Can Effective String Become Fundamental?

Peter Koroteev

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

THE UNIVERSITY OF CALIFORNIA DAVIS · LET THERE BE LIGHT

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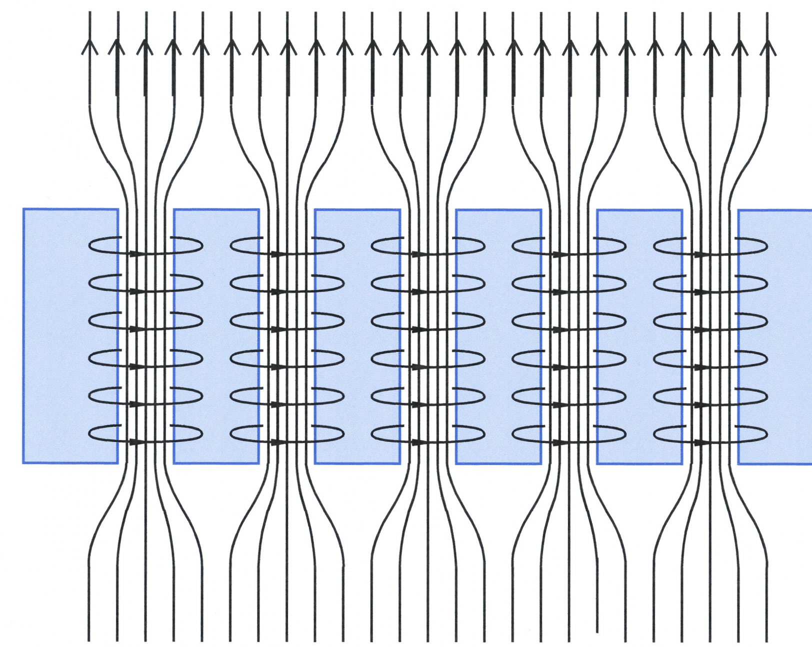
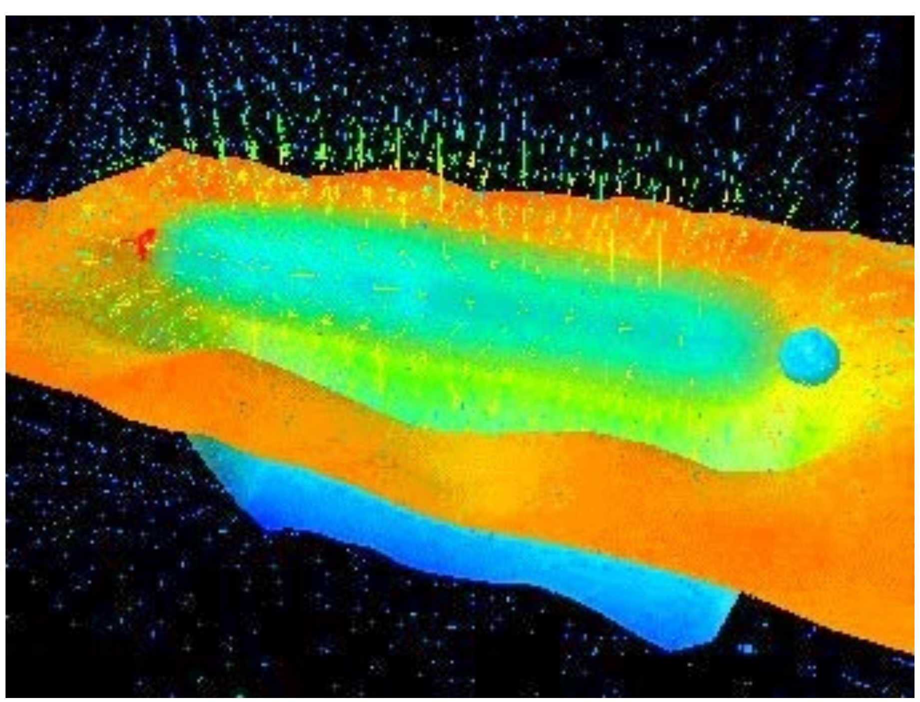
Effective strings

Abrikosov-Nilsen-Olisen (ANO) strings appear as flux tubes inside condensate of Cooper pairs in superconductors of second kind when superconductivity starts to break down. They cary Abelian magnetic flux



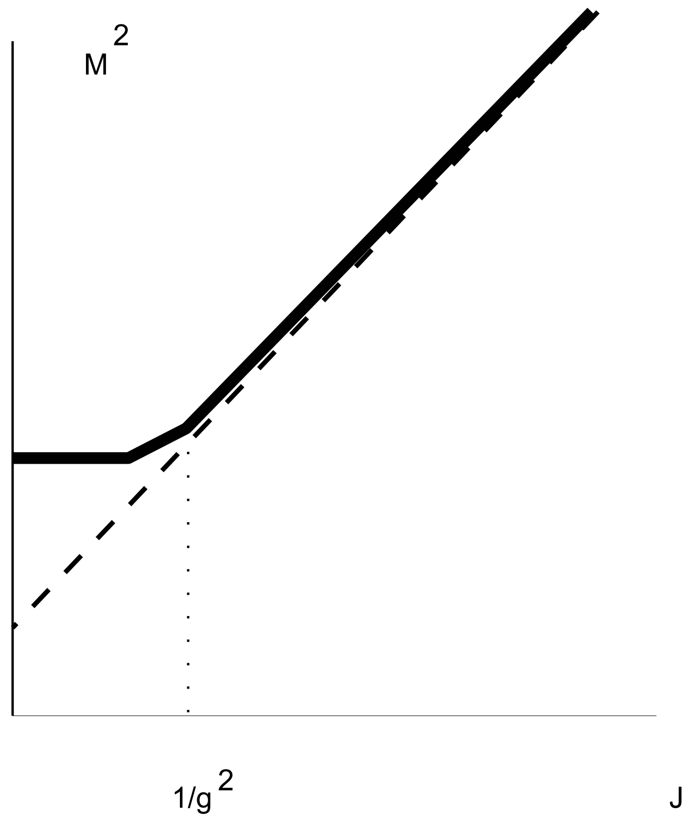


In QCD there are flux tubes stretched between quarks

Effective strings cont'd

These strings are bosonic so complete UV description of quantum spectrum of their excitations only exists in D=26

In noncritical regime quantum corrections completely change the dynamics and the object may not look like a string anymore (it crumples) and states do not obey Regge law for small spins



$$S_{\mathrm{NG}} = T\int d^2\sigma\left\{\sqrt{h} + O\left(\frac{\partial^n}{m^n}\right)\right\}$$

At weak coupling $m \sim g\sqrt{T}$ m-mass scale of bulk excitation

higher derivative terms become large

Thin string

We want to achieve a regime where the string would be thin such that the higher derivative corrections would be parametrically small

The low energy effective theory on the worldsheet should be UV complete

Two requirements

Conformal symmetry on the worldsheet

Critical value of Virasoro central charge

Effective String with SUSY

Thus if we want to find a good candidate for a fundamental string among effective strings criticality must be obeyed

$$T \ll m^2$$

When supersymmetry is present on the worldsheet of an effective string one has more control over the quantum corrections

In this talk we shall discuss strings which are formed as 1/2 BPS objects in four dimensional SQCD with 8 supercharges with gauge group U(N) and 2N (s)quarks

Worldsheet will have (2,2) superconformal symmetry so we should aim for a 10D description, but how?

'ANO' String

It is instructive to look at SQCD with $N_f=N$ first

$U(N)$ **gauge theory with N hypers** $\quad q \to UqV \quad U \in U(N)_G, \quad V \in SU(N)_F$

$$S = \int d^4x \; \mathrm{Tr}\left(\frac{1}{2e^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2\right) + \sum_{i=1}^{N_f} |\mathcal{D}_\mu q_i|^2$$

$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{4}\,\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\, 1_N)^2$$

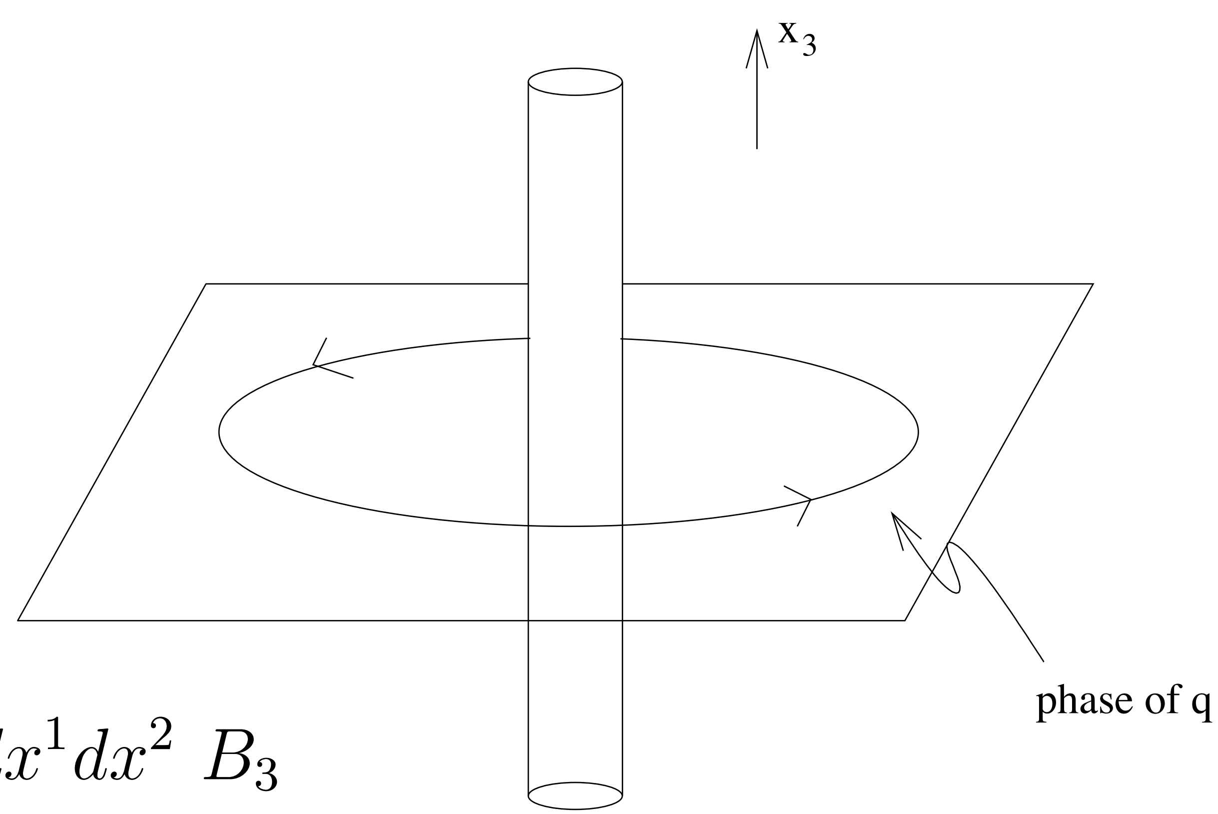
Vacuum

$$\phi = 0 \quad , \quad q^a{}_i = v\delta^a{}_i$$

breaks symmetry

(color-flavor locking)

$$U(N)_G \times SU(N)_F \to SU(N)_{diag} \times U(1)$$



winding at infinity

$$q_N \sim q\,\mathrm{e}^{ik\theta}$$

$$A_\theta \sim \frac{k}{\rho}$$

$$2\pi k = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} i\partial_\theta q\; q^{-1} = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} A_\theta = \mathrm{Tr}\int dx^1 dx^2\; B_3$$

Non-Abelian Vortices

ANO U(1) vortex has two collective coordinates-translations in x,y directions

U(N) vortex has more moduli parameterized by rotations

$$A_z = \begin{pmatrix} A_z^\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \quad , \qquad q = \begin{pmatrix} q^\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Moduli space (k=1)

$$SU(N)_{\rm diag}/S[U(N-1)\times U(1)] \cong \mathbb{CP}^{N-1}$$

$$\mathcal{V}_{1,N} \cong \mathbf{C} \times \mathbb{CP}^{N-1}$$

translational+ orientational

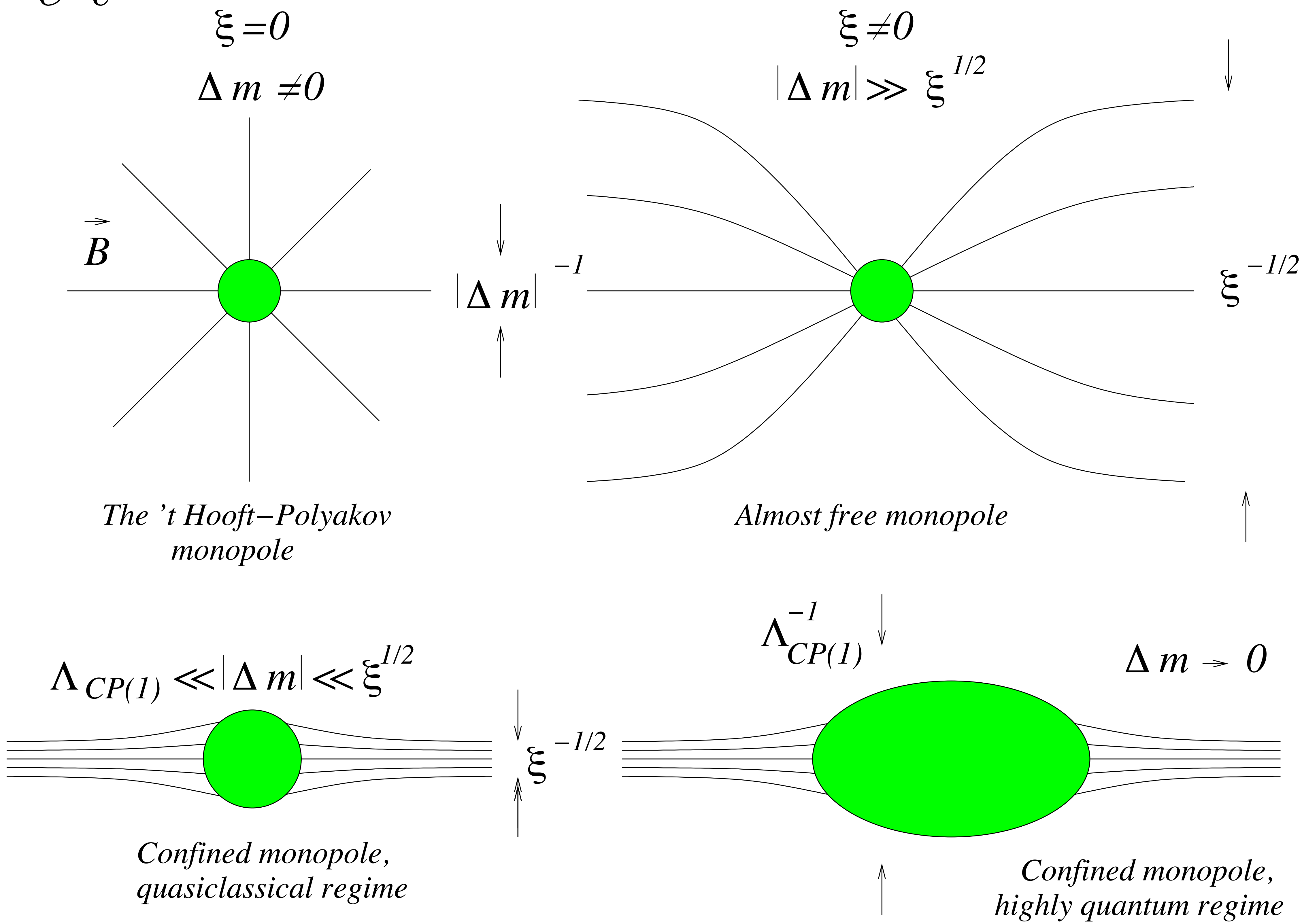
For higher k

$$\dim(\mathcal{V}_{k,N}) = 2kN$$

Confined monopoles

[Shifman Yung]

$$\xi = e^2 v^2$$



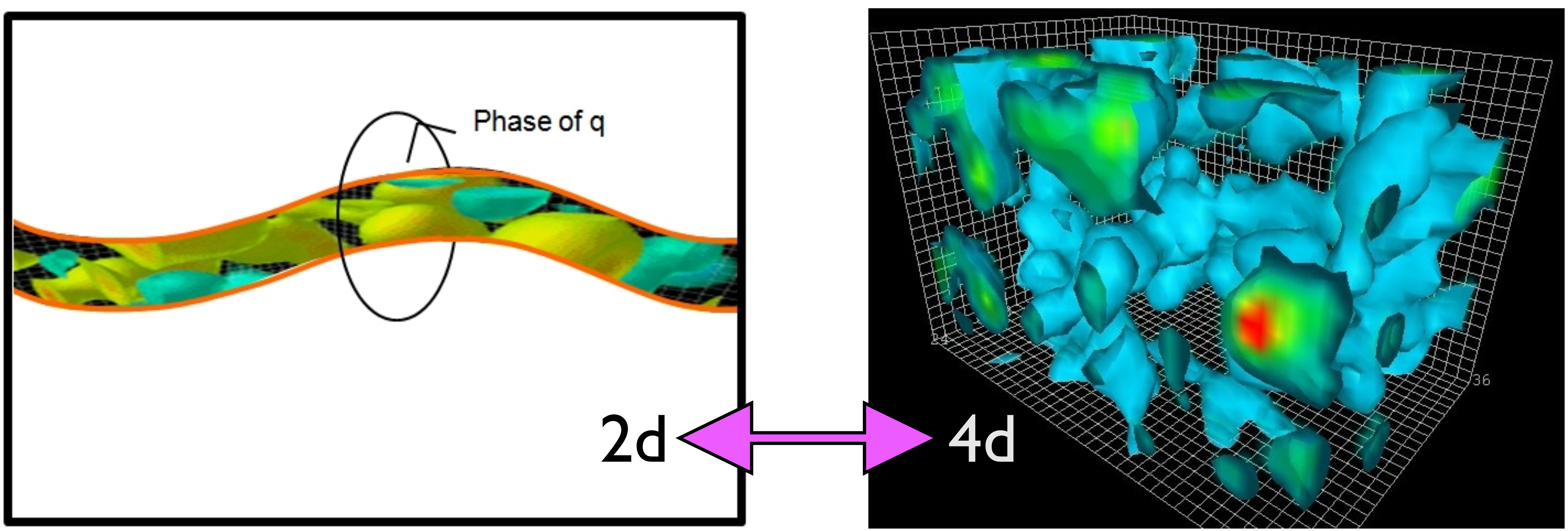
$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term β

4d/2d Duality

$N_f = N$ color-flavor locked phase $U(N)_G \times SU(N)_F \to SU(N) \times U(1)$

local vortex $$\frac{SU(N)}{SU(N-1) \times U(1)} = \mathbb{CP}^{N-1}$$

Duality between two strongly coupled theories



N_f=2N SQCD

Moduli space will still have the compact CP(N-1) part. But since it is not possible to Higgs all matter fields there will be noncompact moduli

Those 'semilocal' vortices are described by (2,2) sigma gauge linear sigma-model

$$
\begin{aligned}
S_1 \;&=\; \int d^2\sigma \sqrt{h}\,\Big\{ h^{\alpha\beta}\left(\tilde{\nabla}_\alpha \bar{n}_P\, \nabla_\beta\, n^P + \nabla_\alpha \bar{\rho}_K\, \tilde{\nabla}_\beta\, \rho^K\right) \\
&+\; \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - \beta\right)^2 \Big\} + \text{fermions}\,,
\end{aligned}
$$

$$
\nabla_\alpha = \partial_\alpha - iA_\alpha\,, \qquad \tilde{\nabla}_\alpha = \partial_\alpha + iA_\alpha
$$

Symmetry

$$n^P \leftrightarrow \rho^K$$

$$\beta \to \beta_D = -\beta$$

Unbroken global symmetry in 4d $\quad \mathrm{SU}(N)_{C+F} \times \mathrm{SU}(\widetilde{N}) \times \mathrm{U}(1)$

Its target manifold is a rank-N bundle over CP(N-1)

String action

In addition we have translational moduli

$$S_0 = \frac{T}{2}\int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\partial_\alpha x^\mu\, \partial_\beta x_\mu + \text{fermions}$$

Full action is translational + orientational $\quad S = S_0 + S_1$

To make a UV complete description the string needs to be infinitely thin $\quad T \ll {\rho_0}^{-2}$ **[Polchinski Strominger]**

For our string $\quad \frac{\beta}{T} \sim \rho_0^2 \quad$ so at weak coupling $\quad \beta = \frac{4\pi}{g_{2d}} \gg 1$

At strong coupling $\quad \beta = 0$

This maps to the fixed point of the S-duality of the 4d theory

$$\tau \to \tau_D = -\frac{1}{\tau}\,, \qquad \tau = i\frac{4\pi}{g^2} + \frac{\theta_{4D}}{2\pi}$$

Criticality

2d sigma model is conformal. The dimension of the full target space is 2(2N-1)+D

For N=2 and D=4 we get 10 dimensional target space

Checking Virasoro central charge $c_{\rm Vir} = \frac{3}{2}(D + 2\hat{c}_0 - 10)$

Here $\hat{c}_0 = 2N - 1\,, D = 4$ $c_{\rm Vir} = 0$ for N=2

The two conditions from Polchinski-Strominger are satisfied

The resulting target space is $\mathbb{R}^4 \times Y$

Y – resolved conifold $|n^P|^2 - |\rho^K|^2 = \beta.$

Spacetime SUSY

In static gauge

$$S_{\rm tr} = \frac{T}{2}\int d^2x\{\partial_k x^i\,\partial_k x^i + \bar{\zeta}_L\,\partial_R\,\zeta_L + \bar{\zeta}_R\,\partial_R\,\zeta_R\}$$

$\sigma_1 = x_0,\ \sigma_2 = x_3$

x^i transversal translational moduli

fermions obey $\bar{n}_P\xi^P_{L,R} - \bar{\rho}_K\chi^K_{L,R} = 0$

There are 4(2N-1) = 12 orientational fermions

Overall 16 fermionic degrees of freedom, precisely as many as the number of θ in the static gauge

Type IIA on conifold

We obtained critical superstring in ten dimensions. Which type is it IIA or IIB?

Our starting point — 4d SCQD is a vector-like theory which preserves parity. Therefore the string has to by of Type IIA

parity $\psi^{\alpha} \to \bar{\tilde{\psi}}_{\dot{\alpha}}$

$x_{1,2,3} \to -x_{1,2,3}$

$n \to -n, \; \rho \to -\rho$

$\zeta_R \to -\bar{\zeta}_L, , \quad \xi^P_R \to -\xi^P_L$

explicit profiles

$\bar{\psi}_{\dot{2}} \sim (x_1 + i x_2)\zeta_L,$

$\bar{\tilde{\psi}}_{\dot{1}} \sim (x_1 - i x_2)\zeta_R$

worldsheet action is invariant

We can now unload the machinery of string compactifications on CY threefolds to study the effective 4d theory (a different 4d theory). We shall study how the 4d spectrum depends on β

Resolved Conifold

Parameter beta describes deformations of Kahler structure of the CY which are enumerated by cohomology $H^{1,1}$

For conifold $h^{1,1} = 1$

So if normalizable there should be a single vectormultiplet coming from such reduction

Since it lies in the same supergravity multiplet with graviton, existence of such state would imply presence of massless gravitons which is problematic [Winberg-Witten]

Fortunately this mode is non-normalizable

Compactification

Massless 10d bosonic fields of IIA are

NS-NS $G_{MN}\,,\quad \phi\,,\quad B_{MN}$ R-R $A_M\,,\quad C_{MNK}$

Graviton obeys

$$D_A D^A \delta G_{MN} + 2R_{MANB}\delta G^{AB} = 0$$

4+6 factorization

$$\delta G_{\mu\nu} = \delta g_{\mu\nu}(x)\,\phi_6(y), \qquad \delta G_{\mu i} = B_\mu(x)\,V_i(y), \qquad \delta G_{ij} = \phi_4(x)\,\delta g_{ij}(y)$$

Massless states have zero eigenvalue

$$-\Delta_6 g_6(y) = \lambda_6 g_6(y)$$

First option $-D_i D^i \phi_6 = -D_i \partial^i \phi_6 = 0$

Only constant solution is available thus graviton mode nonnormalizable

Physics of non-normalizable modes

Can be treated as coupling constants (background values)

4d metric

2d coupling (related to 4d coupling)

Modes are unstable and delocalize from the string

U(1)xSU(N) is broken by squark condensation so gauge bosons become massive $m \approx g\sqrt{\xi}$

All states (N=2 multiplet) appear as singlets or adjoints and in bifundamental rep of $\mathrm{SU(N)}_{C+F} \times \mathrm{SU}(\widetilde{N})$

$$\left(\bar{\mathbf{N}},\, \widetilde{\mathbf{N}},\, \frac{N_f}{2\widetilde{N}}\right), \quad \left(\mathbf{N},\, \bar{\widetilde{\mathbf{N}}},\, -\frac{N_f}{2\widetilde{N}}\right)$$

After compactification non-normalizable states can decay into massless perturbative excitations on the Higgs branch of the 4d theory

$$\lambda_6 > 0 \qquad \dim\mathcal{H} = 4N\tilde{N} = 16.$$

Deformed Conifold

Something interesting happens at beta=0. The conifold develops a singularity and we cannot use SUGRA

However, we can deform further past the singularity into a different topology — *deformed conifold*

$$w_\alpha = \frac{1}{2}\,\mathrm{Tr}\left[(\bar{\sigma}_\alpha)_{KP}\, n^P \rho^K\right] \qquad\qquad \sum_{\alpha=1}^{4} w_\alpha^2 = b$$

In other words we opened a new mode parameterized by b

Deformed Conifold

Effective action

$$S(b) = T \int d^4x \, h_b |\partial_\mu b|^2,$$

volume of the conifold

$$(\mathrm{Vol})_{Y_6} \sim \int \left| \frac{dw_2 dw_3 dw_4}{w_1} \right|^2$$

metric

$$h_b \sim \frac{\partial}{\partial b} \frac{\partial}{\partial \bar{b}} \int \left| \frac{dw_2 dw_3 dw_4}{w_1} \right|^2$$

log-behavior

$$h_b = (4\pi)^3 \, \frac{4}{3} \, \log \frac{\widetilde{r}^2_{\mathrm{max}}}{|b|}$$

Thus b-mode is **log-normalizable**

same true for wave-functions of size moduli ρ

Physics of b-mode

b-mode is related to the deformations of complex structure of the conifold described by Dolbeault cohomology $H^{2,1}$

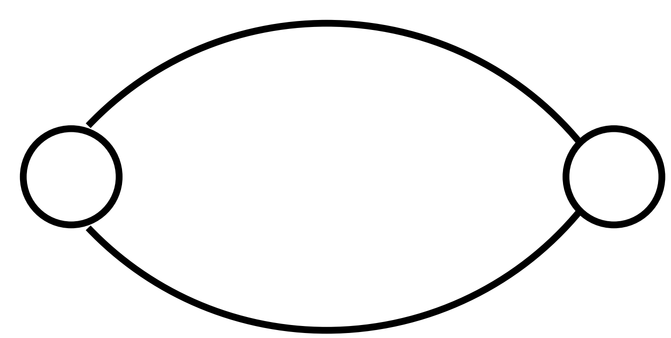
For conifold $h^{2,1} = 1$

so there should be a one-(complex)dimensional branch parameterized by a VEV of some *hypermultiplet*

Global symmetry $SU(2) \times SU(2) \times U(1)$

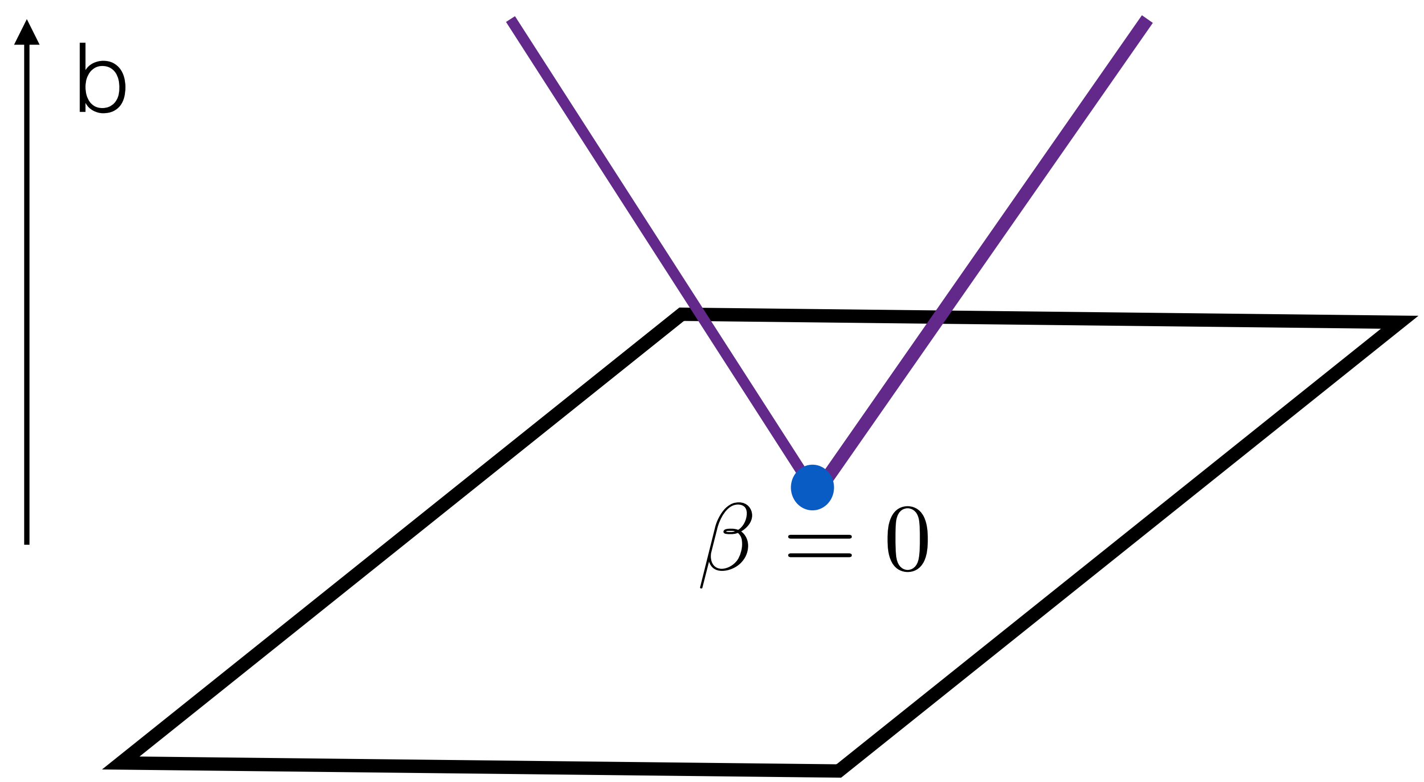
Since $w_\alpha = \frac{1}{2} \mathrm{Tr} \left[(\bar{\sigma}_\alpha)_{KP} n^P \rho^K \right]$ b-state transforms as (1,1,2)

monopole-monopole baryon



Recap

4d SQCD —> 2d sigma model —> Type IIA superstring
—> effective 4d theory

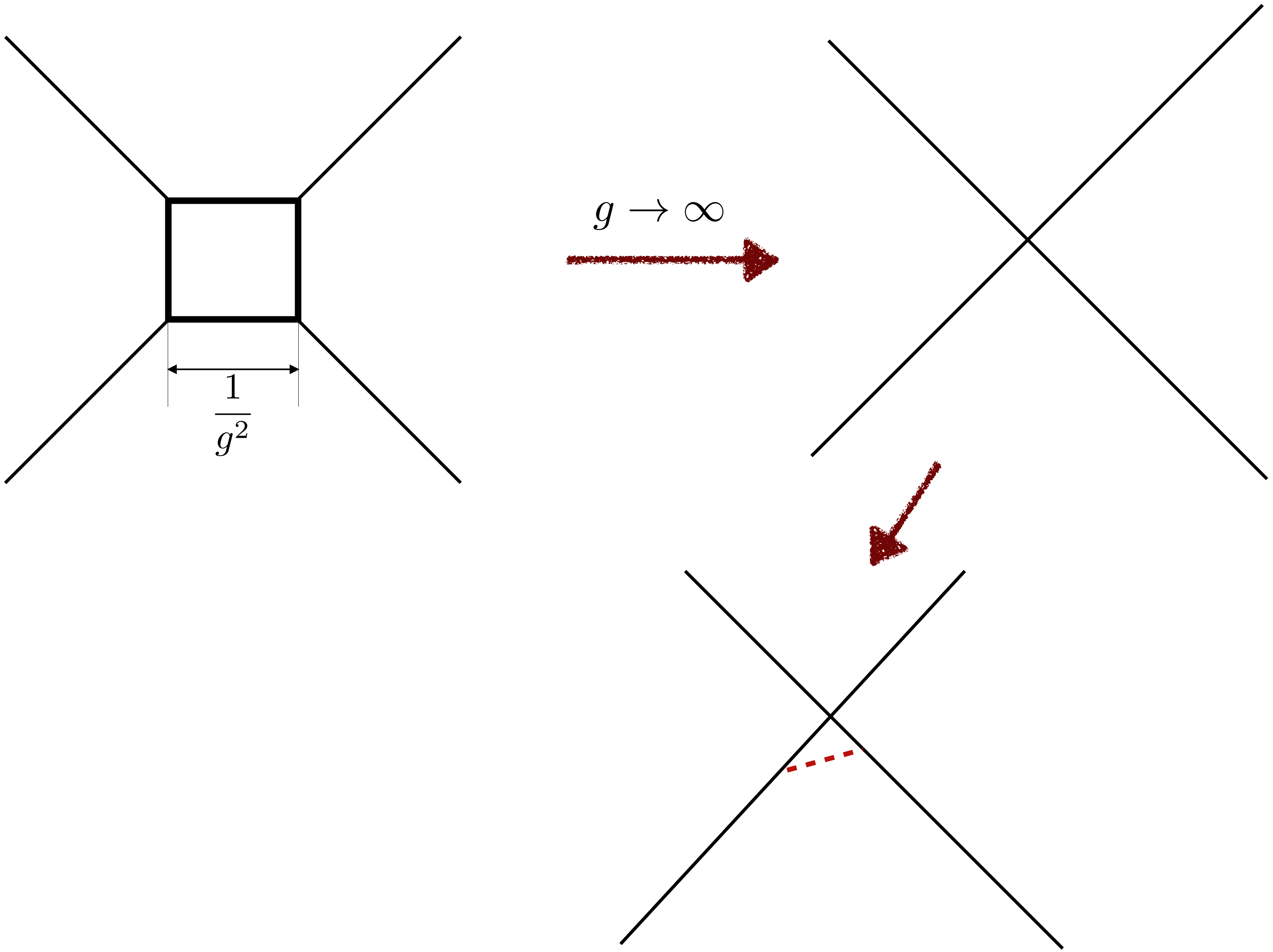


Mon/Mon Baryon
=
n-rho Kink
=
Hyper

String states describe non-perturbative physics at strong coupling

Have we seen this before?

Yes — ***exotic*** Higgs branches in 5d theories



Future Directions of ‘micro string theory’

- Connections to Little String Theory (and its double scaling limit)
- Other examples in higher ranks and different theories
- CFT description of exotic Higgs phases
- Heterotic String
- Membranes