A QCD Analogy for Quantum Gravity

--- Revisit Quadratic Gravity in Analogy with QCD

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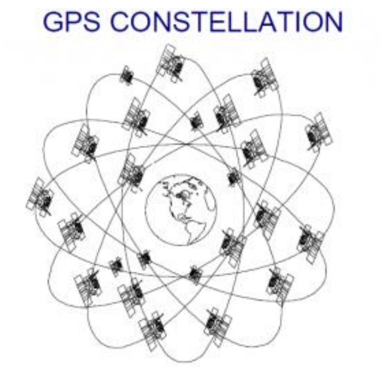
Based on Bob Holdom, JR, arXiv: 1512.05305

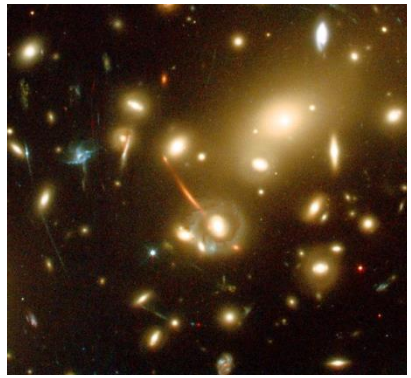
General Relativity

- 100 years of General relativity: huge success in large scale!

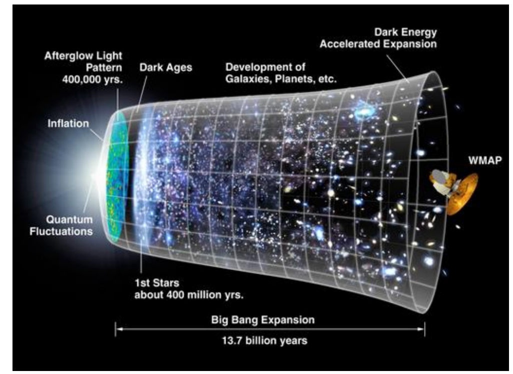
Daily life



Astrophysics



Cosmology



- Quantum mechanics + General relativity??
  - Non-renormalizable, treat as an effective field theory

$$S_{\rm GR} = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2\left(-\Lambda + \frac{1}{2}R\right) + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \ldots\right]$$

Gravitational interactions get strong at M_{Pl}, need UV completion.

  - Decades of efforts for quantum gravity: string, loop, asymptotic safety…

Quadratic Gravity

- Generalization with quadratic curvature terms $R^2,\ R^{\mu\nu}R_{\mu\nu},\ R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$

$$S_{\rm QG} = \int d^4x\,\sqrt{-g}\left(\frac{1}{2}M^2R - \frac{1}{2f_2^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \frac{1}{3f_0^2}R^2\right)$$

- Quadratic gravity is renormalizable and asymptotically free
  - Perturbative renormalizable: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $1/k^4$ propagator softens the UV divergence [Stelle, PRD 16, 953 (1977)]
  - Asymptotically free [Fradkin, Tseytlin, NPB 201, 469 (1982); Avramidi, Barvinsky, PLB 159, 269 (1985)]

$$\frac{df_2^2}{dt} = -\left(\frac{133}{10} + a_m\right)f_2^4, \quad \frac{1}{f_2^2}\frac{dw^2}{dt} = -\left[\frac{5}{12} + w\left(5 + \frac{133}{10} + a_m\right) + \frac{10}{3}w^2\right] \quad w = f_2^2/f_0^2$$

- f_2^2 always asymptotically free: $a_m > 0$ (constructive interference)
- Two roots of the ratio $w_2 < w_1 < 0$: $w = w_1$ is UVFP, $(w_2, 0)$ is UV attractive

QUE: Why NOT a UV completion of quantum gravity?

ANS: Because of "the ghost problem"!

The Ghost Problem

Extract the perturbative spectrum from $h_{\mu\nu}$ propagator on a flat background with gauge-fixing [Stelle, PRD 16, 953 (1977)]

$$D_{\mu\nu\rho\sigma} = i\left(\boxed{-\frac{2f_2^2 P^{(2)}_{\mu\nu\rho\sigma}}{k^2(k^2-M_2^2)}} + \frac{f_0^2 P^{(0)}_{\mu\nu\rho\sigma}}{2k^2(k^2-M_0^2)} + GF\right) \qquad M_2^2 = \frac{1}{2}f_2^2 M^2,\quad M_0^2 = \frac{1}{4}f_0^2 M^2$$

$$\frac{-i}{k^2(k^2-M_2^2)} = \frac{1}{M_2^2}\left(\frac{i}{k^2} - \frac{i}{k^2-M_2^2}\right)$$

$$\xrightarrow[i\epsilon \text{ prescription}]{\text{wrong sign??}}$$

- **Negative energy**: vacuum instability
- **Negative norm**: no probability interpretation, unitarity violation

Ostrogradski instability in Hamiltonian (ADM) formalism [Ostrogradski, Mem. Ac. St. Petersbourg VI (1850) 385.]

- For non-degenerate higher derivative theory, the Hamiltonian has linear dependence on some canonical variable, i.e. unbounded from below
- For quadratic gravity, it is the case when Weyl term C^2 is present

HOWEVER, note one caveat

- These arguments are based on tree-level propagator or classical Hamiltonian.
- Implicit assumption that the perturbative analysis reflects the true physical spectrum for any physical process that manifests the ghost problem.

$$S_{QG} = \int d^4x\sqrt{-g}\left(\frac{1}{2}M^2R - \frac{1}{2f_2^2}C^2 + \frac{1}{3f_0^2}R^2\right)$$

- When M is large, f_2^2, f_0^2 remain weak, the perturbative analysis makes sense.
- BUT when M is small, f_2^2, f_0^2 get strong at some low energy scale $\Lambda_{QG} > M$. Perturbative poles fall into the nonperturbative region. Do we still suffer from "the ghost problem"?

QCD and Quadratic Gravity Analogy

- **QCD**: gluon is not in the physical spectrum, but useful to describe hard process involving high virtuality (far off-shell gluon).
- **Proposal**: when $M^2 < \Lambda_{QG}^2$, the quadratic gravity enters into a distinctive phase where the ghost mode is absent in the physical spectrum. We conjecture that it may define a healthy theory.

QCD	Quadratic Gravity
AF at UV, g_s^2 gets strong at Λ_{QCD}	AF at UV, f_0^2, f_2^2 get strong at $\Lambda_{\rm QG}$
Perturbative transverse gluon removed from physical spectrum	Perturbative ghost pole removed from physical spectrum
Color singlet states described by Chiral Lagrangian in the IR	$M = 0$, massless graviton described by General relativity in the IR, $M_{Pl} \sim \Lambda_{QG}$
(Current quark mass gives pion mass)	New scale M controls graviton mass gap

- Use path integral as a nonperturbative definition of a quantum theory: a nontrivial measure may cure the problem of classical action

The rest of the talk

- Analogy based on propagators

  -- *how does the analogy proceed and where do we go?*

- A focus on the measure

  -- *how could gravity be like QCD anyway?*

- Implications and speculations

  -- *what physics do we expect from this picture?*

- Summary

Analogy based on Propagators

-- How does the analogy proceed?

QCD in the IR

- Confinement problem

  "Why quark and gluon confined in the hadrons?" still far from solved, but some general features known and studied in many different approaches: lattice, Schwinger-Dyson equation, Gribov copies …

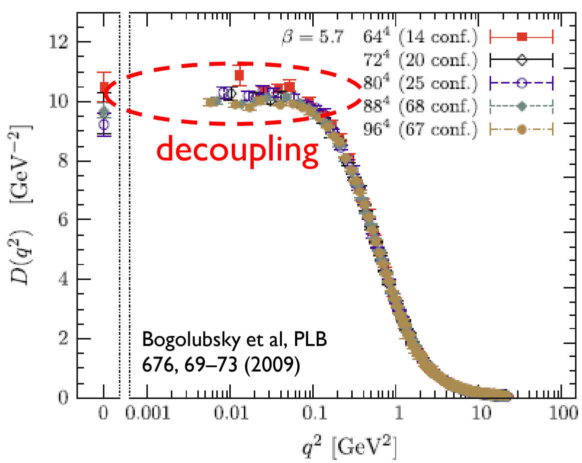
- "Gluon is unphysical" in view of propagator
  - A consequence of confinement, but more directly understood by the **full gluon propagator** (gauge dependent but should provide self-consistent picture)
  - Traditionally studied by Schwinger-Dyson equations, propagator suppressed in the IR and no poles anymore. Find solutions different in deep IR,

    scaling (non-integer anomalous power) v.s. decoupling (constant)
  - Later on lattice opens up the black box by gauge fixing. Confirmed the decoupling solution in Landau gauge.
  - As the gluon develops a mass gap, the IR Landau pole is removed.

Transverse Gluon Propagators

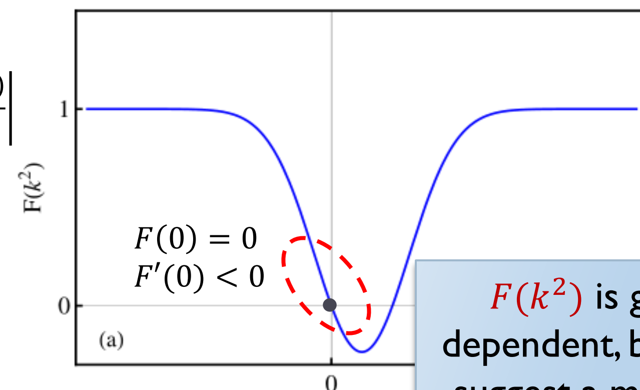
- The full propagator: $F(k^2)/k^2$ × (tensor factor) × (perturbative correction)
  - $F(k^2) \to 1$ for $k^2 \to \pm\infty$; $F(k^2)$ is only nontrivial in the IR

- Lattice data in Landau gauge ($\partial_\mu A^\mu = 0$)



$$D(q^2) = \left|\frac{F(k^2)}{k^2}\right|$$

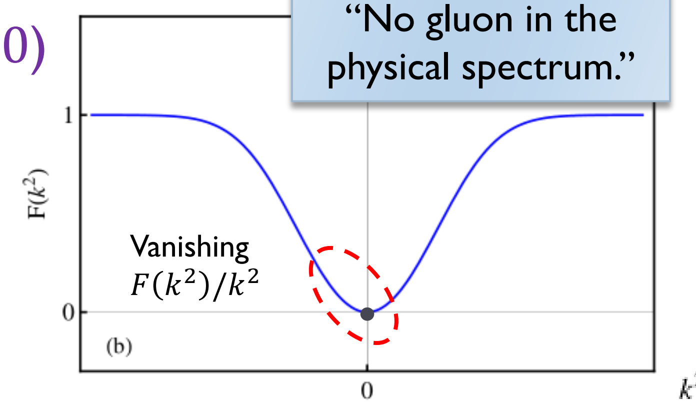
$$q^2 = -k^2$$



$F(k^2)$ is gauge dependent, but both suggest a mass gap, positivity violation. "No gluon in the physical spectrum."

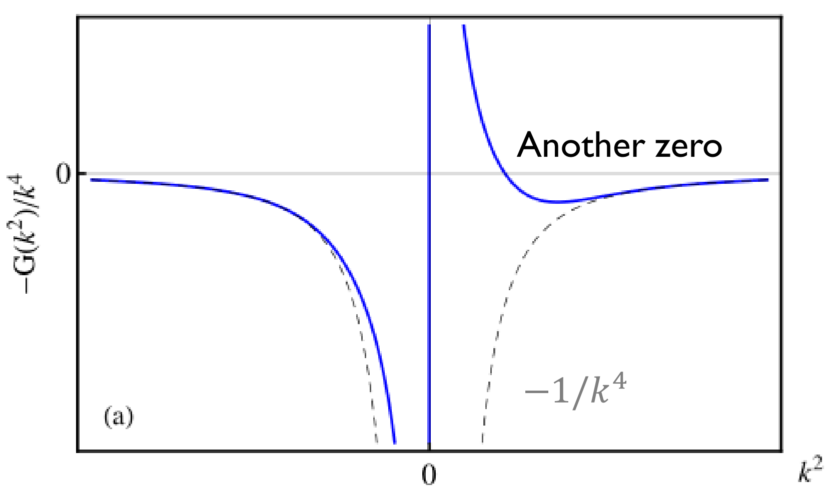
- Lattice data in Coulomb gauge ($\partial_i A^i = 0$)

Evidence less clear, but some lattice results (also Gribov picture) do suggest vanishing $F(k^2)/k^2$ in deep IR

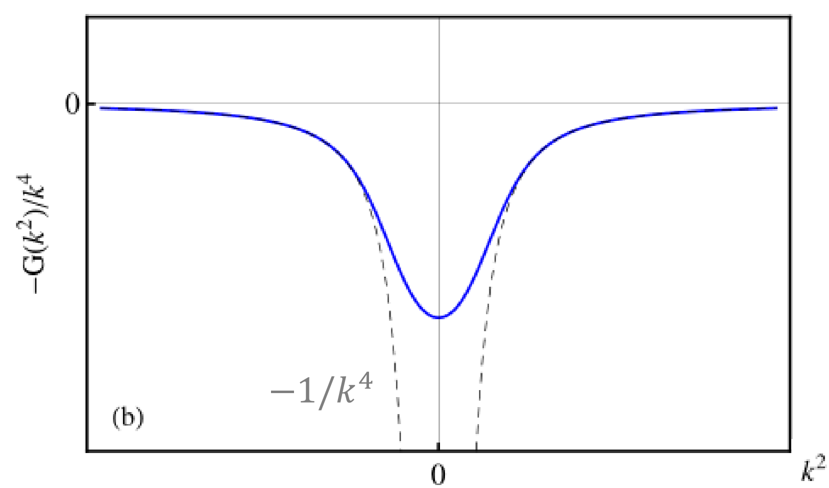


Graviton Propagators ($M^2 = 0$)

- First focus on pure quadratic gravity on flat background
- The full propagator: $-G(k^2)/k^4$ × (tensor factor) × (perturbative correction)
- **What if** the nonperturbative effects in quadratic gravity operate in a way similar to QCD? Consider the same two possibilities for $G(k^2)$ as found for $F(k^2)$ from lattice QCD. Plot $-G(k^2)/k^4$.

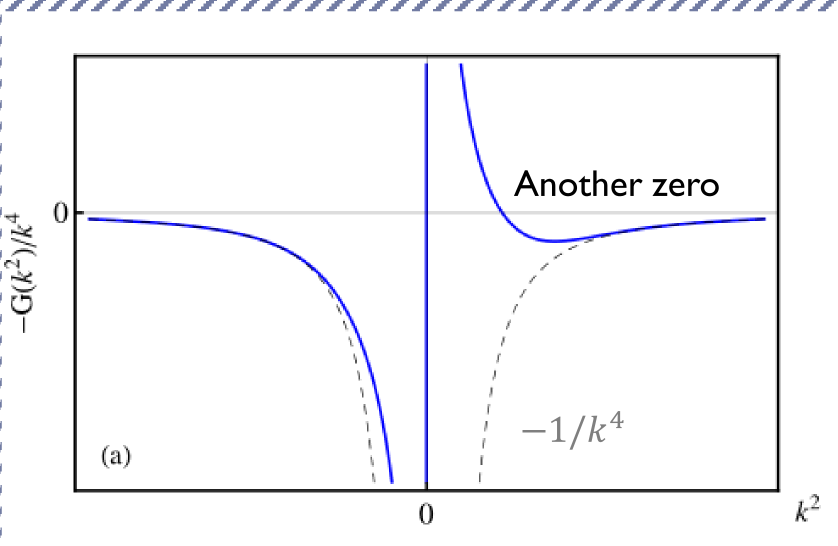


$\lim_{k^2\to 0} G(k^2) \sim -k^2$: $-1/k^4$ softened to a $1/k^2$ pole (positive sign), i.e. massless state.



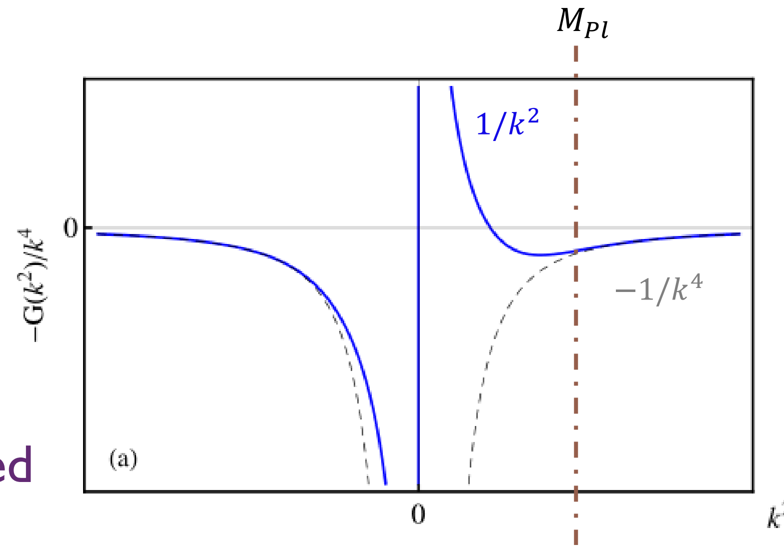
$\lim_{k^2\to 0} G(k^2) \sim k^4$: a mass gap develops, no propagating graviton. More like QCD.

Graviton Propagators ($M^2 = 0$)



More on the Case (a)

- Only a $1/k^2$ pole, implies a massless spin-2 particle, on-shell graviton, naive ghost pole removed from physical spectrum
- UV linear rising potential ($\sim 1/k^4$) is modified as IR Newtonian potential ($\sim 1/k^2$)



- Given the general covariance in the strong phase, the massless graviton is described by General relativity with $M_{Pl}^2 = -1/G'(0) \sim \Lambda_{QG}^2$. UV physics is encoded in the derivative expansion of curvature terms

$$S_{\rm EFT} = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\rm Pl}^2 R + c_1 R^2 + c_2 C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} + \ldots \right)$$

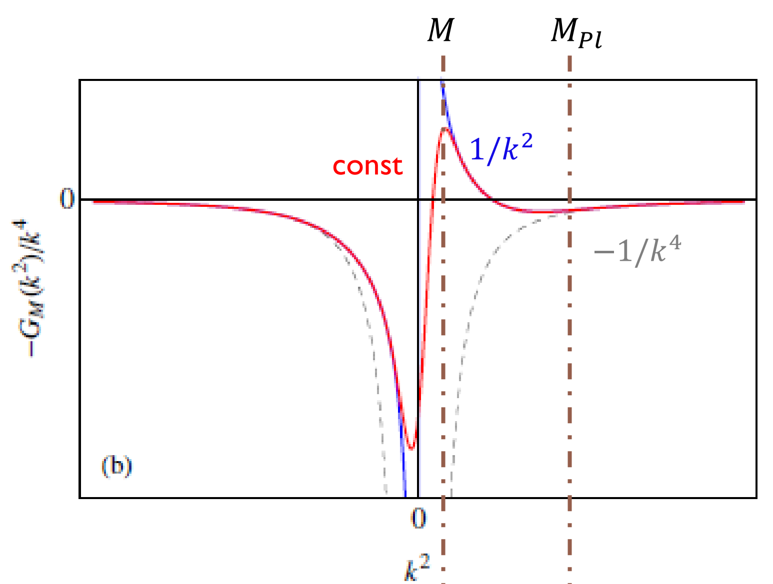
Gravitational interaction is non-renormalizable, weak. A new way to remove Landau pole without a mass gap.

(Different from QCD, the same field appears in both UV and IR)

A New Mass Scale $M^2 < M_{\rm Pl}^2$

$$S_{\rm QG} = \int d^4x\,\sqrt{-g}\left(\boxed{\frac{1}{2}M^2R} - \frac{1}{2f_2^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \frac{1}{3f_0^2}R^2\right)$$

- Mass gap as a massive graviton pole (assuming $G(k^2)$ insensitive to M)
  - May imply dynamical symmetry breaking of diffeomorphism (no known UV completion of massive gravity)
  - May indicate a spin-0 ghost (no Fierz-Pauli tuning) (fatal vacuum instability)
- Mass gap more like in QCD
  - A confining phase: all perturbative modes removed from the physical spectrum
  - Mass gap controlled by another M
  - When $M^2 \ll M_{Pl}^2$, three regions with different propagator behavior (red line)



A Focus on the Measure

-- How could gravity be like QCD anyway?

Path Integral in Gauge Theory

Fundamental similarity between gravity and QCD regarding the nontrivial measures in the path integral!

- Gauge symmetry as a redundancy

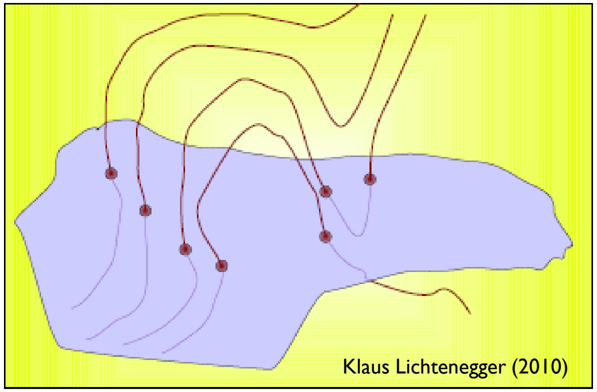
  Formally, define path integral in physical configuration space, i.e. the space of gauge orbits.

  In practice, use Faddeev-Popov procedure to extend path integral to the full configuration space.

- Gribov copies problem

  In sensible gauges, i.e. Landau and Coulomb gauge, a gauge orbit intersects with gauge-fixing surface more than once

  [Gribov, Nucl. Phys. B139 (1978) 1; Singer, Commun. Math. Phys. 60, 7 (1978).]



Gribov Copies in Gauge Theory

- Generalized generating function [Gribov, Nucl. Phys. B139 (1978) 1]

$$Z = \int \mathcal{D}A \frac{1}{1+N_F(A)} \delta(F(A^U)) \left|\det M_F(A)\right| e^{iS(A)}$$

Number of copies (defined for gauge orbits)

  - Small fluctuation (perturbative): $N_F(A) = 0, \det M_F > 0$, goes back to FP formalism
  - Large fluctuation (nonperturbative): $N_F(A) \neq 0, \det M_F = 0$ (horizon) or $\det M_F < 0$
  - In any gauge with copies, copies are effects built into the theory, could be essential for the correct nonperturbative description.

- Impact of $1/(1 + N_F(A))$ on gluon propagator $F(k^2)$ factor [Holdom, PRD 79, 085013 (2009)]
  - The effects of copies turn on at the strong scale Λ_{QCD}. Copies are important only if $A_k^2 \gtrsim k^2/\Lambda_{QCD}^4$ (typical size $A_k^2 \sim 1/k^2$)
  - At high k^2, exponentially small corrections, $F(k^2) \sim 1 + O(exp(-k^4/\Lambda^4))$
  - In deep IR, easily sample configurations above critical value, $N_F(A)$ grows fast, $F(k^2)$ suppressed (explicit form sensitive to how $N_F(A)$ grows)

Path Integral of Quadratic Gravity

- Gauge transformation of the metric perturbation: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$\delta_\xi h_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} = \bar{\nabla}_\nu \xi_\mu + \bar{\nabla}_\mu \xi_\nu + \xi^\rho \bar{\nabla}_\rho h_{\mu\nu} + h_{\mu\rho} \bar{\nabla}_\nu \xi^\rho + h_{\rho\nu} \bar{\nabla}_\mu \xi^\rho$$

- Generalized generating function

$$Z = \int \mathcal{D}h \, \frac{1}{1 + N_F(h)} \delta(F(h)) \left|\det M_F(h)\right| \, e^{iS_{QG}(g)},$$

If there are Gribov copies for gravity, nontrivial measure comes in.

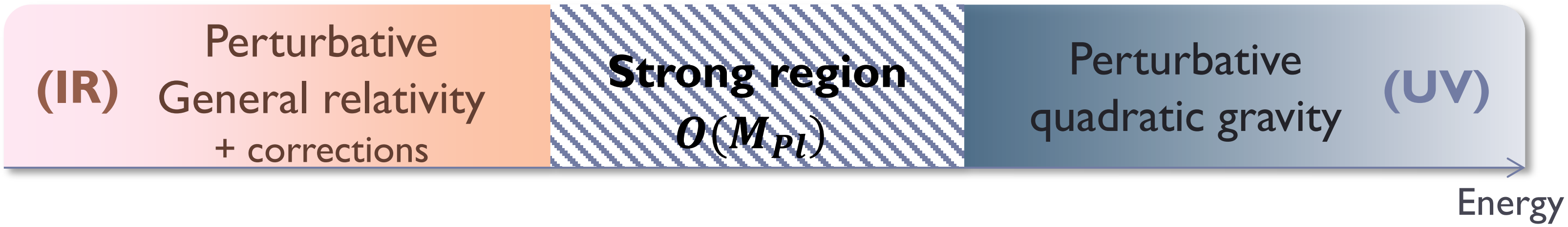
- Gribov copies in gravity: the infinitesimal version
  - Gribov horizon equation with different background $\bar{g}_{\mu\nu}$: find solution of ξ with $\det M_F(h) = 0$ for $F(h) = 0$.
  - Flat background: no solution at $h_{\mu\nu} = 0$, but do have solution for nontrivial $h_{\mu\nu}$, similar to QCD

- Although far from proving the demanded similarity, similar nontrivial infrared effects are built in in path integral of both theories

Implications and Speculations

--What physics do we expect in this picture?

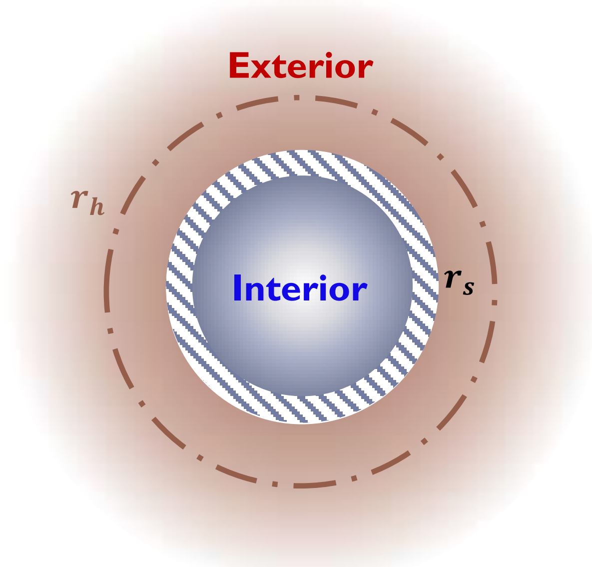
The General Picture at $M^2 = 0$



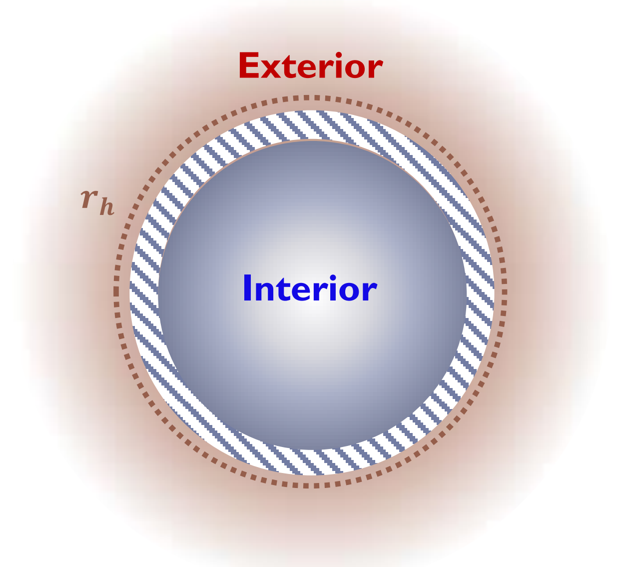
- IR region: general relativity as EFT with derivative expansion (coefficients might be constrained by the UV completion with a local QFT)
- Intermediate region: large metric fluctuation, semi-classical picture breaks down, not much to say; BUT only expands a limited range
- UV region: asymptotically free quadratic gravity (main difference)
  - Far UV, small couplings and QM fluctuations, the theory could be fundamentally defined around flat spacetime
  - Background with typical curvature scale well above M_{Pl}, i.e. sourced by a heavy probe, the effective action at the scale receives small QM correction
  - Classical solution of quadratic gravity is good approximation for high curvature region, no higher derivative corrections

Static spherically symmetric vacuum solutions ($M_S \gg M_{Pl}$)

Schd Black Hole



Horizonless Object

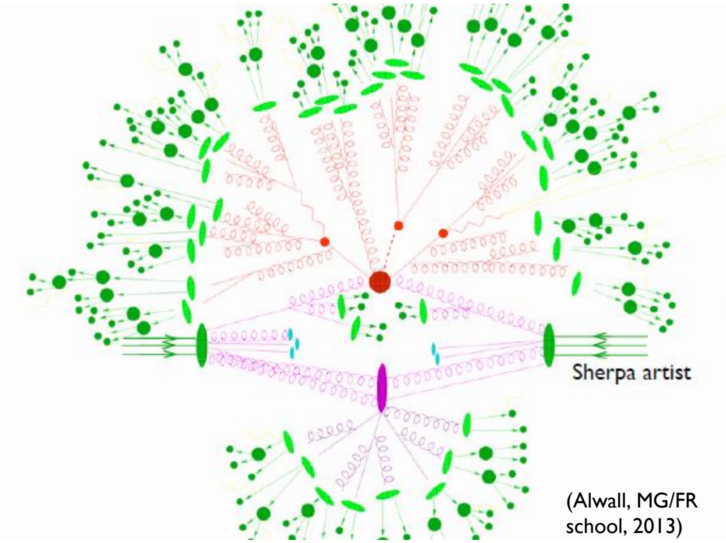


	Schd Black Hole	Horizonless Object
Exterior region (effective theory)	Schwarzchild metric with horizon $r_h \sim M_S/M_{Pl}^2$	Yukawa type correction (small) to Schd metric
Strong transition region (shell)	derivative expansion breaks down at $r_s \sim (M_S/M_{Pl}^4)^{1/3}$	curvature blows up close to would-be horizon r_h
Interior region (quadratic gravity)	spacelike singularity	Naked, timelike singularity (nonsingular one with matter)

[Stelle, Gen. Rel. Grav. **9**, 353 (1978); Holdom, PRD **66**, 084010 (2002)]

SuperPlanckian High Energy Collider

- In analogy to QCD, in principle, we may collide graviton/matter with $\sqrt{s} \gg M_{Pl}$ to probe the UV region.



(Alwall, MG/FR school, 2013)

- Factorization theorem (ansatz): hard process + parameterized strong regime
- Useful tools: PDF, parton shower, hard process, fragmentation function…
- All particles interact gravitationally, no analogy of electron in QCD

- Difference from QCD: strong gravity object produced ($\sqrt{s} \gg M_{Pl}$)
  - A close trapped surface forms, may insensitive to UV physics [Eardley, Giddings, PRD **66**, 044011 (2002)]
  - May form semiclassical BH or horizonless objects?

Implication for Matter Sector

Motivate asymptotically free extension of the SM

- When matter couplings run through the limited range of strong gravity region, mild effects, i.e. $O(1)$ multiplicative factor
- The SM structure cannot persist in far UV region due to UV Landau pole problems of $U(1)$ gauge coupling, scalar quartic couplings
- Solve problems within matter sector
  - Stable Asymptotically Free Extensions (SAFEs) of the SM   Holdom, JR, Zhang, JHEP 1503, 028 (2015)
  - Non-Abelian gauge couplings drive yukawa, scalar quartic couplings asymptotically safe

$$(4\pi)^2\beta_y = a_y y^3 - a_g g^2 y \qquad (4\pi)^2\beta_\lambda = a_{\lambda\lambda}\lambda^2 - a_{\lambda g}\lambda g^2 + a_{gg}g^4 \qquad a_y, a_g, a_{\lambda\lambda}, a_{\lambda g}, a_{gg} > 0)$$

  - The simplest possibility is Pati-Salam model with one (4,2,1), SAFEs require $2n_F + n_f = 21$

Fields	Number	$SU(4)$	$SU(2)_L$	$SU(2)_R$
F_L	n_F	4	2	1
F_R	n_F	4	1	2
$f_{L,R}$	n_f	4	1	1
Φ	1	4	2	1

Instability within $M = 0$ Theory?

Theory has fatally unstable when M is large. Is it possible that the $M = 0$ theory could be pushed into instability within the theory in certain circumstance?

- A universe compactified with size $\ll 1/M_{Pl}$ or in high $T \gg M_{Pl}$ phase: there is an IR cutoff that eliminates the strong dynamical effects that protect us from the ghost.

- Background with naked, timelike singularity
  - Gribov copies exist for $h_{\mu\nu} = 0$, perturbative description breaks down
  - A high curvature region develops in gravitational collapse (around timelike singularity) $\Rightarrow$ vacuum decay, negative modes confined in high curvature region, positive modes escape as normal graviton $\Rightarrow$ matter density reduced and high curvature region removed $\Rightarrow$ a burst of energy, might be a source of high energy cosmic ray

Summary

- We conjecture that the quadratic gravity with $M < M_{Pl}$ might not suffer from the ghost problem by incorporating nonperturbative effects. In analogy with QCD, general relativity, or a modification that depends on M, can emerge in the IR.
- Both QCD and quadratic gravity are based on path integrals over space of orbits — similar nontrivial infrared effects are built in.
- Although strong gravity around M_{Pl} still unknown, existence of UV asymptotically free region has interesting implications.
- Open questions…
  - Other approaches of strong gravity: lattice gravity, SD equation??
  - Implication of no fundamental holographic picture??
  - Conceptual issue of gravity that has no analogy in QCD??

Thank You!

Literature on The Ghost Problem

- Remove ghost pole by quantum effects
  - Large matter loop corrections transform real ghost pole into complex conjugate poles, implement Lee-Wick prescription [Tomboulis, 1977]
  - Anomalous running at non-Gaussian UVFP [Salam et al, 1978; Benedetti et al, 2009]

- Alternative quantization schemes
  - PT symmetric Hamiltonian, negative Dirac-norm → positive PT-norm [Mannheim, 2007]
  - Negative-norm (Dirac-Pauli) coordinate representation [Salvio, Strumia, 2015]

- Require ghost-free at classical level
  - Summing up all higher derivative terms: non-local classical action [Tomboulis, 1997; Biswas et al, 2011]
  - Horava gravity: breaking Lorentz symmetry at the beginning [Horava, 2009]
  - Implement boundary condition to remove ghost solution [Maldacena, 2011]
  - Implement constraint but break Lorentz symmetry [Chen et al, 2014]

Nonperturbative Form Factor

Two examples of nonperturbative multiplicative factor $F(k^2)$

$$\text{(a)} \quad \frac{k^2}{k^2 + b/(k^2 - a)}, \qquad \text{(b)} \quad \frac{\operatorname{erf}(a) - \operatorname{erf}(bk^2) - \operatorname{erf}(-bk^2 + a)}{\operatorname{erf}(a)}$$

- In both cases, $a, b > 0$ corresponds to $F(k^2)$ in Landau gauge, while $a \to 0$ corresponds to $F(k^2)$ in Coulomb gauge.
- Case (a) has complex conjugate poles. $a, b > 0$ corresponds to refined Gribov-Zwanziger propagator.
- Case (b) shows an entire function, with exponentially small effect in the UV as we expected.

Gribov Horizon Equation for Gravity

- Gribov horizon equation with different background $\bar{g}_{\mu\nu}$: find solution of ξ with $\det M_F(h) = 0$ for $F(h) = 0$.

- Instead of de-Donder gauge, doing gauge-fixing by functional minimization of the norm (norm gauge)

$$\mathcal{N} = \int d^3x \sqrt{-\bar{g}} h_{\mu\nu} h^{\mu\nu}$$

In QCD, Landau gauge is directly derived from norm function

  - Flat background: no solution at $h_{\mu\nu} = 0$, but do have solution for nontrivial $h_{\mu\nu}$, similar to QCD

  - For background with naked timelike singularity, there are even copies for vanishing $h_{\mu\nu}$. Perturbative theory breaks down?

- Still much unknown: the shape of Gribov horizons, the nature of the space of gauge orbit…