Effective Field Theory of Dissipative Fluids

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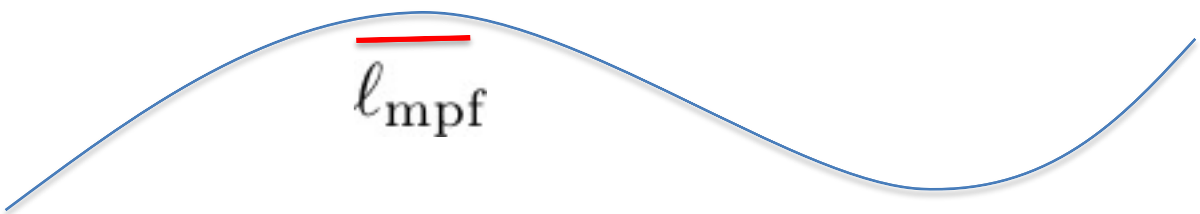
arXiv: 1511.03646

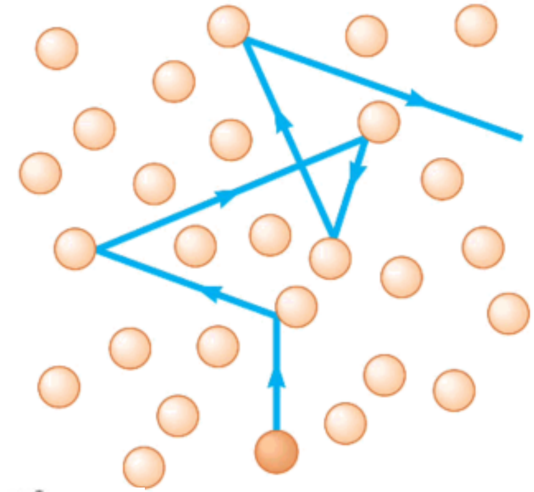
Plan

- Motivations
- Formalism
- Examples
- Conclusions

Conserved quantities

Consider a long wavelength disturbance of a system in thermal equilibrium





non-conserved quantities: relax locally, $\tau_{\rm relax} \sim \ell_{\rm mfp}$

conserved quantities: cannot relax locally, can only via transports

$$\lambda \to \infty, \quad \Rightarrow \quad \tau_{\rm relax} \to \infty$$

$$\text{sound}: \quad \omega = v_s k, \quad \text{diffusion}: \quad \omega = -iDk^2$$

Conserved quantities → Gapless modes (only ones in thermal equilibrium)

They control low energy physics: Low energy effective theory?

Hydrodynamics

Thermal equilibrium: $\rho = \frac{1}{Z} \exp \left[\int d\vec{x}\, \beta (u_\mu T^{0\mu} + \mu J^0) \right]$

Promote these quantities to dynamical variables: (local equilibrium)

$$\beta(t, \vec{x}),\ u^\mu(t, \vec{x}),\ \mu(t, \vec{x})$$

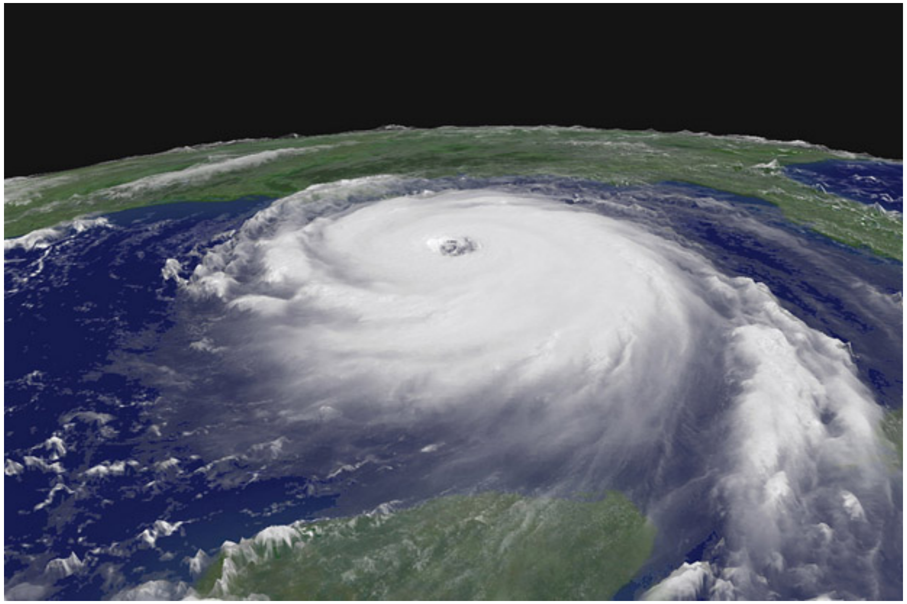
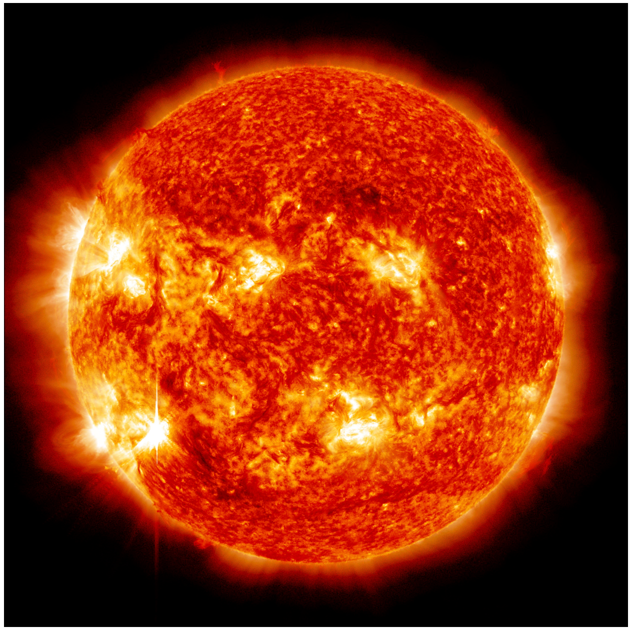
slowly varying functions of spacetime

Express expectation values of the stress tensor and conserved current in terms of derivative expansion of these variables: the so-called constitutive relations.

Equations of motion:

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0, \qquad \partial_\mu \langle J^\mu \rangle = 0$$

d+1 variables, d+1 equations


Despite the long and glorious history of hydrodynamics

It does not capture fluctuations.

Fluctuations

There are always statistical fluctuations .....

Important in many contexts:

molecular (hydro)dynamics, dynamical aspects of phase transitions, non-equilibrium stead states, turbulence, ...

At low temperatures, quantum fluctuations are also important.

Phenomenological level: stochastic hydro

$$\partial_\mu \langle T^{\mu\nu} \rangle = \xi^\nu, \quad \partial_\mu \langle J^\mu \rangle = \zeta \qquad \xi^\mu, \zeta$$: Gaussian noises

Does not capture:

1. interactions between dynamical variables and noises
2. fluctuations of dynamical variables themselves

Constraints

Current formulation of hydrodynamics is awkward.

Constitutive relations : Expand $\langle T^{\mu\nu}\rangle$, $\langle J^{\mu}\rangle$ in derivatives of $\beta(t,\vec{x}), u^{\mu}(t,\vec{x}), \mu(t,\vec{x})$

$$\partial_{\mu}\langle T^{\mu\nu}\rangle = 0, \qquad \partial_{\mu}\langle J^{\mu}\rangle = 0$$

Naively one should just write down the most general expansion consistent with symmetries. But that turns out to be wrong.

Phenomenological constraints: solutions should satisfy

1. Entropy condition $\partial_{\mu} S^{\mu} \geq 0$ $\quad S^{\mu}$: Entropy current constructed out of dynamical variables

2. Onsager relations: linear response matrix must be symmetric

Microscopic explanation? Are these complete?

awkward: use solutions to constrain equations of motion

Hopefully an effective field theory (EFT) based on action principle can address these issues.

Results

Long standing problem, dating back at least to G. Herglotz in 1911 …

It turns out: Easiest to start with a full quantum system in curved spacetimes.

1. Hydrodynamics with classical statistical fluctuations

   is described by a (supersymmetric) quantum field theory

$$\hbar_{\text{eff}} \propto \frac{1}{s} \qquad s : \text{entropy density}$$

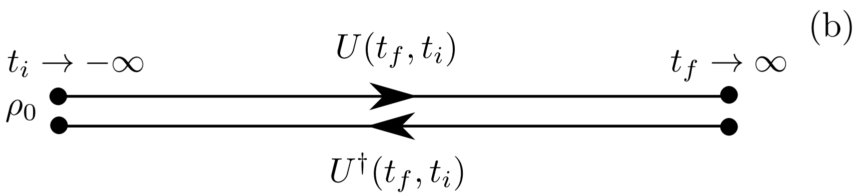
2. Hydrodynamics with quantum fluctuations also incorporated

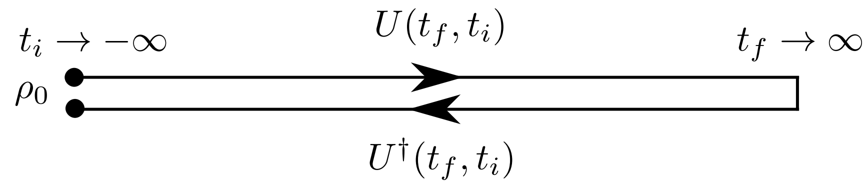
   is described by a “quantum-deformed” (supersymmetric) quantum field theory.

Part II: formulation

Transition amplitudes v.s. expectation values

We are interested in an effective theory describing nonlinear dynamics around a state.





$$\rho(t_f) = U(t_f, t_i)\rho_0 U^\dagger(t_f, t_i) \qquad\qquad \mathrm{Tr}\left(\rho_0 \cdots\right)$$

Closed time path (CTP) or Schwinger-Keldysh contour

Should be contrasted with EFT describing transition amplitudes, e.g. the Pion theory.

$$\langle f| \cdots |i\rangle$$

Effective field theory

Microscopic Schwinger-Keldysh path integral:

$$\mathrm{Tr}\left(\rho_0 \cdots\right) = \int_{\rho_0} D\psi_1 D\psi_2 \, e^{iS[\psi_1]-iS[\psi_2]} \ldots$$

$$\rho_0 = \frac{1}{Z} e^{-\beta(H-\mu Q)}$$ thermal density matrix

Integrate out all massive modes: gapless hydrodynamic modes

$$\mathrm{Tr}\left(\rho_0 \cdots\right) = \int_{\rho_0} D\chi_1 D\chi_2 \, e^{iS_{\mathrm{hydro}}[\chi_1,\chi_2]} \ldots$$

EFT approach:

1. What are χ ? $\beta(t,\vec{x}), u^\mu(t,\vec{x}), \mu(t,\vec{x})$ do not work

2. What are the symmetries of S_{hydro} ?

Dynamical variables (I)

Toy example: a single conserved current J^μ

$$e^{W[A_{1\mu}, A_{2\mu}]} = \text{Tr}\left(\rho_0 \mathcal{P} e^{i\int d^dx\, A_{1\mu} J_1^\mu - i\int d^dx\, A_{2\mu} J_2^\mu}\right),$$

1. Current conservation:

$$W[A_{1\mu}, A_{2\mu}] = W[A_{1\mu} + \partial_\mu \lambda_1, A_{2\mu} + \partial_\mu \lambda_2]$$

2. W should contain non-derivative terms of $A_{1\mu}, A_{2\mu}$

→ (i) W must be nonlocal

(ii) Non-locality due to integrating out hydro modes

Need to un-integrate hydro modes

guide: conservation and locality

Proposal:

$$e^{W[A_{1\mu},A_{2\mu}]} = \int D\phi_1 D\phi_2 \, e^{iI[B_{1\mu},B_{2\mu}]}$$

$$B_{1\mu} \equiv A_{1\mu} + \partial_\mu \phi_1, \quad B_{2\mu} \equiv A_{2\mu} + \partial_\mu \phi_2$$

$I[B_{1\mu}, B_{2\mu}]$ is a local action. $\phi_{1,2}$: hydro modes

Satisfy the following consistency requirements:

1. $W[A_{1\mu}, A_{2\mu}] = W[A_{1\mu} + \partial_\mu \lambda_1, A_{2\mu} + \partial_\mu \lambda_2]$

2. Eoms of $\phi_{1,2}$ are equivalent to current conservations.

Dynamical variables (II)

For stress tensor, we put the system in a curved spacetime

$$e^{W[g_{1\mu\nu}, g_{2\mu\nu}]} = \mathrm{Tr}\left[U_1(+\infty, -\infty; g_{1\mu\nu})\rho_0 U_2^\dagger(+\infty, -\infty; g_{2\mu\nu})\right]$$

Conservation of stress tensor:

$$W[g_1, g_2] = W[\tilde{g}_1, \tilde{g}_2] \qquad \tilde{g}_{1\mu\nu}(x) = \frac{\partial y_1^\sigma}{\partial x^\mu} g_{1\sigma\rho}(y_1(x)) \frac{\partial y_1^\rho}{\partial x^\nu}$$

Un-integrate hydro modes: Promote spacetime coordinates to dynamical fields

$$e^{W[g_1, g_2]} = \int DX_1 DX_2 D\tau_1 D\tau_2 \, e^{iI[h_1, \tau_1; h_2, \tau_2]}$$

$$h_{1ab}(\sigma) = e^{-2\tau_1(\sigma)} \frac{\partial X_1^\mu}{\partial \sigma^a} g_{1\mu\nu}(X_1(\sigma)) \frac{\partial X_1^\nu}{\partial \sigma^b}$$

There is an emergent spacetime with coordinates σ^a

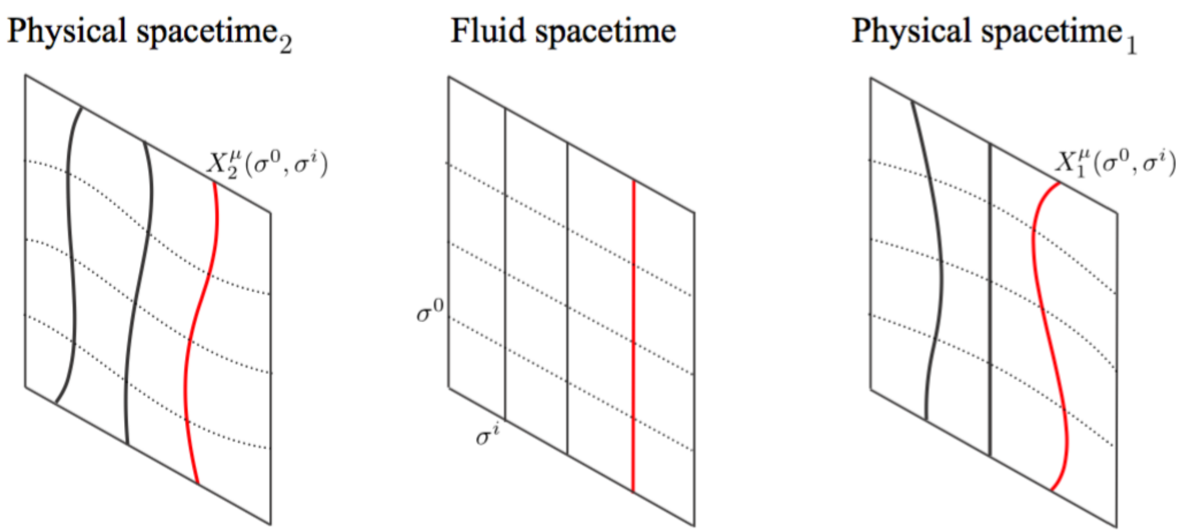
Again with a local action $I[h_1, \tau_1; h_2, \tau_2]$

1. $W[g_1, g_2] = W[\tilde{g}_1, \tilde{g}_2]$

2. X eoms are equivalent to conservation of stress tensor

Interpretation of σ^a : σ^i label individual fluid elements, σ^0 internal time

$X^\mu(\sigma^a)$: motion of a fluid element in physical spacetime



Standard hydro variables (which are now derived quantities)

$$u^\mu = \frac{1}{b}\frac{\partial X^\mu}{\partial \sigma^0}, \quad e^{-\tau} = \frac{T}{T_0}, \quad \mu = u^\mu B_\mu$$

A bit history:

Using a single copy of $\sigma^i(x^\mu)$ as dynamical variable for an ideal fluid action dated back to G. Herglotz in 1911.

Covariant $\sigma^a(x^\mu)$ was used by Taub in 1954.

Rediscovered in 2005 by Dubovsky, Gregoire, Nicolis and Rattazzi in hep-th/0512260 and further developed by Dubovsky, Hui, Nicolis and Son in arXiv:1107.0731 , ...

Nickel and Son showed the covariant version arises naturally from holography (arXiv:1103.2137).

Doubled copies appeared in Haehl eta al arXiv:1502.00636, and Crossley et al arXiv:1504.07611.

The introduction of τ is new.

The current way to motivate them is new.

Symmetries (I)

Now need to specify the symmetries of $I[h_1, \tau_1, B_1; h_2, \tau_2, B_2]$

$$e^{W[g_1,A_1;g_2,A_2]} = \int DX_1 DX_2 D\tau_1 D\tau_2 D\phi_1 D\phi_2 \, e^{iI[h_1,\tau_1,B_1;h_2,\tau_2,B_2]}$$

Note that it is defined in fluid spacetime σ^a

Interpretation of σ^a : σ^i label individual fluid elements, σ^0 internal time

Require the action to be invariant under:

$$\sigma^i \to \sigma'^i(\sigma^i), \quad \sigma^0 \to \sigma^0$$

$$\sigma^0 \to \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \to \sigma^i$$

$$B_{1i} \to B'_{1i} = B_{1i} - \partial_i \lambda(\sigma^i), \quad B_{2i} \to B'_{2i} = B_{2i} - \partial_i \lambda(\sigma^i)$$

It turns out these symmetries do magic for you:

at the level of equations of motion, they ensure all dependence on dynamical variables can be expressed in $\beta(t, \vec{x}), u^\mu(t, \vec{x}), \mu(t, \vec{x})$

Given that EOMs of ϕ, X^μ are equivalent to the conservation of current and stress tensor

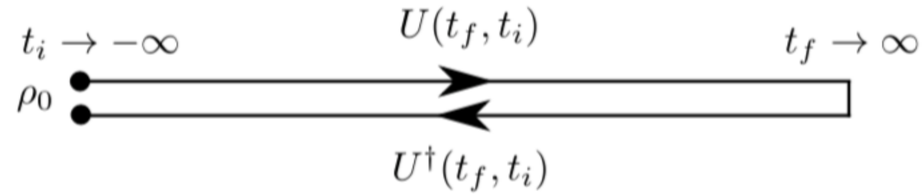
⇨ Recover standard formulation of hydrodynamics

(modulo phenomenological constraints)

This would be the full the story in a usual situation.

Symmetries (II)

We are considering EFT for a system defined with CTP:



The generating functional has the following properties:

$$W^*[g_1, A_1; g_2, A_2] = W[g_2, A_2; g_1, A_1] \qquad (1)$$

$$W[g, A; g, A] = 0 \qquad (2)$$

Thermal ensemble plus time reversal imply KMS condition (Z_2):

$$W[g_1(x), A_1(x); g_2(x), A_2(x)] = W[g_1(-x), A_1(-x); g_2(-t - i\beta_0, -\vec{x}), A_2(-t - i\beta_0, -\vec{x})] \qquad (3)$$

(1) Can be easily achieved:

$$I^*[h_1, \tau_1, B_1; h_2, \tau_2, B_2] = -I[h_2, \tau_2, B_2; h_1, \tau_1, B_1]$$

Complex action and nontrivial positivity conditions

Local KMS conditions

KMS condition:

$$W[g_1(x), A_1(x); g_2(x), A_2(x)] = W[g_1(-x), A_1(-x); g_2(-t - i\beta_0, -\vec{x}), A_2(-t - i\beta_0, -\vec{x})]$$

Note:

$$I[h_1, \tau_1, B_1; h_2, \tau_2, B_2] = I_{\text{source}} + I_{\text{mixed}} + I_{\text{dynamical}}$$

I_{source} : (i) contact terms for W (ii) determines the full action

Proposal (Z_2 symmetry):

$$I_{\text{source}}[g_1, A_1; g_2, A_2] = -I_{\text{source}}[g_1(-x), A_1(-x); g_2(-t - i\beta_0, -\vec{x}), A_2(-t - i\beta_0, -\vec{x})]$$

1. In various cases we have checked W satisfied the KMS condition
2. Lead to entropy constrains in equations of motion
3. New constraints on equations of motion

BRST symmetries

See also Haehl et al arXiv: 1510.02494 1511.07809

$$W[g, A; g, A] = 0$$

is a "topological" condition: i.e. the theory is independent of background metric and external fields when

$$g_1 = g_2 = g, \quad A_1 = A_2 = A$$

Dynamical variables: $\chi_{1,2}$ Background fields: $\phi_{1,2}$

Introduce fermionic partners $c_{1,2}$ (ghost fieds) for dynamical variables and require the action to be BRST invariant:

$$I = Q_B V(\chi_1, \chi_2, c_1, c_2; \phi) \qquad \text{for} \qquad \phi_1 = \phi_2 = \phi$$

$$\delta\chi_r = \epsilon c_r, \quad \delta c_r = 0, \quad \delta c_a = \epsilon \chi_a, \quad \delta\chi_a = 0,$$

The theory is not topological in the absence of background fields as the observables are not BRST invariant.

Supersymmetries

See also Haehl et al, arXiv: 1510.02494, 1511.07809

Given a bosonic action, the BRST symmetry does not completely fix the fermionic part of the action, leading to potential ambiguities.

The ambiguities do not arise at quadratic level of the action.

Here we find an interesting surprise, generalizing an earlier observation in Langevin equations.

One finds that at quadratic level, with local KMS condition imposed, the action possesses an emergent fermionic symmetry

$$\bar{\delta}\chi_r = c_a\bar{\epsilon}, \qquad \bar{\delta}c_r = (\chi_a + \Lambda\chi_r)\bar{\epsilon}, \qquad \bar{\delta}\chi_a = -\Lambda c_a\bar{\epsilon}$$

$$\Lambda = 2\tanh\frac{i\beta_0\partial_t}{2}$$

Combining two fermionic symmetric we have:

$$\delta^2 = 0, \quad \bar{\delta}^2 = 0, \quad [\delta, \bar{\delta}] = \bar{\epsilon}\epsilon 2 \tanh \frac{i\beta_0\partial_t}{2}$$

The currents also transform as irreducible multiplet under the algebra.

Classical limit: $\hbar \to 0$

$$\delta^2 = 0, \qquad \bar{\delta}^2 = 0, \qquad [\delta, \bar{\delta}] = \bar{\epsilon}\epsilon i\beta_0\partial_t$$

become standard supersymmetry in time direction.

Conjecture: in the classical limit supersymmetry can fix the fermionic part action uniquely.

Have checked this to cubic orders

Quantum theory: higher derivative version of SUSY, "quantum-deformed"

Example: nonlinear stochastic diffusion

Consider the theory for a single conserved current, where the relevant physics is diffusion.

Dynamical variables: $\varphi_{1,2}$ (or φ_a, φ_r)

Roughly, φ_r : standard diffusion mode, φ_a: the noise.

$$\mathcal{L} = iT(\partial_i\varphi_a)^2(\sigma + \sigma_1\partial_0\varphi_r) + \partial_0\varphi_a\partial_0\varphi_r(\chi + \chi_1\partial_0\varphi_r) - \partial_i\varphi_a\partial_0\partial_i\varphi_r(\sigma + \sigma_1\partial_0\varphi_r)$$
$$+c_a(\chi\partial_0 - \sigma\partial_i^2)\partial_0 c_r - \chi_1\partial_0 c_a\partial_0\varphi_r\partial_0 c_r - \sigma_1\partial_i^2 c_a\partial_0\varphi_r\partial_0 c_r$$
$$-iT\sigma_1(\partial_i c_a\partial_i\varphi_a\partial_0 c_r + (\partial_0 c_a\partial_i\varphi_a - \partial_i c_a\partial_0\varphi_a)\partial_i c_r),$$

If ignoring interactions of noise

$$(\partial_0 - D\partial_i^2)n - \left(\lambda_1\partial_0 - \frac{\lambda}{2}\partial_i^2\right)n^2 = \xi$$

A variation of Kardar-Parisi-Zhang equation

Nonlinear charged fluids

Full non-linear action requires more apparatus to write down.

Need the most general invariant term with the split σ^a spatial and temporal diffeomorphisms.

Introduce differential geometric structure which captures this, including:

$$D_0,\ D_i,\ R_{ij}^{(1)},\ R_{ij}^{(2)},\ \mathrm{t}_{ij}$$

With all possible terms constructed using this structure, precisely recover the expected constitutive relations.

Summary

An EFT for general dissipative fluids.

Recovers the standard hydrodynamics as equations of motion, constitutive relations, constraints.

Encodes quantum and thermal fluctuations systematically in a path integral expansion.

Full non-linear fluid fluctuating dynamics encoded in non-trivial differential geometry.

Emergent supersymmetry.

Future directions

Formalism:

Non-relativistic limit ,
superfluids,
Anisotropic, inhomogeneous,
“quantum-deformed” Supersymmetry

…….

Applications:

Longtime tails, running of viscosities,

Dynamical aspects of classical and quantum phase transitions

Scaling behavior in hydro behavior via fixed points of QFTs, such as KPZ scaling, turbulence ….

…………

Thank You