

# Testing GR with GWs



**Vítor Cardoso**  
(Técnico & Perimeter)



**1900**

Derives astronomical bounds  
on curvature radius of space:

64 light years if hyperbolic

1600 light years if elliptic

**1914**

Volunteers for war

Belgium: weather station

France, Russia: artillery trajectories

**March 1916**

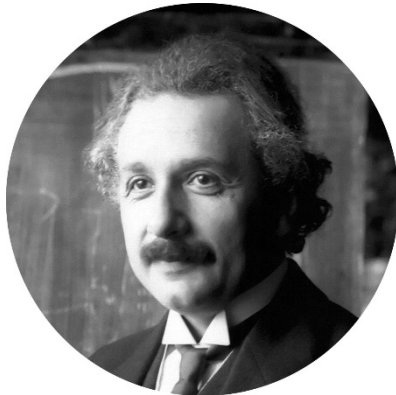
Sent home, ill with pemphigus.

Dies in May.



*“I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: ...”*

K. Schwarzschild to A. Einstein  
(Letter dated 22 December 1915)



Solution re-discovered by many others:

J. Droste, May 1916 (part of PhD thesis under Lorentz): Same coordinates, more elegant

P. Painlevé, 1921, A. Gullstrand, 1922: P-G coordinates (not realize solution was the same)

...and others

**Long, complex  
path to correct  
interpretation**



**Eddington**



**Lemaître**



**Oppenheimer**



**Snyder**



**Wheeler**



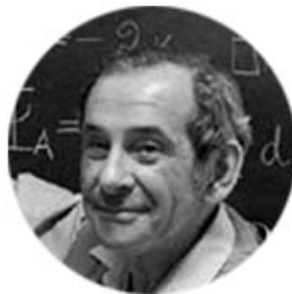
**Finkelstein**



**Kruskal**



**Penrose**



**Israel**

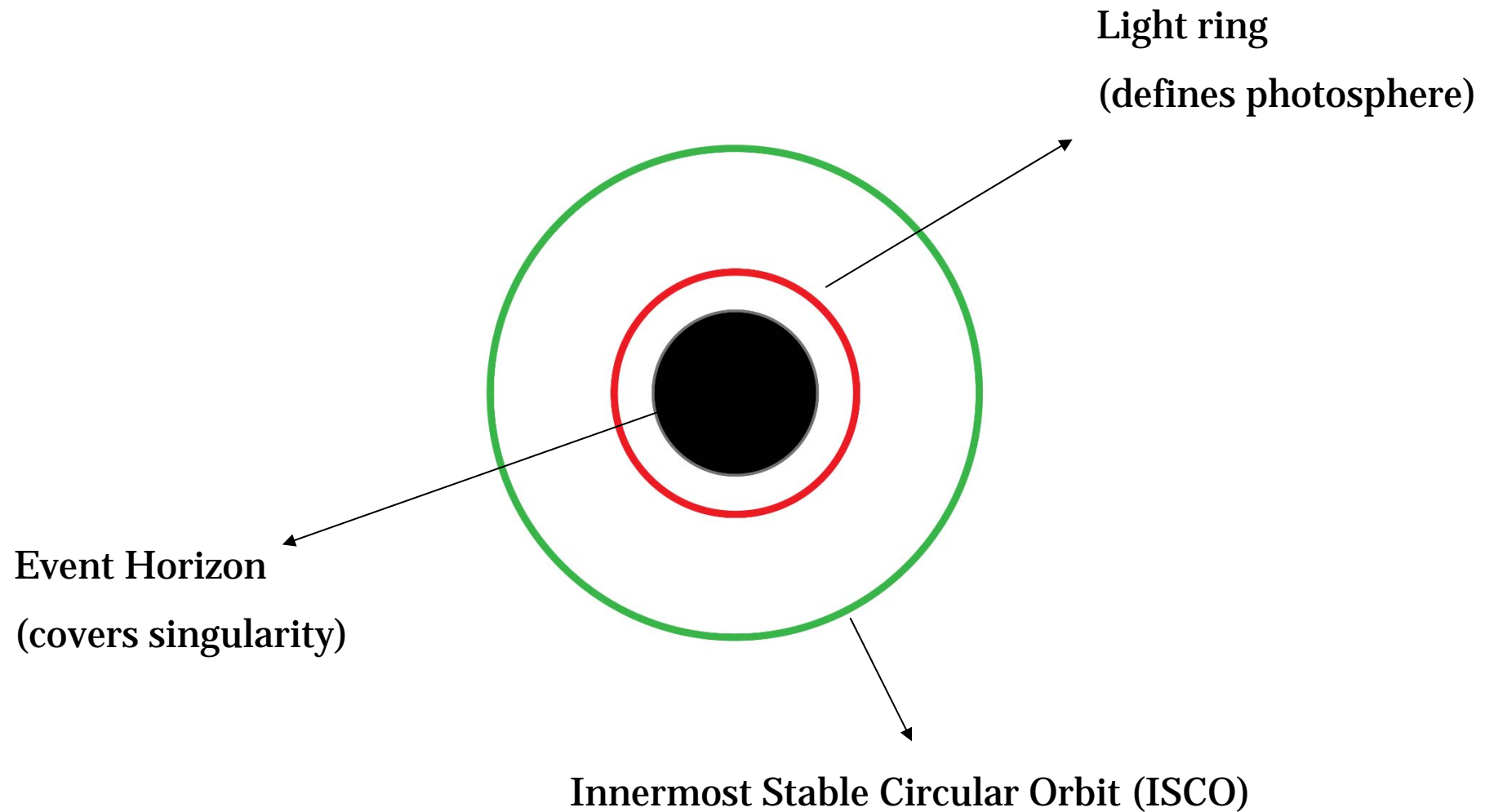


**Carter**



**Hawking**

# Black holes



$$\text{Specific energy} = \frac{2\sqrt{2}}{3} = 0.94$$

# Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975):

A stationary, asymptotically flat, vacuum solution must be Kerr

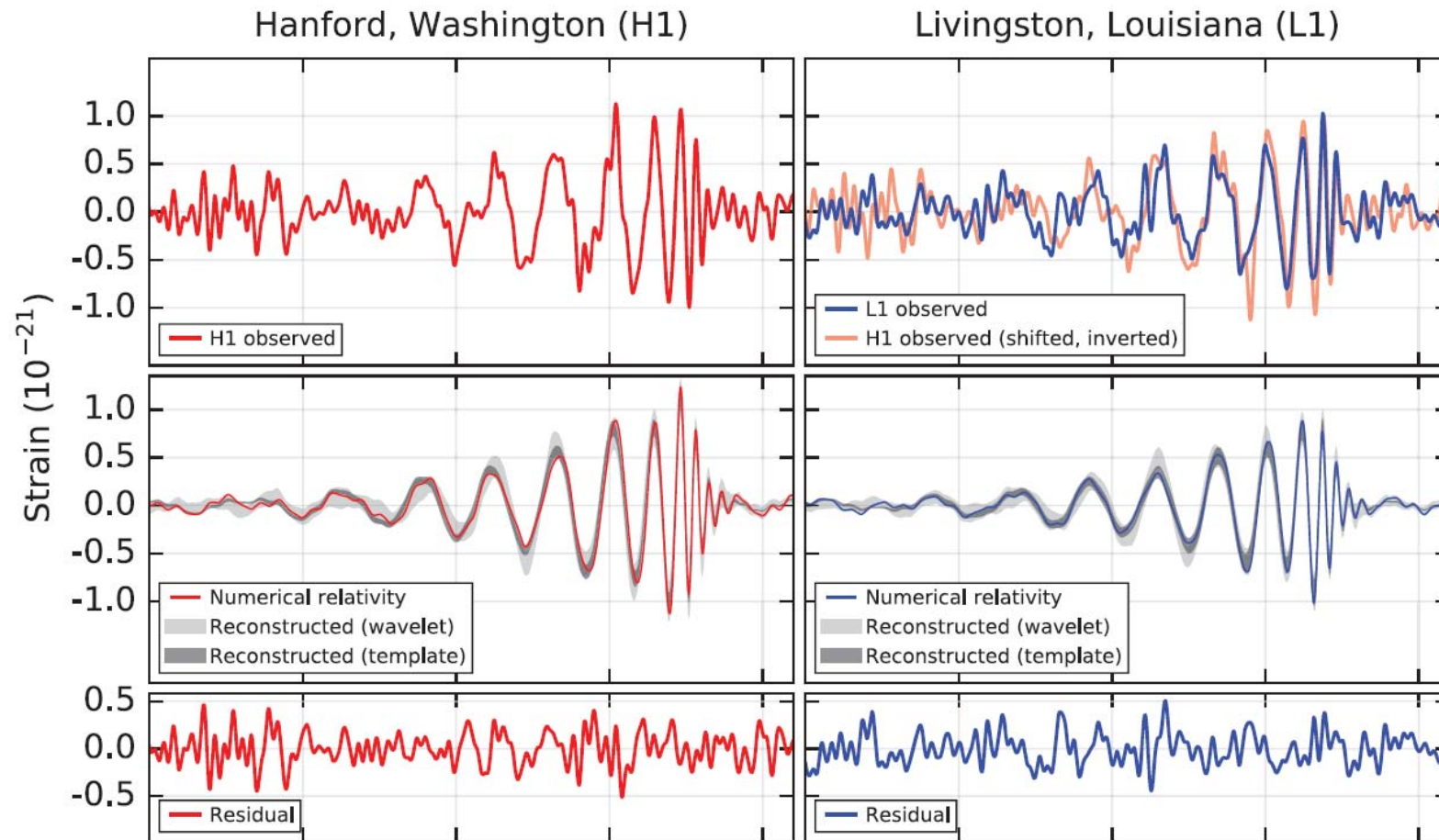
$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta) \sin^2 \theta}{\Sigma} dt d\phi - \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass  $M$  and angular momentum  $J=aM$ ,  $a < M$

“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

*S. Chandrasekhar*, The Nora and Edward Ryerson lecture, Chicago April 22 1975





**FÍSICA  
EINSTEIN TINHA  
RAZÃO: AS ONDAS  
GRAVITACIONAIS  
EXISTEM MESMO**  
Destaque, 2 a 4

Simulação de dois buracos negros em colisão, provocando ondas gravitacionais

"All the News  
That's Fit to Print"

VOL. CLXV . . . No. 57,140

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FRIDAY, FEBRUARY 12, 2016

# The New York Times

**WITH FAINT CHIRP,  
SCIENTISTS PROVE  
EINSTEIN CORRECT**

**A RIPPLE IN SPACE-TIME**

**An Echo of Black Holes  
Colliding a Billion  
Light-Years Away**

**By DENNIS OVERBYE**

A team of scientists announced on Thursday that they had heard and recorded the sound of two black holes colliding, a billion light-years away, a fleeting chirp that fulfilled the last prediction of Einstein's general theory of relativity.

That faint rising tone, physicists say, is the first direct evidence of gravitational waves, the ripples in the fabric of space-time that Einstein predicted a century ago. It completes his vision of a universe in which space and time are intertwined and dynamic, able to stretch, shrink and jiggle. And it is a ringing confirmation of the nature of black holes, the bottomless gravitational pits from which not even light can escape, which were the most foreboding (and unwelcome) part of his theory.

More generally, it means that a century of innovation, testing, questioning and plain hard work after Einstein imagined it on paper, scientists have tapped into the deepest reaches of abstract

**Clinton Paints Sanders Plans As Unrealistic**

**New Lines of Attack at Milwaukee Debate**

**By AMY CHOCICK and PATRICK HEALY**

MILWAUKEE — Hillary Clinton, scrambling to recover from her double-digit defeat in the New Hampshire primary, repeatedly challenged the trillion-dollar policy plans of Bernie Sanders at their presidential debate on Thursday night and portrayed him as a big talker who needed to "level" with voters about the difficulty of accomplishing his agenda.

Foreign affairs also took on unusual prominence as Mrs. Clinton sought to underscore her experience and Mr. Sanders excoriated her judgment on Libya and Iraq, as well as her previous praise of former Secretary of State Henry A. Kissinger. But Mrs. Clinton was frequently on the offensive as well, seizing an opportunity to take about leaders she admired and turning it against Mr. Sanders by bashing his past criticism of President Obama — a remark that Mr. Sanders called a "low blow."

With tensions between the two Democrats becoming increasingly obvious, the debate was full of new lines of attack from Mrs. Clinton, who faces pressure to puncture Mr. Sanders's growing popularity before the next nominating contests in Nevada and South Carolina. She is waging that even voters excited by Mr. Sanders's inspiring message will reconsider their support when they learn of his lack of experi-

A worker installed a baffle in 2010 to control light in the Laser Interferometer Gravitational-Wave Observatory in Hanford, Wash.

**Long in Clinton's Corner, Blacks Notice Sanders**

**Courted Hard in South Carolina, Loyalists Listen Closely**

**By RICHARD FANUSSET**

ORANGEBURG, S.C. — When Helen Duley was asked whom she would vote for in the South Carolina primary, she answered as if the very question were absurd.

"What I'm seeing is a bunch of confusion, hearsay and foolishness," said Ms. Duley, 60, a retired nurse assistant who is African: Hillary Clinton."

But that was late January. Interviewed again Tuesday as Mrs. Clinton's rival, Senator Bernie

**Last Occupier In Rural Oregon Is Coaxed Out**

This article is by **Deve Semmura, Richard Perez-Peña and Kirk Johnson**.

PRINCETON, Ore. — They implored the last holdout in the armed occupation of a wildlife



# LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less  
Agog Over Results of Eclipse  
Observations.

## EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed  
or Were Calculated to be,  
but Nobody Need Worry.

## A BOOK FOR 12 WISE MEN

No More in All the World Could

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# DON'T WORRY OVER NEW LIGHT THEORY

Physicists Agree That It Can Be  
Disregarded for Practical  
Purposes.

## NEWTON'S LAW IS SAFE

At Most It Suffers Only Slight  
Correction, Says Prof. Bum-  
stead of Yale University.

## OTHER PROFESSORS' VIEWS

# EINSTEIN EXPOUNDS HIS NEW THEORY

It Discards Absolutè Time and  
Space, Recognizing Them Only  
as Related to Moving Systems.

## IMPROVES ON NEWTON

Whose Approximations Hold for  
Most Motions, but Not Those  
of the Highest Velocity.

INSPIRED AS NEWTON WAS

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Articles in The Times  
from Nov. 10, 1919,  
left; Nov. 16, 1919,  
center; and Dec. 3,  
1919.

# **EVEN EINSTEIN'S LITTLE UNIVERSE IS BIG ENOUGH**

**Ten Trillion Times as Wide as  
the Orbit of the Earth, Prof.  
Eisenhart Estimates.**

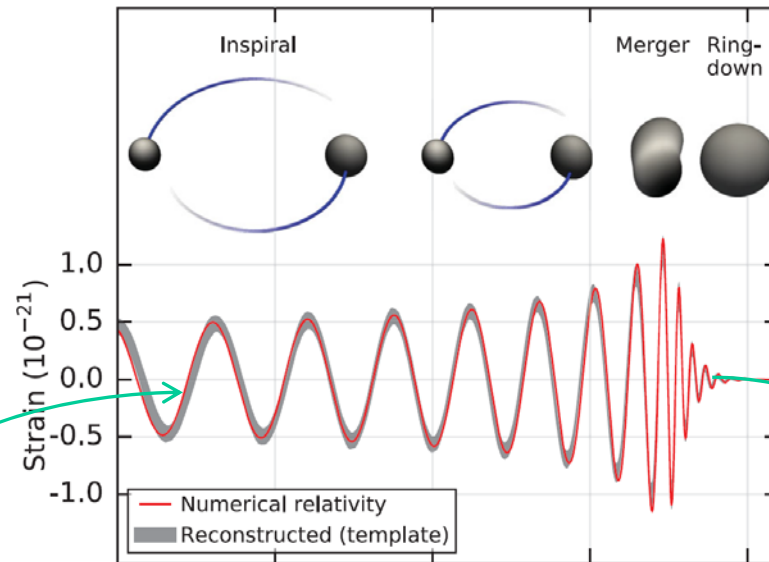
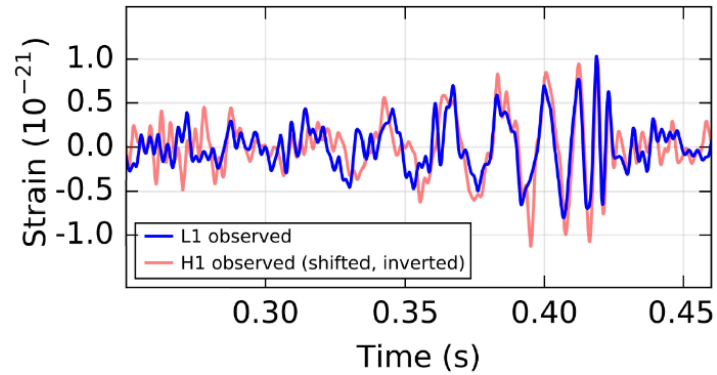
**MAY BE NO SPACE OUTSIDE**

**Light at 186,000 Miles a Sec-  
ond Would Require a Billion  
Years to Go Round It.**

**PROF. EINSTEIN ELUCIDATES**

**(For Those Who Can Understand)  
His Theory That Our Cosmos  
Has Its Limits.**

An article from the  
front page of The  
Times on Feb. 2,  
1921.



Inspiral stage, quasi-Newtonian:

Determine individual masses and spins

Is there dipolar radiation?

Other signs of modified gravity (tidal Love)?

Dark matter imprints?

Breathing of final object (relativistic):

Spectroscopy: measure mass and spin

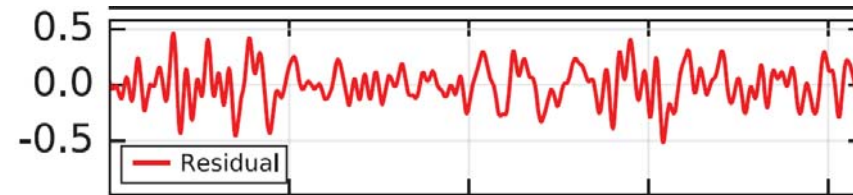
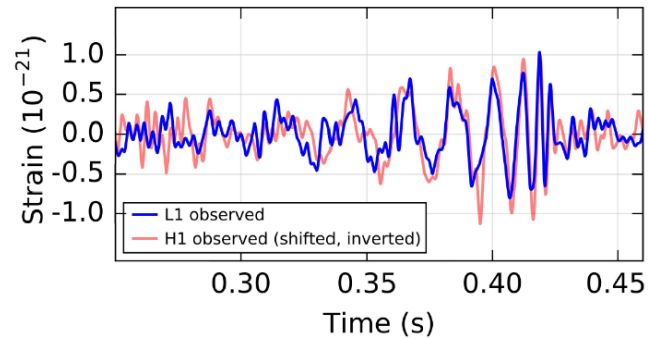
Test GR

Are there event horizons?

# Fundamental questions

- a. Are there hints of CC violation?
- b. Is the final - or initial - object really a black hole?
- c. Are there extra channels of radiation?
- d. Can we use GWs to tests (some) dark matter models?

# Gagging exotica



The residual subtracting the best-fit NR template for a binary BH merger is consistent with noise

Astonishingly simple transition from inspiral to merger

Very rapid “ringdown”

No signs of naked singularities

# Extreme collisions

Maximum luminosity: 0.1 Dyson-Thorne-Gibbons limit ( $c^5/G$ )

Up to, and not more than, 50% of CM is radiated

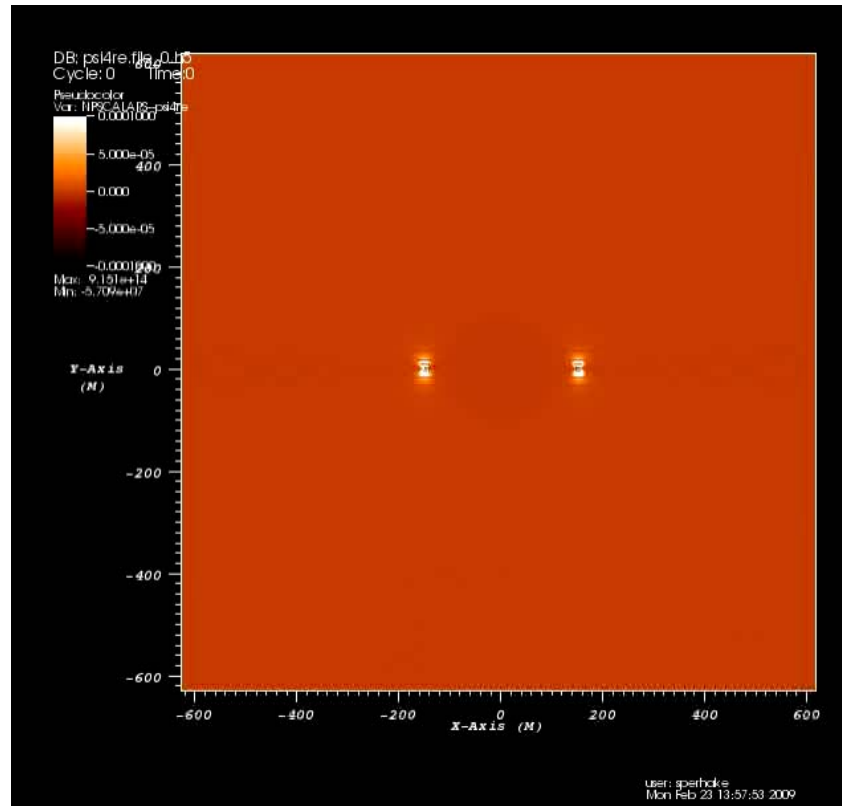
No signs of singularity, even when angular momentum “too” large

Sperhake et al PRL101:161101(2008); PRL103:131102 (2009)

Gundlach et al, PRD86: 084022 (2012); Choptuik & Pretorius PRL104:111101 (2010)

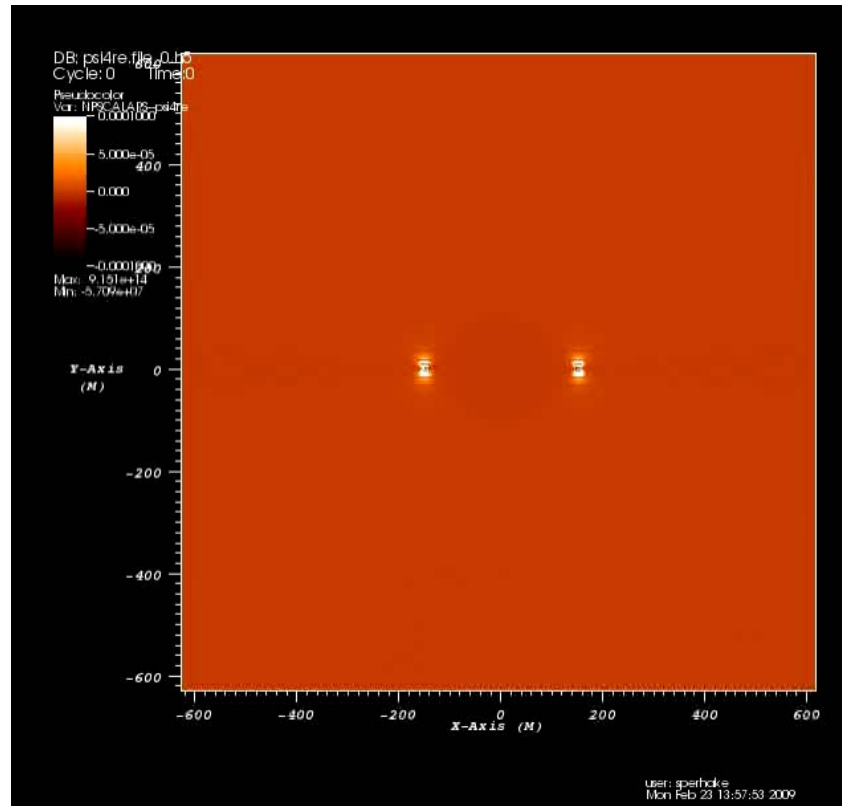
East & Pretorius PRL110:101101 (2013); Rezzolla & Takami CQG30:012001 (2013)

# Extreme CC violation attempts



Sperhake et al, PRL101:161101 (2008); PRL103:131102 (2009)

# Extreme CC violation attempts



Sperhake et al, PRL101:161101 (2008); PRL103:131102 (2009)

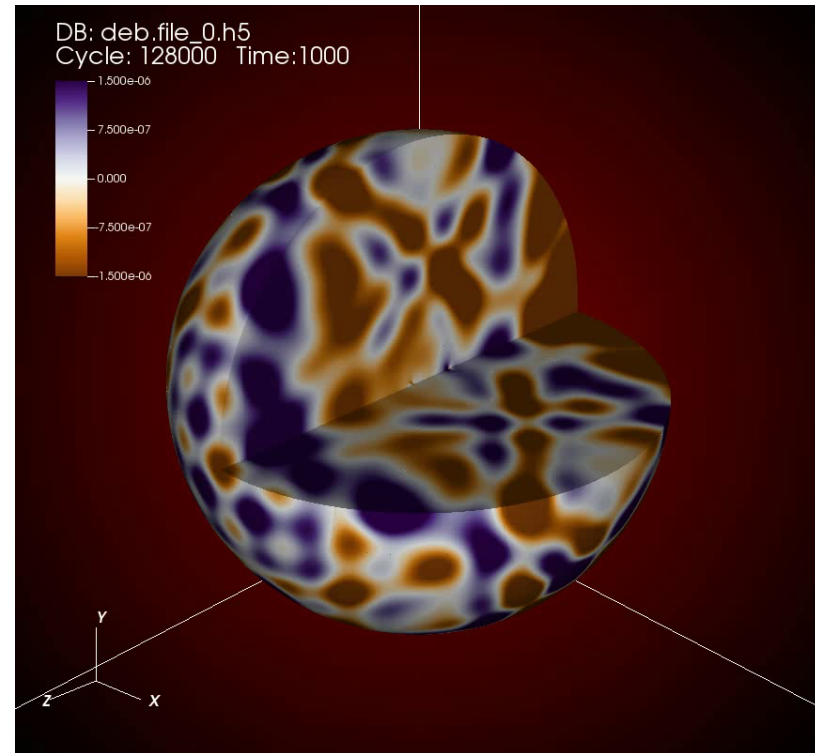


# Bounding dipolar radiation from inspiral

Electric charge (hidden?)

Modified theories of gravity

Scalar charge (perhaps induced?)



*Sperhake et al PRD87:124020 (2013)*

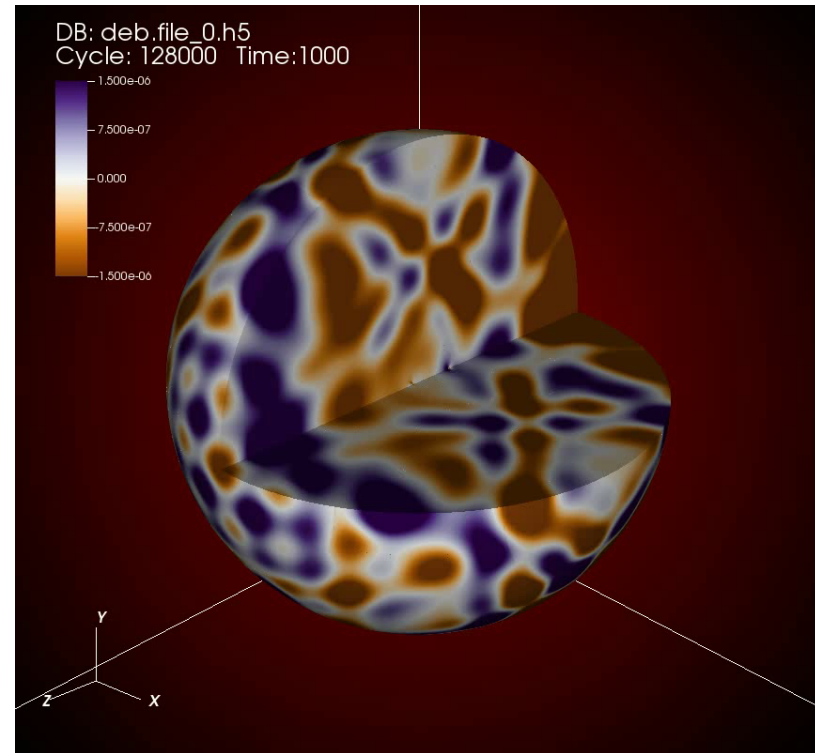
*Cardoso et al, Liv. Rev. Rel. (2014)*

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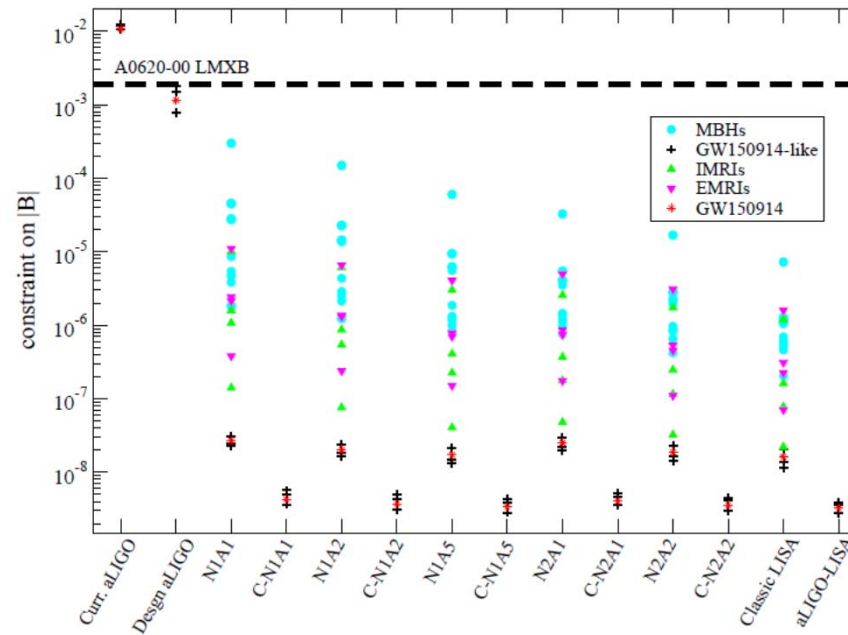
*Sperhake et al PRD87:124020 (2013)*

*Cardoso et al, Liv. Rev. Rel. (2014)*

# Bounding dipolar radiation from inspiral

Barausse, Yunes, Chamberlain, PRL116:241104 (2016)

$$\dot{E}_{\text{GW}} = \dot{E}_{\text{GR}} \left[ 1 + B \left( \frac{Gm}{r_{12}c^2} \right)^{-1} \right]$$

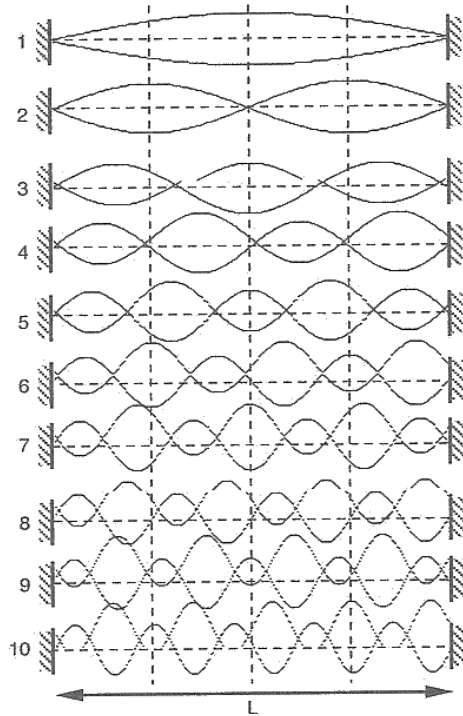


For EM charge or hidden vectors

$$B = \frac{5}{24} \left( \frac{Q_1}{M_1} - \frac{Q_2}{M_2} \right)^2$$

Cardoso et al, JCAP1605:054 (2016)

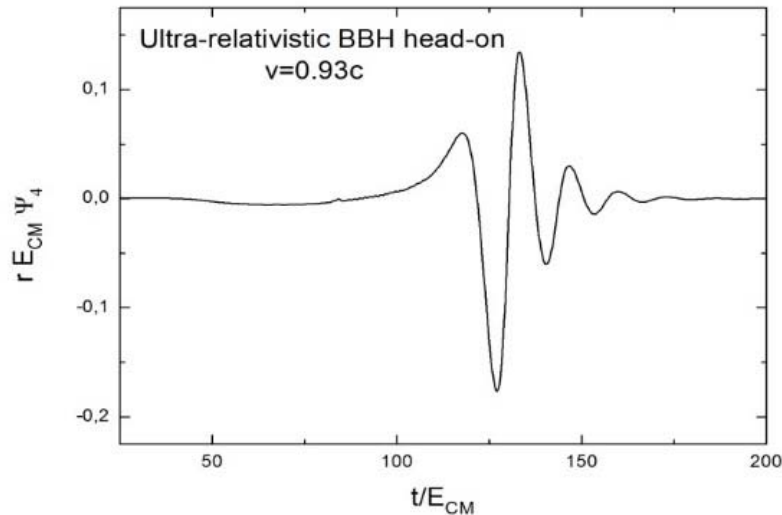
# Tests of the no-hair hypothesis



$$f = \frac{nv}{2L}, \quad n = 1, 2, 3\dots$$

Measure fundamental mode, determine length  $L$ .  
Measure first overtone, test if it's a string...

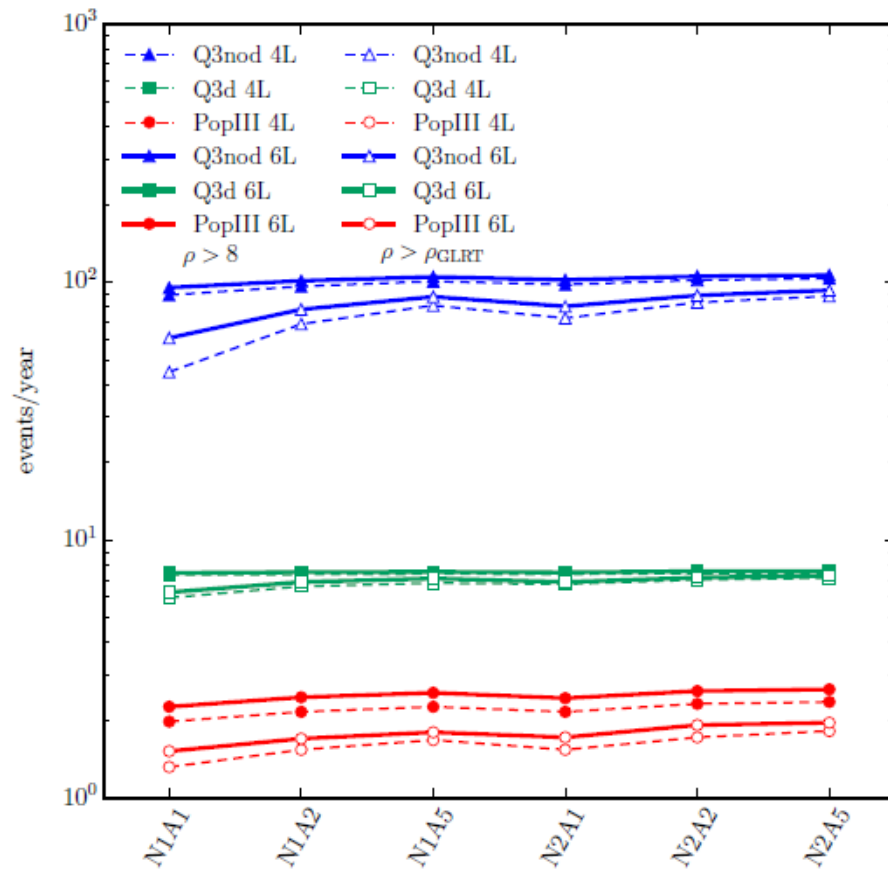
# Tests of the no-hair hypothesis



$$M\omega_{22} = 1.5251 - 1.1568 (1 - j)^{0.1292}$$
$$Q_{22} = 0.7 + 1.4187 (1 - j)^{-0.499}$$

Measure dominant mode, measure mass  
and spin. Measure second mode, test GR...

*Berti, Cardoso, Will PRD73:064030 (2006)*



Rates of binary BH mergers that yield detectable ringdown signals (filled symbols) and allow for spectroscopical tests (hollow symbols) with LISA (6-link (solid) and 4-link (dashed)) configurations with varying armlength and acceleration noise.

# Estimate extra couplings from ringdown

$$\frac{Q}{M} \lesssim 0.1 \sqrt{\frac{100}{\rho}}$$

$$\frac{\alpha}{M^2} \lesssim 0.4 \sqrt{\frac{100}{\rho}}$$

$$\alpha_{\text{DCS}} \lesssim 0.1 \sqrt{\frac{100}{\rho}}$$

Cardoso et al, JCAP 1605: 054 (2016)

Blázquez-Salcedo et al, arXiv:1609.01286

# **GWs and dark matter**



# GWs and dark matter I

Dark matter is not a strong-field phenomenon, but GW observations may reveal a more “mundane” explanation in terms of heavy BHs

Bird et al, PRL116:201301 (2016)  
Cless & Garcia-Bellido, arXiv:1610.08479

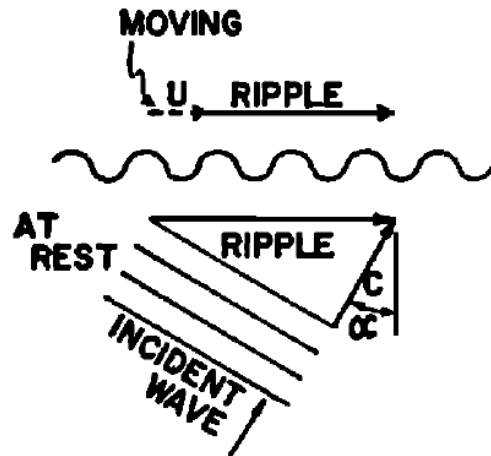
# GWs and dark matter II

Inspiral occurs in dark-matter rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role

Eda et al, PRL110:221101 (2013)

Macedo et al, ApJ774:48 (2013)

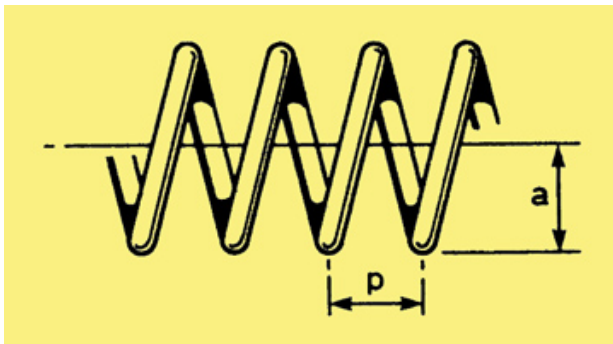
# GWs and DM III: friction & superradiance



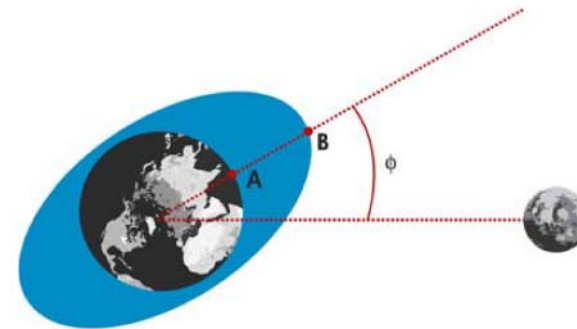
*Ribner, J. Acous. Soc. Amer. 29 (1957)*



*Tamm & Frank, Doklady AN SSSR 14 (1937)*

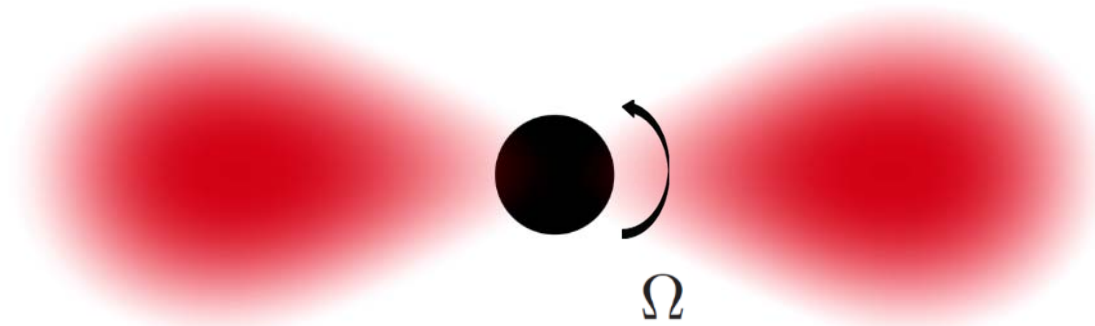


*Pierce (& Kompfner), Bell Lab Series (1947)*  
Ginzburg, anomalous Doppler year



*G. H. Darwin, Philos. Trans. R. Soc. London 171 (1880)*

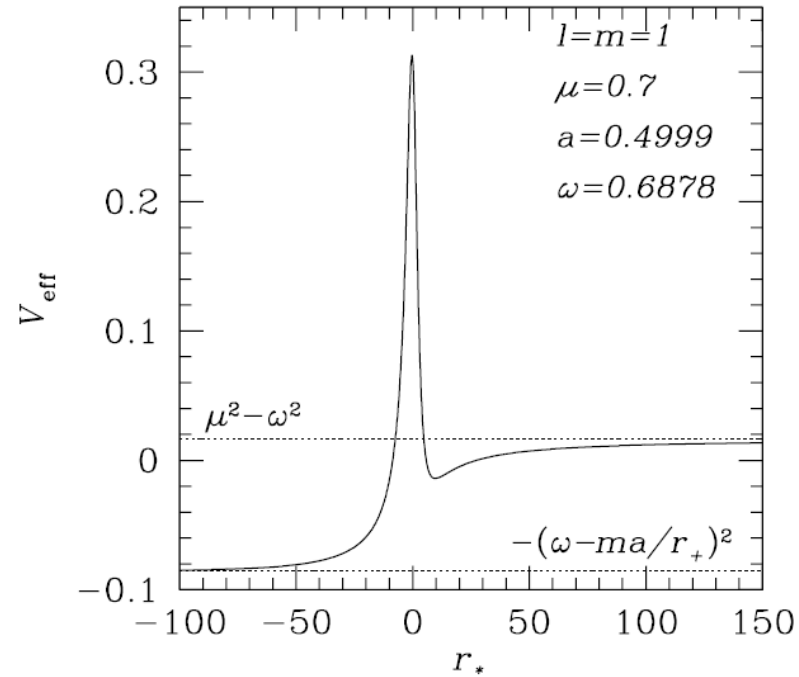
$$\Phi \sim e^{-i\omega t + im\phi} \rightarrow (\text{Angular}) \text{ phase velocity} = \frac{\omega}{m}$$



$$\omega < m\Omega$$

Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. 14 (1971)

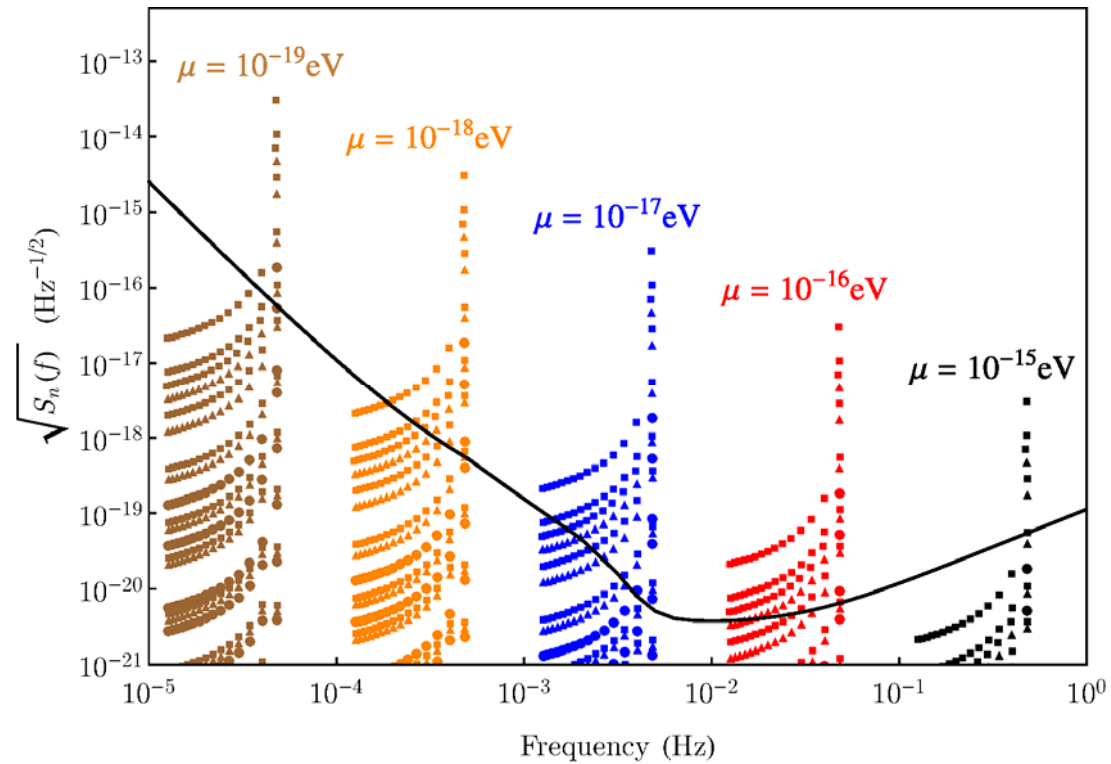
Brito, Cardoso, Pani, *Superradiance*, Lectures Notes in Physics 906 (2015)



$$\tau \sim 100 \left( \frac{10^6 M_{\odot}}{M} \right)^8 \left( \frac{10^{-16} \text{eV}}{\mu} \right)^9 \text{ seconds}$$

**Massive “states” around Kerr are linearly unstable**

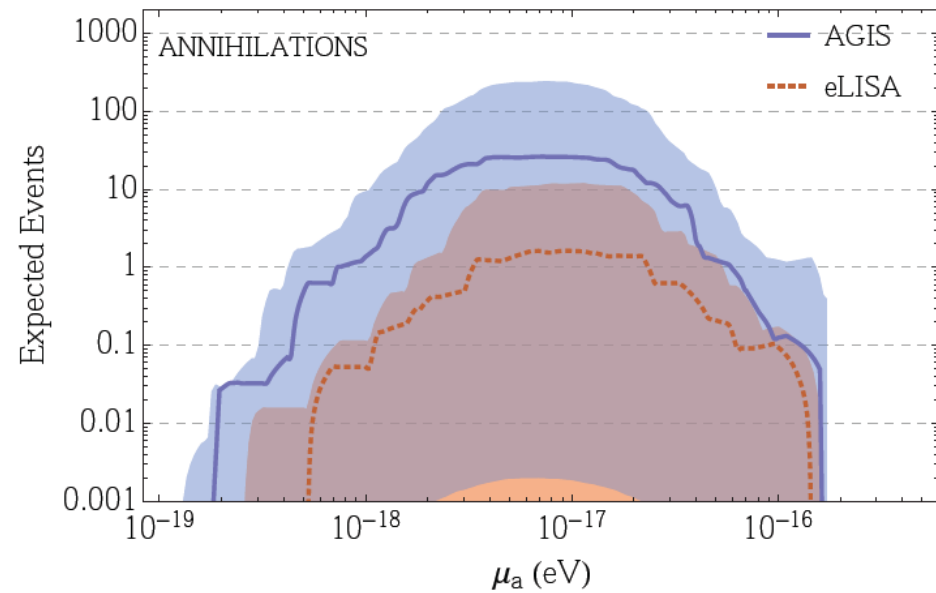
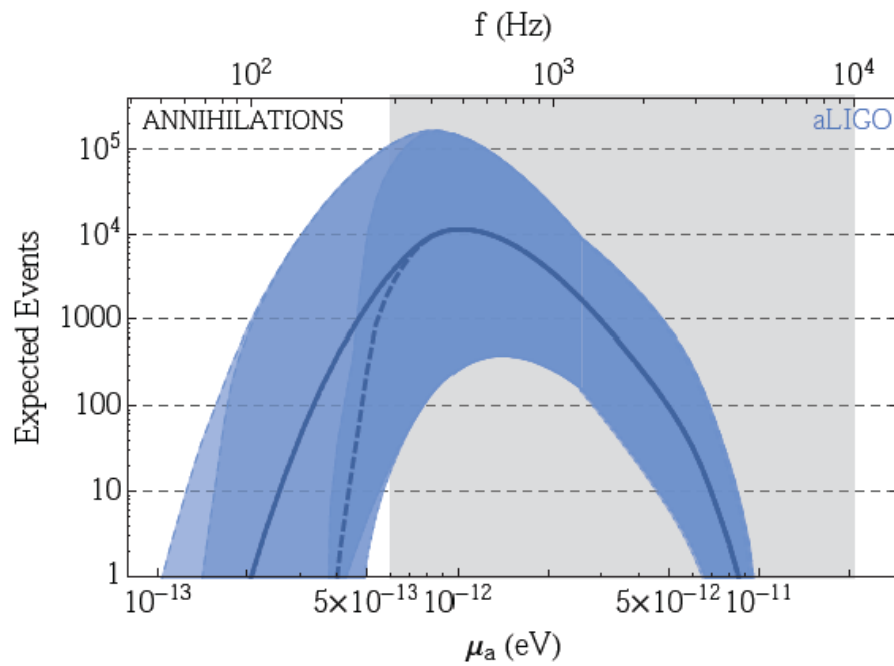
*Damour et al Lett. Nuovo Cimento*15: 257 (1974); *Detweiler PRD*22:2323 (1980);  
*Cardoso & Yoshida JHEP*0507:009 (2005). See review *Brito et al arXiv*:1501.06570



Squares ( $a=0.998$ ), triangles ( $a=0.898$ ) and circles ( $a=0.689$ ).  
 For each set of markers the largest heff means  $uM \sim 0.43$  and  
 going down in a vertical line means decreasing  $\text{Log}_{10}(M_{\text{BH}})$

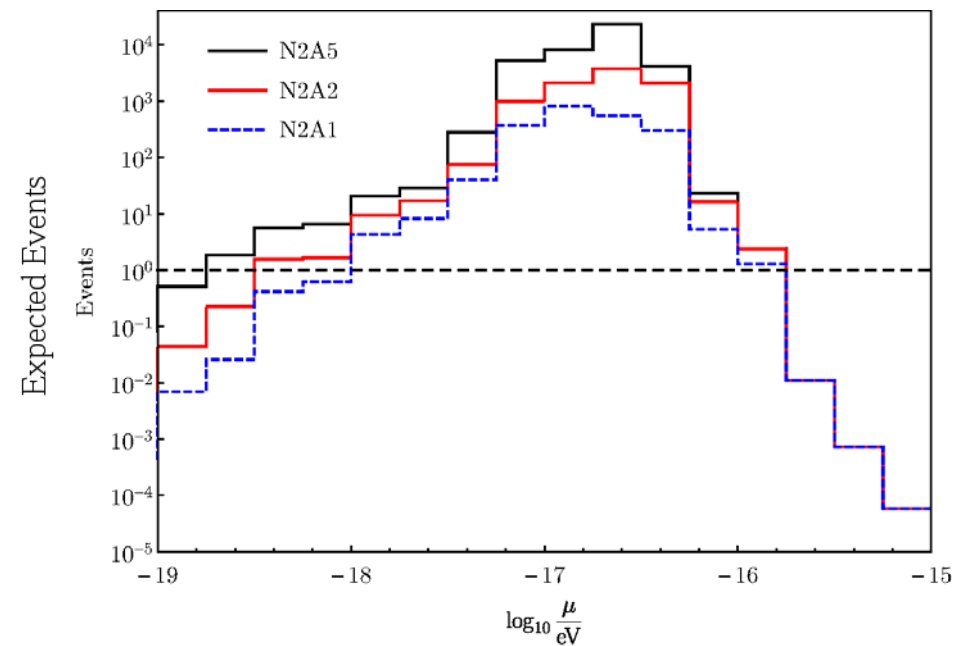
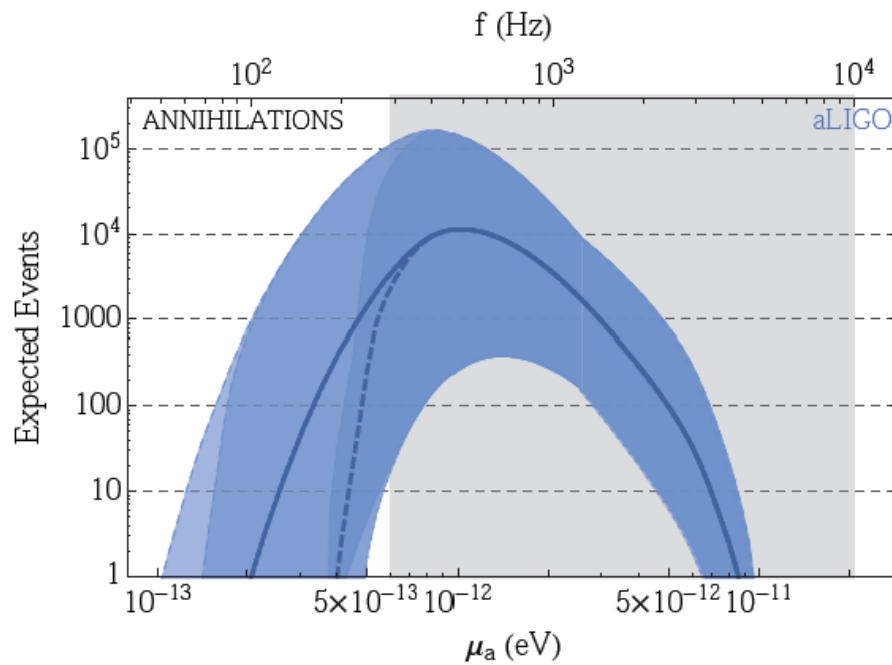
*Brito et al, in preparation (2017)*

# Wonderful sources for different GW-detectors!



*Arvanitaki, Baryakhtar, Huang arXiv:1411.2263*

# Wonderful sources for different GW-detectors!



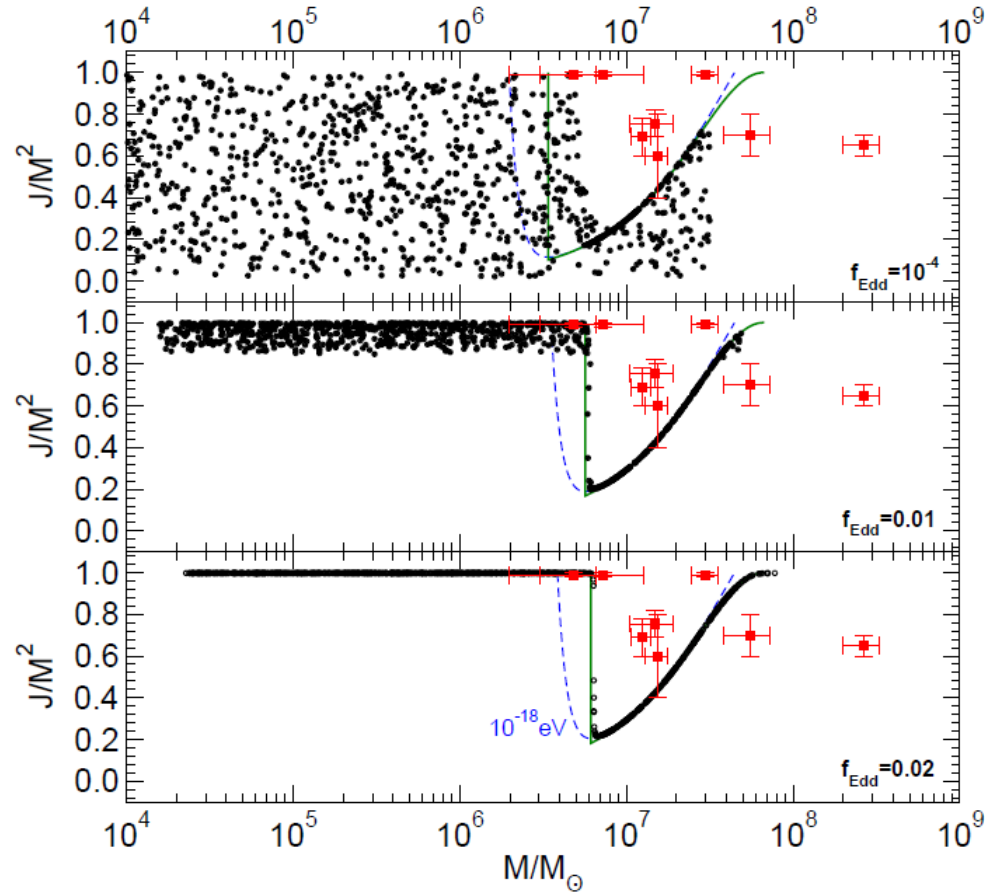
*Arvanitaki, Baryakhtar, Huang arXiv:1411.2263*

*Brito et al, in preparation (warning! preliminary)*



# Holes in Regge plane

*Arvanitaki & Dubovsky 2011; Brito, Cardoso, Pani 2015*

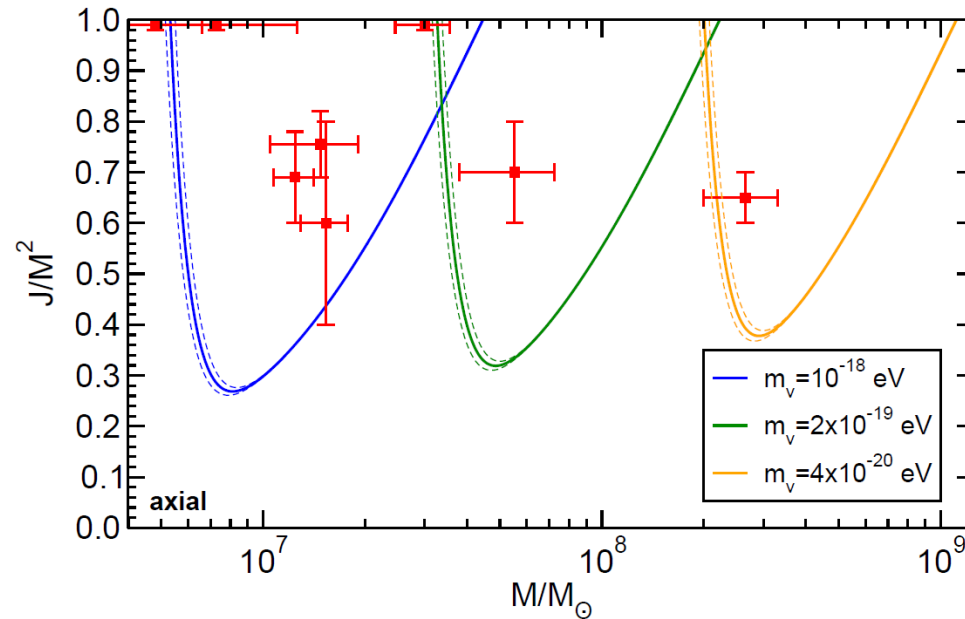


$$\mu = 10^{-18} \text{ eV}$$

Random distributions 1000 BHs, with initial mass between  $\log_{10} M_0 \in [4, 7.5]$  and  $J_0/M_0^2 \in [0.001, 0.99]$  extracted at  $t = t_F$ , with  $t_F$  distributed on a Gaussian centered at  $\bar{t}_F \sim 2 \times 10^9 \text{ yr}$  with width  $\sigma = 0.1 \bar{t}_F$ .

# Bounding the boson mass

*Pani et al PRL109, 131102 (2012)*



Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important... but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23} \text{ eV}$$

*Brito et al PRD88:023514 (2013); Review of Particle Physics 2014*

# Are we really observing black holes?

Strong field intimately connected with some of the deepest mysteries in theoretical physics today such as information loss/firewalls/quantum gravity. It is astonishing that space and time can get so warped to form horizons and singularities.

*Must demand a similar “astonishing” level of evidence.*



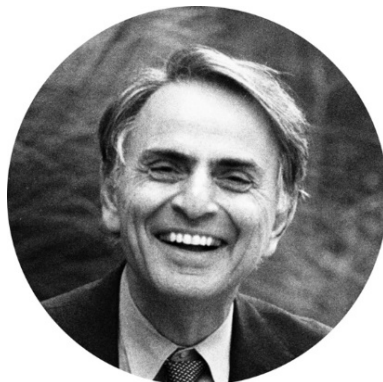
*“Plus un fait est extraordinaire, plus il a besoin d’être appuyé de fortes preuves; car, ceux qui l’attestent pouvant ou tromper ou avoir été trompés, ces deux causes son d’autant plus probables que la réalité du fait l’est moins en elle-même...”*

Laplace, Essai philosophique sur les probabilités 1812



*“No testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous than the fact which it endeavors to establish.”*

David Hume, An Enquiry concerning Human Understanding 1748



*“Extraordinary claims require extraordinary evidence.”*

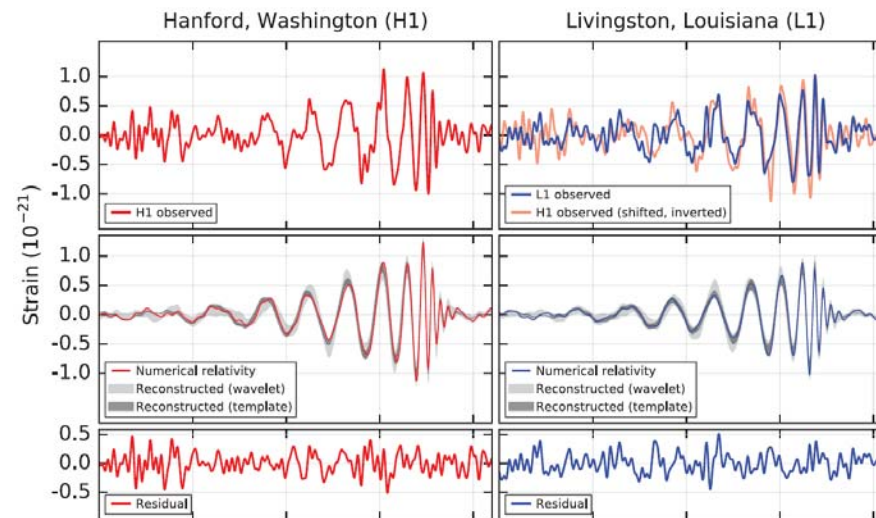
Carl Sagan



“But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future.”

*S. Chandrasekhar*, The Karl Schwarzschild Lecture,  
Astronomischen Gesellschaft, Hamburg, 18 September 1986

## Final state is compact!



# Questions to answer

- i. Are there alternatives?
- ii. Do they form dynamically under reasonable conditions?
- iii. Are they stable?
- iv. What GW signal do they give rise to?

# i. Alternatives

## **Boson stars, fermion-boson stars, oscillatons**

(Kaup 1968; Ruffini, Bonazzolla 1969, Colpi et al 1986, Brito et al 2015)

## **Wormholes**

(Morris, Thorne 1988; Visser 1996)

## **Gravastars**

(Mazur, Mottola 2001)

## **Fuzzballs, Superspinars, Firewalls**

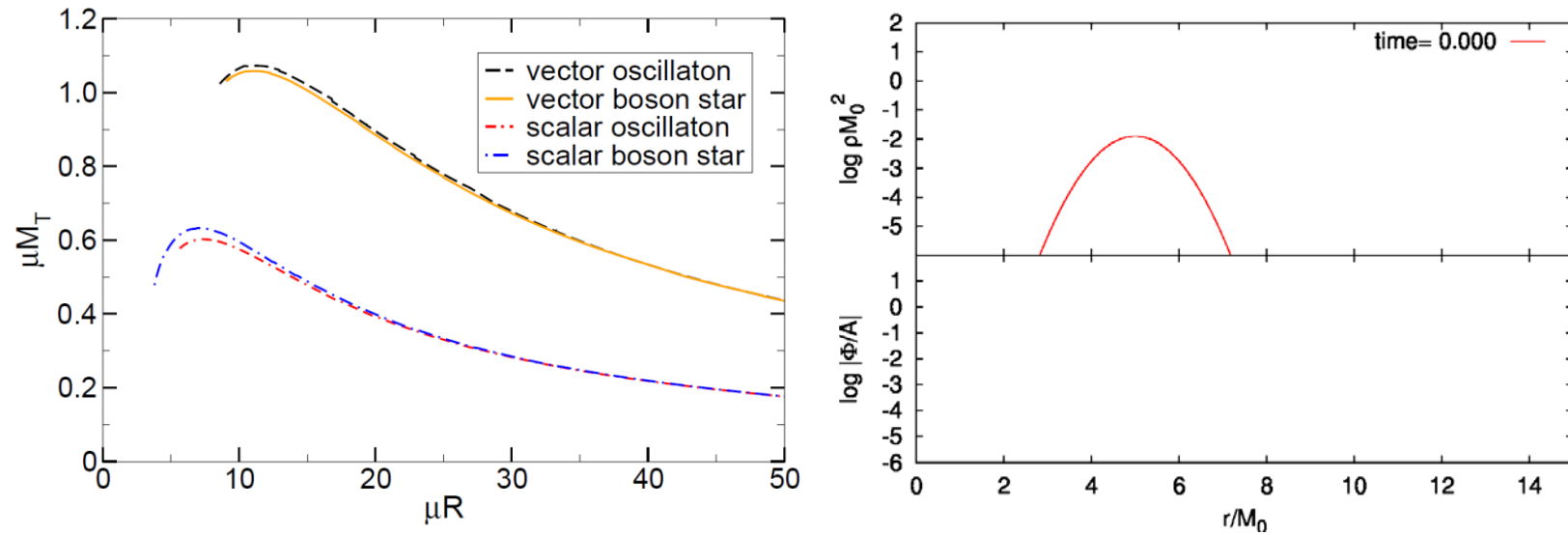
(Mathur 2000; Gimon, Horava 2009; Almheiri, Marolf, Polchinski, Sully 2012)

...

## ii. Formation

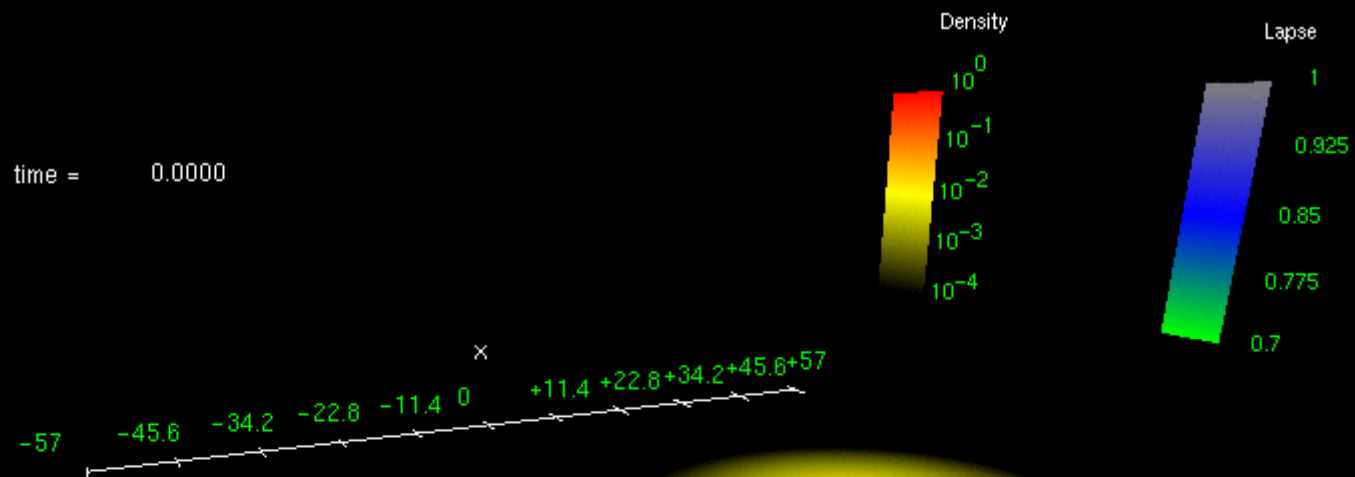
### Boson stars, fermion-boson stars, oscillatons

(Kaup 1968; Ruffini, Bonazzolla 1969; Colpi et al 1986; Okawa et al 2014; Brito et al 2015)



$$\frac{M_{\max}}{M_{\odot}} = 8 \times 10^{-11} \frac{\text{eV}}{m_{BC} c^2}$$



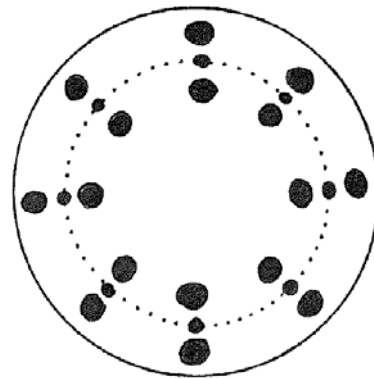


Density and lapse function sub-critical, equal-mass

### iii. Stability of objects with photospheres

Static objects: *No uniform decay estimate with faster than logarithmic decay can hold for axial perturbations of ultracompact objects.*

Keir arXiv:1404.7036; Cardoso et al, PRD90:044069 (2014)



In absence of viscosity,  
Dyson-Chandrasekhar-Fermi  
mechanism might trigger  
nonlinear instabilities

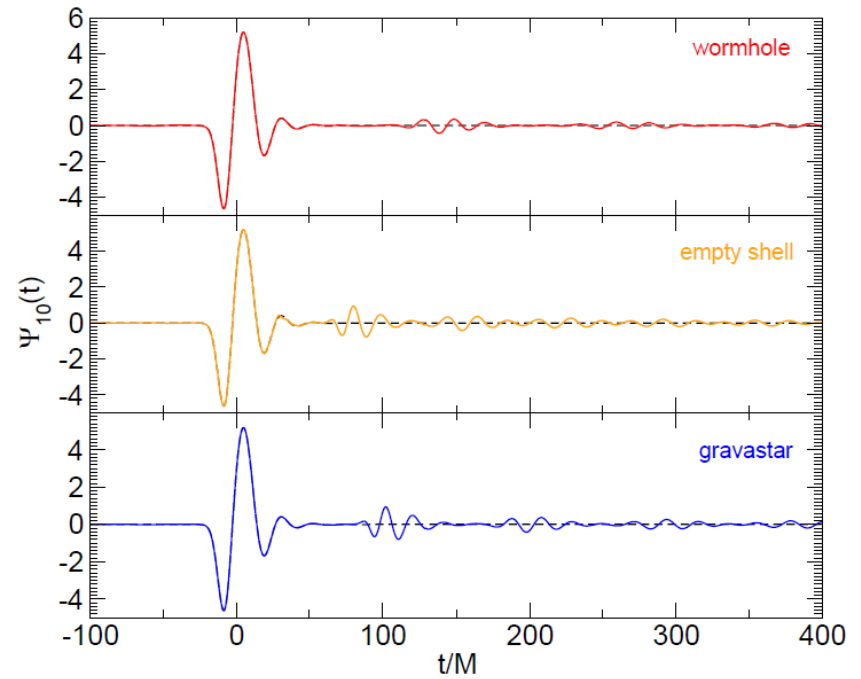
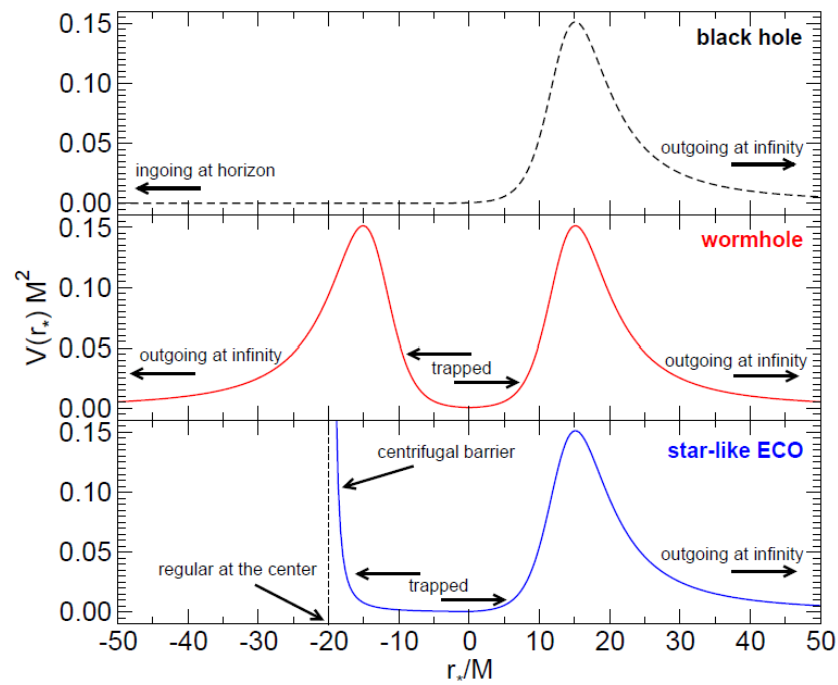
Rotation: *Horizonless objects with ergoregions are linearly unstable*

Friedmann Comm. Math.Phys.63:243, 1978

Most likely objects with photospheres are unstable...but conclusion depends on dissipation mechanisms; decay rates are poorly known.

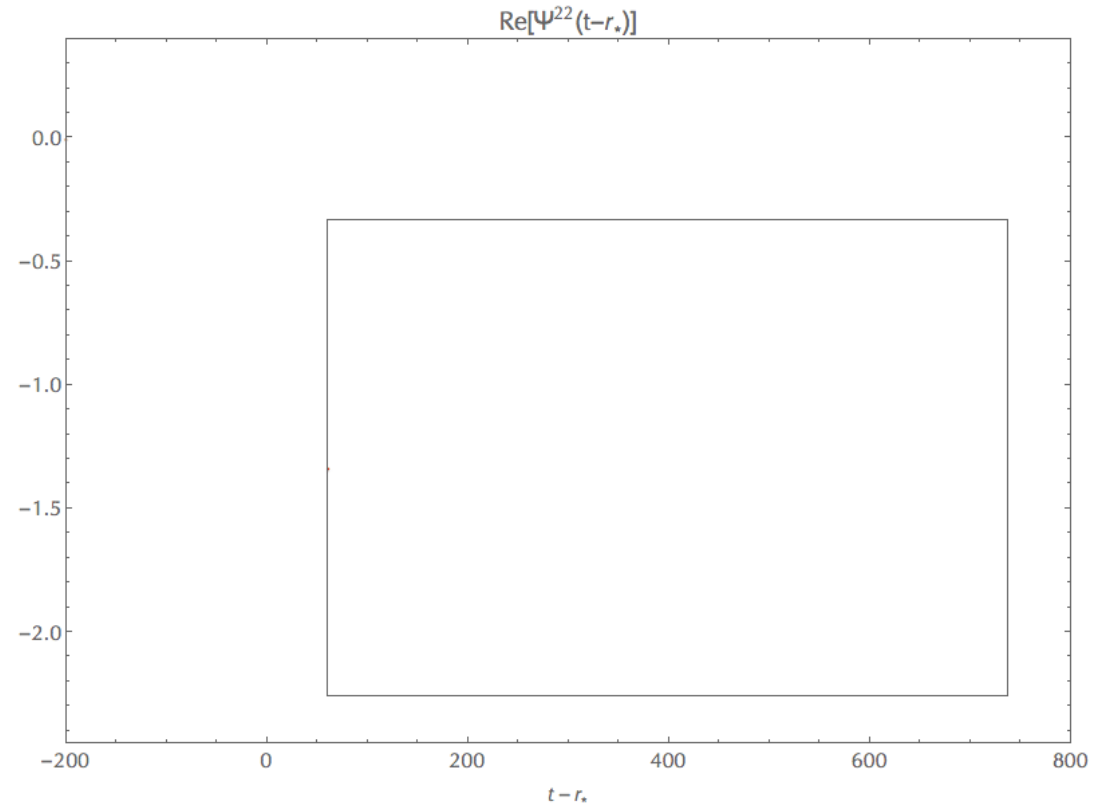
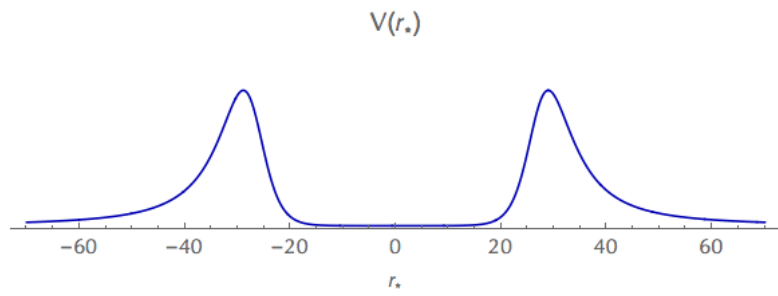
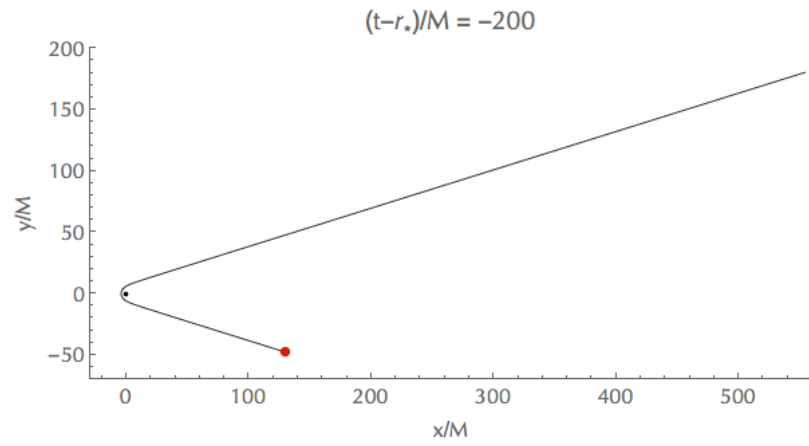
## iv. GW-signal

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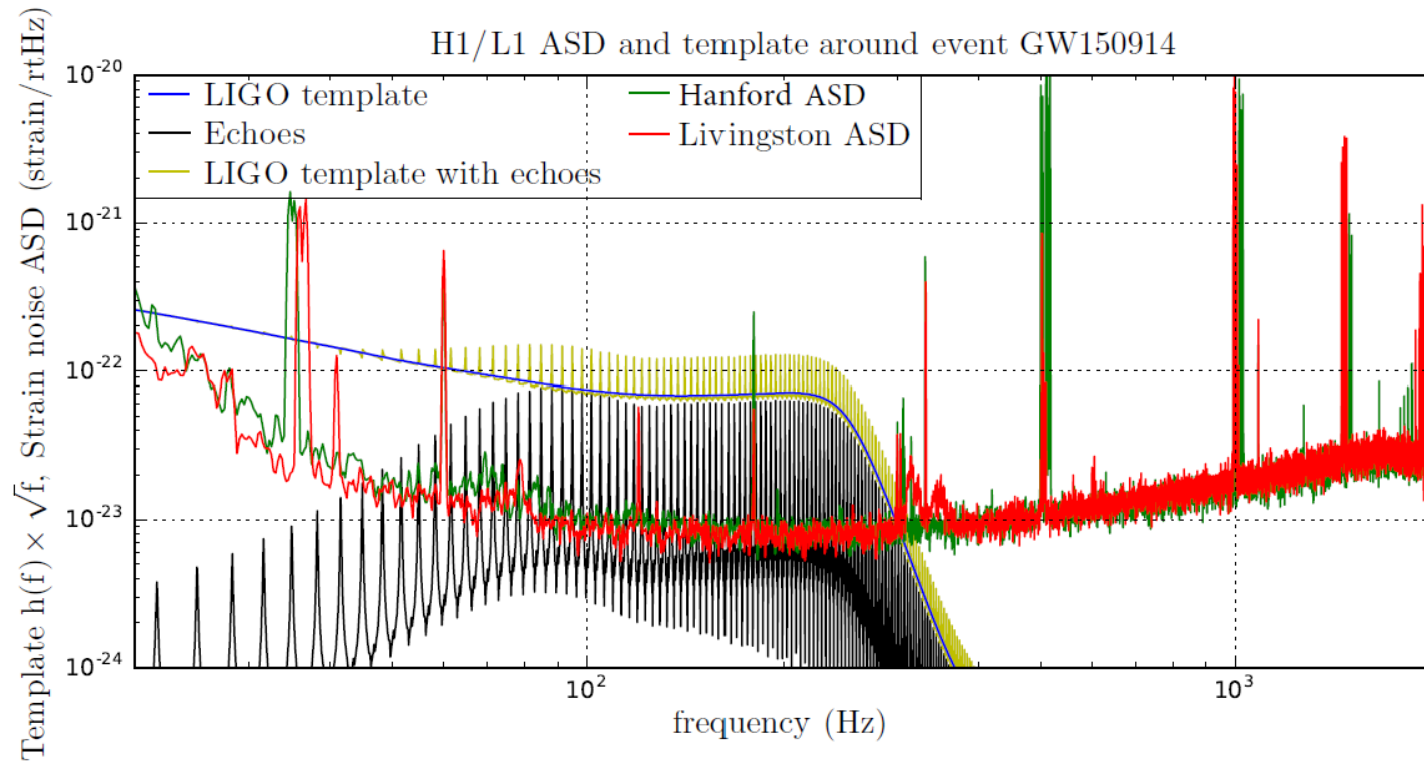
Cardoso, Franzin, Pani, PRL116:171101 (2016)

$\mathcal{E} = 1.5, r_{\min}=4.3M, r_0-2M = 10^{-6}M$



Cardoso, Hopper, Macedo, Palenzuela, Pani 2016

# Have we seen echoes?!



Abedi, Dykaar, Afshordi 2016

## Conclusions: exciting times!

Gravitational wave astronomy *can* become a precision discipline, mapping compact objects throughout the entire visible universe.

Black holes remain the simplest explanation for the observations of dark, massive and compact objects...but one can now test the BH hypothesis... improved sensitivity pushes putative surface closer to horizon... like probing short-distance structure with accelerators.

*“After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of BHs with the same certainty as, say, the 21 cm line identifies interstellar hydrogen”*

(S. Detweiler ApJ239:292 1980)

**Thank you**





# Environment: GW propagation

i. GWs are redshifted and lensed in “usual”, EM way (use geometric optics)

ii. GWs do not couple to perfect, homogeneous fluids

iii. Viscosity:  $L_{att} = \frac{c^6}{32\pi\eta G} = 10^{18} \frac{1poise}{\eta}$  light years

iv. Medium of oscillators  $L_{att} = \frac{1}{n\sigma} = 10^{28}$  light years or so

(if all our galaxy consists of BHs of roughly 10 solar masses)

# Environment: GW generation in inspiral

Correction		$ \delta_{\text{per}} $	$ \delta_{\varphi} [\text{rads}]$
thin disks	planetary migration	—	$10^4$
	dyn. friction/accretion	—	$10^2$
	gravitational pull	$10^{-8}$	$10^{-3}$
	magnetic field	$10^{-8}$	$10^{-4}$
	electric charge	$10^{-7}$	$10^{-2}$
	gas accretion	$10^{-8}$	$10^{-2}$
	cosmological effects	$10^{-31}$	$10^{-26}$
thick disks	dyn. friction/accretion	—	$10^{-9}$
	gravitational pull	$10^{-16}$	$10^{-11}$
DM	accretion	—	$10^{-8} \frac{\text{DM}}{\rho_3}$
	dynamical friction	—	$10^{-14} \frac{\text{DM}}{\rho_3}$
	gravitational pull	$10^{-21} \frac{\text{DM}}{\rho_3}$	$10^{-16} \frac{\text{DM}}{\rho_3}$

# Environment: ringdown properties

Correction	$ \delta_R [\%]$	$ \delta_I [\%]$
spherical near-horizon distribution	0.05	0.03
ring at ISCO	0.01	0.01
electric charge	$10^{-5}$	$10^{-6}$
magnetic field	$10^{-8}$	$10^{-7}$
gas accretion	$10^{-11}$	$10^{-11}$
DM halos	$10^{-21} \rho_3^{\text{DM}}$	$10^{-21} \rho_3^{\text{DM}}$
cosmological effects	$10^{-32}$	$10^{-32}$