} - \partial^a \sigma E^b_{\mu\nu}\end{aligned}$$

where

$$\begin{aligned}E^a_{\mu\nu} &= \nabla_\mu e^a_\nu - \nabla_\nu e^a_\mu = d_\mu e^a_\nu - d_\nu e^a_\mu \\ &= \partial_\mu e^a_\nu + g\omega_\mu^a{}_b e^b_\nu - \partial_\nu e^a_\mu - g\omega_\nu^a{}_b e^b_\mu\end{aligned}$$

The last term causes a lack of conformal invariance (vanishes if metricity assumed)

To compensate define

$$[d_a, d_b] = F_{ab}{}^c\, d_c - i\frac{g}{2} e^\mu_a e^\nu_b J_{cd}\, R^{cd}_{\mu\nu}$$

with

$$d_\mu \equiv \partial_\mu - i\frac{g}{2} J_{ab} \omega^{ab}_\mu \qquad d_a \equiv e^\mu_a d_\mu$$

such that

$$\begin{aligned}F_{ab}{}^c &= \left(d_a e^\mu_b - d_b e^\mu_a\right) e^c_\mu \\ &= e^\lambda_a \left(\partial_\lambda e^\mu_b + g\omega_{\lambda b}{}^d e^\mu_d\right) e^c_\mu - e^\lambda_b \left(\partial_\lambda e^\mu_a + g\omega_{\lambda a}{}^d e^\mu_d\right) e^c_\mu\end{aligned}$$

Then since:

$$E^a_{\mu\nu} \to \Omega E^a_{\mu\nu}$$

$$F_{ab}{}^c \to \Omega^{-1}\left[F_{ab}{}^c + 2(\partial_a\sigma\delta^c_b - \partial_b\sigma\delta^c_a)\right]$$

the new Weyl tensor can be formed using

$$\bar{R}^{ab}_{\mu\nu} = R^{ab}_{\mu\nu} + \frac{1}{2}F^{ab}{}_c E^c_{\mu\nu}$$

and has the form

$$\begin{aligned} D^{ab}_{\mu\nu} &= \bar{R}^{ab}_{\mu\nu} - \frac{1}{2}\left(\bar{R}^a_\mu e^b_\nu - \bar{R}^a_\nu e^b_\mu - \bar{R}^b_\mu e^a_\mu + \bar{R}^b_\nu e^a_\mu\right) \\ &+ \frac{\bar{R}}{6}\left(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu\right) \end{aligned}$$

This is conformally invariant $\quad D^{ab}_{\mu\nu} \to D^{ab}_{\mu\nu}$

and its action is also:

$$S_D = \int d^4x\sqrt{-g}\; D^{ab}_{\mu\nu} D^{\mu\nu}_{ab}$$

Now look at fermion loop effect

Must be conformal and fully covariant

Define $\tilde{w}_d = \epsilon_{abcd} e^{a\mu} \omega_\mu^{bc}$

Direct calculation:

$$\Delta\mathcal{L} = -\frac{1}{384\pi^2\epsilon}\partial_a \tilde{w}_b \partial_{a'} \tilde{w}_{b'} \left[\eta^{aa'}\eta^{bb'} - \eta^{ab'}\eta^{ba'}\right]$$

The full covariant completion, using

$$\begin{aligned} E^c_{\mu\nu} &= \nabla_\mu e^c_\nu - \nabla_\nu e^c_\nu = \partial_\mu e^c_\nu - \partial_\nu e^c_\mu + \omega^c_{\mu d} e^d_\nu - \omega^c_{\nu d} e^d_\mu \\ \tilde{N}_\mu &= \frac{1}{2}\epsilon_{abcd} e^{a\lambda} e^{b\nu} E^c_{\lambda\nu} e^d_\mu \end{aligned}$$

$$\tilde{N}_{\mu\nu} = \partial_\mu \tilde{N}_\nu - \partial_\nu \tilde{N}_\mu$$

then

$$\Delta\mathcal{L} = -\frac{1}{192\pi^2\epsilon}\frac{1}{4}\tilde{N}_{\mu\nu}\tilde{N}^{\mu\nu}$$

Vanishes if metricity is imposed

Other conformal invariants:

The metricity condition is itself conformally covariant

$$\nabla_\mu e^a_\nu \to \Omega \nabla_\mu e^a_\nu \quad .$$

which implies also

$$E^a_{\mu\nu} \to \Omega E^a_{\mu\nu}$$

$$\begin{aligned} E^a_{\mu\nu} &= \nabla_\mu e^a_\nu - \nabla_\nu e^a_\mu = d_\mu e^a_\nu - d_\nu e^a_\mu \\ &= \partial_\mu e^a_\nu + g\omega^a_\mu \;{}_b e^b_\nu - \partial_\nu e^a_\mu - g\omega^a_\nu \;{}_b e^b_\mu \end{aligned}$$

This allows new conformal invariant vectors

$$V^{ab}_{1\mu} = e^{a\nu} \nabla_\mu e^b_\nu = V^{[ab]}_{1\mu}$$

$$V^{ab}_{2\mu} = e^a_\mu \nabla^\nu e^b_\nu$$

$$V^{ab}_{3\mu} = e^{a\nu} E^b_{\mu\nu}$$

$$V^{ab}_{4\mu} = e^{a\nu} \tilde{E}^b_{\mu\nu}$$

along with contractions

$$V_{1\mu} = 0 = \eta_{ab} V_{1\mu}^{ab}$$

$$V_{2\mu} = e_{b\mu} \nabla^{\nu} e_{\nu}^{b} = \eta_{ab} V_{2\mu}^{ab}$$

$$V_{3\mu} = e_{b}^{\nu} E_{\mu\nu}^{b} = \eta_{ab} V_{3\mu}^{ab}$$

$$V_{4\mu} = e_{b}^{\nu} \tilde{E}_{\mu\nu}^{b} = \eta_{ab} V_{4\mu}^{ab}$$

and corresponding conformally invariant field strengths

$$V_{i\mu\nu} = \partial_{\mu} V_{i\nu} - \partial_{\nu} V_{i\mu} \qquad (i \; = \; 2, \; 3, \; 4)$$

The full conformally invariant model

The primary field strength tensors:

$$\mathcal{L}_2 = -\frac{1}{4g_1^2} D^{ab}_{\mu\nu} D^{\mu\nu}_{ab} - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$$
$$+ \ +\alpha C_{\mu\nu\alpha\beta} e^\alpha_a e^\beta_b D^{ab\mu\nu} \quad .$$

Designer propagators:
- also can add terms proportional to the metricity condition

$$S_2 = \int d^4x - \frac{1}{4} \left[\gamma_2 V_{2\mu\nu} V_2^{\mu\nu} + \gamma_3 V_{3\mu\nu} V_3^{\mu\nu} + \gamma_{23} V_{2\mu\nu} V_3^{\mu\nu} + \gamma_4 V_{4\mu\nu} V_4^{\mu\nu}\right]$$

Note: mixing in vierbein and connection propagators is unavoidable

$$< \omega\omega > \sim \frac{1}{p^2} \qquad < ee > \sim \frac{1}{p^4} \qquad < e\omega > \sim \frac{1}{p^4} p^3 \frac{1}{p^2}$$

This could be an advantage in making propagators well behaved

Interaction terms:

Cubic:

$$
\begin{aligned}
\mathcal{L}_3 \;=\; & \sum_{ij} C^{\mu\nu\alpha\beta}\left[a_{ij} e^{\mu}_{e} e^{\alpha}_{f} \eta_{cd} + b_{ij} e^{\mu}_{c} e^{\alpha}_{f} \eta_{de}\right] V^{cd}_{i\nu} V^{ef}_{j\beta} \\
& + \sum_{ij} D^{\mu\nu}_{ab}\left[c_{ij} \delta^{a}_{e} \delta^{b}_{f} \eta_{cd} + d_{ij} \delta^{a}_{c} \delta^{b}_{f} \eta_{de}\right] V^{cd}_{i\mu} V^{ef}_{j\nu} \\
& + \sum_{ij} V^{\mu\nu}_{i}\left[f_{ijk} \eta_{ef} \eta_{cd} + g_{ijk} \eta_{cf} \eta_{de}\right] V^{cd}_{j\mu} V^{ef}_{k\nu}
\end{aligned}
$$

Quartic:

$$
\mathcal{L}_4 = -\sum_{ijkl} \mathrm{Perms}_{ab..h}\; \lambda_{ijkl}\; V^{ab}_{i\mu} V^{cd\mu}_{j} V^{ef}_{k\nu} V^{gh\nu}_{l}
$$

<u>A potentially rich theory:</u>

Too many invariants!

But perhaps not all are needed

For analysis, this is a major complication

But, perhaps this allows freedom to solve potential problems

For example, propagator mixing and ghosts

Much to be done:

Next steps:

Gauge fixing for conformal model without explicit conformal breaking

Beta function calculations

……

Side comment: Alternate possibility

Use metricity condition in reverse to express vierbein in terms of spin connection

$$e^\nu_b \left[\partial_\mu e^a_\nu - \Gamma_{\mu\nu}{}^\lambda e^a_\lambda\right] = -\omega^a{}_{b\mu}$$

with boundary condition

$$e^a_\nu = \delta^a_\nu \qquad \text{when} \qquad \omega^a{}_{b\mu} = 0$$

Then the Lagrangian collapses down to a single term

$$\mathcal{L} = \frac{1}{4g^2} D^{ab}_{\mu\nu} D^{\mu\nu}_{ab} .$$

Of course, it could be tricky dealing with such a constraint

Wilson line representation?

The λ_1 term can serve as a lagrange multiplier

Second side comment: Unimodular gravity

Conformal rescaling can leave a unimodular metric:

$$g_{\mu\nu}(x) = \Omega^2(x)\hat{g}_{\mu\nu}(x) \qquad \text{with} \qquad \det(\hat{g}_{\mu\nu}) = -1$$

For example, consider gauge Lagrangian:
-with rescaling $\sqrt[4]{-g}g^{\mu\nu} = \hat{g}^{\mu\nu}$, one obtains:

$$\int d^4x\, \sqrt{-g}g^{\mu\alpha}g^{\nu\beta}R^{ab}_{\mu\nu}R_{\alpha\beta ab} = \int d^4x\, \hat{g}^{\mu\alpha}\hat{g}^{\nu\beta}R^{ab}_{\mu\nu}R_{\alpha\beta ab}$$

But, PI measure generally not invariant

Could potentially limit invariance to unimodular invariance

Decouples cosmological constant from vacuum energy
- instead – initial condition

Possibility of small c.c from conformal invariant initial condition?

Comments:

The Planck scale may not be the ultimate barrier

- certainly EFT indicates strong coupling
- but can emerge as weak coupling in the UV

If gravity can be a conventional field theory, it probably should look like this

- scale/conformal invariant actions are most promising
- extra conformal symmetry attractive for fundamental gravity

The spin connection can live as an independent field

- most natural as a gauge field

The spin connection (with usual gauge interaction) is asymptotically free

- Confined or condensed?
- weak coupling beyond Planck scale

Dimensional transmutation can yield Einstein action

- weak coupling in the IR

Conformal model should be closed under renormalization